Reliability Modeling of Wind Turbines: Exemplified by Power Converter Systems as Basis for O&M Planning

Reliability Modeling of Wind Turbines: Exemplified by Power Converter Systems as Basis for O&M Planning

Revised Version

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Summary

Cost reductions for offshore wind turbines are a substantial requirement in order to make offshore wind energy more competitive compared to other energy supply methods. During the 20 – 25 years of wind turbines useful life, Operation & Maintenance costs are typically estimated to be a quarter to one third of the total cost of energy. Reduction of Operation & Maintenance costs will result in significant cost savings and result in cheaper electricity production. Operation & Maintenance processes mainly involve actions related to replacements or repair. Identifying the right times when the actions should be made and the type of actions requires knowledge on the accumulated damage or degradation state of the wind turbine components. For offshore wind turbines, the action times could be extended due to weather restrictions and result in damage or degradation increase of the remaining components. Thus, models of reliability should be developed and applied in order to quantify the residual life of the components. Damage models based on physics of failure combined with stochastic models describing the uncertain parameters are imperative for development of cost-optimal decision tools for Operation & Maintenance planning. Concentrating efforts on development of such models, this research is focused on reliability modeling of Wind Turbine critical subsystems (especially the power converter system). For reliability assessment of these components, structural reliability methods are applied and uncertainties are quantified. Further, estimation of annual failure probability for structural components taking into account possible faults in electrical or mechanical systems is considered. For a representative structural failure mode, a probabilistic model is developed that incorporates grid loss failures. Further, reliability modeling of load sharing systems is considered and a theoretical model is proposed based on sequential order statistics and structural systems reliability methods. Procedures for reliability estimation are detailed and presented in a collection of research papers.

Resumé

Omkostningsreduktion er af stor betydning for at havvindmøller kan opnå konkurrencedygtighed i forhold til andre energiforsyningskilder. I løbet af vindmøllers 20-25 års levetid, tegner drift og vedligeholdelses omkostninger sig til at være fra en fjerdedel til en tredjedel af de samlede omkostninger. En reduktion af drift og vedligeholdelses omkostninger kan beløbe sig i betydelige besparelser og resultere i billigere el-produktion. Driften og vedligeholdelses processerne omfatter hovedsageligt vedligehold, reparationer eller udskiftninger af dele. Identificering af det rigtige tidspunkt for en sådan krævet vedligeholdelse og typen af handling kræver viden om komponenternes tilstand, den akkumulerede skade eller den nedbrydning der måtte være pågået vindmøllekomponenterne. For havvindmøller kan behovet for disse påkrævede vedligeholdelses handlinger være øget pga. de barskere vejrforhold på havet eller resultere i øget slid med flere skader i de resterende komponenter. Derfor bør der udvikles pålidelighedsmodeller til at kvantificere komponenternes resterende levetid. Skadesmodeller for fysiske fejl kombineret med stokastiske modeller, der beskriver usikkerheds parametre er afgørende for udvikling af omkostningsoptimale beslutningsværktøjer til planlægning af drift og vedligehold. Fokus i dette projekts forskning har været koncentreret om udviklingen af modeller til pålideligheds modellering af vindmøllens kritiske delsystemer (særligt kraftoverføringssystemet). Til pålidelighedsvurdering af disse komponenter, er der anvendt strukturelle pålidelighedsmetoder og usikkerhederne er kvantificeret. Endvidere er modeller udviklet til estimering af den årlige svigtsandsynlighed for strukturelle komponenter under hensyntagen til eventuelle fejl i de elektriske eller mekaniske systemer. For et repræsentativt strukturelt svigt er der udviklet en probabilistisk model, hvor fejl i nettilkobling er inkorporeret. Endvidere er der opstillet en pålideligheds model for lastfordeling i systemer; i form af en teoretisk model baseret på sekventiel rækkefølge af

svigt ved anvendelse af statistiske og strukturelle systempålideligheds metoder. Procedurer for estimering af pålideligheden er detaljeret beskrevet og præsenteret i en samling videnskabelige artikler.

Reliability modeling of wind turbines – exemplified by power converter systems as basis for O&M planning

by

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List of Published Papers Referred to the Thesis

Paper 1:	Kostandyan E.E., Sørensen J.D., January 2012, "Reliability of Wind Turbine Components-Solder Elements Fatigue Failure", *Proceedings on the 2012 Annual Reliability and Maintainability Symposium (RAMS 2012)*, IEEE *Xplore*, Reno, Nevada, USA, pp. 1-7. DOI: 10.1109/RAMS.2012.6175420.
Paper 2:	Kostandyan E.E., Sørensen J.D., 2012, "Physics of Failure as a Basis for Solder Elements Reliability Assessment in Wind Turbines", *Reliability Engineering and System Safety*, Elsevier, Vol. 108, pp. 100-107. DOI: 10.1016/j.ress.2012.06.020.
Paper 3:	Kostandyan E.E., Sørensen J.D., June 2012, "Structural Reliability Methods for Wind Power Converter System Component Reliability Assessment", *Proceedings on the 16th IFIP WG 7.5 Conference on Reliability and Optimization of Structural Systems*, Yerevan, Armenia, pp. 135-142.
Paper 4:	Kostandyan E.E., Ma K.,2012, "Reliability Estimation with Uncertainties Consideration for High Power IGBTs in 2.3 MW Wind Turbine Converter System", *Microelectronics Reliability*, Elsevier, Vol. 52, pp. 2403–2408. DOI: 10.1016/j.microrel.2012.06.152.
Paper 5:	Kostandyan E.E., Sørensen J.D., January 2013, "Reliability Assessment of IGBT Modules Modeled as Systems with Correlated Components", *Proceedings on the 2013 Annual Reliability and Maintainability Symposium (RAMS 2013)*, IEEE *Xplore*, Orlando, Florida, USA, pp. 1-6. DOI: 10.1109/RAMS.2013.6517663.
Paper 6:	Kostandyan E.E., Sørensen J.D., June 2013, "Reliability Assessment of Offshore Wind Turbines Considering Faults of Electrical / Mechanical Components", *Proceedings on the Twenty-third International Offshore (Ocean) and Polar Engineering Conference (ISOPE 2013)*, Anchorage, Alaska, USA, pp. 402-407.

Other Published Papers

Paper 8:	Kostandyan E.E., Sørensen J.D., 2011, "Reliability Assessment of Solder Joints in Power Electronic Modules by Crack Damage Model for Wind Turbine Applications", *Energies*, MDPI, Vol. 4, pp. 2236-2248. DOI: 10.3390/en4122236.
Paper 9:	Kostandyan E.E., Sørensen J.D., May 2012, "Weibull Parameters Estimation Based on Physics of Failure Model", *Proceedings on the Industrial and Systems Engineering Research Conference (ISERC 2012), 62nd IIE Annual Conference & Expo 2012*, Orlando, Florida, USA, pp. 10.

This thesis has been submitted for assessment in partial fulfillment of the PhD degree. The thesis is based on the submitted or published scientific papers which are listed above. Parts of the papers are used directly or indirectly in the extended summary of the thesis. As part of the assessment, co-author statements have been made available to the assessment committee and are also available at the Faculty. The thesis is not in its present form acceptable for open publication but only in limited and closed circulation as copyright may not be ensured.

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I am thankful to my family and all my friends, who have believed in me and supported me all this time by their cordial words and letters.

With Regards,
Erik E. Kostandyan.

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CHAPTER 1. RESEARCH PROBLEM FORMULATION

1.1 Introduction and Problem Statement

Wind turbines (WT) should be designed for 20-25 year lifetimes defined by the International Electrotechnical Commission standards (e.g. IEC 61400-1 (2005)). WTs are complex systems, which consists of electrical, mechanical, hydraulic, structural and software subsystems. The main purpose of WTs is to transform kinetic energy from wind to electrical power. Offshore WTs have the advantage to be green, ecosystem friendly, be located off-shore in deep seas and economically justified for a reasonable period of time. However, all these aspects are influenced by the reliability of the WTs. WTs with low reliability can increase the Operation & Maintenance (O&M) costs and thereby increase the Cost of Energy (CoE). On the contrary, the reliable components can be very expensive but with low O&M costs, resulting in low CoE. The optimal reliability should thus be assessed taking both component costs and O&M costs into account, as well as other cost contributions (e.g. installation costs). Thus, it is important to be able to estimate the reliability of all WT components and design the components such that a cost-optimal reliability level is attained. Different subsystems of WTs can have different levels of reliability. For example, WT blades are designed for an annual probability of failure between 10^{-4} and 10^{-3}. Recently many studies are devoted to the reliability assessment of electrical components in WTs, which shows the high failure rates of electrical systems, typically between 0.05 and 0.2 per year. High failure rates in electrical systems affect profitability via increase in CoE and O&M costs. Electrical systems failure rates can cover both significant (costly) failures and some that are easy to handle and fix (e.g. by remote actions).

Influence of failed systems on survived systems is based on their direct or indirect interactions, resulting on consequences of increasing failure hazard for the survived systems and for the whole system (see Figure 1). Thus, the amount of increase in CoE and O&M costs will directly depend on the electrical systems failure influences on the survived systems (e.g. blades, tower, etc.), resulting on consequences of increasing failure hazard of the survived systems and for the whole WT (and wind farm).

Thus, more detailed understanding of electrical systems main components, their reliability modeling and failure influence on structural components reliability are necessary to be able to decrease the CoE. These issues are addressed in this research.

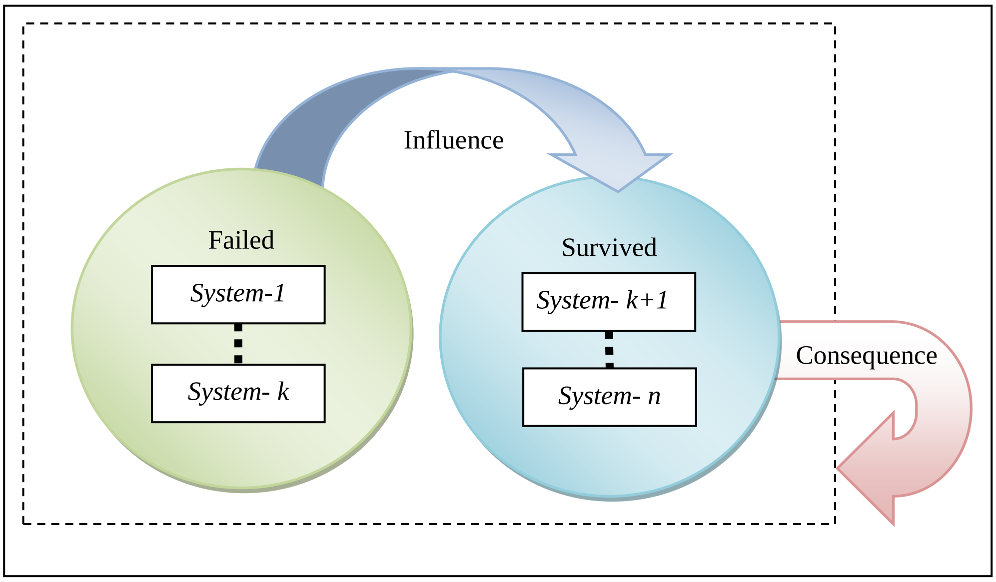

Figure 1: Influence and consequence of subsystems on the whole system

1.2 Operation and Maintenance Planning

O&M planning strategies are related to the decisions that operators / owners of WT should take during the WT life (they are not fixed and might be changed during the WT life by a learning process). Recently many studies are devoted to identify the optimal O&M strategies that can overcome the high cost of unexpected failures. Generally, O&M might be classified into two groups: corrective and preventive O&M strategies. Corrective O&M (COM) is performed after the failure event has been observed, while preventive O&M (POM) is implemented while the failure event is not observed (any time within the start until the time when the failure event occurs). Further, POM might be performed based on usage age, periodically scheduled (calendar), condition based and risk (probability) based maintenance strategies. To determine an optimal O&M strategy, the objective functions should be determined (minimization or maximization) during the service life or infinite time horizon, subject to the model limitations. Objective functions to be minimized might be defined based on costs / downtimes, whereas objective functions to be maximized could be defined based on profits (benefits) / availabilities. Also, it is necessary to have information on the damage level of the critical (electrical) components. This information can be direct information about the damage size or it can be indirect knowledge though indicators. The information can be either deterministic or it can be probabilistically be expressed. An important objective of this research is therefore to formulate deterministic and probabilistic damage measures as function of time related to electrical components

Downtimes play an important part and might influence the choice of O&M strategy. It is important to include them into the considerations, as far as in offshore WT applications time to repair might take significant time. In addition, downtimes could be increased by weather conditions. Altogether, a significant downtime might be observed, during which a parked WT might be affected by the extreme / fatigue wind loads, affecting reliabilities of the WT components, e.g. blades and tower.

1.3 Wind Turbine Components and Failure Statistics

A WT can be considered as a system comprised structural, mechanical and electrical subsystems. Categorization of WT components is necessary to sustain failure statistics and concentrate reliability estimation efforts for critical components / subsystems. Figure 2 illustrates the WT main components and systems.

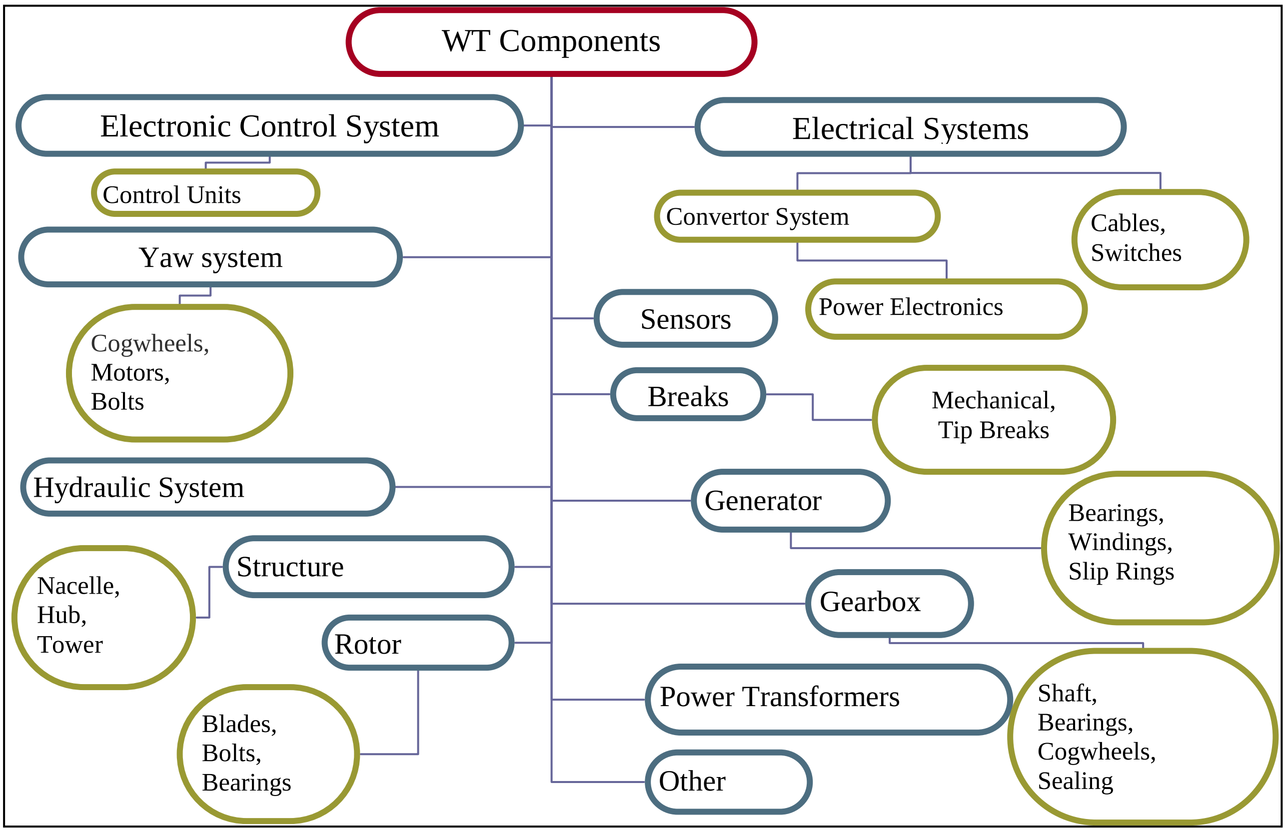

Figure 2: WT main components

Based on main components failure statistics (see Figure 3), electrical systems have the highest annual failure rates among all three WT classes. This analysis was reported by [Faulstich et al., 2008] on "German Wind Energy Report 2008", where data was based on about 1500 German turbines included in the WMEP from 2008.

Further, using this failure statistics, [Isaksson & Dahlberg, 2011] published "2011, Elforsk report 11:18", where failures were represented in a risk matrix, based on likelihood of a failure and the consequences. In "2011, Elforsk report 11:18", consequences were considered on economical (E) as well as health, safety and environment (HSE) aspects, where economical aspect incorporates costs related to opportunity cost (downtime cost while system being maintained) and actual component costs.

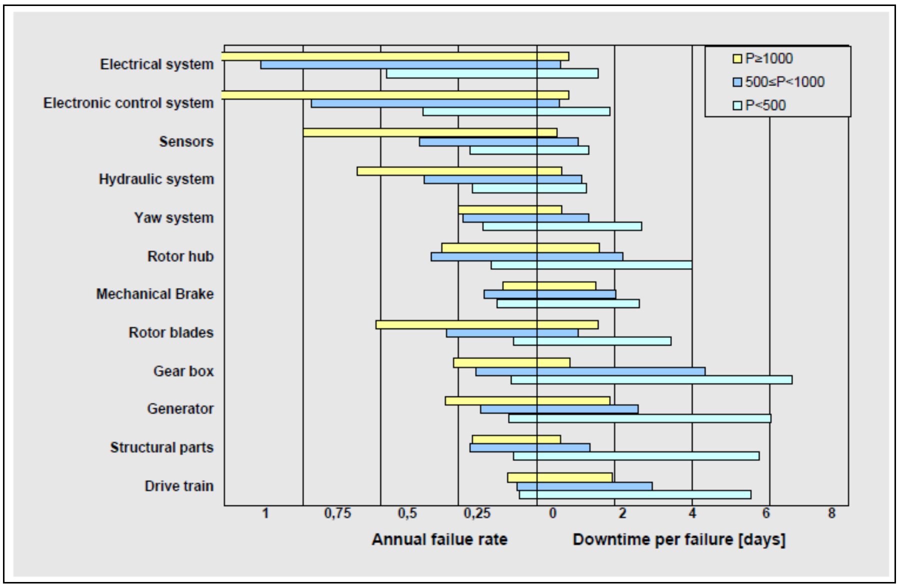

Figure 3: WT failure statistics based on “German Wind Energy Report 2008”, ‘P’ in kW

Likelihood of failure				
	High	Electrical system (E)		
	Medium	Yaw gear box (E) Blade adjustment (E)	Gear box (E)	
	Low		Generator (E) Transformator (E) Machine Foundation (E) Rotor hub (E, HSE)	Rotor blades(E, HSE) Rotor shafts (E, HSE) Bolted joints (E, HSE) Steel tower (E, HSE)
		Low	Medium	High
	Consequence of failure			

Figure 4: Risk matrix based on “2011, Elforsk report 11:18”

As it is seen from Figure 4 (based on “2011, Elforsk report 11:18”), electrical systems have the highest failure likelihood, while its consequence is considered low. This is because in “2011, Elforsk report 11:18”, the electrical systems failure has consequence composed of economical (E) aspect only, and its health, safety, environment (HSE) aspect is negligible.

However, the negligible judgment on health, safety, environment (HSE) aspect for electrical systems failure consequence might result in expensive practical punishment. The electrical systems failure downtimes might influence on WT safety and consequently will increase the hazard, especially for WT blades and tower subsystems. In addition, downtimes might be prolonged due to weather conditions, and consequently the WT will be exposed to breaking as well as damaging (fatigue) loads.

Thus, one part of this research was concentrated on developing reliability models for electrical subsystem and its components. The models were aimed for O&M strategy development and could be integrated with non-destructive evolution techniques (e.g. remotely obtaining information on fatigue measure evolution without damaging the component).

1.4 Wind Turbine Power Converter Systems

Pitch-controlled variable-speed WTs for off-shore applications in practice have a variable generator speed due to variations of the rotor speed. Depending on generator type, different alternating current (AC) at variable frequency will be generated, which has to be adapted to the grid requirements.

Three type of generators are commonly used, which are asynchronous (induction), synchronous or doubly fed induction generators. Mostly asynchronous (induction) generators are used for constant speed WT, where the generated AC is directly coupled to the grid (see Figure 5). Two most common variable speed layouts are full power and partial power conversion systems (see Figure 6 and Figure 7). A converter system is required in order to convert (rectifying) generated variable frequency AC to direct current (DC), then the fluctuating DC to convert back to the grid required AC (inverting). Also some filters are used to smooth the inverted current. For variable speed layout with full power conversion usually synchronous generators are used (even though asynchronous generators could be used as well), while in partial conversion layouts doubly fed induction generators are used.

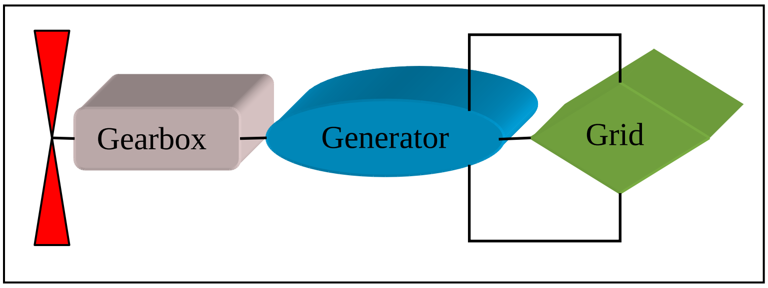

Figure 5: Constant speed layout

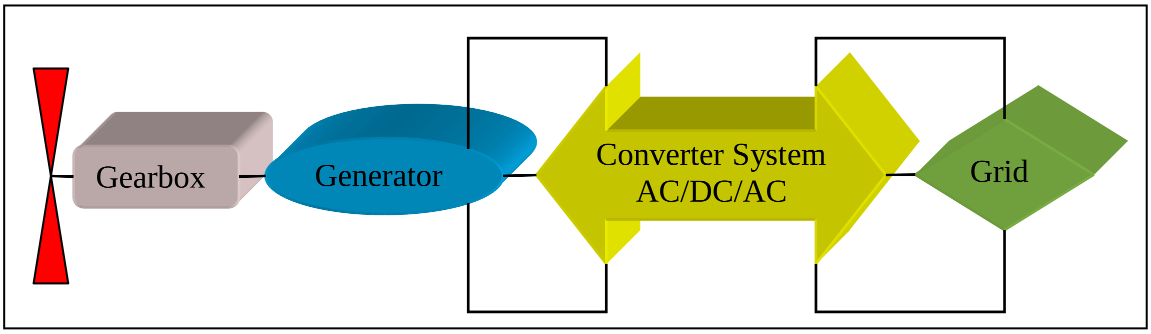

Figure 6: Variable speed layout with full power conversion

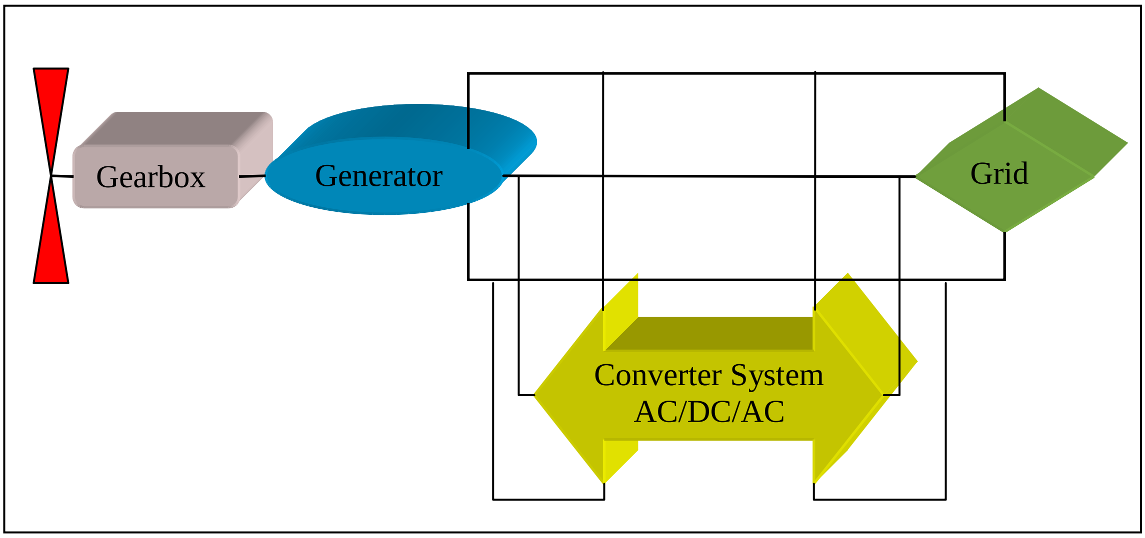

Figure 7: Variable speed layout with partial power conversion

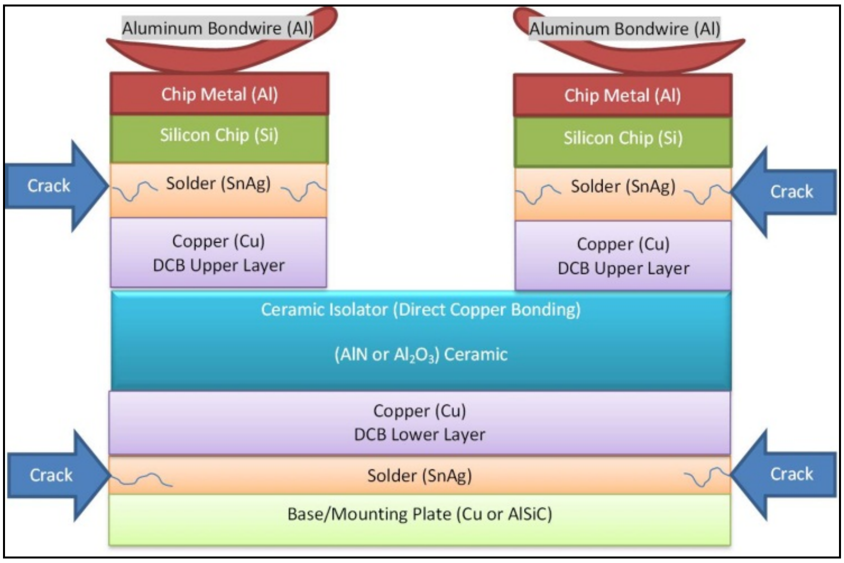

Figure 8: Structural details of IGBT module

A converter system is an electronic circuit composed of power electronics. One of its main components is an insulated-gate bipolar transistor (IGBT) module, which is a three terminal electronic switch, comprised from semiconductors (diodes and IGBT chip), aluminum, copper and ceramics. These components are linked together by soldering, wire bonding and other manufacturing techniques, see [Lu et al., 2009]. Silicon IGBT chips are soldered to the ceramic isolator and to the base plate. The Ceramic isolator has upper and lower layers of copper and on these layers the physical soldering is done (see Figure 8). Usually SnAg lead free solders are used in soldering techniques. Nowadays, SnAg lead free solders are used as a replacement for SnPb lead based solders from the environmental standpoint advantages and restrictions from hazardous substances directives in the EU. In Table 1, thermal expansion linear coefficients for IGBT module layers are shown. During its operation, an IGBT faces power losses in switching of high voltage and current. These causes temperature fluctuations in all layers of the IGBT, which again induces fatigue loads and possible development of fatigue cracks. Thus, temperature profiles are the main loads / stresses that an IGBT is facing during its useful life.

Three dominant failure modes of standard wire-bonded IGBTs are bond wire lift-off, solder joints cracking under the chip and solder joints cracking under the ceramics. This research is focused on the solder cracking failure mode that propagates under the chip and it is predominated by creep-fatigue failure mechanism. The main reason for this is the mismatch in coefficients of thermal expansion between layers of silicon, solder and cupper (see Table 1).

Component/Layer	**Material**	**Thickness (μm)**	**Coefficients of Thermal Expansion at 20 ^{o}C (ppm/ ^{o}C)**
Bond Wire	Al (Aluminum)	300-500 (in diameter)	22-24
Chip Metal	Al (Aluminum)	3	22-24
Silicon Chip	Si (Silicon)	250-300	2.77-3
Solder	SnAg (Tin Silver)	50-100	$21.85+0.02039*T_c$
DCB Upper Layer	Cu (Copper)	280-300	16.8-17.3
Ceramic Isolator	AlN (Aluminum Nitride) Al2O3 (Aluminum Oxide, Alumina)	700-1000	4-4.5 7-8.1
DCB Lower layer	Cu (Copper)	280-300	16.8-17.3
Solder	SnAg (Tin Silver)	100-180	$21.85+0.02039*T_c$
Base/Mounting Plate	Cu (Copper) AlSiC (Aluminum Silicon Carbide)	3000-4000	16.8-17.3 8

Table 1: IGBT layers linear expansion coefficients, where T_c in degree of ^{o}C

CHAPTER 2. RELIABILITY ESTIMATION APPROACHES

2.1 Reliability Estimation

If identical products work under the same conditions, then they will fail or stop functioning at different points of time. Thus, the probabilistic nature of failure exists and reliability should be defined in terms of probability. Reliability is defined as a probability to the event that the product / system will perform its intended purpose in a specified working environment for a specified time. It follows that failure event (inability) of the product / system should be defined based on intended function, working environment and specified time. A failure event is a conceptual notion and could be differently declared among various products. Thus, definition of failure event for a particular product should be clearly stated before any reliability assessment.

Reliability is considered as one of the main characteristics that nowadays' products should fulfill. It helps to evaluate steady duration of functionality in products in the anticipated environment and conditions. The level of required reliability might be determined by consumers and / or being specified during the product / structure design stage. Meeting these requirements will lead to sales volume increase, uphold market domination and keep civil infrastructures safe.

In general, two classes of reliability estimation procedures are defined. One is named as classical reliability estimation approach and another one is known as the structural reliability estimation approach. A distinguish difference in these two approaches is that in structural reliability failure events are mathematically formulated or modeled, uncertain parameters are modeled by stochastic variables, fields or processes, and further analysis lays on probabilistic estimation of the failure events. While in classical reliability approaches failure events are not modeled, but information on failure times is collected based on the physical test results, and further analysis is performed to identify probabilistic nature of the results.

If failure times (T) is considered to be random, then they will follow some failure distribution function, $F_T(t)$ such that $T > 0$, with unconditional failure rate function $f_T(t)$ and conditional failure (hazard) rate function $h_T(t)$. The relationships between them as well as the expected life are determined by:

$$F_T(t) = P(T \le t) = \int_0^t f_T(u)du \tag{1}$$

$$h_T(t) = \frac{f_T(T)}{1 - F_T(t)} \tag{2}$$

$$E(T) = \int_{0}^{+\infty} u f_T(u) du = \int_{0}^{+\infty} \left(1 - F_T(u)\right) du \tag{3}$$

To estimate failure distribution $F_T(t)$, two steps have to be committed, first its distributional form has to be chosen, and second its parameters have to be estimated. To perform these steps structural and / or classical reliability analysis techniques can be applied.

Components / systems usually fail whenever the applied loads are exceeding the materials' strengths (from which the components / systems are made). Martials' strengths are represented by its mechanical properties, the most common ones are:

- Ultimate Tensile strength : before tensile failure maximum stress (MPa)
- Yield strength: measures the level at which material starts deform plastically (MPa)
- Young's modulus: measure of stiffness, deform elastically (MPa)
- Ductility: deform under tensile load (% elongation)
- Shear strength: before shear failure maximum stress (MPa)
- Compressive strength: before compressive failure maximum stress (MPa)
- Fatigue limit (Endurance limit): stress range of cyclic load before initiating fatigue failure (MPa)
- Fracture toughness : in a presence of a crack the ability to resist the crack growth, measured by critical stress intensity factor Kc (J/m^2)
- Etc.

Material failure events are usually distinguished based on failure modes. The most common failure modes are:

- Brittle fracture: mechanical loads exceeds materials' ultimate tensile strength
- Ductile failure: tensile or shear stresses exceed materials' yield strength resulting in original size and shape changes
- Buckling failure: due to compressive or torsional stresses, it depends on materials shape, dimensions and modulus of elasticity
- Creep failure: due to dimensional change, time and temperature dependent, causes fracture failure under the applied loads
- Fatigue failure: due to cyclic loads that are far less of materials' ultimate tensile strength
- Etc.

Thus, failure events might be governed by several failure modes. For each failure mode, a failure mechanism is present, which is a subject to be modeled and analyzed.

2.2 Classical Reliability Estimation Approaches

Depending on the failure type (fatigue, extreme failure, etc.) failure data is fitted to a failure distribution. Fatigue failure times commonly are described based on the Weibull distribution, while extreme failure times are described by Gumbel type distributions. These two distributions are members of the so-called Extreme value distributions family.

2.2.1 Extreme value distributions

If one has observed data points for some stochastic variable, it is important to fit these points to statistical distribution. This will allow using prediction measures for the ranges that has not been observed in the sample. Some stochastic variables make importance from the research standpoint in their extreme values, at lower or upper tails. Extreme value stochastic variables are order statistics (see Appendix D), which depend on the parent distribution. However, as sample size increases and assuming that "Stability Postulate" holds, then extreme values follow Extreme Value Distributions (EVD). The following distributions are considered as an extreme value distributions.

Distributions for the Largest Value

Type I (Gumbel Type Distribution)

$$F_{\max}(x) = P(X_{\max} \le x) = \exp\left(-e^{-\left(\frac{x-\delta}{\theta}\right)}\right) \tag{4}$$

where $\theta > 0$, $-\infty < x < +\infty$.

It is used if the parent distribution is unbounded in the range and direction of the largest value. Commonly assumed parent distributions are Normal, Log Normal, Exponential and Gamma.

Type II (Frechet Type Distribution)

$$F_{\max}(x) = P(X_{\max} \le x) = \exp\left(-\left(\frac{x-\delta}{\theta}\right)^{-\beta}\right) \tag{5}$$

where $\theta > 0$, $\beta > 0$, $\delta \le x < +\infty$.

It is used if the parent distribution is bounded from bellow in the range and direction of the largest value. Commonly assumed parent distribution is Pareto, Log Normal, Exponential and Gamma.

Type III (Weibull Type Distribution)

$$F_{\max}(x) = P(X_{\max} \le x) = \exp\left(-\left(-\frac{x-\delta}{\theta}\right)^{\beta}\right) \tag{6}$$

where $\theta > 0$, $\beta > 0$, $-\infty < x \le \delta$.

It is used if the parent distribution is bounded from above in the range and direction of the largest value. This is also known as a Reverse Weibull Distribution. Commonly assumed parent distribution is Beta(1, alpha). It might be also used to estimate the upper bound of the largest value in worst-case scenarios.

Distributions for the Smallest Value

Type I (Gumbel Type Distribution)

$$F_{\min}\left(x\right) = P(X_{\min} \le x) = 1 - \exp\left(-e^{\left(\frac{x-\delta}{\theta}\right)}\right) \tag{7}$$

where $\theta > 0,\ -\infty < x < +\infty$.

It is used if the parent distribution is unbounded in the range and direction of the smallest value. Commonly assumed parent distribution is Normal.

Type II (Frechet Type Distribution)

$$F_{\min}\left(x\right) = P(X_{\min} \le x) = 1 - \exp\left(-\left(-\frac{x-\delta}{\theta}\right)^{-\beta}\right) \tag{8}$$

where $\theta > 0,\ \beta > 0,\ -\infty < x \le \delta$.

It is used if the parent distribution is bounded from above in the range and direction of the smallest value.

Type III (Weibull Type Distribution)

$$F_{\min}\left(x\right) = P(X_{\min} \le x) = 1 - \exp\left(-\left(\frac{x-\delta}{\theta}\right)^{\beta}\right) \tag{9}$$

where $\theta > 0,\ \beta > 0,\ \delta \le x < +\infty$.

It is used if the parent distribution is bounded from below in the range of the smallest value. It might be used to estimate the lower bound of the smallest value in worth case scenarios. Commonly assumed parent distributions are Pareto, Log Normal, Exponential and Gamma. This is also a well-known Weibull distribution. If parent distribution is Exponential distribution, then the smallest value from Exponential distribution has Weibull distribution or Smallest Type III distribution.

2.2.2 Data fitting and estimation procedures

One of the estimation procedures of the distribution parameters is a Maximum Likelihood Estimation (MLE) technique, where covariance matrix of these estimates can be numerically calculated based on the Hessian matrix.

Based on $x_1, x_2, ..., x_r$ realizations from the sample of size *'n'* (assuming censoring is observed after the largest observation, so *'n-r'* observations are right censored), the Weibull distribution MLE's for shape and scale parameters are (see Appendix A):

$$\hat{\theta} = \left(\frac{\sum_{i=1}^{r} {}^{*} x_i^{\beta}}{r} \right)^{\frac{1}{\beta}} \tag{10}$$

$$\frac{\sum_{i=1}^{r} Ln(x_i)}{r} = \frac{\sum_{i=1}^{r} {}^{*} x_i^{\hat{\beta}} Ln(x_i)}{\sum_{i=1}^{r} {}^{*} x_i^{\hat{\beta}}} - \frac{1}{\hat{\beta}} \tag{11}$$

where $\sum_{i=1}^{r} {}^{*} y_i = \sum_{i=1}^{r} y_i + (n-r) y_r$,

and covariance matrix of the estimated MLEs from log likelihood function could be calculated via the Hessian matrix based on the following relationship:

$$Cov\left[\hat{\beta}, \hat{\theta}\right] = \left[-H\right]^{-1} = \begin{bmatrix} \frac{\partial^2 Ln\left(L(\theta, \beta / x_i)\right)}{\partial \beta^2} & \frac{\partial^2 Ln\left(L(\theta, \beta / x_i)\right)}{\partial \beta \partial \theta} \\ \frac{\partial Ln\left(L(\theta, \beta / x_i)\right)}{\partial \beta \partial \theta} & \frac{\partial^2 Ln\left(L(\theta, \beta / x_i)\right)}{\partial \theta^2} \end{bmatrix} \tag{12}$$

where,

$$\frac{\partial^2 Ln\left(L(\theta, \beta / x_i)\right)}{\partial \beta^2} = -\frac{r}{\beta^2} - \sum_{i=1}^{r} \frac{x_i^{\beta}}{\theta^{\beta}} \left(Ln\left(\frac{x_i}{\theta}\right) \right)^2 - (n-r) \frac{x_r^{\beta}}{\theta^{\beta}} \left(Ln\left(\frac{x_r}{\theta}\right) \right)^2$$

$$\frac{\partial^2 Ln\left(L(\theta, \beta / x_i)\right)}{\partial \theta^2} = \frac{r\beta}{\theta^2} - \frac{\beta(\beta+1)}{\theta^2} \left(\sum_{i=1}^{r} \frac{x_i^{\beta}}{\theta^{\beta}} + (n-r) \frac{x_r^{\beta}}{\theta^{\beta}} \right)$$

$$\frac{\partial Ln\left(L(\theta, \beta / x_i)\right)}{\partial \beta \partial \theta} = \frac{1}{\theta} \left(-r + \sum_{i=1}^{r} \frac{x_i^{\beta}}{\theta^{\beta}} + (n-r) \frac{x_r^{\beta}}{\theta^{\beta}} \right) + \frac{\beta}{\theta} \left(\sum_{i=1}^{r} \frac{x_i^{\beta}}{\theta^{\beta}} Ln\left(\frac{x_i}{\theta}\right) + (n-r) \frac{x_r^{\beta}}{\theta^{\beta}} Ln\left(\frac{x_r}{\theta}\right) \right)$$

Another technique to estimate distribution parameters is based on Least Square Estimation (LSE) technique via regression analyses (see Appendix B). Using the inverse transformation of the cumulative distribution function, the (linear) relationship between the observed and empirical cumulative probabilities is found. Non-parametric Kaplan-Meier (see Appendix C for derivations) and / or Rank distribution (see Appendix E for derivations) methods could be used for the empirical cumulative probabilities estimation.

The expected cumulative probabilities based on non-parametric Kaplan-Meier is given by:

$$1-\widehat{F}(x)=\prod_{r\in I_x}\frac{n-r}{n-r+1} \tag{13}$$

where, 'r' is the rank of the ordered uncensored observation, I_x is all positive integers 'r', such that $x(r)\le x$ and $x(r)$ is uncensored.

Based on the Rank distribution, the expected cumulative probabilities could be estimated via mean rank or median rank and are given by, respectively:

$$\widehat{F}(x_{r:n})=\frac{r}{n+1} \tag{14}$$

$$\widehat{F}(x_{r:n})\approx\frac{r-\frac{1}{3}}{n+\frac{1}{3}} \tag{15}$$

where, 'r' is the rank of the ordered uncensored observation and censoring is observed after 'r'.

The (leaner) relationship is plotted against the data and LSE technique via regression analysis is carried out to estimate parameters and their correlations. It should be noted that reciprocals of the estimated variances of the empirically estimated cumulative probabilities might be used as weights and Weighted Least Square Estimation (WLSE) technique via regression analysis could be carried out for the parameters and correlation estimations.

Based on $x_1,x_2,...,x_r$ realizations (in increasing order $x_{1:n},x_{2:n},...,x_{r:n}$) from the sample of size 'n' (assuming censoring is observed after the largest observation, so 'n-r' observations are right censored), for the defined Weibull distribution in (9) with shape 'β' and scale 'θ' parameters (assuming the location parameter is zero), the leaner relations for direct and inverse regressions are:

$$\ln\left(-\ln\left((1-\widehat{F}(x_{i:n})\right)\right)=-\beta\ln\theta+\beta\ln(x_{i:n}) \tag{16}$$

$$\ln(x_{i:n})=\ln\theta+\frac{1}{\beta}\ln\left(-\ln\left((1-\widehat{F}(x_{i:n})\right)\right) \tag{17}$$

where $i=1,\cdots,r$, and $\left(1-\widehat{F}(x_{i:n})\right)$ is empirically estimated via Kaplan-Meier by (13), or via mean rank / median rank of $\widehat{F}(x_{i:n})$ by (14) or (15).

Also, for the defined Gumbel (Type I) distribution defined by (4) with location 'u' and scale 'b' parameters, the leaner relations for direct and inverse regressions are:

$$\left(-\ln\left(-\ln\left((\widehat{G}(y_{i:n})\right)\right)\right)=-\frac{u}{b}+\frac{1}{b}y_{i:n} \tag{18}$$

$$y_{i:n} = u + b\left(-\ln\left(-\ln\left((\widehat{G}(y_{i:n})\right)\right)\right) \quad (19)$$

where $i = 1,\cdots,r$, and $\widehat{G}(x_{i:n})$ is empirically estimated via one minus of Kaplan-Meier estimate by (13), or mean rank / median rank of $\widehat{G}(x_{i:n})$ by (14) or (15).

The transformation between Gumbel (Type I) and Weibull distributions are based on the $Y = -\ln X$ relationship, with $u = -\ln\theta$ and $b = 1/\beta$.

Based on estimated ($\hat{\theta},\hat{\beta}$) or ($\hat{u},\hat{b}$) parameters and their correlations, conditional and unconditional failure functions, as well as desired quantile levels are estimated.

2.3 Structural Reliability Estimation Approaches

Structural reliability approaches are based on the so-called limit state and design situation formulations. Limit state defines the boundary of separation between desired states from undesired states (success from failure event(s)) of the structure. While design situation is duration of time with physical conditions, during which the defined limit state of the structure is not exceeded.

Based on ISO 2394 (General principles on reliability for structures), the following main types of limit states and design situations are defined:

- Ultimate limit state: a state associated with collapse, or with other similar forms of structural failure. Ultimate limit states include failure modes such as:
  - Internal failure or excessive deformation of the structure or structural members,
  - Failure or excessive deformation of the ground / foundation,
  - Loss of static equilibrium of the structure,
  - Fatigue failure of the structure or structural members,
- Serviceability limit state: a state that corresponds to conditions beyond which specified service requirements for a structure or structural element are no longer met,
- Persistent situation: normal condition of use for the structure, generally related to its design working life,
- Transient situation: provisional condition of use or exposure for the structure,
- Accidental situation: exceptional condition of use or exposure for the structure.

Further, the analysis is carried out with focus on probability estimation of the defined limit state during the selected design situation. Structural reliability estimation techniques for simple problems can be based on theoretical solutions. For more complicated problems (where computers are required to make calculations) simulation, first order or second order reliability approximation methods are generally used. The accuracy of the results depends on the non-linearity of the limit state equations, the distribution functions and the dependency structure of the random variables that constitute the limit state equation. Random variables are transformed into the standard normal random variables and dependences can be handled by the Nataf transformation (independent on ordering and assuming normal copula structure between variables) or Rosenblatt transformation (assumes conditional structure and it depends on ordering).

2.3.1 Reliability estimation by First and Second Order Reliability Methods

In structural reliability theory, reliability is estimated based on the formulation of a failure function or limit state equation. The failure function (limit state equation) is a function that separates the space into two distinct subspaces, termed failure and survival subspaces. The failure function is typically formulated based on the strengths, the loads and the mechanisms of failure by taking into the account the physical, geometrical, mechanical, etc. properties of the component. The limit state equation becomes a function of the stochastic variables $\mathbf{X}^{\mathbf{T}} = (X_1, \ldots, X_m)$ and is denoted by $g(\mathbf{X})$ such that the failure subspace is defined whenever $g(\mathbf{X}) \leq 0$. It follows from the formulation of the limit state equation that the time-independent probability of failure is:

$$P_f = P\left[g(\mathbf{X}) \leq 0\right] \tag{20}$$

The mathematical formulation of $g(\mathbf{X})$ might not be unique, but the estimated probability of failure by (20) should be unique. $g(\mathbf{X})$ might be transformed into the standardized Normal domain by some transformation function $\mathbf{X} = T(\mathbf{Z})$, where $\mathbf{Z}$ is *'m'* dimensional column vector of the mutually independent standard Normal random variables, so $Cov(Z_i, Z_j) = 0 \ \forall i, j;\ i \neq j$. This means that the limit state equation can be represented in the $\mathbf{Z}^{\mathbf{T}} = (Z_1, \ldots, Z_m)$ standardized domain. Therefore, the probability of failure will be defined by:

$$P_f = P\left[g(\mathbf{X}) \leq 0\right] = P\left[g(T(\mathbf{Z})) \leq 0\right] \tag{21}$$

Depending on the linearity of $g(T(\mathbf{Z}))$, the exact or an approximate probability of failure might be computed or estimated.

If $g(T(\mathbf{Z}))$ is linear, then the limit state equation represented by $\mathbf{Z}^{\mathbf{T}} = (Z_1, \ldots, Z_m)$ in standardized domain becomes a hyperplane in $\mathbb{R}^m$, thus the limit state equation can be written:

$$g(T(\mathbf{Z})) = \beta - \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{Z} \tag{22}$$

where β is the reliability index and $\boldsymbol{\alpha}$ is a unit normal column vector directed towards to the failure subset, so $\|\boldsymbol{\alpha}\| = 1$.

Expectation and variance of the linearly defined limit state equation $g(T(\mathbf{Z}))$ will be $E\left[g(T(\mathbf{Z}))\right] = \beta$ and $Var\left[g(T(\mathbf{Z}))\right] = 1$.

The time-independent probability of failure, based on the linearly defined $g(T(\mathbf{Z}))$, becomes:

$$P_f = P\left[\beta - \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{Z} \le 0\right] = \Phi(-\beta) \tag{23}$$

where $\Phi(\ )$ is the standard normal distribution function.

The reliability index β is defined as the shortest distance from the origin to the $\beta - \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{Z}$ hyperplane, and it is known as Hasofer and Lind reliability index [Madsen et al., 1986].

If $g(T(\mathbf{Z}))$ is non-linear, then the limit state equation represented by $\mathbf{Z}^{\mathbf{T}} = (Z_1,\ldots,Z_m)$ in the standardized domain becomes a hypersurface defined in $\mathbb{R}^m$ and the smallest distance from the origin to the hyper-surface can be found by iteration algorithms. Next, the failure surface can be linearized in that point and this linear surface can be used as an approximation. This method is termed the First Order Reliability Method (FORM) and the estimated shortest distance is the reliability index. Thus, the formulation will be:

$$g(T(\mathbf{Z})) \approx \beta - \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{Z} \tag{24}$$

$$P_f = P\left[g(\mathbf{X}) \le 0\right] = P\left[g(T(\mathbf{Z})) \le 0\right] \approx P\left[\beta - \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{Z} \le 0\right] = \Phi(-\beta) \tag{25}$$

At the design point of a non-linear failure surface, $g(T(\mathbf{Z}))$ a second order Taylor series expansion could be performed as well, and the failure surface approximated by a paraboloid. This method is known as the Second Order Reliability Method (SORM). Based on an appropriate rotation of the coordinate system in standardized domain, the probabilistic content under the paraboloid can be calculated.

Suppose the design point ($\mathbf{z}^* = \beta\boldsymbol{\alpha}$) is known or estimated by FORM, then the second order Taylor series expansion of the limit state equation at this point would be:

$$g(T(\mathbf{Z})) \approx g(T(\mathbf{z}^*)) + \nabla g(T(\mathbf{z}^*))^T(\mathbf{Z} - \mathbf{z}^*) + \frac{1}{2}(\mathbf{Z} - \mathbf{z}^*)^T\mathbf{H}(\mathbf{Z} - \mathbf{z}^*) \tag{26}$$

where $g(T(\mathbf{z}^*)) = 0$, $\nabla g(T(\mathbf{z}^*))$ and $\mathbf{H}$ are the gradient vector and the Hessian matrix of the limit state equation at the design point $\mathbf{z}^*$, respectively.

After some manipulation of (26), it becomes:

$$g(T(\mathbf{Z})) \approx \frac{1}{2}\left[a_0 + 2\mathbf{a}_1^{\mathbf{T}}\mathbf{Z} + \mathbf{Z}^T\mathbf{H}\mathbf{Z}\right] \tag{27}$$

where $a_0 = 2\left|\nabla g(T(\mathbf{z}^*))\right|\beta + \beta^2\boldsymbol{\alpha}^{\mathbf{T}}\mathbf{H}\boldsymbol{\alpha}$, $\mathbf{a}_1^{\mathbf{T}} = \left|\nabla g(T(\mathbf{z}^*))\right|\boldsymbol{\alpha}^{\mathbf{T}}\left(\mathbf{I} - \dfrac{\beta}{\left|\nabla g(T(\mathbf{z}^*))\right|}\mathbf{H}\right)$.

As far as the Hessian matrix is real and symmetric, then an orthogonal Eigen transformation based on $\mathbf{H} = \mathbf{S}\boldsymbol{\Lambda}\mathbf{S}^{\mathbf{T}}$ will be $\mathbf{Z} = \mathbf{S}\mathbf{Y}$, and (27) can be written:

$$g(T(\mathbf{S}\mathbf{Y})) \approx \frac{1}{2}\left[a_0 + 2\mathbf{A}_1^T\mathbf{Y} + \mathbf{Y}^T\boldsymbol{\Lambda}\mathbf{Y}\right] \tag{28}$$

where, $\mathbf{A_1} = \mathbf{S}^T \mathbf{a_1}$.

The probability of failure by SORM will then be given by:

$$P_f = P\left[g(T(\mathbf{YZ})) \le 0\right] \approx P\left[\sum_{i=1}^{m} \frac{\left(A_i + \lambda_i Y_i\right)^2}{\lambda_i} \le \sum_{i=1}^{m}\left(\frac{A_i}{\lambda_i^{1/2}}\right)^2 - a_0\right] = \\ = P\left[\sum_{i=1}^{m} \lambda_i \frac{\left(A_i + \lambda_i Y_i\right)^2}{\lambda_i^2} \le \sum_{i=1}^{m}\left(\frac{A_i}{\lambda_i^{1/2}}\right)^2 - a_0\right] \tag{29}$$

where $\frac{\left(A_i + \lambda_i Y_i\right)^2}{\lambda_i^2}$ is a r.v. with a non-central Chi-square distribution with 1 degree of freedom and non-centrality parameter $\left(\frac{A_i}{\lambda_i}\right)^2$, while $\sum_{i=1}^{n} \lambda_i \frac{\left(A_i + \lambda_i Y_i\right)^2}{\lambda_i^2}$ has a general Chi-square distribution, see [Provost & Rudiuk, 1996], [Lee et al., 2012]. One of the approximations to the probability content defined by (29) is given by [Hohenbichler & Rackwitz, 1988] by:

$$P_f \approx \Phi(-\beta) \prod_{i=2}^{m} \left(1 - \frac{\varphi(-\beta)}{\Phi(-\beta)} k_i\right)^{1/2} \tag{30}$$

where k_i is eigenvalues (principal curvatures) of the rotated paraboloid such that '$\mathbf{z^*}$' is lying on the first axis.

In cases when a non-linear failure surface is complicated (e.g. composed of islands), simulation techniques should generally be used, e.g. crude Monte Carlo Simulation. An efficient simulation method is the Importance Sampling technique, which is based on design point solution from the FORM. Sampling is done such that points are close to the design point and probability of failure is calculated using appropriate weights for each observation.

For time-dependent reliability problems the probability of failure as a function of time, (20) should be reformulated to be dependent on time. If the limit state equation as a function of both time 't' and stochastic variables $\mathbf{X}^{\mathbf{T}} = (X_1, \ldots, X_m)$ is denoted by $g(\mathbf{X}, t)$ and then the time-dependent 'point-in-time' probability of failure will be defined as:

$$P_f(t) = P\left[g(\mathbf{X}, t) \le 0\right] = P\left[g(T(\mathbf{Z}), t) \le 0\right] \tag{31}$$

In this formulation, the limit state equation depends on time explicitly (e.g. not through random processes). All the above-mentioned procedures (FORM, SORM, simulation) will be performed in the same manner for each time step by fixing the time and treating it as a constant, and the corresponding reliability indexes and probabilities of failures as a function of time can be estimated.

If e.g. the load is modeled by a stochastic process and failure is defined by the first time when the load exceeds the resistance, then probability of failure within a small time interval has to be estimated by solving a so-called first passage time reliability problem. This reliability problem is generally very difficult to solve and instead an approximate failure probability can be estimated based on the out-crossing rate of the load exceeding the resistance. The out-crossing rate might be estimated e.g. analytically, asymptotically or by PHI2 methods, see [Madsen et al., 1986], [Andrieu-Renaud et al., 2004].

CHAPTER 3. FATIGUE FAILURE

3.1 Fatigue Reliability Estimation

Fatigue failure is a failure type under cyclic loading of mechanical or thermal stresses. A significant property of fatigue failure is that the applied stresses are much lower in magnitude than those required to initiate a failure for a single cycle. Low cycle and high cycle fatigues are termed to distinguish between fatigue failures that require few number of cycles to failure (usually 10^{-3}) versus high number. The applied cyclic / alternating loading combined with a change of environmental temperature causes creep-fatigue failure. This fatigue failure mode can be critical for IGBTs solder joint crackling under the chip (more details are provided in the thesis).

Fatigue failure mode is governed by fatigue cracking failure mechanism and divided into cyclic hardening / softening, crack nucleation, micro crack growth, macro crack growth and final failure stages, with crack initiation and prorogation periods or stages (see Figure 9).

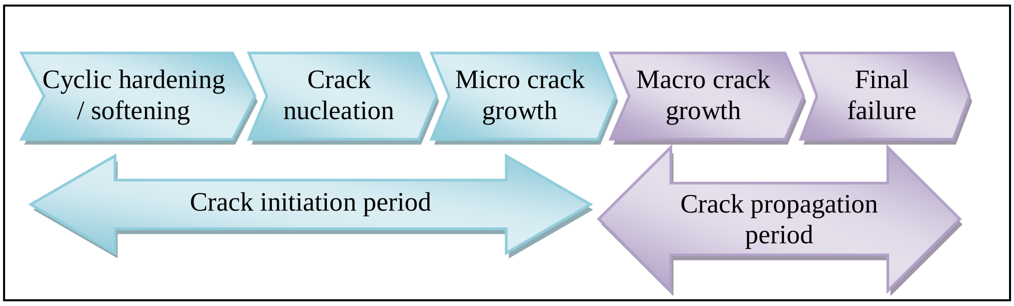

Figure 9: Fatigue crack stages

To quantify the material fatigue resistance, physical tests are often performed and the number of cycles versus load level are recorded. This approach is termed the S-N curve approach (see Appendix F). In the S-N curve approach no separation on crack nucleation and propagating periods and no crack growth information are recorded, only final times to fracture are recorded. Further, MLE or LSE methods could be used to estimate model parameters and their correlations, which can quantify the S-N curve model uncertainties.

To estimate damage level from the given alternating stress history, the Palmgren-Miner accumulated linear damage rule (see Appendix F) can be applied. The drawback of this method is that sequence effects and interactions of consecutive loads are neglected. However, due to its simplicity it is widely used in reliability analysis.

Another approach to quantify the fatigue reliability is based on the Fracture Mechanics (FM) approach via crack propagation length. FM evaluates the crack growth rate in the presence of a crack or a flaw, where material conditions during fatigue crack growth are assumed to be linear elastic (Linear Elastic Fracture Mechanics), or if not, then elastic-plastic FM approaches can be used. By this method, the stress intensity factor is estimated and it is compared to the critical stress intensity factor K_C (J/m^2), while exceedance results in unstable crack growth. Crack extension / growth is usually driven by the opening (Mode I), in plane shearing (Mode II) and tearing (Mode III) modes of stresses. Mode I is the most common in fatigue crack growth processes and its stress intensity factor K_I describes how fast the stress at the crack tip tends to infinity.

Based on the FM approach, K_I is a function of the geometry function Y(a), the far field stress 'σ' and the crack length 'a', and given by:

$$K_I = \sigma Y(a)\sqrt{\pi a} \tag{32}$$

As far as fatigue failure is a failure type under cyclic loading, then the stress range ($S = \sigma_{\max} - \sigma_{\min}$) is used in (32) and stress intensity factor range in opening mode is obtained from:

$$\Delta K_I = SY(a)\sqrt{\pi a} \tag{33}$$

The applied stress ranges have influences on the number of cycles to failure, crack lengths at failure and crack growth rates. E.g. if two cyclic stresses with ranges S_1 and S_2 are applied on identical specimens with the initial crack length a_0, such that $S_1 > S_2$, then:

- Failure times described by number of cycles to failure (n_1 and n_2) will be less $n_1 < n_2$,
- Crack lengths at failure will be shorter $a_1 < a_2$
- Crack growth rates at the given crack length will be higher $da/dn_1 > da/dn_2$.

A log-log plot of crack growth rates versus Mode I stress intensity factor ranges reveals sigmoidal shape with three distinguished regions (see Figure 10).

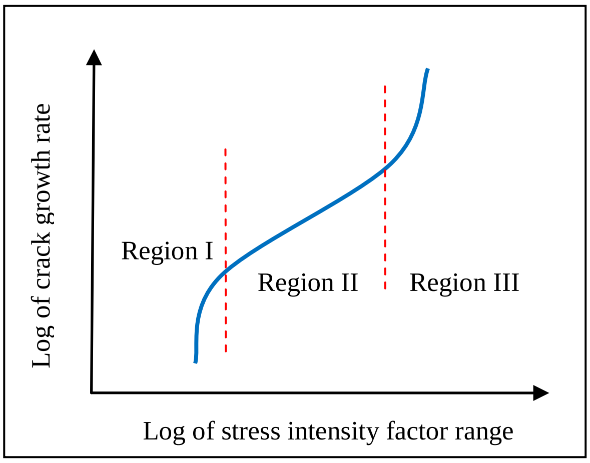

Figure 10: Sigmoidal behavior of fatigue crack growth rate

Region I is a near threshold region described by ΔK_I^{th}, below which no crack growth is observed. Region II is described by linear $\log(\Delta K_I^{II})$ behavior where crack growth rate is constant and Paris-Erdogan law can be applied in this region. Region III is governed by high stress intensity factor ranges with unstable fatigue crack growth rates if $\Delta K_I^{III} \approx K_{IC}$, where K_{IC} is fracture toughness in opening mode.

Paris and Erdogan crack grows equation suggests plotting crack growth rates versus stress intensity factor ranges based on the relationship:

$$\frac{da}{dn} = A\left(\Delta K\right)^m \tag{34}$$

where A and m are experimentally estimated constants.

Combining (33) and (34) will give dn/da relationship and integrating it will give general fatigue life equation:

$$N_f = \int_0^{N_f} dn = \frac{1}{AS^m \pi^{\frac{m}{2}}} \int_{a_0}^{a_f} \frac{da}{\left(Y(u)\sqrt{u}\right)^m} \tag{35}$$

where N_f is the number of cycles to failure and a_f is the crack length at failure, which could be determined by (33) and K_{IC} for constant amplitude uniaxial cyclic loading:

$$a_f = \frac{1}{\pi}\left(\frac{K_{IC}}{Y(a)\sigma_{\max}}\right)^2 \tag{36}$$

It is seen from (35) that $N_f S^m = const.$ at the failure time, which is in agreement with S-N curve approach, and variabilities / uncertainties of the parameters should be included into (35) for fatigue reliability analysis.

The above mentioned methods could be used in relation to the power electronic components and they are applied for IGBT module reliability estimation.

3.2 Linking Structural and Classical Reliability Approaches for Fatigue Reliability Estimation

Usually, the statistical analysis of test data for modeling an S-N curve is carried out by regression analysis (see Appendix B). Thereby uncertainties can be included in a reliability analysis using a limit state equation based on the SN-curve. In general, the uncertainties are divided into aleatory and epistemic uncertainties. Aleatory uncertainty is an inherent variation associated with the physical system or the environment, and it can be characterized as irreducible uncertainty or random uncertainty. Epistemic uncertainty is uncertainty due to lack of knowledge of the system or the environment and includes model, statistical and measurement uncertainties. It can be characterized as uncertainty, which can be reduced by better models, more data, etc. It is noted that some aleatory uncertainties “change” to epistemic uncertainties when the system is realized.

One of the models to incorporate these uncertainties is to define a limit state equation such that those uncertainties will be accounted. It is possible to define a limit state equation as:

$$g(\Delta m, \mathbf{X}, t) = \Delta m - \sum_{i \in [0:t)} d_i(\mathbf{X}, t) \tag{37}$$

where Δm models the model uncertainty related to Miner’s rule, $d_i(\mathbf{X}, t)$ is a partial damage induced by the time ‘t’ and $\mathbf{X}$ is a vector of random variables associated with the quantification of the partial damage.

As far as Miner’s rule has the drawback of not accounting for the sequence and interaction effects of the stresses / loads, then the relevance to use it should be justified and uncertainties related to its shortcoming are supported by Δm. Also, to increase estimation accuracy, in (37) some calibration parameters could be introduced.

For the time dependent limit state equation (37), the unconditional failure rate at time t with reference time interval Δt (typically one year) can be estimated by:

$$f(t, \Delta t) = \frac{P\left(g(\Delta m, \mathbf{X}, t) \le 0\right) - P\left(g(\Delta m, \mathbf{X}, t - \Delta t) \le 0\right)}{\Delta t} \tag{38}$$

and the corresponding conditional failure (hazard) rate at time t for a given time interval Δt given survival at time t might be estimated by:

$$h(t, \Delta t) = \frac{f(t, \Delta t)}{P\left(g(\Delta m, \mathbf{X}, t - \Delta t) > 0\right)} \tag{39}$$

Generally, WT components are divided in two groups:

- Electrical and mechanical components, where the reliability is estimated using either classical reliability models or physics of failure based models,
- Structural members, such as tower, mainframe, blades and foundation, where limit state equations can be formulated by defining failure or unacceptable behavior. E.g. Failure of the foundation could be overturning, failure of a blade could be large deflections with nonlinear effects and delamination.

Failure of electrical and / or mechanical components can influence failure of structural components, since the loads on these can increase dramatically. E.g., loss of torque due to failure in control system may cause problems in blades or tower-nacelle motion, which again may imply large edgewise vibrations in the blades. Therefore, the reliability of electrical and / or mechanical components should be included in a reliability assessment of the whole WT system.

In this research, faults of the converter system are considered. Converter system failure causes grid connection failure (grid loss), such that it is indirectly causing an increase of the damage level of the structural components (e.g. fatigue failure) or provide a risk for extreme failure, which is critical for offshore WT applications.

In general, if the annual failure probability of the i-th failure mode of a selected structural component is defined by $p_i = P(F_i)$ such that $F_i \cap F_j = \varnothing$ for all $i \neq j$, and a partition of the sample space of failure for the considered component is $F = \{F_1, F_2, \ldots, F_n\}$, then by considering the annual probability of grid loss, the annual probability of failure of the selected structural component will be given by:

$$P_F = \left\{ \sum_i P\left(F_i \left| grid\ \ loss\right.\right) \right\} \cdot P_{annual}\left(grid\ \ loss\right) \tag{40}$$

where $P_{annual}\left(grid\ \ loss\right)$ is estimated by (38) with one year reference period, $\Delta t = 1$ year.

Alternatively, if the mean annual failure rate $\lambda^m_{grid\ \ loss}$ of grid loss is estimated by a classical reliability approaches, then the mean annual failure rate of the considered structural component $\lambda^m_{F \cap grid\ \ loss}$ at the time of the grid loss, by considering mean wind speed (W) and the blade positions (Pos), is estimated by:

$$\lambda^m_{F \cap grid\ \ loss} = \left\{ \sum_i \sum_{W_i \in W^I} \sum_{Pos_i \in Pos^I} \left[\begin{array}{l} P\left(F_i \left| \text{grid loss} \cap W_i \in W^I \cap Pos_i \in Pos^I \right.\right) \times \\ P\left(W_i \in W^I\right) P\left(Pos_i \in Pos^I\right) \end{array} \right] \right\} \lambda^m_{grid\ \ loss} \tag{41}$$

where W^I is a wind speed interval, which could be obtained by discretization, e.g. $\left[0:15\right) \cup [15:25) \cup [25:\infty)$ [m/s]; Pos^I is the blades position (Pos) sample space, which is determined by the relative angle (θ) of a blade to the tower, $\theta \sim U[0:2\pi]$. θ also could be discretized by disjoint intervals e.g. with $\pi/4$ steps. The mean annual failure rate of grid loss $\lambda^m_{grid\ \ loss}$ should be estimated based on the observed data and it can be highly site dependent.

The annual failure probability of the considered component can then be approximately estimated based on the mean annual failure rate $\lambda^m_{F \cap grid\ \ loss}$ by:

$$P_F = 1 - \exp\left(-\lambda^m_{F \cap grid\ \ loss}\right) \tag{42}$$

The above mentioned approaches might be used for consideration of failures in power converter systems, resulting in grid loss, particularly due to the failures of IGBTs. The developed IGBTs reliability estimation methods can be used together with the structural components reliability estimation models to estimate the reliability of the structural components under the grid loss situations.

CHAPTER 4. RELIABILITY ON SYSTEMS LEVEL

4.1 Background for Systems Configuration

Systems, where components / subsystems are in parallel arrangement, are termed as parallel systems. Such a system can be represented by time-independent or time-dependent models. In the time-independent model representation, the components / subsystems reliabilities are considered constant with time, and some base period is implied for modeling of the uncertainties. Whereas, representing the system reliability by time-dependent model indicates that components / subsystems reliabilities are varying as a function of time.

A parallel system with *'n'* components is a system, which fails if all *'n'* components / subsystems fail(s). For the time-independent model, the parallel system unreliability consisting of *'n'* components is given by:

$$P^{p}_{unrel.} = P\left[E_1 \quad \cap \quad E_2 \quad \cap \quad \cdots \quad \cap \quad E_n\right] \tag{43}$$

where E_i is the event that i^{th} subsystem / component operates unsuccessfully.

For the time-dependent model, the parallel system unreliability consisting of *'n'* components is given by:

$$P^{p}_{unrel.}(t) = P\left[E_1(t) \quad \cap \quad E_2(t) \quad \cap \quad \cdots \quad \cap \quad E_n(t)\right] \tag{44}$$

where $E_i(t)$ is the event that i^{th} subsystem / component operates unsuccessfully by the time *'t'* (from zero till time *'t'*).

A series system with *'n'* components is such system, which fails if at least one of the components / subsystems fail(s). If series system unreliability consisting *'n'* components are considered, then time-independent and time-dependent models will be given by:

$$P^{s}_{unrel.} = P\left[E_1 \quad \cup \quad E_2 \quad \cup \quad \cdots \quad \cup \quad E_n\right] \tag{45}$$

$$P^{s}_{unrel.}(t) = P\left[E_1(t) \quad \cup \quad E_2(t) \quad \cup \quad \cdots \quad \cup \quad E_n(t)\right] \tag{46}$$

where E_i is the event that i^{th} subsystem / component operates unsuccessfully, and $E_i(t)$ is the event that i^{th} subsystem / component operates unsuccessfully by the time *'t'*.

The assumption, that the lifetime of each component follows some continuous cumulative distribution function (c.d.f.), is implying that continuous damage accumulation exists and cumulative damage increases by the usage time propagation. Let Y_i be the random lifetime of the component 'i' with $F_{Y_i}(t)$ c.d.f., implying that:

$$F_{Y_i}(t) = P(Y_i \le t) = P\left[E_i(t)\right] \tag{47}$$

and $0 \le F_{Y_i}(t) \le 1$ for any $t \ge 0$.

If two or more components comprise the parallel system, then the parallel system unreliability by the time *‘t’* will be given based on the components failure joint distribution function defined as:

$$F^{p}_{unrel.}(t) = F_{Y_1,\cdots,Y_n}(t,\ldots,t) = P^{p}_{unrel.}(t) \tag{48}$$

If two or more components comprise the series system, then the series system unreliability by the time *‘t’* will be given via the components survival joint distribution function defined as:

$$F^{s}_{unrel.}(t) = 1 - \overline{F_{Y_1,\cdots,Y_n}(t,\ldots,t)} = P^{s}_{unrel.}(t) \tag{49}$$

where $\overline{F_{Y_1,\cdots,Y_n}(t,\ldots,t)} = P\left[Y_1 > t \quad \cap \quad Y_2 > t \quad \cap \quad \cdots \quad \cap \quad Y_n > t\right]$ is components survival joint distribution function.

4.2 Systems Reliability Estimation by Classical Reliability Approach

Systems reliability estimation by the classical reliability approach is generally based on the assumption of statistical independence. Thus, it is assumed that components / subsystems failure times (lifetimes) are statistically independent among each other and no influence is considered upon of failure of either one. This implies that all subsystems are activated when system is activated and failures do not influence on the reliability of survived components / subsystems. It should be noted that by classical reliability approach, the independence assumption applies to both time-independent and time-dependent models. Based on independence assumption, the parallel (48) and series (49) systems unreliability by the time *‘t’* will be given by:

$$P^{p}_{unrel.}(t) = \prod_{i=1}^{n} F_{Y_i}(t) \tag{50}$$

$$P^{s}_{unrel.}(t) = 1 - \prod_{i=1}^{n}\left(1 - F_{Y_i}(t)\right) \tag{51}$$

4.3 Systems Reliability Estimation by Structural Reliability Approach

Structural reliability estimation is based on the mathematical formulation of the failure event by the time *‘t’* via limit state equation. If i-th component failure event by the time *‘t’* is defined by the limit state equation $g_i(\mathbf{X},t)$, where $i = 1,\cdots,n$, the parallel (44) and series (46) systems unreliability by the time 't' will be given by:

$$P^{p}_{unrel.}(t) = P\left[\bigcap_{i=1}^{n}\left\{g_i(\mathbf{X},t) \le 0\right\}\right] \tag{52}$$

$$P^{s}_{unrel.}(t) = P\left[\bigcup_{i=1}^{n}\left\{g_i(\mathbf{X},t) \le 0\right\}\right] \tag{53}$$

If the limit state equation $g_i(\mathbf{X},t)$ is linearly defined and based on an independent random vector $\mathbf{X}$, then exact probabilities are calculated by (114) and (118) (see Appendix G). If independence of the random vector $\mathbf{X}$ is not satisfied, but marginal distributions and linear correlations are available, then e.g. the Nataf transformation can be applied.

If the limit state equation $g_i(\mathbf{X},t)$ is not linearly defined, then FORM, SORM or simulation methods could be used to evaluate (52) and (53) probabilities. E.g. FORM approximate solution to the systems unreliability will be given by:

$$P^{p}_{unrel.}(t) \approx \Phi_m\left(-\boldsymbol{\beta}(t), \boldsymbol{\rho}^{(t)}\right) \tag{54}$$

$$P^{s}_{unrel.}(t) \approx 1 - \Phi_m\left(\boldsymbol{\beta}(t), \boldsymbol{\rho}^{(t)}\right) \tag{55}$$

where $\boldsymbol{\rho}^{(t)} = \boldsymbol{\alpha}^{\mathbf{T}}(t)\boldsymbol{\alpha}(t)$, (see Appendix G for derivation and details).

CHAPTER 5. RELIABILITY AND OPERATION & MAINTENANCE

5.1 Reliability Estimation Procedures Aimed for Operation & Maintenance Strategies Development

The environment and loads under which the components are utilized are directly influencing the reliability of the components. Thus, it is important to build reliability models, which take into account loads and environmental conditions, and can be used as a basis for optimal O&M strategies development.

The general framework and procedure for such type reliability modeling is illustrated in Figure 11. The first step is to identify failure modes for the corresponding system. Failure modes are type of failures, which have been observed, perceived and seen (e.g. mechanical failure modes are fatigue, corrosion, wear, erosion, etc.). The next step is to understand failure mechanisms for each failure mode, which are processes that govern the failures (e.g. crack initiation and propagation, etc.). Next step is the mathematical modeling of the selected failure mechanism for the observed failure mode by considering material, geometry, interaction, physical properties and affects.

Further step is to use the mathematical model to establish limit state equations based on degradation or damage models. Next step is to expose the site-specific loads and stresses to the limit state models and analyze outputs by structural reliability methods (e.g. FORM, SORM, etc.), resulting in reliability levels determination.

If the goal is to develop reliability estimation models aiming for O&M strategy development, then each component / system in the WT should be considered. This will require enormous time and financial resources, thus this research was focused on developing reliability estimation models for the WT critical components.

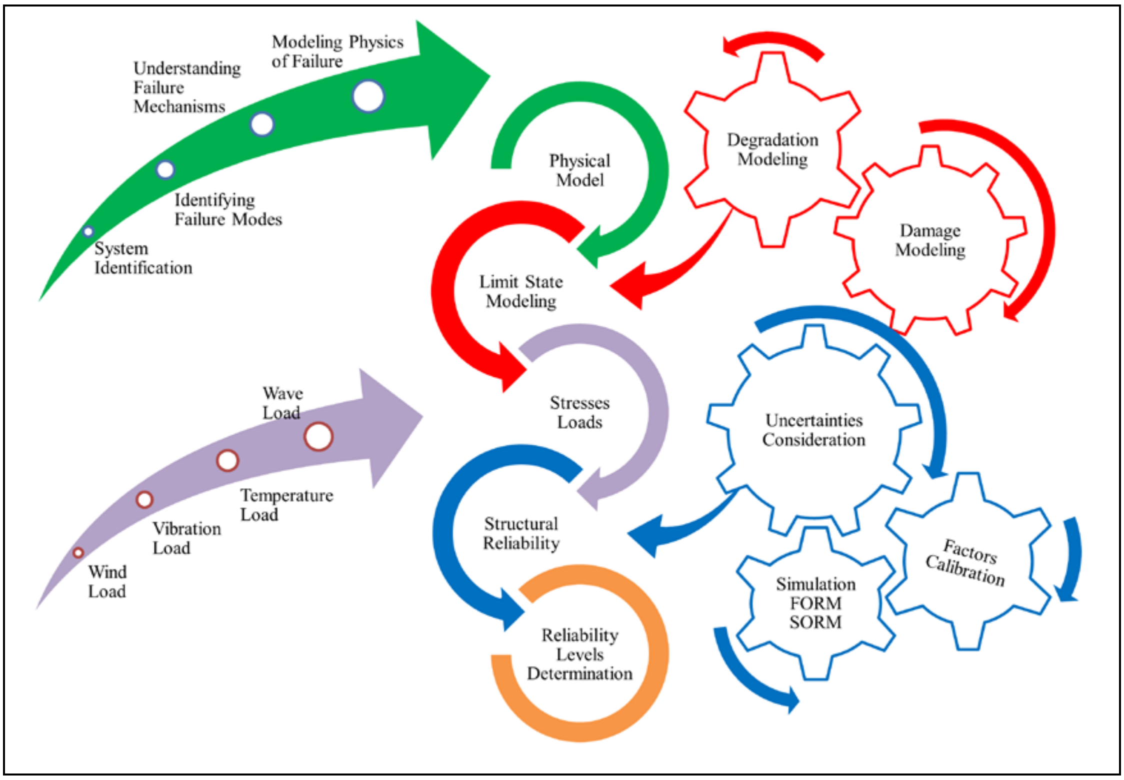

Figure 11: Reliability estimation procedure by considering loads

5.2 Risk Based Operation & Maintenance Planning

At the design stage, a decision on the optimal, initial design parameters $\mathbf{z} = (z_1, ..., z_N)$ is made which generally should maximize the total expected benefits minus costs during the whole lifetime such that safety requirements are fulfilled at any time. In practice, requirements from standards and actual costs of materials are used to determine the optimal design. Further, upon implementation of it into and usage during WT lifetime, a continuous monitoring of WT critical components or details is performed. The critical components are subject to (correlated) uncertain exposure / quantities such as wind and wave climate loads, strengths, degradation parameters, model and measurement uncertainties and be modeled by $\mathbf{X} = (X_1, ..., X_n)$ stochastic variables. Thus, it is necessary to identify the O&M strategy parameters (see Figure 12), which could be:

- Total number of inspections N during the service life T_L,
- Actual times $\mathbf{T} = (T_1, T_2, ..., T_N)$ of each inspections,
- The inspection methods (qualitative measures) $\mathbf{q} = (q_1, q_2, ..., q_M)$
- Possible maintenance actions (assuming as good as new after maintenance), which are modeled by a decision rule $d(S)$, based on anticipated inspection

observations $\mathbf{S} = \left(S_1, ..., S_{N_M}\right)$, which are emerged due to realizations of the $\mathbf{X} = \left(X_1, ..., X_n\right)$ stochastic variables.

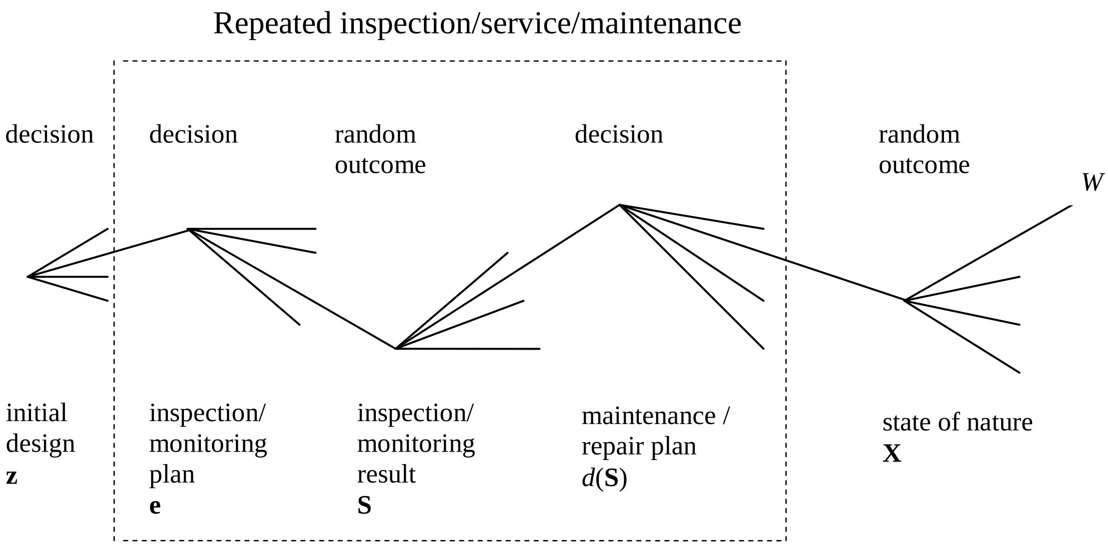

Figure 12: Decision tree for optimal O&M planning, [Sørensen, 2009]

Thus, at time zero no information about the system is available, and it is necessary to identify the above-mentioned O&M strategy parameters, written as $\mathbf{e} = (N, \mathbf{T}, \mathbf{q})$. Based on [Sørensen, 2009], a risk based O&M strategy might be determined via the following optimization problem:

$$\begin{aligned} \max_{\mathbf{z},\mathbf{e},d} \quad & \mathrm{W}(\mathbf{z},\mathbf{e},d) = B(\mathbf{z},\mathbf{e},d) - C_{\mathrm{I}}(\mathbf{z},\mathbf{e},d) - \\ & C_{IN}(\mathbf{z},\mathbf{e},d) - C_{REP}(\mathbf{z},\mathbf{e},d) - C_F(\mathbf{z},\mathbf{e},d) \\ \text{s.t.} \quad & \Delta P_{F,t}(t,\mathbf{z},\mathbf{e},d) \le \Delta P_F^{\max}, \qquad t = 1, 2, ..., T_L \end{aligned} \tag{56}$$

where, $W(z, \mathbf{e}, d)$ is the total expected capitalized benefits minus costs in the service lifetime T_L, B is the expected benefits, C_I is the initial costs, C_{IN} is the expected inspection or service costs, C_{REP} is the expected costs of maintenance or repair action, C_F is the expected failure costs, $\Delta P_{F,t}$ is an annual probability of failure in the year t, $\Delta P_F^{\max}$ is a maximum acceptable annual (or accumulated) failure probability (which are restrictive measures and identified by standards and codes).

Solution to (56) might be achieved based on yearly failure probabilities (or reliabilities), estimated by structural reliability methods and via stochastic variables distributions, estimated based on experience and historical data. That is why developing methods for the components / systems reliability estimation are critical for O&M planning strategy determination.

CHAPTER 6. RESEARCH OUTCOMES

6.1 Reliability Models for IGBTs and Wind Turbine Systems

The aim of this research was to develop reliability model(s), which will be integrated into O&M strategies planning and development. This means that reliability models should be formulated such that failure could be described via critical thresholds of the accumulated damage or degradation paths, and the failure probability as function of time could be estimated.

Development of a reliability model for IGBT application is described in Paper 1. In late 90s by [Held et al., 1997], an extensive test was performed to investigate the solder fatigue cracking failure mechanism due to fluctuating temperature cycles. Failure was defined by a predetermined percentage increase in collector-emitter voltage and lifetime dependency from both mean temperatures (by a thermally activated mechanism) and temperature range for the selected IGBT module was observed. Later, in [Lu et al., 2007] a lifetime model was proposed by considering accumulated plastic strain per cycle and number of cycles of failure, where the chip interconnected area degradation was defined as failure criteria (represented by a crack length). In addition, model parameters were estimated for 20% interconnected area shrinkage used as failure criteria. In [Yin et al., 2008] a finite element model was used to estimate the accumulated plastic strain per cycle for a variety of combinations of temperature means and temperature ranges.

In this research, the development of a failure model was based on the consideration that it should result in failure criteria to allow for O&M strategy development. Therefore, the model should be based on accumulated plastic strain per cycle, mean temperatures as well as temperature ranges and failure criteria will be defined by chip interconnected area degradation. Combining these, a model was developed and presented in Paper 1. This model was coupled with Rainflow counting [Nieslony, 2009] and Miner's rule for linear damage accumulation and a limit state equation was proposed. Some parameters in a model were estimated via published data, while assumptions were made regarding others where no data was available.

Further, an application of the developed model for reliability analysis was described in Paper 2. At this stage, a simulation based approach was used to estimate reliability via structural reliability methods. A detailed reliability research was conducted including censored data and the results were analyzed based on non-parametric reliability approaches. In addition, an application was explored for using structural reliability approaches for safety factor calibration, including the effect of changes in load profile behavior on reliability levels and damage accumulation.

Next, First Order Reliability Method (FORM) was applied and an extensive computer code in MATLAB environment was developed for this purpose. Comparison of the results from (crude Monte Carlo) simulation and FORM were presented in Paper 3. Close agreement of the reliability estimates were observed. This indicates that FORM is an efficient tool for such type problems reliability assessment and the developed MATLAB code is valid. Further, the advantage of FORM over (crude Monte Carlo) simulation in short computational times was observed. Next, the ability to integrate the code with the WT operating system was proposed, which will allow estimating current reliability level of the component from the measured load history and indicate the state of the risk, which is a direct application for O&M strategy development.

In Paper 4, representative temperature loads were estimated for grid side IGBTs in a variable speed layout for a full power converter system for 2.3MW WT. Junction temperatures were estimated and presented as a function of wind speeds. Based on the estimated load profiles, grid side IGBT's chip solder joint reliability was estimated. It was noted that the model parameters need to be estimated accurately and junction temperatures should be considered in shorter time steps in order to obtain sufficient accurate reliability estimates.

Further, if IGBT module be considered as a parallel system consisting of IGBT chips then system level reliability estimation needs to be developed. Distinguish difference of this system is that it is a load sharing system. Upon failure of either component, the load on surviving components will be redistributed and consequently will be much higher. In Paper 5, such a situation was considered. It was assumed that the three parallel connected IGBT chips comprise the IGBT module and upon failure of either chip a 20% of load increase would be observed on each survived chips. An approximate method was proposed for the system reliability estimation. The proposed method is simple and has advantage of providing conservative estimate of the system reliability.

As it has been discussed in Chapter 3, another important aspect of WT reliability estimation is the situation where a fault occurs e.g. due to grid loss resulting from the converter system failure. Reliability of structural components was examined under such situations and theoretical background was developed in Paper 6. Such situations are quite common and have vital importance especially for off-shore WTs.

Advancing the topic of dependent / load sharing system reliability estimation, a theoretical background was developed. A method was proposed based on the sequential order statistics theory. This is an ongoing research and some preliminary result on reliability estimation of the components from load sharing system is presented below.

6.2 Depended Systems Reliability Estimation by Structural Reliability Approaches

Correlations between components may have significant influence for systems reliability estimation. The classical reliability approaches often assume that components / subsystems are statistically independent and upon failure, no influence will be impacted on the survived components / subsystems. However, in many real world systems this is not true. An example for a structural system could be suspension or cable-stayed bridges where failure of any cable will increase the load on the remaining cables and increase the hazard of failure. Examples utilizing mechanical systems could be twin-engine tanks or helicopters considered as a system, or suspension systems in mechanical vehicles. An example in electrical systems could be wire-bonding lift-off failure modes (on chips or processors). For WT systems, it could be bolts or welded joints failures, or mooring cables failures in floating wind turbines applications, etc. As it is seen, many real world systems require a more detailed approach for the system reliability modeling taking correlation into account.

Correlation between components / subsystems can be taken into account through statistical correlation and by mechanical (physical failure-effect) dependency. In this investigation, the statistical correlation was assumed negligible and only physical failure effect correlation is considered. This type of correlation is modeled by sequential order failure system and combined by structural reliability approaches for subsequent failure time estimation. It is assumed that the component / subsystem fails due to fatigue (or generally by a continuously increasing damage function), governed by the accumulated damage during the usage time. Thus, the lifetime of each component / system will follow some continuous and non-negative cumulative distribution function.

6.2.1 Theoretical Background for Ordinary and Sequential Ordered Random Variables

Ordered random variables are a special class of random variables in statistical and probability theory. Behavior of such a random sample has been studied throughout the years. It has applications in description of engineering systems and reliability estimation of the so-called *'r-th out n'* systems, non-parametric distribution estimations, etc. However, the derivation of the distribution of order random variables assumes identical distribution and independence within the sample (see Appendix D). These assumptions lead to the class of ordinary order statistics [David & Nagaraja, 2004], [Balakrishnan & Cohen, 1991], [Bartoszynski & Niewiadomska-Bugaj, 2007].

The distribution function of the ordered random variable $X_{\mathrm{r:n}}$, from $\left(X_1, X_2, ..., X_n\right)$ random sample, where X_{i}'s are independent and identically distributed from parent distribution function $F_X(t)$ and corresponding density function $f_X(t)$, is given by (see Appendix D):

$$O_{X_{r:n}}(t) = P(X_{r:n} \le t) = I_{F_X(t)}(r, n-r+1) = \\ = \frac{\Gamma(n+1)}{\Gamma(r)\Gamma(n-r+1)} \int_0^{F_X(t)} p^{r-1}(1-p)^{n-r} dp \tag{57}$$

$$o_{X_{r:n}}(t) = \frac{n!}{(r-1)!(n-r)!} F_X(t)^{r-1}(1-F_X(t))^{n-r} f_X(t) \tag{58}$$

where $I_p(\alpha,\beta)$ is a regularized incomplete beta function and $\Gamma(\ )$ is a gamma function.

The joint density function of the ordered random variables $\left(X_{1:n}, X_{2:n}, ..., X_{r:n}\right)$ from the parent distribution $F_X(t)$ will form a multinomial distribution and can be written as:

$$o_{(X_{1:n}, X_{2:n}, ..., X_{r:n})}(t_1, t_2, \ldots, t_r) = \frac{n!}{(n-r)!}\left[\prod_{w=1}^{r} f_X(t_w)\right]\left(1-F_X(t_r)\right)^{n-r} \tag{59}$$

Given information on preceding failure times $\{X_{r-i:n} = t_{i-1} / i = 1, \ldots, (r-1)\}$, the conditional distribution of $X_{r:n}$ given the preceding failure times is only depending on $\{X_{r-1:n} = t_{r-1}\}$. This means that ordinary order statistics form a Markov chain with transition probabilities:

$$P\left(X_{r:n} > t_r / X_{r-1:n} = t_{r-1}\right) = \left(\frac{1-F_X(t_r)}{1-F_X(t_{r-1})}\right)^{n-r+1} \tag{60}$$

In ordinary order statistics, it is assumed that failure of either component does not influence on the life of the surviving components. If a failure in the sample influencing on the reaming components / subsystems lifetime, then the scheme of sequential order statistics is used to represent this pattern. The concept of sequential order statistics was introduced by Udo Kamps. In this paper, the logic and derivation of sequential order statistics are described in more details, see [Kamps, 1995], [Kamps, 1996].

Introducing the random sample of size *'n'* by $\left(X_1^{(1)}, X_2^{(1)}, ..., X_n^{(1)}\right)$ where all $X_i^{(1)}$ are independent and identically distributed (i.i.d) and continuous random variables with $f_{X^{(1)}}(t)$ probability density function (p.d.f) and $F_{X^{(1)}}(t)$ cumulative distribution function (c.d.f.), respectively. After failure of the minimum $X_{1:n}^{(1)} = \min\left(X_1^{(1)}, X_2^{(1)}, ..., X_n^{(1)}\right)$ in the sample of $\left(X_1^{(1)}, X_2^{(1)}, ..., X_n^{(1)}\right)$, the new i.i.d. sample of size *'n-1'* from the remaining components lifetimes are composed $\left(X_1^{(2)}, X_2^{(2)}, ..., X_{n-1}^{(2)}\right)$ and their distribution function is assumed to be conditional on $X_{1:n}^{(1)} = s_1$. By using $F_{X^{(2)}}(t)$ instead of $F_{X^{(1)}}(t)$, the influence of the first failure time on the remaining lifetimes is specified. So, conditional on $X_i^{(2)} > s_1, i = 1, \cdots, n-1$, the distribution function of i.i.d. $X_i^{(2)}, i = 1 \cdots n-1$, is:

$$F_{X^{(2)}/X_{1:n}^{(1)}=s_1}(t / s_1) = \frac{F_{X^{(2)}}(t) - F_{X^{(2)}}(s_1)}{1 - F_{X^{(2)}}(s_1)} \tag{61}$$

Next, the second failure corresponding to the minimum $X_{1:n-1}^{(2)} = \min\left(X_1^{(2)}, X_2^{(2)}, ..., X_{n-1}^{(2)}\right)$, where $X_i^{(2)} \overset{i.i.d.}{\sim} F_{X^{(2)}/X_{1:n}^{(1)}=s_1}(t / s_1)$ $i = 1 \cdots n-1$, is taken into account. Observe that cumulative distribution function of the minimum $X_{1:n-1}^{(2)} = \min\left(X_1^{(2)}, X_2^{(2)}, ..., X_{n-1}^{(2)}\right)$ from the $F_{X^{(2)}/X_{1:n}^{(1)}=s_1}(t / s_1)$ parent distribution function is:

$$F_{X_{1:n-1}^{(2)}/X_{1:n}^{(1)}=s_1}(t / s_1) = 1 - \left(1 - F_{X^{(2)}/X_{1:n}^{(1)}=s_1}(t / s_1)\right)^{n-1} = 1 - \left(\frac{1 - F_{X^{(2)}}(t)}{1 - F_{X^{(2)}}(s_1)}\right)^{n-1} \tag{62}$$

The sample $\left(X_1^{(3)}, X_2^{(3)}, ..., X_{n-2}^{(3)}\right)$ of size *'n-2'* from the remaining components lifetimes is constructed after the second failure. The distribution function of the i.i.d.

$X_i^{(3)} > s_2 > s_1, i = 1, \cdots, n-2$ is conditional on $X_{1:n-1}^{(2)} = s_2$ only (portraying the Markov property).

The sequential failures are continued until the last component $\left(X_1^{(n)}\right)$ fails based on the condition that $X_{1:2}^{(n-1)} = s_{n-1}$ and the $F_{X^{(n)}}(t)$ distribution function. The revealed ordered sequence of $\left(X_{1:n}^{(1)}, X_{1:n-1}^{(2)}, ..., X_{1:n-i}^{(i+1)}, ..., X_{1:2}^{(n-1)}, X_{1:1}^{(n)}\right)$ is called sequential order statistics based on $\left(F_{X^{(1)}}, F_{X^{(2)}}, ..., F_{X^{(i+1)}}, ..., F_{X^{(n-1)}}, F_{X^{(n)}}\right)$.

Noting that the sequential order statistics are conditionally independent, the joint distribution of the first 'r' sequential order statistics $\left(X_{1:n}^{(1)}, X_{1:n-1}^{(2)}, ..., X_{1:n-i}^{(i+1)}, ..., X_{1:n-r+2}^{(r-1)}, X_{1:n-r+1}^{(r)}\right)$ is:

$$g_{\left(X_{1:n}^{(1)}, X_{1:n-1}^{(2)}, ..., X_{1:n-i}^{(i+1)}, ..., X_{1:n-r+2}^{(r-1)}, X_{1:n-r+1}^{(r)}\right)}(t_1, t_2, \ldots, t_r) = = \frac{n!}{(n-r)!}\left[\prod_{i=1}^{r} \frac{f_{X^{(i)}}(t_i)}{R_{X^{(i)}}(t_{i-1})}\left(\frac{R_{X^{(i)}}(t_i)}{R_{X^{(i)}}(t_{i-1})}\right)^{n-i}\right] \tag{63}$$

where, $t_1 < t_2 < \ldots < t_r$, $s_0 = -\infty$, $F_{X^{(1)}}(s_0) = 0$, $R(\bullet) = 1 - F(\bullet)$.

6.2.2 Systems Reliability in Sequential Order Representation by Structural Reliability Approach

A truncated distribution function in sequential order representation will be given by:

$$F_{X^{(i)}/X_{1:n-i+2}^{(i-1)}=s_{i-1}}(t/s_{i-1}) = \frac{F_{X^{(i)}}(t) - F_{X^{(i)}}(s_{i-1})}{1 - F_{X^{(i)}}(s_{i-1})} \tag{64}$$

$$f_{X^{(i)}/X_{1:n-i+2}^{(i-1)}=s_{i-1}}(t/s_{i-1}) = \frac{f_{X^{(i)}}(t)}{R_{X^{(i)}}(s_{i-1})} \tag{65}$$

where, $i = 1 \cdots n$, $X_{1:n+1}^{0} = s_0 = -\infty$, $F_{X^{(1)}}(s_0) = 0$, $s_{i-1} < t$, $R(\bullet) = 1 - F(\bullet)$.

Thus, the distribution function of the corresponding sequential order statistic is given by:

$$F_{X_{1:n-i+1}^{(i)}/X_{1:n-i+2}^{(i-1)}=s_{i-1}}(t/s_{i-1}) = 1 - \left(\frac{1 - F_{X^{(i)}}(t)}{1 - F_{X^{(i)}}(s_{i-1})}\right)^{n-i+1} \tag{66}$$

$$f_{X^{(i)}_{1:n-i+1}/X^{(i-1)}_{1:n-i+2}=s_{i-1}}(t/s_{i-1})=(n-i+1)\left(\frac{R_{X^{(i)}}(t)}{R_{X^{(i)}}(s_{i-1})}\right)^{n-i}\frac{f_{X^{(i)}}(t)}{R_{X^{(i)}}(s_{i-1})} \tag{67}$$

where, $i=1\cdots n,\ X^{0}_{1:n+1}=s_0=-\infty,\ F_{X^{(1)}}(s_0)=0,\ s_{i-1}<t$.

Observe that (60) with *'r=1'* is comparable with (66). However the sample size in (60) when *'r=1'* will be changed with each observed sequential order failure. This leads to the formulation: if an i.i.d. sample $Y^{(i)}_1,\ldots,Y^{(i)}_{n-i+1}$ with size '$n-i+1$' was selected from $F_{Y^{(i)}}$ at level 'i', $i=1\cdots n$, such that $P(Y^{(i)}_k=Y^{(i)}_m)=0$ where $k\neq m, k=1,\cdots,n-i+1$ and $m=1,\cdots,n-i+1$, then the distribution function of the sequential order statistic $X^{(i)}_{1:n-i+1}$ at level 'i' based on the $F_{X^{(i)}}$ parent distribution function, such that $F_{X^{(i)}}=F_{Y^{(i)}}$, and given that failure at level '$i-1$' was s_{i-1} ($s_0=-\infty$), can be written as:

$$\begin{aligned} &F_{X^{(i)}_{1:n-i+1}/X^{(i-1)}_{1:n-i+2}=s_{i-1}}(t/s_{i-1})=P\left(X^{(i)}_{1:n-i+1}\leq t/X^{(i-1)}_{1:n-i+2}=s_{i-1}\right)= \\ &=P\left(Y^{(i)}_{1:n-i+1}\leq t/Y^{(i)}_{1:n-i+1}>s_{i-1}\right)=F_{Y^{(i)}_{1:n-i+1}/Y^{(i)}_{1:n-i+1}>s_{i-1}}(t/s_{i-1}) \end{aligned} \tag{68}$$

This relationship has an important representation in methodical sense and its application from structural reliability viewpoint is explored in the following: Left side of (68) describes the distribution function of the minimum distribution from the truncated c.d.f. based on the $F_{X^{(i)}}$ parent distribution (see left side of Figure 13), whereas right side of (68) describes the distribution function of the truncated distribution from the minimum c.d.f. based on the $F_{Y^{(i)}}=F_{X^{(i)}}$ parent distribution (see right side of Figure 13).

This result is useful in failure dependent system applications as far as estimation of the parent distribution function in sequential order representation is hard to achieve in practice, whereas estimation of the minimum c.d.f. is practically observable. It is proposed that the distribution function of the minimum in sequential order representation to be computed / estimated by a series system representation. The series systems reliability estimation by structural reliability approaches is proposed to be used for these purposes, see Figure 13. This estimation procedure is non-destructive and robust, as far as it is based on limit state equations, describing each component / subsystem failure behavior and it allows estimating reliability in engineering designs by parameters / variables modification or introduction of new parameters.

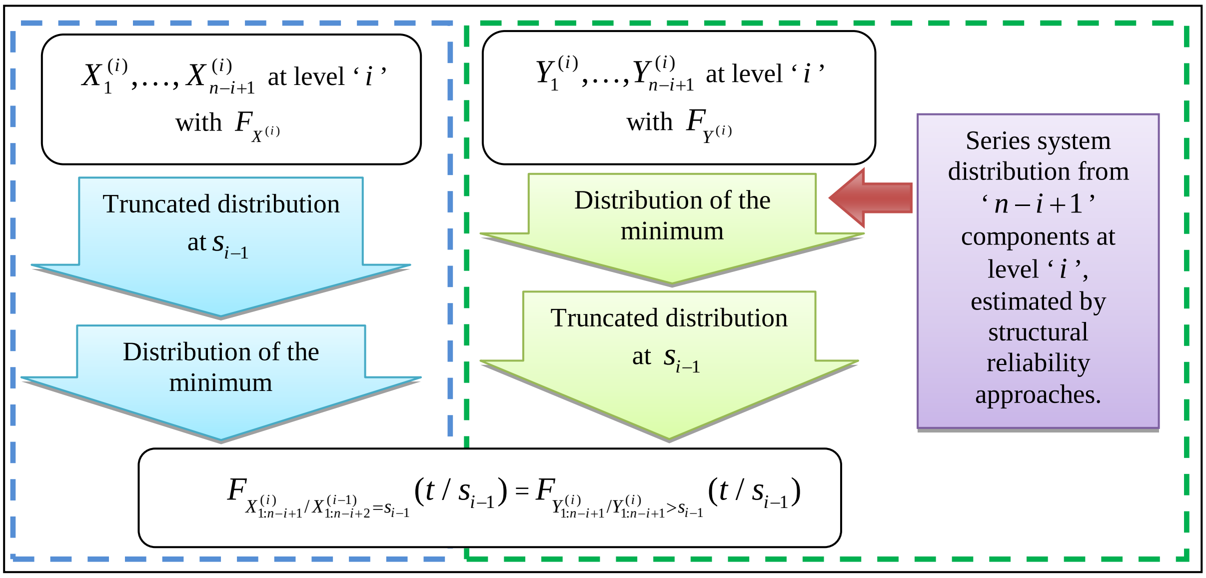

Figure 13: Illustration of the procedure defined by (68)

Thus in summary the following approach is formulated: suppose a load sharing system from *'n'* components is considered at time *'0'* and level *'1'*. Exactly after '$i-1$' total failures, the new load sharing system will be formed from the '$n-i+1$' survived components at level 'i'. Next, the failure time in the parallel system from '$n-i+1$' components is modeled by the truncated distribution function of the series system from '$n-i+1$' components at level 'i'. The estimation of the series system probability of failure is carried out by structural reliability approaches, see Appendix G. The failure time at level 'i' is predicted from the estimated truncated distribution for the desired quantile and it is used to estimate the subsequent truncated distribution, as well as the desired quantile time at level '$i+1$'. The procedure is continued until the truncated distribution at level *'n'* is estimated, which corresponds to the lifetime distribution of the initial *'n'* component load sharing system (failure-effect correlated components).

6.2.3 Example of Sequential Order Representation by Structural Reliability

A small illustrative example of the application of the above theory is presented in the following. Suppose a three-component load sharing system is under the consideration. Failure of either component is due to the cumulative damage and might be represented by a limit state equation. The limit state equation will be formulated to describe a component failure time, which is a time when the load is higher than the strength of the component. It will be assumed that the component strength degrades by the usage time and a constant load (stochastic but time invariant) is applied on the component. Upon failure of the weakest one, the survived components will be exposed a higher load level (by load redistribution), thus three load levels will be considered. In this example, it is assumed that information on the first (weakest) failure under load level *'1'* is observed at time *'200'*. As far as this example is for an illustration purposes, no units are attached to the variables. An objective is to use this information to estimate the next (2^{nd}) failure in the system, the corresponding distribution function and to use its median time (50% quantile) to estimate subsequent (3^{rd})

failure distribution. The latter will describe an anticipated failure distribution of the three component load sharing system given that the first component fails at the time *'200'* and the second component fails at its median time.

Let the random variables $X1, X2, X3, X4, X5, X6$ define the limit state equations for each component in the following way:

$$g_1(\mathbf{X}(t), t, LoadLevel)) = X_1(t) - X_4(LoadLevel) \tag{69}$$

$$g_2(\mathbf{X}(t), t, LoadLevel)) = X_2(t) - X_5(LoadLevel) \tag{70}$$

$$g_3(\mathbf{X}(t), t, LoadLevel)) = X_3(t) - X_6(LoadLevel) \tag{71}$$

where distributions and parameters of $X1, X2, X3, X4, X5, X6$ are defined in Table 2. It is assumed that all stochastic variables are statistically independent of each other, for the sake of simplicity.

Note: 't' represents time and 'L' represents load level={1,2,3}	Distribution	Mean	Coefficient of Variation
X1, X2, X3	Normal	150*exp(-0.002*t)	0.2*exp(0.0002*t)
X4, X5, X6	Normal	50+10*(L-1)	0.2

Table 2: Parameters of the stochastic variables

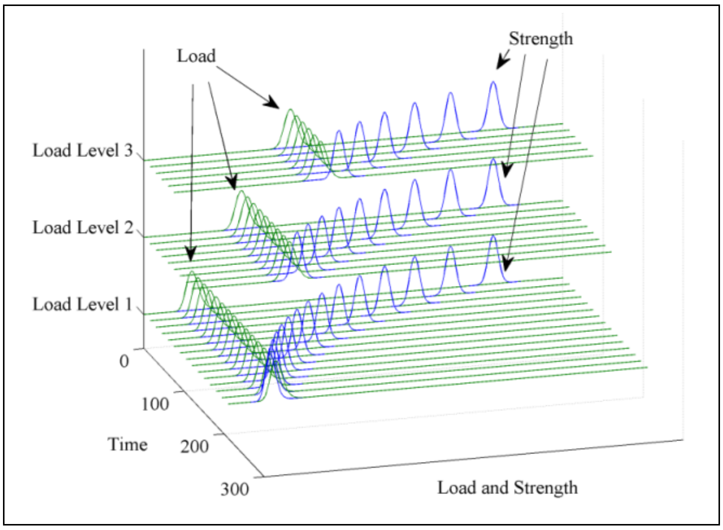

Figure 14: Illustration of density functions of loads and strengths as function of time

The random variables $X1, X2, X3$ are describing the strength of the material for each component and they are functions of time *'t'*. By the usage time, their expected values are decaying and coefficients of variations are increasing (see Figure 14 and Table 2). The random variables describing loads are $X4, X5, X6$, with increasing mean values as a function of load level and constant coefficients of variations, see Table 2. Thus, the limit state equations will be functions of load levels and time.

Step 1: Parent Distributions

It is expected that the probability of failure should increase by the increase of load levels. Using the FORM (structural reliability) method (with difference quotient in numerical differentiation and error for reliability index estimation of 10^{-8} and 10^{-5}, respectively), the component probability of failure for each load level at a given time step was estimated (see Figure 15). It is seen that probability of failure is increasing by the increase of load levels, which is typical behavior of load sharing (failure dependent) systems.

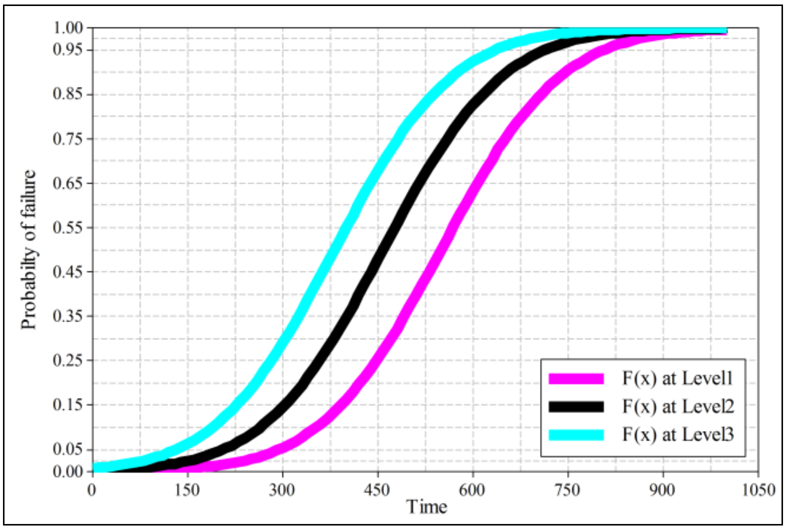

Figure 15: Parent distributions

To estimate representative probabilistic distributions and associated parameters, a classical reliability analysis was performed. An adequate fit was obtained using the Normal distribution (as it was expected, as far as the limit state equations are linear combination of Normal random variables). The Anderson-Darling (AD) test statistics p-values were reported with high level of significance. The location and scale parameters for the parent distributions were estimated (see Figure 16).

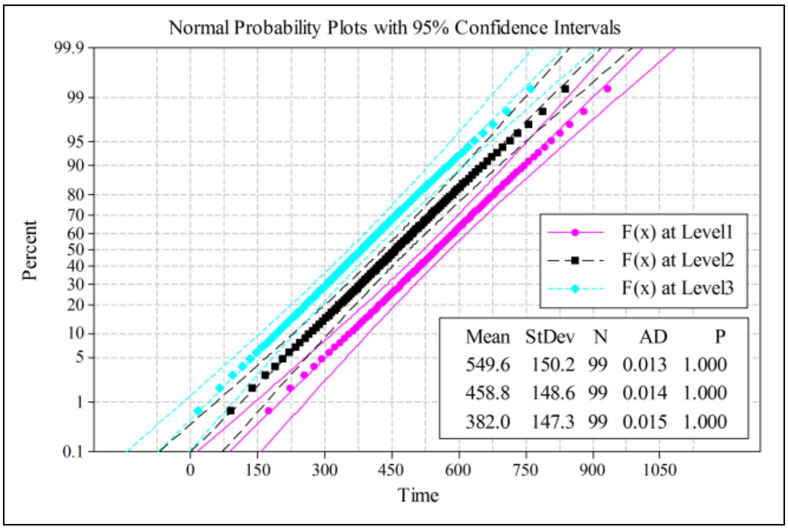

Figure 16: Parent distributions represented as a Normal

Estimation of representative probabilistic distributions are not necessary in practice, as soon as parent distributions are estimated. It is done to clarify and lead the reader to the sequential order statistics formulation.

Next, a load sharing parallel system comprised of three components is formed at load level *'1'*. Once a failure of the weakest component out of three components at load level *'1'* is observed ($X_{1:3}^{(1)}$), the parallel system composed of the two survived components will be formed and undergo to the load level *'2'*. The second failure time will be determined within the two component parallel system, which is the failure of the weakest component out of two at load level *'2'*, ($X_{1:2}^{(2)}$). Once $X_{1:2}^{(2)}$ is observed, the last component undergoes to the load level *'3'* and its failure time will be defined by $X_{1:1}^{(3)}$, which is the third failure time. So, based on sequential order statistic theory, the failure times $X_{1:3}^{(1)}, X_{1:2}^{(2)}, X_{1:1}^{(3)}$ will form the sequential order statistics based on the distributions $F_{X^{(1)}} \sim N(549.6;150.2)$, $F_{X^{(2)}} \sim N(458.8;148.6)$ and $F_{X^{(3)}} \sim N(382;147.3)$.

Step 2: Illustration of the relationship in (68) and Figure 13

Suppose the weakest component out of three components at load level *'1'* fails at the time *'200'*. So, this failure time corresponds to $X_{1:3}^{(1)} = s_1 = 200$.

Left side of (68): Based on the parent distribution function $F_{X^{(2)}}$ (see I in Figure 17), the distributions of $X^{(2)} / X_{1:3}^{(1)} = 200$ and $X_{1:2}^{(2)} / X_{1:3}^{(1)} = 200$ are computed by (64) and (66) (see II and III in Figure 17 and left side of Figure 13).

Right side of (68): Based on the parent distribution $F_{X^{(2)}}$ and using (57) the distribution of $X_{1:2}^{(2)}$ is computed. Then, based on the $X_{1:2}^{(2)}$ distribution, the distribution of $X_{1:2}^{(2)} / X_{1:3}^{(1)} = 200$ is estimated via (60) (see IV and V in Figure 17 and right side of Figure 13).

As it is seen from Figure 17, the truncated distributions are identical and this is due to the relationship illustrated by (68).

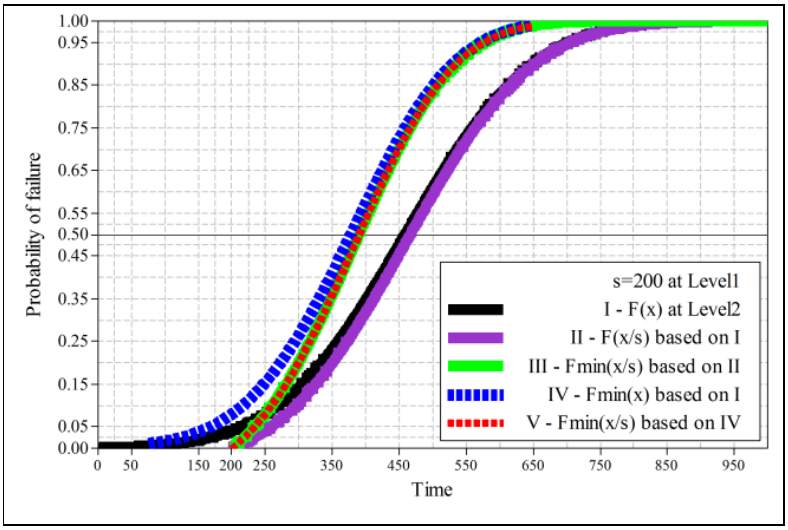

Figure 17: Estimated probability of failure distributions based on order statistics

Step 3: Series systems for the minimum distribution estimation

Next, series system structural reliability techniques are applied (see Appendix G). Particularly using (116) and implementing FORM for the two component series system, the probability of failure under load level *'2'* for each time step was estimated (see I in Figure 18). Also, based on the parent distribution $F_{X^{(2)}}$ and using (57) the minimum distribution at load level *'2'* was estimated (see II in Figure 18). It is seen that they are identical and thus the series system structural reliability techniques are proposed to be used for the minimum distribution estimation for the given load level.

Step 4: Estimation of the consecutive failure times or load sharing system lifetime

The minimum distribution at load level *'2'* was estimated based on the two component series system by structural reliability method in Step 3 (see I in Figure 18). If it is assumed that the weakest component out of three components at load level *'1'* fails at time *'200'*, then based on the minimum distribution at load level *'2'* the truncated distribution of $X_{1:2}^{(2)} / X_{1:3}^{(1)} = 200$ is estimated (see III in Figure 18, also identical with both III and V in Figure 17). The median time for the next failure is estimated to be 390 (see III in Figure 18), also any desired reliability level quantiles for the next failure time could be estimated from the graph.

Further, the distribution of $X_{1:1}^{(3)} / X_{1:2}^{(2)} = 390$ is estimated via the above described procedures where the parent distribution is $F_{X^{(3)}}$ (see IV in Figure 18), and it is lifetime of three component parallel system with failure-effect correlated components, given that the first component fails at the time *'200'* and the second component fails at its median time.

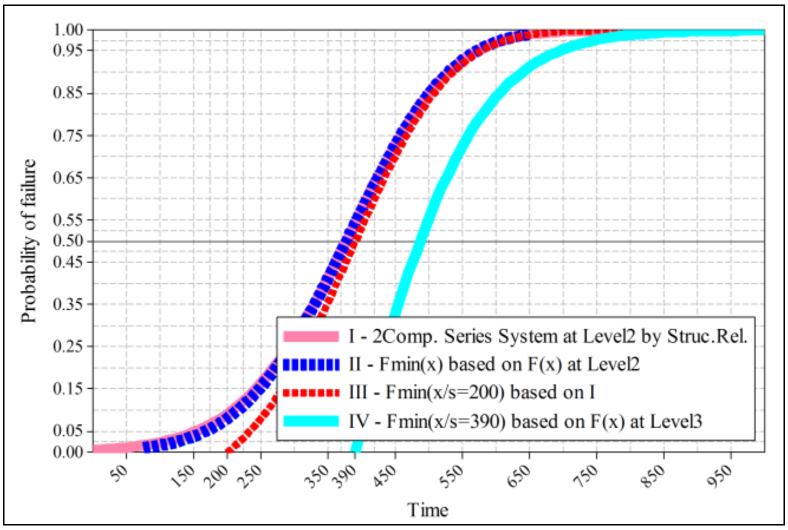

Figure 18: Estimated probability of failure distributions based on structural reliability approach

6.2.4 Conclusion

In section 6.2, it is shown how the reliability of a load sharing systems can be estimated. The aspect of not taking into account failure dependencies is the main deficiency of classical reliability theory in systems reliability estimation. The sequential orders of failures were considered and the influences from the load changes were taken into account. It was assumed that components are statistically independent and share the dependences only by mechanical structure, termed as mechanical correlation or failure effect correlation. An important representation in methodical sense was revealed, which is expressing next failure time in a parallel system (with failure effect correlations) at the given load level by a truncated distribution of the series system distribution. Where, the latter is estimated by structural reliability methods for the given load level. The parent distribution at the given load level is not necessary to be known, even though it is possible to estimate. The described technique could be used for calibration of limit state equations based on the test data availability. It could be used for the decision-making and residual life estimation for dependent systems as well as implemented for the optimal operation and maintenance strategy determination considering both the severity of system failures and data from service visits and / or condition monitoring systems, etc. A limitation of this study is that only failure effect correlation is considered and for further development of this work, the statistical correlation together with failure effect correlation should be considered. Also for further research, it is possible to use the estimated probability of failures from the truncated distribution as weights for estimating the subsequent components failure distributions.

CHAPTER 7. CONCLUSIONS AND FURTHER RESEARCH

Reliability estimation procedures for WT components divided in structural and electrical / mechanical components are discussed. The main concepts of two different approaches were presented, one was structural reliability methods and the other one was classical reliability estimations approaches. A definition of failure was required for the reliability analysis, especially when the goal was to develop optimal O&M strategies. Widely held reliability models aiming for O&M strategies development should follow the procedure described in Figure 11. Application and methods used to achieve reliability models incorporating loads and environmental conditions were applied in this research. Fatigue reliability has been investigated for the component level reliability, and comprehensive models linking classical and structural reliability approaches was presented.

A collection of papers was presented for the IGBT solder cracking fatigue failure mode analysis. The developed limit state equations might be extended and adjusted for different thermally activated mechanisms for fatigue reliability estimation. The reliability models were based on Miner's rule for linear damage accumulation, where physical and model uncertainties were incorporated. The limitation of Miner's rule, in a context of neglecting load interactions and sequence effects, was imitated by introducing a stochastic variable for the Miner's rule uncertainty. The advantages of the proposed methods were discussed especially for the physics of failure based reliability modeling, where variations of loads / stresses were directly affecting the reliability assessment.

The considered reliability models were applied for one failure mode and other dominating failure modes should be considered in future research. However, the proposed procedures and logic might be extended for these purposes.

Failed WT components / subsystems might significantly damage other systems / subsystems, even when the WT is parked. Such a situation was considered and a general model was developed.

System interaction via load sharing systems was considered. Reliability estimation procedures and methods for such situations are under investigation. Some preliminary results were discussed and presented on this topic.

Priorities of concentrating efforts on different subsystems reliability assessments could be different depending on WT type (on-shore vs. off-shore) and O&M costs would be different as well. It is mainly explained by the load / stress changes and the high costs of transportation, prolonged by environmental barriers. In general, WT operators collect data on failure statistics conditional on the exploitation environment. After data analysis, it is possible to prioritize systems and subsystems for the further reliability centered analysis. Unfortunately, revealing outcomes from this stage to the research community has a weak link in current days.

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APPENDICES

Appendix A: Maximum Likelihood and Covariance (Hessian) Matrix Estimation for the Weibull Distribution Parameters in a Group with Type II Right Censored Data

Let $x_1, x_2, ..., x_r$ be the 'r' realization of the test of 'n' components from Weibull distribution defined in (9) with shape 'β' and scale 'θ' parameters. Rearranging the realizations in order of magnitude the $x_{1:n}, x_{2:n}, ..., x_{r:n}$ sample will be denoted.
Then the likelihood function will be defined as:

$$L(\theta, \beta / x_i) = \frac{n!}{(n-r)!}\left(\prod_{i=1}^{r} \frac{\beta}{\theta^\beta} x_i^{\beta-1} e^{-\left(\frac{x_i}{\theta}\right)^\beta}\right)\left(1 - F(x_{r:n})\right)^{n-r} \quad (72)$$

Taking the natural logarithm of the likelihood function (Ln LF) we will have:

$$\begin{aligned} Ln\left(L(\theta, \beta / x_i)\right) &= rLn(\beta) - r\beta Ln(\theta) + \beta \sum_{i=1}^{r} Ln(x_i) - \\ &- \sum_{i=1}^{r} Ln(x_i) - \frac{\sum_{i=1}^{r} x_i^\beta}{\theta^\beta} - (n-r)\frac{x_r^\beta}{\theta^\beta} + const. \end{aligned} \quad (73)$$

Differentiating of the Ln LF with respect to θ and equating to zero will lead to:

$$\frac{\partial\left(Ln\left(L(\theta, \beta / x_i)\right)\right)}{\partial \theta} = -\frac{r\beta}{\theta} + \beta \frac{\sum_{i=1}^{r} x_i^\beta}{\theta^{\beta+1}} + \beta \frac{(n-r)x_r^\beta}{\theta^{\beta+1}}$$

$$\frac{\beta}{\theta}\left(-r + \frac{\sum_{i=1}^{r} x_i^\beta}{\theta^\beta} + \frac{(n-r)x_r^\beta}{\theta^\beta}\right) = 0 \Rightarrow \qquad \theta^\beta = \frac{\sum_{i=1}^{r} x_i^\beta + (n-r)x_r^\beta}{r}$$

$$\hat{\theta} = \left(\frac{\sum_{i=1}^{r} x_i^\beta + (n-r)x_r^\beta}{r}\right)^{\frac{1}{\beta}} \quad (74)$$

If $\sum_{i=1}^{r}{}^{*} y_i = \sum_{i=1}^{r} y_i + (n-r)y_r$ then (74) becomes:

$$\hat{\theta} = \left(\frac{\sum_{i=1}^{r} {}^{*} x_i^{\beta}}{r} \right)^{\frac{1}{\beta}} \quad (75)$$

Differentiating of the Ln LF with respect to β and dividing by r, substituting (74) and equating to zero will lead to:

$$\frac{\partial Ln\left(L(\theta,\beta / x_i)\right)}{\partial \beta} = \frac{r}{\beta} - rLn(\theta) + \sum_{i=1}^{r} Ln(x_i) - \sum_{i=1}^{r} \frac{x_i^{\beta}}{\theta^{\beta}} Ln\left(\frac{x_i}{\theta}\right) - (n-r)\frac{x_r^{\beta}}{\theta^{\beta}} Ln\left(\frac{x_r}{\theta}\right)$$

$$\frac{1}{\beta} + \frac{\sum_{i=1}^{r} Ln(x_i)}{r} - \frac{\sum_{i=1}^{r} x_i^{\beta} Ln\left(x_i\right) + (n-r)x_r^{\beta} Ln\left(x_r\right)}{\sum_{i=1}^{r} x_i^{\beta} + (n-r)x_r^{\beta}} = 0$$

$$\frac{\sum_{i=1}^{r} Ln(x_i)}{r} = \frac{\sum_{i=1}^{r} x_i^{\hat{\beta}} Ln\left(x_i\right) + (n-r)x_r^{\hat{\beta}} Ln\left(x_r\right)}{\sum_{i=1}^{r} x_i^{\hat{\beta}} + (n-r)x_r^{\hat{\beta}}} - \frac{1}{\hat{\beta}} \quad (76)$$

If $\sum_{i=1}^{r} {}^{*} y_i = \sum_{i=1}^{r} y_i + (n-r)y_r$ then (76) becomes:

$$\frac{\sum_{i=1}^{r} Ln(x_i)}{r} = \frac{\sum_{i=1}^{r} {}^{*} x_i^{\hat{\beta}} Ln\left(x_i\right)}{\sum_{i=1}^{r} {}^{*} x_i^{\hat{\beta}}} - \frac{1}{\hat{\beta}} \quad (77)$$

So, MLEs for $\hat{\beta}$ and $\hat{\theta}$ are given in (75) and (77). Procedure is the following, one should use (77) to find the closest value for the MLE of $\hat{\beta}$, then by using the estimated $\hat{\beta}$ in (75) the MLE of $\hat{\theta}$ is found.

Covariance matrix of the estimated MLEs from Ln LF could be calculated via Hessian matrix based on the following relationship:

$$Cov\left[\hat{\beta},\hat{\theta}\right] = \left[-H\right]^{-1} = \begin{bmatrix} \frac{\partial^2 Ln\left(L(\theta,\beta / x_i)\right)}{\partial \beta^2} & \frac{\partial^2 Ln\left(L(\theta,\beta / x_i)\right)}{\partial \beta \partial \theta} \\ \frac{\partial Ln\left(L(\theta,\beta / x_i)\right)}{\partial \beta \partial \theta} & \frac{\partial^2 Ln\left(L(\theta,\beta / x_i)\right)}{\partial \theta^2} \end{bmatrix} \quad (78)$$

where,

$$\frac{\partial^2 Ln\left(L(\theta,\beta / x_i)\right)}{\partial\beta^2} = -\frac{r}{\beta^2} - \sum_{i=1}^{r}\frac{x_i^\beta}{\theta^\beta}\left(Ln\left(\frac{x_i}{\theta}\right)\right)^2 - (n-r)\frac{x_r^\beta}{\theta^\beta}\left(Ln\left(\frac{x_r}{\theta}\right)\right)^2$$

$$\frac{\partial^2 Ln\left(L(\theta,\beta / x_i)\right)}{\partial\theta^2} = \frac{r\beta}{\theta^2} - \frac{\beta(\beta+1)}{\theta^2}\left(\sum_{i=1}^{r}\frac{x_i^\beta}{\theta^\beta} + (n-r)\frac{x_r^\beta}{\theta^\beta}\right)$$

$$\frac{\partial Ln\left(L(\theta,\beta / x_i)\right)}{\partial\beta\partial\theta} = \frac{1}{\theta}\left(-r + \sum_{i=1}^{r}\frac{x_i^\beta}{\theta^\beta} + (n-r)\frac{x_r^\beta}{\theta^\beta}\right) + \frac{\beta}{\theta}\left(\sum_{i=1}^{r}\frac{x_i^\beta}{\theta^\beta}Ln\left(\frac{x_i}{\theta}\right) + (n-r)\frac{x_r^\beta}{\theta^\beta}Ln\left(\frac{x_r}{\theta}\right)\right)$$

Appendix B: General Linear Regression Model

A model with intercept is given by:

$$\underset{n\times 1}{\mathbf{Y}} = \underset{n\times p}{\mathbf{X}}\,\underset{p\times 1}{\boldsymbol{\beta}} + \underset{n\times 1}{\boldsymbol{\varepsilon}} \tag{79}$$

where, $\mathbf{Y}$ is a vector of responses, $\mathbf{X}$ is a matrix of constants, $\boldsymbol{\beta}$ is a vector of parameters and $\boldsymbol{\varepsilon}$ is a vector of independent Normally distributed random variables with, expectation $E[\boldsymbol{\varepsilon}] = 0$ and covariance matrix $Cov[\boldsymbol{\varepsilon}] = \sigma^2\mathbf{I}$.

Least square estimator of $\boldsymbol{\beta}$ is $\mathbf{b} = \left(\mathbf{X}'\mathbf{X}\right)^{\mathbf{-1}}\left(\mathbf{X}'\mathbf{X}\right)\mathbf{Y}$, where $E[\mathbf{b}] = \boldsymbol{\beta}$ and $Cov[\mathbf{b}] = \sigma^2\left(\mathbf{X}'\mathbf{X}\right)^{\mathbf{-1}}$, and $E\left[\sigma^2\right] = MSE$ (Mean Square Error or Residual Mean Square).

For the given $\underset{p\times 1}{\mathbf{X_h}} = \begin{bmatrix} 1 \\ \vdots \\ X_h \end{bmatrix}$, estimated mean response $\widehat{Y_h}$ has expectation $E\left[\widehat{Y_h}\right] = \mathbf{X_h'b}$ and variance $Var\left[\widehat{Y_h}\right] = \sigma^2\mathbf{X_h'}\left(\mathbf{X}'\mathbf{X}\right)^{\mathbf{-1}}\mathbf{X_h}$, while estimated new observation $\widehat{Y}_{h(new)}$ has $E\left[\widehat{Y}_{h(new)}\right] = \mathbf{X_h'b}$ and variance $Var\left[\widehat{Y}_{h(new)}\right] = \sigma^2\left[1 + \mathbf{X_h'}\left(\mathbf{X}'\mathbf{X}\right)^{\mathbf{-1}}\mathbf{X_h}\right]$.

Appendix C: Non-Parametric Kaplan-Meier Estimates (Product Limit Estimates)

Let $t_1,\cdots,t_n$ be a random sample of size '*n*' with survival function $S(t)$. Recall that an estimate of survival function is:

$$\hat{S}(t) = \frac{\text{Number of } (t_1,\cdots,t_n) \geq t}{n} \quad (80)$$

For estimating $S(t)$ for cases including ties (more than one observation at time '*t*') and censoring (right), the Kaplan-Meier estimation method might be used to estimate empirical cumulative survival function. It will be derived based on the following simple example.
Suppose sample of 10 products are tested during 2-year period. During 1st year, 6 items were failed and during 2nd year 3 items were failed. In addition, at the end of 1st year the second sample of 20 items were placed on test, and during the following year, 15 items were failed. One possible estimate of the survival function might be so called 'reduced sample estimate', which will ignore information form the second sample and an estimated survival function for the 2nd year will be $\hat{S}(2) = 1/10 = 0.1$. To include the second sample information the following events might set up:

$\hat{S}(2) = P(\text{item survives} \geq 2 \text{ years}) = P(\text{item survives } 1st \text{ year} \cap \text{item survives } 2nd \text{ year})=$

$=P(\text{item survives } 1st \text{ year}) * P(\text{item survives } 2nd \text{ year / item survives } 1st \text{ year})=$

$=((4+5)/(10+20))*(1/4) = 0.075$

By generalizing the above logic, the following product limit estimate could be introduced for censored and tied data. First, the $t_1,\cdots,t_r,\cdots,t_k$ observed failure times from the sample of '*n*', $k \leq n$, are putted in increasing order and $t_{1:n},\cdots,t_{r:n},\cdots,t_{k:n}$ is obtained. If uncensored and censored data are tied, then the uncensored data should appear first and its rank is used for calculations. If two or more observations are uncensored and tied, then the rank of the highest one is used for the calculations. An estimated empirical cumulative survival function is given by:

$$\hat{S}(t) = \prod_{r \in I_t} \frac{n-r}{n-r+1} \quad (81)$$

where, '*r*' is the rank of the ordered uncensored observation, I_t is all positive integers '*r*', such that $t(r) \leq t$ and $t(r)$ is uncensored.
This technique is appealing due to the fact that Kaplan-Meier estimates are MLEs of $S(t)$. Also, it should be noted that if the highest observation is censored then Kaplan-Meier estimates is defined only up to this last observation. Estimated $\hat{S}(t)$ by (81) is asymptotically Normal distributed, with expectation and variance of:

$$E\left[\hat{S}(t)\right] \approx S(t) \quad (82)$$

$$Var\left[\hat{S}(t)\right] \approx \left(\hat{S}(t)\right)^2 \sum_{r \in I_t} \frac{1}{(n-r)(n-r+1)} \tag{83}$$

If no censored observation is exist in the sample, then Kaplan-Meier estimates are given by:

$$\hat{S}(t) = \frac{n-r}{n} \tag{84}$$

In this case (no censored observation is exist in the sample), the rank distribution is more appropriate to use and the estimated expectation and variance of $S(t)$ are exact values.

Appendix D: Cumulative Probability or Probability Distribution Functions of the *'r-th out of n'* Order Random Variable

Let the random sample of size n be $t_1, \cdots, t_n$, where all t's are independent, identically distributed and continuous random variables with f(t) probability density and F(t) cumulative distribution functions, respectively (parent distribution).
Consider the vector $t_{1:n}, \cdots, t_{n:n}$ of random variables, which is composed of the t's random variables, where $t_{i:n}$ is the *i-th* in magnitude, so that $t_{1:n} \leq, \cdots \leq, t_{n:n}$. Then the $t_{1:n}, \cdots, t_{n:n}$ random variables would have $G_{r:n}(t)$ cumulative distribution and $g_{r:n}(t)$ probability density functions, respectively, where $G_{r:n}(t) = P(t_{r:n} \leq t)$, $r \leq n$.

If the condition is $t_{r:n} \leq t$, so *'r'* or more elements from $t_1, \cdots, t_n$ sample should satisfy the condition $t_i \leq t$ and since each $t_i \leq t$ has a binomial distribution with the probability of success of $\mathrm{F}(\mathrm{t}) = \mathrm{P}(t_i \leq t)$, then:

$$G_{r:n}(t) = P(t_{r:n} \leq t) = \sum_{w=r}^{n} \binom{n}{w} F(t)^w (1 - F(t))^{n-w} \tag{85}$$

To find $g_{r:n}(t)$, the derivative of $G_{r:n}(t)$ with respect to *'t'* should be taken and using the properties that
$f(t) = dF(t)/dt$ and $duv/dx = v(du/dv) + u(dv/du)$, the $g_{r:n}(t)$ will be given by:

$$g_{r:n}(t) = \sum_{w=r}^{n} \binom{n}{w} wF(t)^{w-1}(1 - F(t))^{n-w} f(t) - \sum_{w=r}^{n} \binom{n}{w} F(t)^w (n-w)(1 - F(t))^{n-w-1} f(t) =$$

$$= \sum_{w=r}^{n} \binom{n}{w} wF(t)^{w-1}(1 - F(t))^{n-w} f(t) - \sum_{w=r}^{n-1} \binom{n}{w} F(t)^w (n-w)(1 - F(t))^{n-w-1} f(t)$$

Note that for the second term *'w=r:n-1'*, the variable *'w'* can be changed to *'v'*, so that *'v=r+1:n'*.

$$= \sum_{w=r}^{n} \binom{n}{w} wF(t)^{w-1}(1-F(t))^{n-w} f(t) - \sum_{v=r+1}^{n} \binom{n}{v-1} F(t)^{v-1}(n-v+1)(1-F(t))^{n-v+1-1} f(t) =$$

$$= \sum_{w=r}^{n} \binom{n}{w} wF(t)^{w-1}(1-F(t))^{n-w} f(t) - \sum_{v=r+1}^{n} \binom{n}{v-1} F(t)^{v-1}(n-v+1)(1-F(t))^{n-v} f(t) =$$

$$= \sum_{w=r}^{n} \frac{n!}{w!(n-w)!} wF(t)^{w-1}(1-F(t))^{n-w} f(t) - \sum_{v=r+1}^{n} \frac{n!}{(v-1)!(n-v+1)!} F(t)^{v-1}(n-v+1)(1-F(t))^{n-v} f(t) =$$

$$= \sum_{w=r}^{n} \frac{n!}{(w-1)!(n-w)!} F(t)^{w-1}(1-F(t))^{n-w} f(t) - \sum_{v=r+1}^{n} \frac{n!}{(v-1)!(n-v)!} F(t)^{v-1}(1-F(t))^{n-v} f(t) =$$

$$= \frac{n!}{(r-1)!(n-r)!} F(t)^{r-1}(1-F(t))^{n-r} f(t) = n\binom{n-1}{r-1} F(t)^{r-1}(1-F(t))^{n-r} f(t)$$

So,

$$g_{r:n}(t) = n\binom{n-1}{r-1} F(t)^{r-1}(1-F(t))^{n-r} f(t) \qquad (86)$$

To summarize the above statement, if $t_1, \cdots, t_n$ are each independent, identically distributed and continuous random variables with f(t) probability density and F(t) cumulative distribution functions respectively, then the order random variable $t_{r:n}$ would have $g_{r:n}(t)$ probability density and $G_{r:n}(t)$ cumulative distribution functions, given by (86) and (85).

Appendix E: The Rank Distribution

Given that the parent distribution $F_T(t), f_T(t)$ and consecutively probability distribution as well as density functions of the r-th order statistic out of n are known and may be represented by (86) and (85). Let p_c be the percentage of the population below some time t_c, that is $p_c = P(T \le t_c)$. If $t_c = t_{r:n}$ then $p_{r:n} = P(T \le t_{r:n})$ would be the percentage of population below the observed r-th out of n ordered variable. If the r-th out of n ordered variable $t_{r:n}$ is not known (suppose the sample is not observed yet), then it will be treated as a random variable $T_{r:n}$ and $p_{r:n} = P(T \le T_{r:n})$ becomes a random variable as well. Also, holding the reverse relationship that $T_{r:n} = F_T^{-1}(p_{r:n})$.

Observing that:

$$G_{r:n}(a) = P(T_{r:n} \le a) = P\left(F_T^{-1}(p_{r:n}) \le a\right) = P\left(p_{r:n} \le F_T(a)\right) = Q_{r:n}(p_a) \qquad (87)$$

where $p_a = F_T(a)$.

From (86) it follows that:

$$G_{r:n}(a) = \int_{-\infty}^{a} g_{r:n}(t)dt = \int_{-\infty}^{a} n\binom{n-1}{r-1} F_T(t)^{r-1}(1-F_T(t))^{n-r} f_T(t)dt \qquad (88)$$

Let $p = P(T \le t) = F_T(t)$, then $dp = f_T(t)dt$, also passing the limits as $t = -\infty, \Rightarrow p = 0$ and $t = a, \Rightarrow p = p_a$, then based on (87) and (88) it becomes,

$$Q_{r:n}(p_a) = P(p_{r:n} \le p_a) = \int_{0}^{p_a} n\binom{n-1}{r-1} p^{r-1}(1-p)^{n-r} dp \qquad (89)$$

and

$$q_{r:n}(p_a) = n\binom{n-1}{r-1} p_a^{\,r-1}(1-p_a)^{n-r} = \frac{n!}{(r-1)!(n-r)!} p_a^{\,r-1}(1-p_a)^{n-r} \qquad (90)$$

where $Q_{r:n}(p_a)$ and $q_{r:n}(p_a)$ are cumulative distribution and probability density functions of the random variable $p_{r:n}$, and it is defined within $0 \le p_{r:n} \le 1$.

One could recognize that $q_{r:n}(p_a)$ is the well known Beta distribution given by:

$$F_X(x) = \int_0^x \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} u^{\alpha-1}(1-u)^{\beta-1} du \qquad (91)$$

$$f_X(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1} \qquad (92)$$

where $0 \le X \le 1$.
Expectation, median and variance of Beta distributed 'X' random variable are give by:

$$E(X)=\frac{\alpha}{\alpha+\beta} \qquad (93)$$

$$Median \approx \frac{\alpha-\frac{1}{3}}{\alpha+\beta-\frac{2}{3}}, \alpha \ge 1, \beta \ge 1 \qquad (94)$$

$$Var(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \qquad (95)$$

Using the fact that Γ(n)=(n-1)! and letting $\alpha = r,\ \beta = n-r+1$, then (89) and (90) becomes Beta distribution. Expectation, median and variance of $p_{r:n}$ random variable will be defined:

$$E(p_{r:n})=\frac{r}{n+1} \qquad (96)$$

$$Median \approx \frac{r-\frac{1}{3}}{n+\frac{1}{3}}, \alpha \ge 1, \beta \ge 1 \qquad (97)$$

$$Var(p_{r:n})=\frac{r(n-r+1)}{(n+1)^2(n+2)} \qquad (98)$$

Application of the Rank distribution is found in computing the order statistic time for a desired quantile. If one would like to know the ($1-\alpha$) level quantile *'a'* of the *'r-th out of n'* order statistic from parent distribution $F_T(t), f_T(t)$, that is: $1-\alpha = P(T_{r:n} \le a) = G_{r:n}(a)$, then the inverse transformation of (85) might be applied, $a = G_{r:n}^{-1}(1-\alpha)$. This calculation is difficult to do manually and is more suitable to a computer algorithm. Another approach is to use the Rank distribution. Recall that (90) is a Beta distribution and p_a can be computed for a ($1-\alpha$) quantile level if an inverse transformation of the incomplete Beta distribution defined in (89) is applied:

$$p_a = Q_{r:n}^{-1}(1-\alpha) \qquad (99)$$

but $p_a = P(T \le a) = F_T(a)$, therefore the inverse of it will be:

$$a = F_T^{-1}(p_a) \tag{100}$$

which is the $(1-\alpha)$ level quantile of the the *'r-th out of n'* order statistic with parent distribution $F_T(t), f_T(t)$.

Appendix F: S-N Curve Approach and Miner-Palmgren Rule

Usually forces are described through the stresses ranges (S), while fatigue life is described by the resistance of metal to some stress level. S-N curve (also known as Wöhler curve) is widely used to portray resistance and endurance of metal to stress level (see Figure 19: S-N Curve).

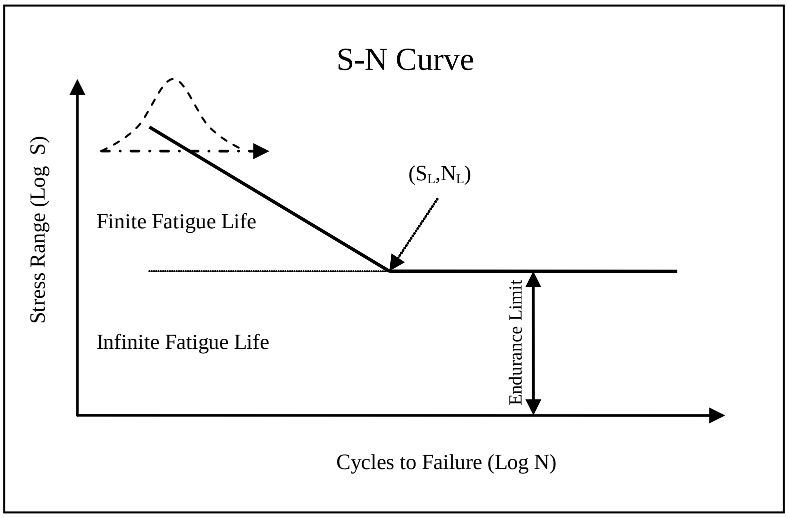

Figure 19: S-N Curve

Endurance limit S_L considers a limit for a stress range, bellow which no failure occurs and component life is infinite (or at least very long in comparison with component intended usage life). Any stress range above S_L limit will initiate a fatigue or damage accumulation, which eventually will cause a component to fail.

Let's define S_i and $N_i=N_i(S_i)$ as an arbitrary stress range level and corresponding number of cycles at which a component will fail based on its S-N curve. "Work" that enough to cause a failure is a function of stress range and number of cycles. As far as the failure of component is possible to formulate in many S and N pairs and assuming that energy function causing a failure has a form of $E = N(S) * S^m$, then a product of any load level (S) with its corresponding number of cycles to failure (N) will be determined by the following relationship:

$$N_i(S_i) * S_i^m = N_L(S_L) * S_L^m \tag{101}$$

where 'm' is a slope of S-N curve in log representation. The notation of $N_i(S_i)$ was intentionally used to emphasis that the number of cycles to failure N_i determines from the stress range level S_i.

In practice, there are situations where random loads or stresses are observed and a component is still working and has not fail. These stresses have exhausted the component and accumulated fatigue process has started. Useful information would be to know how much these stress ranges damage the component and how long it might last for some fixed stress range level.

Miner's and Palmgren rule states that if some stress range S_i occurs n_i times and the number of cycles to failure for that stress level is N_i, then partial damage produced on the component determines by:

$$d_i = \frac{n_i(S_i)}{N_i(S_i)} \tag{102}$$

Some of partial damages will determine the total cumulative damage caused by different stress range levels, thus the cumulative damage would be defined as:

$$D = \sum_i \frac{n_i(S_i)}{N_i(S_i)} \tag{103}$$

Failure occurs if accumulated damage will exceed unity.

From (101) and (103) it follows

$$D = \sum_i \frac{n_i(S_i) * S_i^{\ m}}{N_L(S_L) * S_L^{\ m}} = \frac{1}{N_L(S_L) * S_L^{\ m}} \sum_i n_i(S_i) * S_i^{\ m} \tag{104}$$

Many techniques are developed to amount the damage level from observed stress range history e.g. the number of cycles n_i at stress range level S_i.
The $\sum_i n_i(S_i) * S_i^m$ is a cumulative fatigue damage during the observed time. This might be equivalent to fatigue determined by some stress range level S_c and the number of cycles to failure $N_c(S_c)$ from S-N curve. Of course this assumption is valid if $S_c > S_L$. Then,

$$\sum_i n_i(S_i) * S_i^{\ m} = N_c(S_c) * S_c^{\ m} \tag{105}$$

$$D = \frac{N_c(S_c) * S_c^{\ m}}{N_L(S_L) * S_L^{\ m}} \tag{106}$$

Usually, it is assumed that values for $N_c(S_c)$ are $5*10^6$ or 10^7. This allow to compare different S_c's from different materials and / or different designs by:

$$S_c = \left(\frac{\sum_i n_i(S_i) * S_i^{\ m}}{N_c(S_c)} \right)^{\frac{1}{m}} \tag{107}$$

Also, $N_c(S_c)$ might be represented from frequency stand point and if it is assumed that 1 full cycle occur in 1 unit time (second), then 1 Hz frequency would be a conversion unit and $N_c(S_c) = T$.

The assumption that cumulative fatigue might be represented by stress range level S_c and the number of cycles to failure $N_c(S_c)$ from S-N curve leads to relative damage distribution.

From (101) and (102)

$$d_i = \frac{n_i(S_i)}{N_i(S_i)} = \frac{n_i(S_i) * S_i^{\ m}}{N_L(S_L) * S_L^{\ m}} \tag{108}$$

And from (106) and (108) follows that:

$$d_{rel} = \frac{d_i}{D} = \frac{n_i(S_i) * S_i^{\ m}}{N_L(S_L) * S_L^{\ m}} * \frac{N_L(S_L) * S_L^{\ m}}{N_c(S_c) * S_c^{\ m}} = \frac{n_i(S_i) * S_i^{\ m}}{N_c(S_c) * S_c^{\ m}} \tag{109}$$

If we assume 1 Hz frequency per unit time, then (109) becomes:

$$d_{rel}(t) = \frac{t_i * S_i^{\ m}}{t_c * S_c^{\ m}} \tag{110}$$

where 't' represents duration of the observed stress range S.

Appendix G: Structural Reliability Approaches for Systems Reliability Estimation

Parallel systems reliability estimation by structural reliability approach

If the parallel system consisting of *'n'* subsystems / components is considered and the time-dependent limit state equation for each component is defined by $g_i(\mathbf{X},t)$ where $i=1,\cdots,n$ and it is based on the same random vector $\mathbf{X}^{\mathbf{T}}=(X_1,\dots,X_m)$, then system unreliability defined by (44) will be defined as:

$$P^{p}_{unrel.}(t)=P\left[\bigcap_{i=1}^{n}\{g_i(T(\mathbf{Z}),t)\le 0\}\right] \tag{111}$$

As it was mentioned, if $g_i(T(\mathbf{Z}),t)$ is linearly defined then in standardized domain it becomes a hyperplane in $\mathbb{R}^m$, and will be defined as:

$$g_i(T(\mathbf{Z}),t)=\beta_i(t)-\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\mathbf{Z} \tag{112}$$

So, (111) will be written as:

$$P^{p}_{unrel.}(t)=P\left[\bigcap_{i=1}^{n}\left\{\mathbf{-}\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\mathbf{Z}\le-\beta_i(t)\right\}\right] \tag{113}$$

The expectation, correlation and covariance of linearly defined $\mathbf{-}\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\mathbf{Z}$ will be $E\left[\mathbf{-}\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\mathbf{Z}\right]=0$, $Var\left[\mathbf{-}\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\mathbf{Z}\right]=1$, $Cov\left[\mathbf{-}\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\mathbf{Z},\left(\mathbf{-}\boldsymbol{\alpha}_{\mathbf{j}}^{\mathbf{T}}(t)\mathbf{Z}\right)^{T}\right]=\boldsymbol{\alpha}_{\mathbf{i}}^{\mathbf{T}}(t)\boldsymbol{\alpha}_{\mathbf{j}}(t)$.

By introducing $\boldsymbol{\beta}(t)=\left[\beta_1(t)\quad\beta_2(t)\quad\cdots\quad\beta_n(t)\right]^T$ and

$$\boldsymbol{\alpha}(t)=\left[\boldsymbol{\alpha}_{\mathbf{1}}(t)\quad\boldsymbol{\alpha}_{\mathbf{2}}(t)\quad\cdots\quad\boldsymbol{\alpha}_{\mathbf{n}}(t)\right]=\begin{bmatrix}\alpha_{1,1}(t) & \alpha_{1,2}(t) & \cdots & \alpha_{1,n}(t)\\ \alpha_{2,1}(t) & \alpha_{2,2}(t) & \cdots & \alpha_{2,n}(t)\\ \vdots & \vdots & \ddots & \vdots\\ \alpha_{m,1}(t) & \alpha_{m,2}(t) & \cdots & \alpha_{m,n}(t)\end{bmatrix}$$, correlation matrix

will be determined as $\boldsymbol{\rho}^{(t)}=\boldsymbol{\alpha}^{\mathbf{T}}(t)\boldsymbol{\alpha}(t)$ and (113) will be given as:

$$P^{p}_{unrel.}(t)=\Phi_m\left(-\boldsymbol{\beta}(t),\boldsymbol{\rho}^{(t)}\right) \tag{114}$$

where is $\Phi_m(\ )$ *m*-dimensional multivariate standard normal distribution.

Based on (114), the generalized parallel system reliability index $\beta_G^p(t)$ will be introduced and be written as:

$$\beta_G^p(t) = -\Phi\left[\Phi_m\left(-\boldsymbol{\beta}(t), \boldsymbol{\rho}^{(t)}\right)\right] \tag{115}$$

If the limit state equations $g_i(\mathbf{X},t)$ for $i = 1,\cdots,n$ in parallel system are not linearly defined, then FORM or SORM (Second Order Reliability Method) approximation method might be used to determine system unreliability for the given time step and corresponding generalized parallel system reliability index $\beta_G^p(t)$.

Series systems reliability estimation by structural reliability approach

A series system consisting of '*n*' subsystems / components is considered and the time-dependent limit state equation for each component is defined by $g_i(\mathbf{X},t)$ where $i = 1,\cdots,n$ and it is based on the random vector $\mathbf{X}^\mathbf{T} = (X_1,\ldots,X_m)$. If $\mathbf{X}$ be transform to the standardized domain by some transformation function, $\mathbf{X} = T\left(\mathbf{Z}\right)$, such that $\mathbf{Z}$ is '*m*' dimensional column vector of the mutually independent standard normal random variables, then system unreliability by the time '*t*' defined by (46) can be written as:

$$P_{unrel.}^s(t) = P\left[\bigcup_{i=1}^{n}\{g_i(T(\mathbf{Z},t) \le 0\}\right] \tag{116}$$

(116) will be written for the linearly defined (112) as:

$$P_{unrel.}^s(t) = 1 - P\left[\bigcap_{i=1}^{n}\left\{\boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z} \le \beta_i(t)\right\}\right] \tag{117}$$

The expectation, correlation and covariance of $\boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z}$ will be $E\left[\boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z}\right] = 0$, $Var\left[\boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z}\right] = 1$, $Cov\left[\boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\mathbf{Z}, \left(\boldsymbol{\alpha}_\mathbf{j}^\mathbf{T}(t)\mathbf{Z}\right)^T\right] = \boldsymbol{\alpha}_\mathbf{i}^\mathbf{T}(t)\boldsymbol{\alpha}_\mathbf{j}(t)$. It is seen that the correlation matrix will not be changed and can be determined by $\boldsymbol{\rho}^{(t)} = \boldsymbol{\alpha}^\mathbf{T}(t)\boldsymbol{\alpha}(t)$.

So, unreliability of the series system as well as the generalized $\beta_G^s(t)$ reliability index will be given by:

$$P_{unrel.}^s(t) = 1 - \Phi_m\left(\boldsymbol{\beta}(t), \boldsymbol{\rho}^{(t)}\right) \tag{118}$$

$$\beta_G^s(t) = -\Phi^{-1}\left[1 - \Phi_m\left(\boldsymbol{\beta}(t), \boldsymbol{\rho}^{(t)}\right)\right] \tag{119}$$

If $g_i(T(\mathbf{Z})),t)$ is non-linearly defined then FORM or SORM approximation method can be used and unreliability of the series system for the given time step and corresponding generalized series system reliability index $\beta_G^s(t)$ can be determined

In many instances from the real life problems, the formulation of the limit state equations such that all the failure mechanisms are incorporated is a challenging task. This requires multidisciplinary understanding of the failure modes and modeling of primary failure mechanisms (corrosion, erosion, fatigue and overload) that causes to failure through deterioration processes. However, test data might be collected for the leading failure modes and the described limit state equations of the failure mechanisms could be calibrated such that accurate estimations are achieved.

LIST OF PUBLISHED PAPERS REFERRED TO THE THESIS

Paper 1:	Kostandyan E.E., Sørensen J.D., January 2012, "Reliability of Wind Turbine Components-Solder Elements Fatigue Failure", *Proceedings on the 2012 Annual Reliability and Maintainability Symposium (RAMS 2012)*, IEEE *Xplore*, Reno, Nevada, USA, pp. 1-7. DOI: 10.1109/RAMS.2012.6175420.
Paper 2:	Kostandyan E.E., Sørensen J.D., 2012, "Physics of Failure as a Basis for Solder Elements Reliability Assessment in Wind Turbines", *Reliability Engineering and System Safety*, Elsevier, Vol. 108, pp. 100-107. DOI: 10.1016/j.ress.2012.06.020.
Paper 3:	Kostandyan E.E., Sørensen J.D., June 2012, "Structural Reliability Methods for Wind Power Converter System Component Reliability Assessment", *Proceedings on the 16th IFIP WG 7.5 Conference on Reliability and Optimization of Structural Systems*, Yerevan, Armenia, pp. 135-142.
Paper 4:	Kostandyan E.E., Ma K.,2012, "Reliability Estimation with Uncertainties Consideration for High Power IGBTs in 2.3 MW Wind Turbine Converter System", *Microelectronics Reliability*, Elsevier, Vol. 52, pp. 2403–2408. DOI: 10.1016/j.microrel.2012.06.152.
Paper 5:	Kostandyan E.E., Sørensen J.D., January 2013, "Reliability Assessment of IGBT Modules Modeled as Systems with Correlated Components", *Proceedings on the 2013 Annual Reliability and Maintainability Symposium (RAMS 2013)*, IEEE *Xplore*, Orlando, Florida, USA, pp. 1-6. DOI: 10.1109/RAMS.2013.6517663.
Paper 6:	Kostandyan E.E., Sørensen J.D., June 2013, "Reliability Assessment of Offshore Wind Turbines Considering Faults of Electrical / Mechanical Components", *Proceedings on the Twenty-third International Offshore (Ocean) and Polar Engineering Conference (ISOPE 2013)*, Anchorage, Alaska, USA, pp. 402-407.

PAPER 1

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