2

The Dynamic Characteristics of a Manipulator with Parallel Kinematic Structure Based on Experimental Data

S. Osadchy[1], V. Zozulya[2] and A. Timoshenko[3]

[1,2]Faculty of Automation and Energy, Kirovograd National Technical University, Ukraine
[3]Faculty of Automation and Energy, Kirovograd Flight Academy National Aviation University, Ukraine
Corresponding author: S. Osadchy <srg2005@ukr.net>

Abstract

The chapter presents two identification techniques which the authors found most useful in examining the dynamic characteristics of a manipulator with a parallel kinematic structure as an object of control. These techniques emphasize a frequency domain approach. If all input/output signals of an object can be measured, then the first one of such techniques may be used for identification. In the case when all disturbances can't be measured, the second identification technique may be used.

Keywords: Manipulator with parallel kinematics, structural identification, control system.

2.1 Introduction

Mechanisms with parallel kinematics [1, 2] compose the basis for the construction of single-stage and multi-stage manipulators. A single-stage manipulator consists of an immobile basis, a mobile platform and six guide rods. Each rod can be represented as two semi rods A_{ij} and an active kinematics pair B_{ij} (Figure 2.1).

Advances in Intelligent Robotics and Collaborative Automation, 27–48.

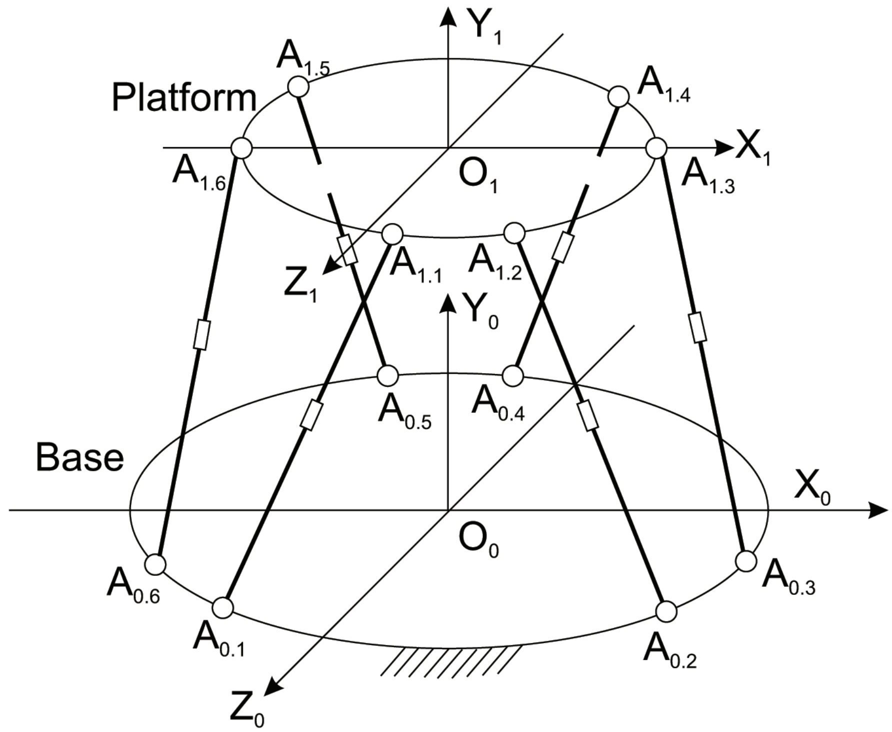

Figure 2.1 Kinematic diagram of single-section mechanism.

We will consider two systems of co-ordinates: inertial $O_0X_0Y_0Z_0$ with the origin in the center of the base O_0 and mobile $O_1X_1Y_1Z_1$, with the origin O_1 in the platform center of mass. From Figure 2.1 it is evident, that such mechanisms consist of thirteen mobile links and eighteen kinematics pairs. That is why, in accordance with [2], the number of its possible motions equals six.

Let us propose the following definitions ($j\in[1:6]$): l_{1j} – length of rod number j; M_x,M_y,M_z– projections of the net resistance moment vector on the axes of the co-ordinate system $O_0X_0Y_0Z_0$.

Obviously, while lengths $l_{1,j}$ are changing, then the co-ordinates of the platform's center of mass and the projections of the resistance moment vector are changing too.

From the point of view of automatic control theory, the mechanism with parallel kinematics belongs to the array of mobile control objects with two multidimensional entrances (control signals and disturbances) and one output vector (platform co-ordinates).

2.2 Purpose and Task of Research

The main purpose of this research is to construct a mathematical model which characterizes the interrelation between control signals, disturbances and co-ordinates of the platform center of mass on the base of experimental data.

If one assembles the length of rod changes in the vector of control signals u_1, the projections of force resistance moment changes in the disturbance vector ψ and the coordinates of the platform center of mass changes in the output vector x

$$u_1 = \begin{bmatrix} l_{1,1} \\ \vdots \\ l_{1,6} \end{bmatrix}, \psi = \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}, x = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}, \tag{2.1}$$

then the block diagram of the mechanism with parallel kinematics can be represented as shown on the Figure 2.2 where W_u is an operator which characterizes the influence of the control signals vector u on the output vector x and W_ψ is an operator which describes the influence of the disturbance vector ψ on the output vector x. In this case, in order to find the mathematical model, it is necessary to define these operators. If we want to find such operators

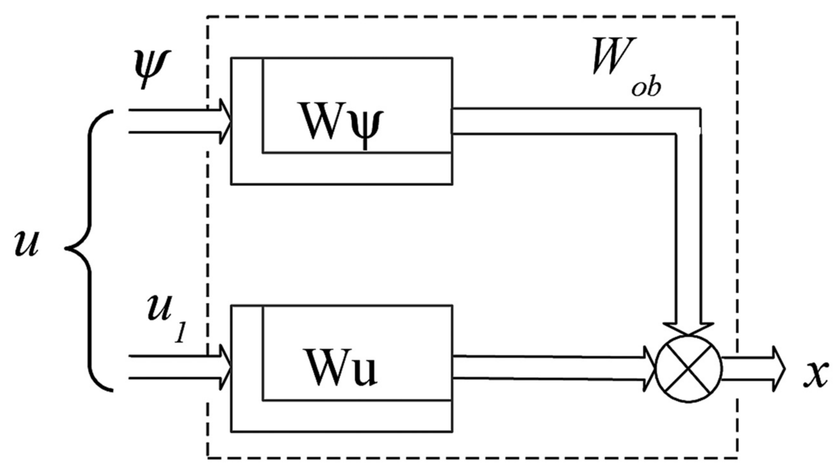

Figure 2.2 Block diagram of the mechanism with parallel kinematics.

based on experimental data, then two variants of the research task can be enunciated.

The first variant will be applied if the components of vectors u_1, x and ψ can be measured fully (the complete data). The second variant will be applied in the case when only the components of vectors u1 and x can be measured (the incomplete data).

So the research on the dynamics of the mechanism with parallel kinematics can be formulated as follows: to find transfer function matrices W_u, W_ψ and also to estimate the influence of vectors u_1 and ψ on vector x on the base of known complete or incomplete experimental data.

The solution of such a problem has been found as a result of three stages of:

- The development of algorithms for the structural identification of a multivariable Dynamic object with the help of complete or incomplete data;
- Collecting and processing experimental data about vectors u_1, x and ψ;
- The verification of the results of the structural identification.

2.3 Algorithm for the Structural Identification of the Multivariable Dynamic Object with the Help of the Complete Data

Let's suppose the identification object dynamics is characterized by a transfer function matrix W_{ob} (Figure 2.2), which may have unstable poles. Suppose that as a result of the processing of regular components of vectors *u* and *x,* the Laplace transformations $\overset{\frown}{U}_p$and $\overset{\frown}{X}_p$ are defined

$$\overset{\frown}{U}_p = L\left\{u\right\} = L\left\{\begin{bmatrix} u_1 \\ \psi \end{bmatrix}\right\}, \overset{\frown}{X}_p = L\left\{x\right\}. \tag{2.2}$$

Thus, the Laplace transformation of output vector $\overset{\frown}{X}_p$ has unstable poles of vector $\overset{\frown}{U}_p$ and unstable poles of matrix W_{ob}. Therefore, it is possible to remove all unstable poles from $\overset{\frown}{X}_p$[3] which differ from the unstable poles of $\overset{\frown}{U}_p$ and to define a diagonal polynomial matrix W_2 such as

$$\overset{\frown}{Y}_p = W_2 \cdot \overset{\frown}{X}_p . \tag{2.3}$$

In this case, the interdependence between vectors $\overset{\frown}{Y}_p$ and U_p is expressed with the help of equation

$$\overset{\frown}{Y}_p = F_{1p} \cdot \overset{\frown}{U}_p, \tag{2.4}$$

where F_{1p} is a transfer function matrix, all the poles of which are located in the left half-plane (LHP) of complex variables. It is equal to

$$F_{1p} = W_2 \cdot W_{ob}. \tag{2.5}$$

Consequently, the identification problem consists in determining a physically implemented matrix F_{1p} that minimizes a quality functional

$$J = \frac{1}{2 \cdot \pi \cdot j} \cdot \int_{-j\infty}^{j\infty} tr(\varepsilon \cdot \varepsilon_* \cdot A) \cdot ds, \tag{2.6}$$

where ε – identification error, which is equal to

$$\varepsilon = F_{1p} \cdot \overset{\frown}{U}_p - \overset{\frown}{Y}_p, \tag{2.7}$$

A– is a positively defined polynomial weight matrix.
To solve this problem, the ratio (2.7) must be submitted in a vector-matrix form

$$\varepsilon = \begin{bmatrix} F_{1p} & -E_n \end{bmatrix} \cdot \begin{bmatrix} \overset{\frown}{U}_p \\ \overset{\frown}{Y}_p \end{bmatrix}. \tag{2.8}$$

The Hermitian conjugated vector ε_* from Equation (2.6) is equal to

$$\varepsilon_* = \begin{bmatrix} \overset{\frown}{U}_{p*} & \overset{\frown}{Y}_{p*} \end{bmatrix} \cdot \begin{bmatrix} F_{1p}* \\ -E_n \end{bmatrix}. \tag{2.9}$$

After introducing the expressions (2.8) and (2.9) into the Equation (2.6), the quality functional can be shown as follows:

$$J = \tfrac{1}{2 \cdot \pi \cdot j} \cdot \int\limits_{-j\infty}^{j\infty} tr \left\{ \begin{bmatrix} F_{1p} & -E_n \end{bmatrix} \times \right.$$
$$\left. \times \begin{bmatrix} \overset{\frown}{U}_p \cdot \overset{\frown}{U}_{p*} & \overset{\frown}{U}_p \cdot \overset{\frown}{Y}_{p*} \\ \overset{\frown}{Y}_p \cdot \overset{\frown}{U}_{p*} & \overset{\frown}{Y}_p \cdot \overset{\frown}{Y}_{p*} \end{bmatrix} \cdot \begin{bmatrix} F_{1p}* \\ -E_n \end{bmatrix} \cdot A \right\} \cdot ds. \tag{2.10}$$

Thus, the problem of structural identification is reduced to the minimization of the functional (2.10) on the class of a steady variation matrix of transfer functions F_{1p}. Such minimization has been carried out as a result of the application of the Wiener-Kolmogorov procedure. In accordance with such procedure [5], the first variation of the quality functional (2.10) has been defined as

$$\delta J = \frac{1}{2 \cdot \pi \cdot j} \cdot \int tr\{A_{0*} \cdot [A_0 \cdot F_{1p} \cdot D - (H_0 + H_+ + H_-)] \cdot L \cdot D_* \times \delta F_{1p*} + \delta F_{1p} \cdot D \cdot L \cdot [D_* \cdot F_{1p*} \cdot A_{0^*} - (H_0 + H_+ + H_-)_*] \times A_0\} \cdot ds, \tag{2.11}$$

where A_0 is a result of the factorization [4] of the matrix *A* the determinante pf which has zeros with the negative real parts

$$A = A_{0^*} \cdot A_0; \tag{2.12}$$

D is a fraction-rational matrix with particularities in the left half-plane (LHP) which is defined on the basis of the algorithms in articles [3, 4] from the following equation

$$D \cdot L \cdot D_* = \overset{\frown}{U}_p \cdot \overset{\frown}{U}_{p*}, \tag{2.13}$$

where *L* is a singular matrix, each element of which is equal to one; bottom index * designates the Hermitian conjugate operation; $H_0+H_++H_-$ is a fraction-rational matrix which is equal to

$$H_0 + H_+ + H_- = A_0 \cdot \overset{\frown}{Y}_p \cdot \overset{\frown}{U}_{p*} \cdot D_*^{-1} \cdot L^+; \tag{2.14}$$

L^+ is the pseudo inverse to matrix *L* [5]; matrix H_0 is the result of the division; H_+ is a proper fractional rational matrix with poles that are analytic only in the right half-plane (RHP); H_- is a proper fractional rational matrix with poles that are analytic in LHP. In accordance with the chosen minimization procedure, a steady and physically realized variation F_{1p} which delivers a minimum to the functional (2.10) is equal to

$$F_{1p} = A_0^{-1} \cdot (H_0 + H_+) \cdot D^{-1}. \tag{2.15}$$

If one takes into account matrices W_2, F_{1p} from Equations (2.3) and (2.15), then an unknown object transfer function matrix W_{ob} can be identified with the help of the following expression

$$W_{ob} = W_2^{-1} \cdot F_{1p}. \tag{2.16}$$

The separation [4] of the transfer function matrix (2.16) makes it possible to find the unstable part of the object transfer function matrix with the help of equation

$$W_{ob2} = W_-, \tag{2.17}$$

where W_- is a fraction-rational matrix with particularities in the RHP.

An algorithm for the structural identification of the multivariable dynamic object with an unstable part on the base of the vectors u and x emplees the implementation of the following operations:

- Search the matrix W_2 as a result of the left-hand removal of the unstable poles from X_p, which differ from the poles of U_p;
- Factorization of the weight matrix *A* from (2.12);
- Identification of the analytical complex variable matrix *D* Equation (2.13);
- Calculation of H_0 + H_+ as a result of the separation (2.14);
- Calculation of F_{1p} on the basis of the Equation (2.15);
- Identifying W_{ob2} by the separation of the product (2.16).

In this way, we have substantiated the algorithm for the structural identification of the multivariable dynamic object with the help of the complete experimental data.

2.4 Algorithm for the Structural Identification of the Multivariable Dynamic Object with the Help of Incomplete Data

Let's suppose that the identification object dynamics is characterized by a system of ordinary differential equations with constant coefficients. The Fourier transformation of such system, subject to the zero initial conditions, can be shown as follows:

$$P \cdot x = M \cdot u_1 + \psi, \tag{2.18}$$

where *P* and *M* are polynomial matrices of the appropriate dimensions; ψ is the Fourier image of a centered multivariable stationary random process with the unknown spectral densities matrix $S_{\psi\psi}$. Let us admit also that vectors *u* and *x* are the centered multivariable stationary random processes with the matrices of the spectral and cross-spectral densities S_{xx}, S_{uu}, S_{xu}, S_{ux} known as a result of the experimental data processing. It is considered that the random process ψ can be formed by a filter with the transfer function matrix Ψ and is equal to

$$\psi = \Psi \cdot \Delta, \tag{2.19}$$

where Δ is the vector of the single δ-correlated "white" noises.

If one takes into account expression (2.19), then Equation (2.18) can be rewritten as follows:

$$x = P^{-1} \cdot M \cdot u_1 + P^{-1} \cdot \Psi \cdot \Delta \tag{2.20}$$

and a transfer function matrix which must be identified can be defined as the expression

$$\phi = \left[\begin{array}{cc} \phi_{11} & \phi_{12} \end{array} \right] = \left[\begin{array}{cc} P^{-1} \cdot M & P^{-1}\Psi \end{array} \right]. \tag{2.21}$$

So, the Equation (2.20) can be simplified to the form

$$x = \phi \cdot y, \tag{2.22}$$

where y is an extended vector of the external influences

$$y = \left(\begin{array}{cc} u' & \Delta' \end{array} \right)'. \tag{2.23}$$

Thus, the identification problem can be formulated as follows. Using the records of the vectors *x* and *y*, choose the sectional matrix string ϕ (2.21) that provides minimum to the following quality functional

$$J = \frac{1}{j} \int_{-j\infty}^{j\infty} tr\{S'_{\varepsilon\varepsilon} R\} ds, \tag{2.24}$$

where J is equal to the sum of the identification errors variances as the elements of the identification errors vector ε

$$\varepsilon = x - \phi \cdot y. \tag{2.25}$$

$S'_{\varepsilon\varepsilon}$ is a transposed matrix of the identification errors spectral densities

$$S'_{\varepsilon\varepsilon} = S'_{xx} - S'_{yx}\phi_* - \phi S'_{xy} + \phi S'_{yy}\phi_*; \tag{2.26}$$

$$S'_{yx} = \left(\begin{array}{cc} S'_{ux} & S'_{\Delta x} \end{array} \right); \tag{2.27}$$

$$S'_{yy} = \left[\begin{array}{cc} S'_{uu} & O_{m\times n} \\ O_{n\times m} & S'_{\Delta\Delta} \end{array} \right], \tag{2.28}$$

S'_{xx}, S'_{uu}, S'_{yy} are the transposed spectral density matrices of the vectors *x*, *u, y*; S'_{xy}, S'_{yx}, S'_{ux} is the transposed cross-spectral density matrices between vectors *x* and *y, y* and *x, u* and *x*; $S'_{\Delta x}$ is a transposed cross-spectral density matrix which is found on the basis of the Wiener's factorization [3] of the additional connection equation

$$S_{x\Delta}S_{\Delta x} = S_{xx} - S_{xu}S_{uu}^{-1}S_{ux}, \tag{2.29}$$

R is a positively defined polynomial weight matrix.

An algorithm for the set problem decision, which is grounded in [8] and allows defining the sought after matrix ϕ which minimizes the functional (2.24), has the following form

$$\phi = R_0(K_0 + K_+)D^{-1}, \tag{2.30}$$

in which matricees R_0 and *D* are results of the Wiener's factorization [3] of matrices *R* and S'_{yy} so that

$$R_{0*}R_0 = R; DD_* = S'_{yy}; \tag{2.31}$$

K_0+$K_+$is a transfer function matrix with the stable poles, which is defined as a result of the following equation right part separation [7]

$$K_0 + K_+ + K_- = R_0S'_{yx}D_*^{-1}. \tag{2.32}$$

An algorithm for the structural identification of a multivariable dynamic object with the help of the stochastic stationary components of vectors u_1 and *x* implies the following operations:

- Search for the spectral and cross-spectral densities matrices S_{xx}, S_{uu}, S_{yy}, S_{ux}, S_{xu} on the base of the experimental data processing;
- Factorization of the weight matrix *R* from (2.31);
- Factorization of the additional connection Equation (2.29);
- Factorization of the transposed spectral densities matrix (2.28);
- The Wiener's separation of the matrix (2.32);
- Calculation of matrix ϕ based on Equation (2.30);
- Identification of matrices ϕ_{11} and $\phi_{12}$with the help of Equation (2.21).

In this way, we substantiate the algorithm for the structural identification of the multivariable object on the base of the incomplete experimental data.

2.5 The Dynamics of the Mechanism with a Parallel Structure Obtained by Means of the Complete Data Identification

To identify the models of a dynamics, we used tracks of changes in the components of vectors *u*, ψ and *x*, obtained as a result of the behavior of a modeling platform using a virtual model. The case was considered when the motion platform center of mass O_1 remained in the plane of the manipulator symmetry $O_0X_0Y_0$. Thus, it is evident that in this case, instead of six rods only three (Figure 2.1) may be considered and the dimension of vector *u* (2.1) is equal 3.

As a result of the computational experiment, all the above vectors' components were obtained and all graphs of their changes (Figures 2.3–2.5) were built.

For solving of the identification problem, the control and perturbations vectors were combined into a single vector *u* of the input signals

$$u = \begin{bmatrix} l_{1,1} & l_{1,2} & l_{1,3} & M_x & M_y & M_z \end{bmatrix}^T. \qquad (2.33)$$

In this case, the identification problem is to estimate the order and the parameters of a differential equations system which characterizes the mechanism motion.

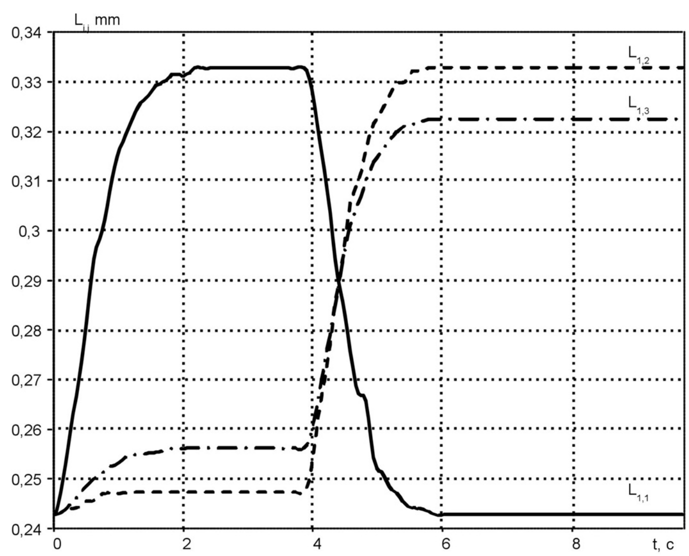

Figure 2.3 Graphs of changes in the length of the rods.

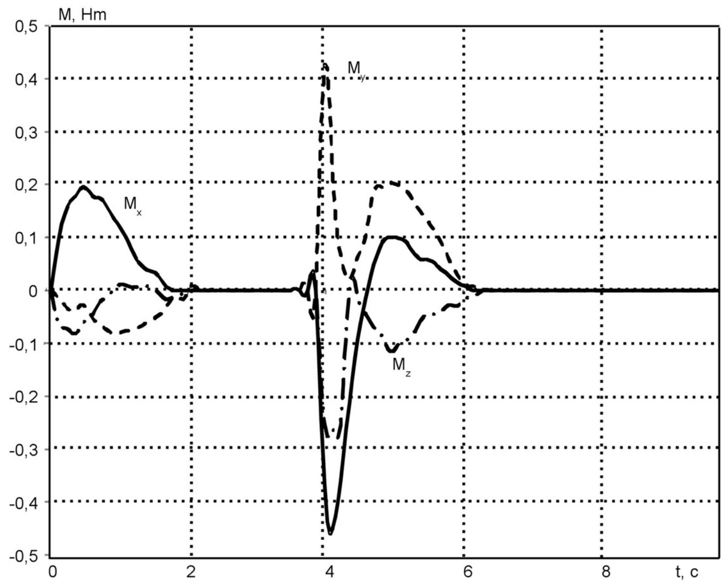

Figure 2.4 Graphs of changes in the projections of the resistance moments.

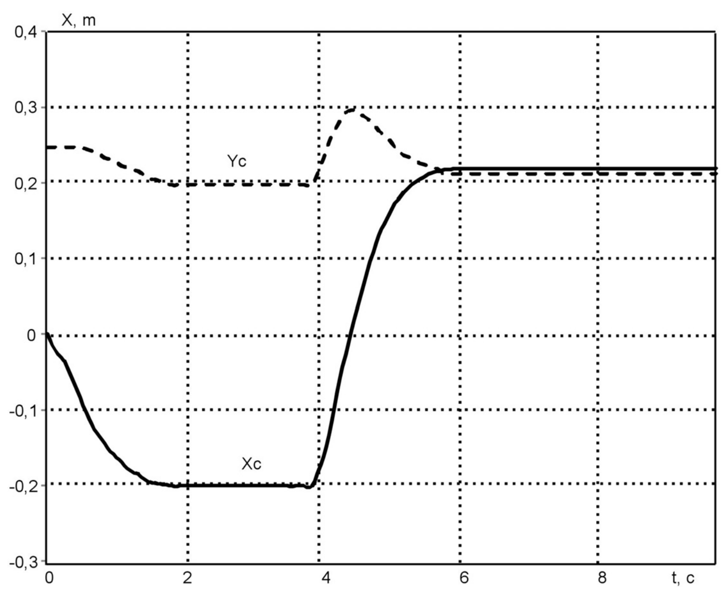

Figure 2.5 Graphs of chances in the coordinates of the platform's center of mass.

The state space dynamic model of the mechanism *is defined with the help of the* System Identification Toolbox of the Matlab environment. Considering the structure of vector *u*, defined by (2.33), allows to obtain the equation of the hexapod's state as follows:

$$\dot{y}(t) = Ay(t) + B_u u(t) + B_\psi \psi(t),$$
$$x(t) = Cy(t) + D_u u(t) + D_\psi \psi(t), \qquad (2.34)$$

where the matrices B_u, B_ψ, D_u and D_ψ are easily determined.

The analysis of the obtained model of dynamics shows that the moving object is fully controllable and observable.

As a result, the Laplace transformation of the left and right parts of the Equations (2.34) obtained the following relations

$$\begin{cases} (sE_n - A)y(s) = B_u u(s) + B_\psi \psi(s) \\ x(s) = Cy(s) + D_u u(s) + D_\psi \psi(s), \end{cases} \qquad (2.35)$$

where *y(s)*, *x(s)*, *u(s)*, $\psi(s)$ – the Laplace image of the vector, E_n – the identity matrix, *s* – the independent complex variable.

After solving the system of Equations (2.35) with respect to the vector of the initial coordinates of the mechanism *x,* the following matrices of the transfer functions W_u and W_ψ (Figure 2.2) were obtained

$$W_u = C\,(sE_n - A)^{-1}\,B_u + D_u, \qquad (2.36)$$

$$W_\psi = C\,(sE_n - A)^{-1}\,B_\psi + D_\psi. \qquad (2.37)$$

Substituting the appropriate numerical matrices *C*, B_u, B_ψ, D_u, D_ψ, in expressions (2.36), (2.37) allowed determining that

$$W_u = \begin{bmatrix} \frac{-0.37(s+1.606)(s^2+21.03s+790.2)}{(s+1.55)(s^2+3.463s+117.8)} & \frac{-0.599(s-57.51)(s^2-2.215s+8.456)}{(s+1.55)(s^2+3.463s+117.8)} \\ \frac{0.29(s+2.232)(s^2+21.65s+529.7)}{(s+1.55)(s^2+3.463s+117.8)} & \frac{2.23(s-14.01)(s^2-0.67s+87.82)}{(s+1.55)(s^2+3.463s+117.8)} \end{bmatrix}$$

$$\begin{matrix} \frac{0.92796(s+0.4741)(s^2-8.066s+404.6)}{(s+1.55)(s^2+3.463s+117.8)} \\ \frac{0.36433(s+75.92)(s^2-2.029s+91.39)}{(s+1.55)(s^2+3.463s+117.8)} \end{matrix} \Bigg], \qquad (2.38)$$

$$W_\psi = \begin{bmatrix} \frac{-0.006(s-2.782)(s^2+10.9s+450.5)}{(s+1.55)(s^2+3.463s+117.8)} & \frac{0.0023(s+43.94)(s^2-3.357s+8.447)}{(s+1.55)(s^2+3.463s+117.8)} & \frac{0.0049(s+57.79)(s+8.592)(s-2.702)}{(s+1.55)(s^2+3.463s+117.8)} \\ \frac{0.046892(s-10.1)(s^2+3.372s+133.1)}{(s+1.55)(s^2+3.463s+117.8)} & \frac{0.059(s+0.162)(s^2-2.875s+124.3)}{(s+1.55)(s^2+3.463s+117.8)} & \frac{-0.014(s+22.92)(s^2+1.572s+174.3)}{(s+1.55)(s^2+3.463s+117.8)} \end{bmatrix}. \quad (2.39)$$

The analysis of the matrix structure of (2.38) and (2.39) and the Bode diagrams (Figure 2.6) shows that this mechanism can be classified as a multi-resistant mechanical filter with the input signals and the disturbances energy bands lying in the filter spectral band pass. The eigen frequency of such a filter is close to *11*s^{-1} and depends on the moments of inertia and the mass of the moving elements of the mechanism (Figure 2.1).

The ordinary differential equations dynamics model of the system can be obtained if you present the transfer function matrices W_u and W_ψ as a product of the polynomial matrices P, M and M_ψ with the minimum order polynomials:

$$W_u = P^{-1}M, \quad (2.40)$$

$$W_\psi = P^{-1}M_\psi. \quad (2.41)$$

To find the polynomial matrices *P* and *M* with the minimum order polynomials, we propose the following algorithm:

- By moving the poles to the right [3], the transfer function matrix W_u should be introduced as follows:

$$W_u = N_R D_R^{-1} \quad (2.42)$$

and the diagonal polynomial matrix D_R should be found;

- On the basis of the polynomial matrices N_R and D_R found by the CMFR algorithm, substantiated in [7], the unknown matrices *P* and *M* should be identified

$$P^{-1}M = N_R D_R^{-1}; \quad (2.43)$$

- From Equation (2.41) and the known matrices *P* and W_ψ the polynomial matrix M_ψ should be found

$$M_\psi = PW_\psi. \quad (2.44)$$

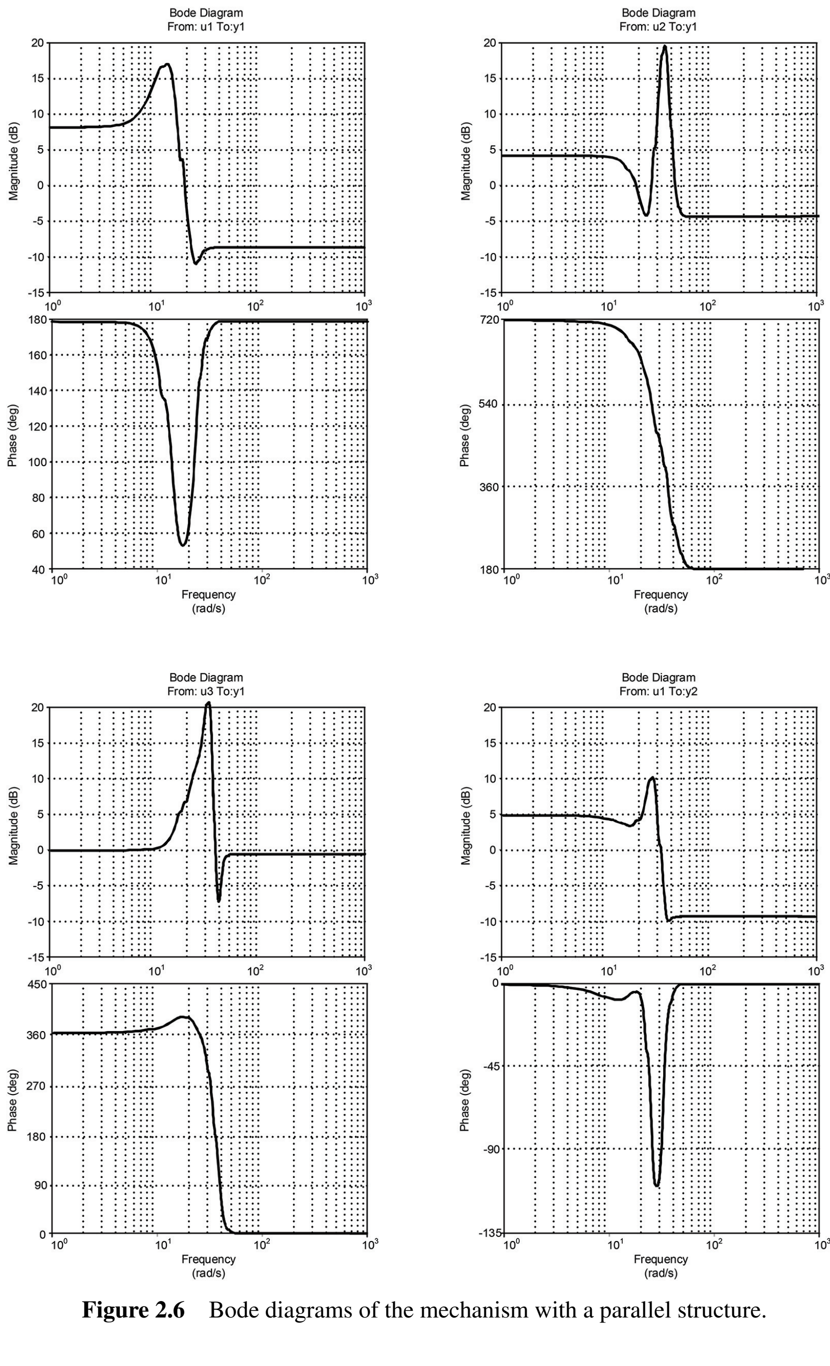

Figure 2.6 Bode diagrams of the mechanism with a parallel structure.

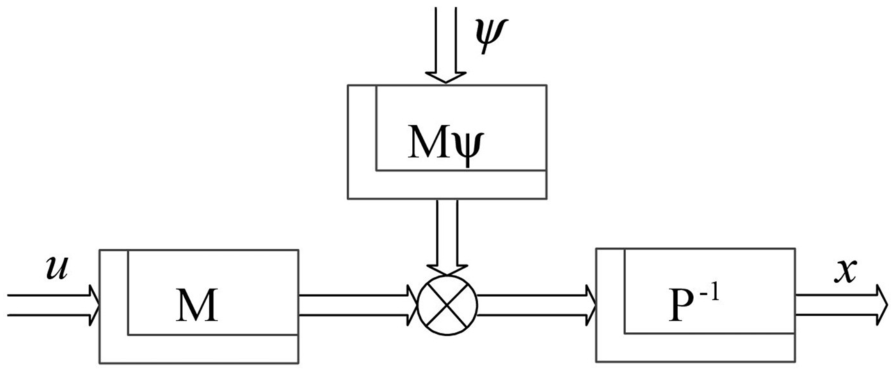

Figure 2.7 Block diagram of the mechanism with a parallel kinematics.

The application of the algorithms (2.42) and (2.43) to the original data represented by expressions (2.38), (2.39) made it possible to obtain the polynomial matrices *P*, *M*, M_ψ. Then, application of the inverse Laplace transform under the zero initial conditions allowed determining the following system of ordinary differential equations

$$P_0\ddot{x}+P_1\dot{x}+P_2x = M_0\ddot{u}+M_1\dot{u}+M_2u+M_{\psi 0}\ddot{\psi}+M_{\psi 1}\dot{\psi}+M_{\psi 2}\psi, \quad (2.45)$$

where P_i, M_i, ψ_i – the numeric matrices.

Representing the dynamic model (2.45) allowed to reconstruct a block diagram of a parallel kinematics mechanism in the standard form (Figure 2.7), where P^{-1} is an inverse of matrix *P*.

2.6 The Dynamics of the Mechanism with a Parallel Structure Obtained by Means of the Incomplete Data Identification

The incomplete experimental data arises when not all entrance signals of vector *u* (Figure 2.2) can be measured and recorded. Such a situation appears during the dynamics identification of a manipulator with controlled diode motor-operated drive (Figure 2.8). In this case, only signals of the platforms center of mass set position which form vector u_1 and signals of the platform's center of mass current position which form vector *x* are accessible for measuring. Thus, the task of the identification is to define matrices ϕ_{11}, ϕ_{12} from Equation (2.21) by records of vectors u_1 and *x*.

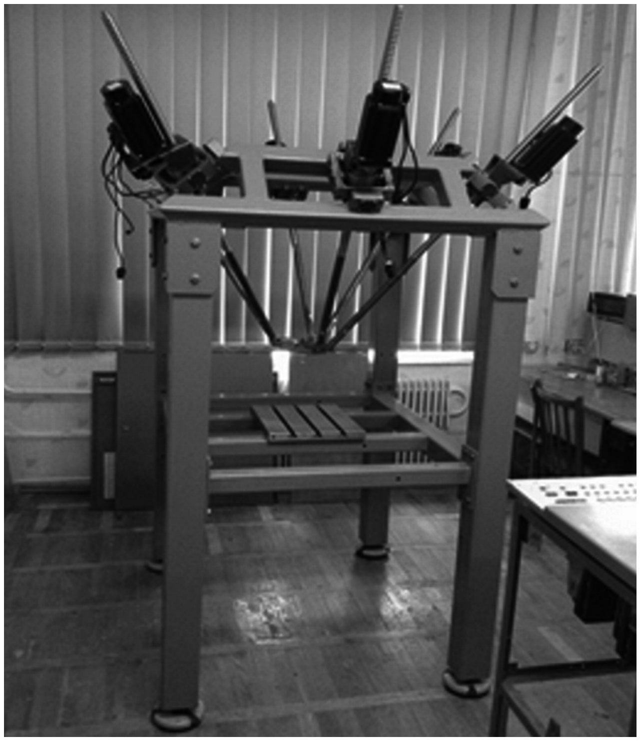

Figure 2.8 Manipulator with a controlled diode motor-operated drive.

Solving this task is achieved as a result of algorithm (2.30–2.32) applied to the estimations of the spectral and cross-spectral density matrices S_{uu}, S_{xx}, S_{ux} and $S_{\Delta x}$. For the illustration of this algorithm application, we used the records of the «input-output» vectors *u, x* with the following structure

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}; x = [Z_c], \tag{2.46}$$

where u_1 – the required value of the manipulators' platform center of mass O_1 projection on the axis O_0X_0 (Figure 2.1); u_2 – the required value of the manipulators platform center of mass O_1 projection on the axis O_0Y_0; u_3 – the required value of the manipulators' platform center of mass O_1 projection on the axis O_0Z_0 (Figure 2.1).

As a result of the experiment, all the above vectors' (2.46) components were obtained and all the graphs of their changes (Figures 2.9, 2.10) were built.

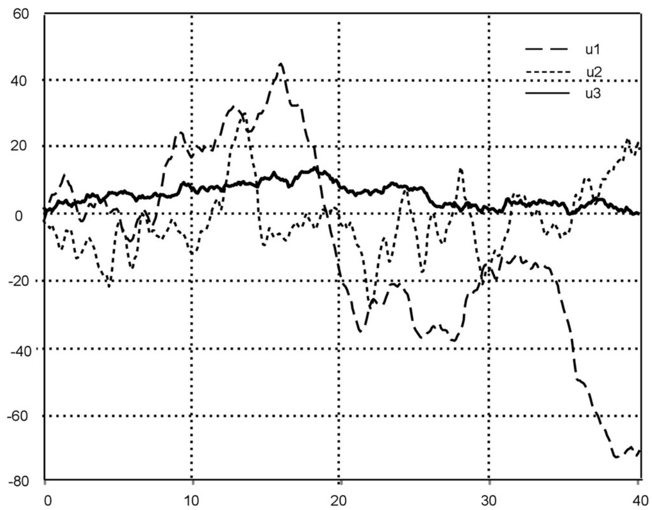

Figure 2.9 Grapas of the vector u componente changes.

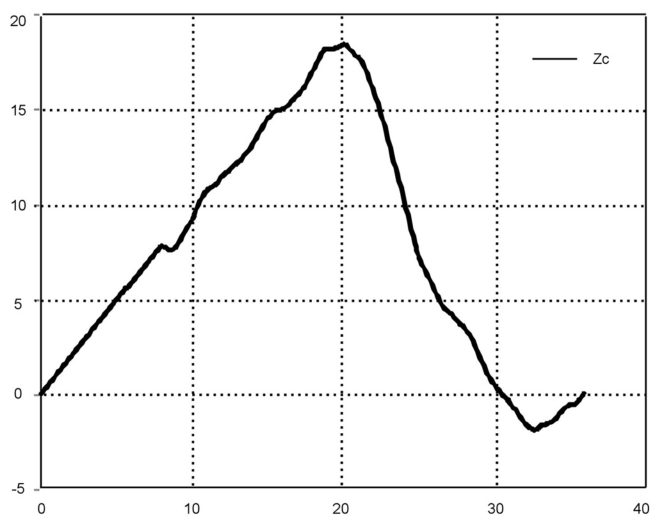

Figure 2.10 Grapas of vector x component changes.

In accordance with this algorithm (2.26–2.32) on the first stage of the calculations, estimations of matrices S_{uu}, S_{xx}, S_{ux} were found.

Approximation of such estimations made it possible to construct the following spectral densities matrices with the help of the logarithmic frequency descriptions method [4]

$$S'_{uu} = \begin{bmatrix} \frac{7.87|s+0.12|^2}{|s^2+0.29s+0.034|^2} & 0 & \frac{12(s+0.095)(-s+0.15)}{|s^2+0.29s+0.034|^2} \\ 0 & \frac{45.5|s+0.075|^2}{|s^2+0.64s+0.16|^2} & 0 \\ \frac{12(-s+0.095)(s+0.15)}{|s^2+0.29s+0.034|^2} & 0 & \frac{2.88|s+0.095|^2}{|s^2+0.29s+0.034|^2} \end{bmatrix};$$

$$S'_{xx} = \frac{4.59\,|(s+0.08)\,(s+1.3)|^2}{|(s+0.8)\,(s^2+0.29s+0.034)|^2};$$

$$S'_{ux} = \left[\ \frac{4.89(-s+0.1)(s+0.2)(-s+2.1)}{(s+0.8)|s^2+0.29s+0.034|^2} \quad 0 \right.$$

$$\left. \frac{1.2\,(s+3)\,|s+0.095|^2}{(s+0.8)|s^2+0.29s+0.034|^2} \right]. \qquad (2.47)$$

The introduction of the found matrices (2.47) into the Equation (2.29) and its factorization made it possible to find the cross spectral density $S_{\Delta x}$

$$S_{\Delta x} = \frac{0.89\,(s+1.6)\,(s+0.031)}{(s+0.8)\,(s^2+0.29s+0.034)}. \qquad (2.48)$$

The factorization [4] of the transposed spectral densities matrix S'_{yy} from expression (2.31) allowed finding the following matrix D

$$D = \begin{bmatrix} \frac{8.87(s+0.1)}{s^2+0.29s+0.034} & 0 & \frac{-0.54}{s^2+0.29s+0.034} & 0 \\ 0 & \frac{6.75(s+0.075)}{s^2+0.64s+0.16} & 0 & 0 \\ \frac{1.34(s+0.11)}{s^2+0.29s+0.034} & 0 & \frac{1.04(s-0.057)}{s^2+0.29s+0.034} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (2.49)$$

Taking into account the dimension of the output co-ordinates vector x (2.46), we have accepted that matrix R is equal to the identity matrix. At that rate matrix R_0 also equals 1. Substitution of the results (2.47–2.49) in expression (2.32) and its separation allowed defining that

$$K_0 + K_+ = \begin{bmatrix} \frac{-0.44(s-2.57)(s+0.12)}{(s+0.8)(s^2+0.29s+0.034)} \\ 0 \\ \frac{1.95(s+1.16)(s-0.04)}{(s+0.8)(s^2+0.29s+0.034)} \\ \frac{0.89(s+1.6)(s+0.03)}{(s+0.8)(s^2+0.29s+0.034)} \end{bmatrix}'. \qquad (2.50)$$

Substitution of matrices (2.49), (2.50) in expression (2.30) and taking into account the vectors *u* and *x* made it possible to solve the problem and to find such matrices ϕ_{11} and ϕ_{12} as

$$\phi_{11} = \begin{bmatrix} \frac{-0.33(s+0.61)(s-0.034)}{(s+0.03)(s+0.8)} \\ 0 \\ \frac{1.87(s+1.12)(s-0.008)}{(s+0.03)(s+0.8)} \end{bmatrix}'; \tag{2.51}$$

$$\phi_{12} = \frac{0.89\,(s+1.6)\,(s+0.031)}{(s+0.8)\,(s^2+0.29s+0.034)} \tag{2.52}$$

Taking into account the flow diagram on Figure 2.2 and the physical sense of matrices ϕ_{11} and ϕ_{12} made it possible to formulate the equation

$$W_u = \phi_{11}; \quad W_\psi = \phi_{12}. \tag{2.53}$$

For the definition of the incomplete data identification error, the Equation (2.26) is used and the error spectral density is found in the form which is shown below:

$$S'_{\varepsilon\varepsilon} = \frac{0.023}{|s+0.037|^2}. \tag{2.54}$$

The identification error mathematical mean is equal to zero and its relative variances is equal to

$$E_\varepsilon = \frac{\int_{-j\infty}^{j\infty} S_{\varepsilon\varepsilon} ds}{\int_{-j\infty}^{j\infty} S_{xx} ds} = 0.0157. \tag{2.55}$$

Obviously it is clear that the main part of the error ε oscillations power density is concentrated in the area of the infrasonic frequencies. The presence of such an error is explained by the limited duration of the experiment.

2.7 Verification of the Structural Identification Results

Verification of the identification results was implemented with the help of the modeling tool SIMULINK from Matlab. The *p*rinciple of the verification of the identification results exactness was based on the comparison of vector x records to the results of the platform center of mass position simulation.

On Figure 2.11, the simulation model block u is designed to create the set of the input signals. The block m generates a set of projections of the net

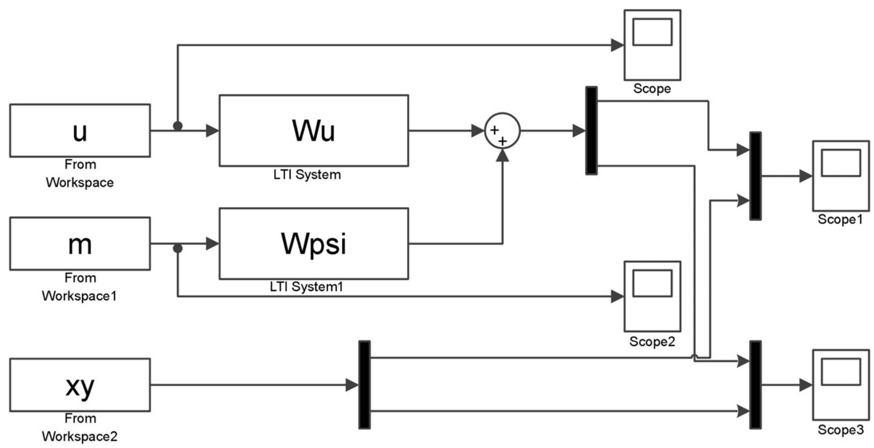

Figure 2.11 The scheme simulation model of the mechanism with parallel kinematics.

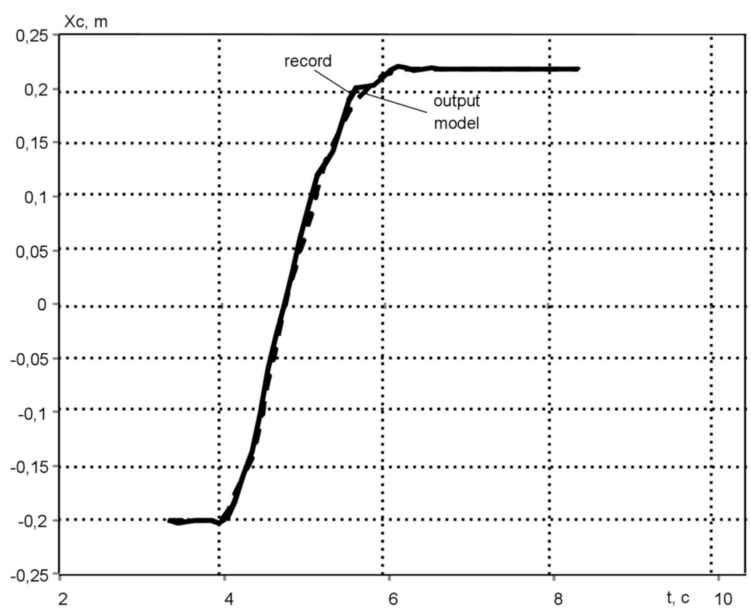

Figure 2.12 Graphs of the change of coordinates X of the center of mass of the platform.

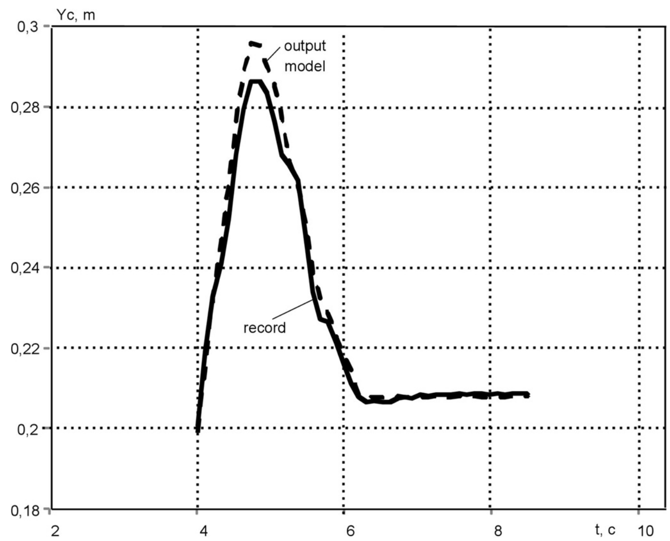

Figure 2.13 Graphs of thee changes of Y coordinates of the center of mass of the platform.

resistance moment vector on the axis of the co-ordinate system $O_0X_0Y_0Z_0$. The output of the block xy formed the vector x of records. The blocks Wu and Wpsi are designed for storing the matrices of the transfer functions W_u and W_ψ.

According to the simulation results, the grapas (Figures 2.12 and 2.13) are built. Analysis of these graphs show that they are close enough.

2.8 Conclusions

The conducted research on the mechanism with a parallel structure dynamics made it possible to obtain the following scientific and practical results:

- Substantiate two new algorithms for the structural identification of the multivariable moving object dynamic models. The first one of them allows to define the structure and parameters of a transfer function matrix of the object with unstable poles using the regular vectors "input-output". The second one allows identifying not only the model of a mobile object but also the model of the non-observed stationary stochastic disturbance;
- Three types of models which characterize the dynamics of the manipulator with a parallel kinematics are identified. This allows to use different modern multidimensional optimal control systems synthesis methods for designing the optimal mechatronic system;

- It is shown that the mechanism with a parallel kinematics as an object of control is a multi-resistant mechanical filter.

References

[1] J. P. Merlet, 'Parallel robots. Solid mechanics and its application', V.74 – Kluwer Academic Publishers, 2000, pp. 394

[2] S. V. Volkomorov, J. P. Hagan, A. P. Karpenko, 'Modeling and optimization of some parallel mechanisms', Information Technology. Application 2010, No. 5. pp.1–32 (in Russian).

[3] M. C. Davis, 'Factoring the spectral matrix', IEEE Trans. Automat. Cointr. – 1963.- AC-8, N 4, pp. 296–305.

[4] V. N. Azarskov, L. N. Blokhin, L. S. Zhitetsky, 'The methodology of constructing optimal systems stochastic stabilization', Monograph, Ed. Blokhin L. N. - K.: NAU Book Publishers, 2006, pp. 440 - Bibliography: pp. 416–428 (in Russian).

[5] F. R. Gantmakher, 'Theory matrits.-4th ed', Nauka, 1988. p. 552 (in Russian).

[6] V. Kucera 'Discrete line control: the polynomial equation approach'. Praha: Akademia, 1979, pp. 206

[7] F. A. Aliev, V. A. Bordyug, V. B. Larin, 'Time and frequency methods for the synthesis of optimal regulators', Baku: Institute of Physics of the Academy of Sciences, 1988, pp. 46 (in Russian).