

Structural Health Monitoring, Metamaterials, Metastructures and Additive Manufacturing and Dynamics of Biomedical and Bioinspired Systems, Volume 10

Dario Di Maio
Matthew Allen
Sheyda Davaria
Babak Moaveni



Proceedings of the 44th IMAC, A Conference and
Exposition on Structural Dynamics 2026



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Dario Di Maio · Matthew Allen · Sheyda Davaria · Babak Moaveni
Editors

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Preface

Structural Health Monitoring, Metamaterials, Metastructures and Additive Manufacturing and Dynamics of Biomedical & Bioinspired Systems represents one of ten volumes of technical papers presented at the 44th IMAC, A Conference and Exposition on Structural Dynamics, organized by the Society for Experimental Mechanics, and held January 19-22, 2026. The full proceedings also include volumes on Nonlinear Structures & Systems; Dynamics of Civil Structures; Model Validation and Uncertainty Quantification; Dynamic Substructuring & Transfer Path Analysis; Special Topics & Experimental Techniques in Structural Dynamics; Computer Vision & SLDV for Structural Dynamics; Dynamic Environments and Durability Testing; Topics in Modal Analysis & Parameter Identification in Structural Dynamics; and Data Science in Engineering.

Each collection presents early findings from analytical, experimental and computational investigations on an important area within Structural Dynamics. Structural Health Monitoring, Metamaterials, Metastructures and Additive Manufacturing and Dynamics of Biomedical & Bioinspired Systems are areas which cover topics of interest of several disciplines in engineering and science.

Structural Health Monitoring, Metamaterials, Metastructures and Additive Manufacturing and Dynamics of Biomedical & Bioinspired Systems are specific subject areas within the SEM umbrella of technical divisions, including other technical activities devoted to structural analysis, testing, monitoring, and assessment.

The organizers would like to thank the authors, presenters, session organizers, and session chairs for their participation in this track.

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Chapter 1

Applying a Viscoelastic Model to Multiple Unique Additively-Manufactured Trapped-Powder Dampers

Jon Black, Jacob Cox, Derek Koski, Carlos Inca Roca, Brooklyn Andrus, and Matthew S. Allen

Introduction

Metal additive manufacturing processes facilitate the fabrication of geometrically complex metal structures. When used for part consolidation, additive manufacturing can result in decreased damping due to lost frictional interfaces. Other potential additively manufactured parts, such as turbine blades [1], are destined for high-vibration environments where increased damping is highly desirable. One proposed technique for increasing damping in additively manufactured parts, and therefore delaying high-cycle fatigue failure, involves strategically incorporating pockets of unfused metal powder into the structure [1]. This technique has demonstrated the potential to dramatically increase the inherent damping without significantly affecting the strength and stiffness of a structure or requiring additional assembly or post-processing steps. Nevertheless, reliably modeling this phenomenon is an active area of research due to complex effects of powder state, magnitude-dependent damping, and other factors. While some success has been had in modeling powder particles using discrete element codes [2], this approach is very computationally expensive, making it cumbersome for designer use.

Black et al. [3] recently proposed approximating trapped powders as linear viscoelastic materials (VEM), with the material properties adjusted to capture the powder at a set vibration amplitude. This method is computationally efficient and accessible via standard finite-element packages. This work reviews the degree to which Black's model was able to capture the measured damping of the samples that were used to derive the equivalent material properties for the powder. Those samples were all beams with powder pockets of differing dimensions. Those same viscoelastic material properties are then used to predict the damping of several tuning forks and the results are evaluated. In addition, the viscoelastic model is extended to include kinetic-energy-based damping, to see if this can address its deficiencies.

Methods

The amplitude-dependent damping and frequency behavior of the tuning forks was found using impact modal testing. The test setup and geometries for the tuning forks are shown in Figure 1. Amplitude-dependent frequency and damping were extracted from accelerometer data using a Hilbert-based method. In contrast with the beams tested by Black [3], the tuning forks were hung such that the length of the tines was in the direction of gravity. This affects the way in which the powder settles and may have impacted the results.

Black's viscoelastic model utilizes the complex-stiffness method in ABAQUS finite element software [3]. Since both the beams and tuning forks were made of 316L Steel, the fused and powder material properties found by Black were directly applied to the tuning forks.

In addition to the original viscoelastic modeling, a modal strain and kinetic energy (MSKE) model [4] was used to predict the trapped powder behavior. This model linearizes the damping calculations and extends the model to include damping due to kinetic energy-based effects resulting in Eq. 1. In this equation, the first two terms represent the damping due to strain for both the fused and powder material while the third term represents kinetic-energy-based damping of the powder. In each term, η_i is a material property and $F_{i,mat}$ is the fraction of strain or kinetic energy within the given material. In these

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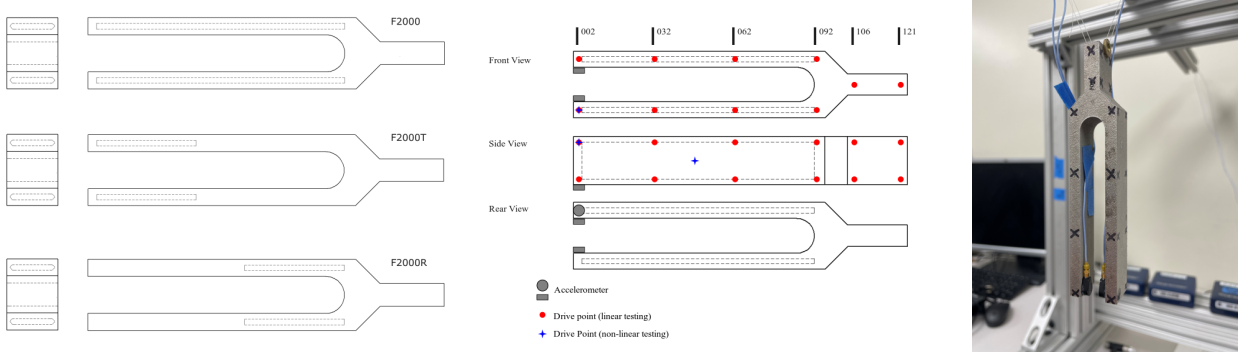


Fig. 1 Visual description of the tuning fork geometries and impact testing set-up

calculations, the $F_{i,mat}$ and frequency values are outputs of a real-valued modal analysis in ABAQUS. MSKE fused powder properties were chosen to match those used by the VEM model, assuming that $\eta_{s,powder}$ is roughly equivalent to the shear loss factor.

$$\eta_{tot} = \eta_{s,powder} F_{s,powder} + \eta_{s,fused} F_{s,fused} + \eta_{k,powder} F_{k,powder} \quad (1)$$

The MSKE parameters were optimized using a different approach than was used for the VEM model. The VEM parameters were optimized for the B1000 beam (1mm thick pocket throughout the beam) using the same properties for all modes [3]. The MSKE parameters were optimized for the two-pocket beam (which has two 20mm long, 1 mm thick pockets placed at the antinodes of the second bending mode). Furthermore, different MSKE pocket parameters were used for the soft direction modes and the other modes. This is consistent with the suggestion made in [5] that high aspect ratio pockets should be modeled as anisotropic. Interestingly, if VEM properties are optimized separately for soft and non-soft modes, little to no improvement is seen in our model. Meanwhile, the MSKE method benefits greatly from this separation, showing very different kinetic-energy damping behavior for soft and non-soft modes. However, this approach could not be directly applied to the tuning forks, because their modes tended to mix soft, stiff, and torsional movements, making them difficult to categorize. Therefore, both soft and non-soft MSKE parameters were applied to the tuning fork, and the results for each mode are reported separately.

Results and Discussion

A sample of the frequency and damping results for the beams and tuning forks is shown in Figure 2. The MSKE and VEM properties used are given in Table 1.

Table 1 Table of optimized VEM and MSKE properties for high amplitude modal behavior

VEM Parameter	VEM	MSKE Parameter	MSKE Soft	MSKE Stiff/Tors
Young's Modulus (MPa)	21624	Young's Modulus (MPa)	5274	21704
Poisson's Ratio	0.45	Poisson's Ratio	0.12	0.05
Shear loss factor	0.45	$\eta_{s,powder}$	0.12	0.51
Bulk loss factor	0.0	$\eta_{k,powder}$	0.15	0.0

For the beam data, the VEM model appears to capture the stiff and torsional modes quite well, with most errors falling within one standard deviation. Notably, the optimized MSKE parameters for stiff and torsional modes closely parallel those used in the VEM model with similarly accurate results, despite being optimized on separate beams. The exception is Poisson's ratio, which the beams appear not to be sensitive to. This parallel, along with the model's accuracy, suggests that the strain-based loss behavior of the VEM model is sufficient to model the stiff and torsional modes of the beams.

At the same time, the VEM model struggles to capture the trend of the soft direction modes. The most drastic example is the two-pocket beam's second soft direction mode as seen in Figure 2b. Although the two-pocket beam was designed to damp the second soft-bending mode, the viscoelastic model predicts very low damping. The MSKE model aims to address this deficiency by including a kinetic energy damping mechanism. In this case, the kinetic energy factor is shown to drastically

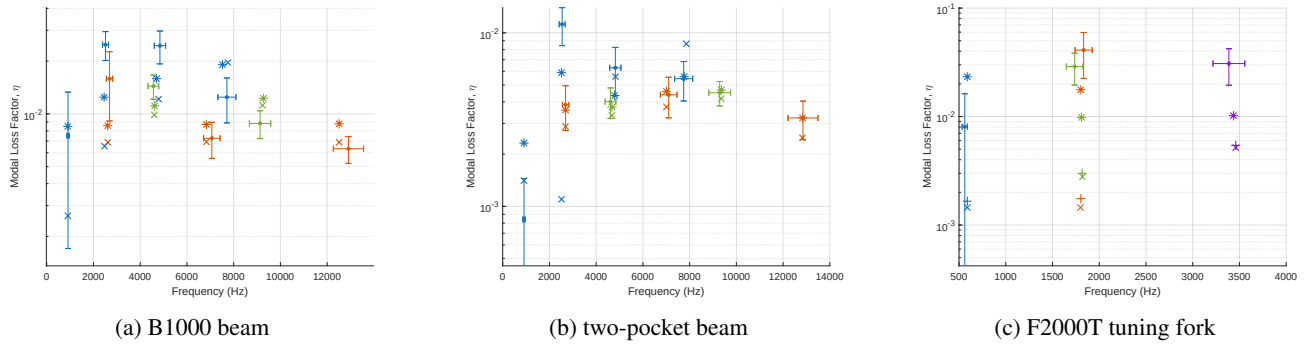


Fig. 2 Damping and frequency results for three geometries. Color refers to mode or modetype. For the beams, blue refers to soft modes, orange to stiff modes, green to torsional modes. Error bars report one standard deviation of the experimental data. Marker types refer to the model, × for the VEM model, * for the MSKE model. For the tuning fork, * refers to the MSKE Soft model while + denotes the MSKE Stiff/Torsional model (see Table. 1)

improve our estimates of this mode. Similarly, this kinetic energy factor also improves the soft direction estimates of B1000 in Figure 2a. Hence, by optimizing the soft and non-soft directions separately and including a kinetic energy factor, the MSKE model shows promise for capturing both soft and stiff direction behavior of the beams.

However, the kinetic energy factor negatively impacts our estimate of the two-pocket beam's first soft-direction mode, resulting in an absolute error of 175%. In contrast, the maximum absolute damping error for the viscoelastic model (soft mode 2) was only 90%. Nevertheless, the average absolute damping error for both models is similar despite this outlier, 31% and 35% for MSKE and VEM, respectively.

Generally, the tuning fork data in Figure 2c is not well predicted by any of the models, with average errors of roughly 90%. Notwithstanding, the tuning fork data reinforces several observations. As expected, the non-soft MSKE and the VEM models behave similarly. Again, the soft MSKE model, which utilizes the kinetic energy factor, drastically overpredicts the first mode, but improves our estimates of every other mode. Neglecting the first mode, the soft MSKE model reduces the average error by 27%, from 90 to 63%.

The failure to predict the tuning fork results is not surprising. The tuning fork pockets are 2mm thick, where the beam pockets are only 1mm thick. Larger pockets tend to have more unfilled space, significantly influencing damping characteristics [6]. Additionally, the tuning fork pockets have chamfered corners to improve manufacturability. Such changes in pocket geometry affect the particle-to-wall impact behavior that the kinetic energy factor aims to capture [1]. While not yet fully understood, part orientation with respect to gravity during testing also has the potential to affect damping performance. Furthermore, poor estimates of the first mode could be a sign of frequency-dependent behavior, which is common in analogous systems such as particle impact dampers [7].

In summary, trapped-powder dampers were modeled as viscoelastic materials, improving computational time and complexity as compared to discrete element models. Black's VEM model did a good job of capturing the stiff and torsional behavior of the beams for which it was optimized. The MSKE model was proposed and, to some extent, overcame the deficiencies of the VEM model for the beams' soft direction modes. However, neither model precisely predicted the measured damping of the modes of a tuning fork that also included pockets of trapped powder. The conference presentation will show results for the two other tuning forks in Fig. 1, as well as a set of gears with powder in the gear teeth and will discuss the benefits and limitations of each model in more detail.

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Chapter 2

Mobile Optical Sensing for Abdominal Aortic Aneurysm Monitoring

Johannes Maierhofer and Daniel J. Rixen

Abstract Abdominal aortic aneurysms (AAA) remain a life-threatening condition that often progresses without symptoms until rupture. Standard diagnostic imaging methods such as MRI or ultrasound, while effective, are not always ideal for frequent or accessible monitoring due to their cost, infrastructure requirements, and limited mobility. In the BioOptDyn project, we explore contactless optical sensing as a promising alternative for non-invasive AAA assessment.

This contribution presents two key developments. First, we introduce the BioScanRack, a mobile and modular test station designed to accommodate various optical sensor systems. The platform supports rapid deployment in both clinical and experimental environments, and enables flexible integration of laser-based techniques such as laser line triangulation sensors, laser Doppler vibrometry, and digital image correlation.

Second, we report on first in-vivo measurements conducted on patients with diagnosed abdominal aortic aneurysms. Using the BioScanRack, we recorded surface vibrations of the abdomen in a clinical setting, aiming to determine whether these dynamic signatures contain diagnostically relevant information about the aneurysm's mechanical state. We analyze the spatial and temporal characteristics of the measured signals and compare them across individuals and pathological conditions.

The abstract concludes with a critical discussion on the suitability and limitations of current optical techniques in the context of AAA monitoring, including challenges related to signal quality, motion artifacts, and inter-subject variability. These findings contribute to the long-term goal of enabling accessible, non-invasive, and repeatable AAA diagnostics, and lay the groundwork for further clinical studies and algorithmic interpretation of motion data.

Keywords Testing Equipment · Experimental Dynamics · Non-contact Displacement Measurement · Biomechanical Vibration Analysis

Introduction

The demographic shift toward an aging population has led to an increasing prevalence of abdominal aortic aneurysms (AAA), defined as localized dilatations of the abdominal aorta. With progressive enlargement, the risk of rupture rises substantially, often resulting in uncontrolled internal bleeding. The prognosis of rupture remains poor, with mortality rates of up to 90% before surgical intervention can be initiated. Current surveillance strategies rely on imaging modalities such as ultrasound, computed tomography (CT), or magnetic resonance imaging (MRI), while in clinical practice a pulsatile abdominal mass may also be detected during physical examination [5]. The BioOptDyn project was initiated to explore non-contact optical measurement techniques as a novel approach for monitoring biomechanical dynamics of the abdominal wall in AAA patients.

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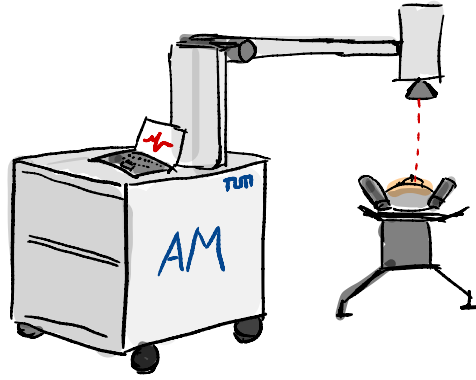


Fig. 1 BioOptDyn - mobile measurement platform to measure the skin motion of the abdomen to monitor abdominal aortic aneurysms.

Previous work in this field has been carried out in recent years through proof-of-concept studies [2, 3, 10]. Several studies have addressed the relationship between aneurysm wall motion and disease state, yet consensus remains elusive. While dynamic measurements of wall compliance and pulsatility have been proposed as potential markers for rupture risk [4, 8], clinical and experimental investigations have yielded conflicting results regarding their predictive value [1, 6]. Consequently, it remains an ongoing discussion whether quantitative assessment of wall motion can provide reliable information on the progression and stability of abdominal aortic aneurysms.

Vision

The long-term vision of the BioOptDyn project is to establish contactless optical sensing as a reliable and accessible tool for the diagnosis and follow-up of abdominal aortic aneurysms (AAA). By combining advanced laser- and camera-based technologies with intelligent signal processing, the project aims to move beyond the limitations of conventional imaging and provide a patient-friendly, mobile solution for routine monitoring. Ultimately, this approach seeks to enable earlier detection of critical changes, reduce dependence on costly imaging infrastructure, and open pathways for broader applications of optical vibration analysis in other areas of medical diagnostics.

Scope of the paper

This paper focuses on the selection of non-contact measurement methods by comparing the requirements with the physical principles. Three methods are selected and combined into a mobile test rig. With that, a pilot study is conducted in collaboration with the clinic on patients (ongoing, at the time of writing). First measurement datasets and corresponding signal pre-processing results are shown and discussed.

Abdominal Aortic Aneurysm

An abdominal aortic aneurysm (AAA) is a dilation of the aorta in the abdominal region. The aorta carries blood from the heart through the chest and abdomen, branching to supply major organs before dividing into the leg arteries. AAAs most often occur below the renal arteries and above the bifurcation to the legs (infrarenal AAAs).

The exact causes are not fully understood, but risk factors include arteriosclerosis, smoking, age, and male sex. Men are about six times more likely to develop AAAs, and prevalence increases with age. AAAs are often asymptomatic until they reach a critical size, when rupture risk rises sharply. Rupture causes severe internal bleeding and is often fatal. For this reason, ultrasound screening is recommended for men over 60. Depending on findings, patients may be monitored or treated.

While the typical diameter of the aorta is around 2 cm, an aneurysm is classified as large or severe if its diameter exceeds 5.5 cm [7].

Aneurysm Repair

The goal of repair is to exclude the aneurysm sac from systemic blood pressure. Two main methods are established:

Open Surgery The diseased section is replaced with a synthetic graft. This approach is durable but highly invasive, requiring major surgery and long recovery.

Endovascular Repair (EVAR) A stent graft is inserted via the leg arteries and positioned inside the aorta to reroute blood flow. EVAR is less invasive and widely used, but carries risks of leakage (endoleaks), requiring lifelong imaging follow-up. CT scans and contrast-enhanced ultrasound are commonly used, though both involve some patient burden.

To reduce the invasiveness of follow-up, alternative monitoring methods are being explored. One idea is to analyze abdominal skin motion with optical techniques, assessing whether surface patterns can indicate complications such as endoleaks without radiation or contrast agents.

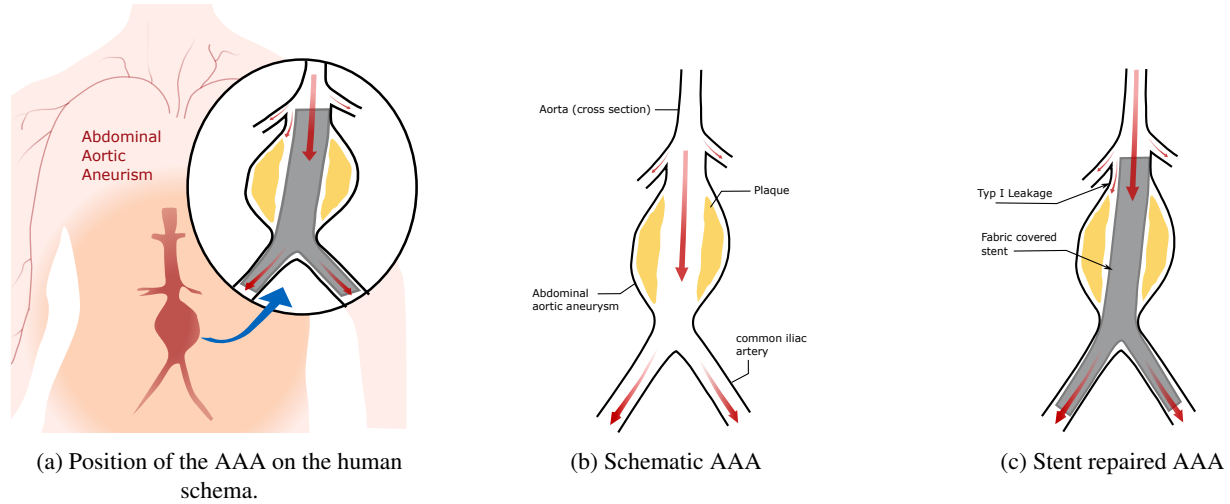


Fig. 2 Overview of AAA location, schematic anatomy, and stent-graft repair.

Physical measurement principles

Non-contact displacement approaches can be broadly divided into mechanical (acoustic) and electromagnetic domains. Acoustic approaches measure surface motion by detecting pressure fluctuations in air. In the electromagnetic domain, radar, infrared, and visible light are applied, with the transition from long to short wavelengths enabling progressively higher spatial resolution. The use of coherent light in laser-based methods further allows interference-based measurements with sub-micrometer precision.

Within the electromagnetic spectrum, different strategies exist: triangulation methods exploit geometrical relations between illumination and detection, whereas interferometric methods rely on coherent superposition of waves (e.g., quadrature demodulation of self-mixing or heterodyne laser signals). Another classification distinguishes active systems, which emit energy and analyze the reflected or transmitted signal, from passive systems, which rely only on the radiation naturally emitted or scattered by the body under investigation.

Selection

For the present study, three representative optical systems have been selected: (i) a Laser Doppler Vibrometer (LDV) with scanhead, enabling highly precise single-point measurements and rapid scanning; (ii) a Laser Line Triangulation (LLT) sensor, providing spatially resolved displacements along a line at medium sampling rates; and (iii) a stereo-camera system for Digital Image Correlation (DIC), which offers full-field displacement information but at the expense of high data volume and the need for surface preparation.

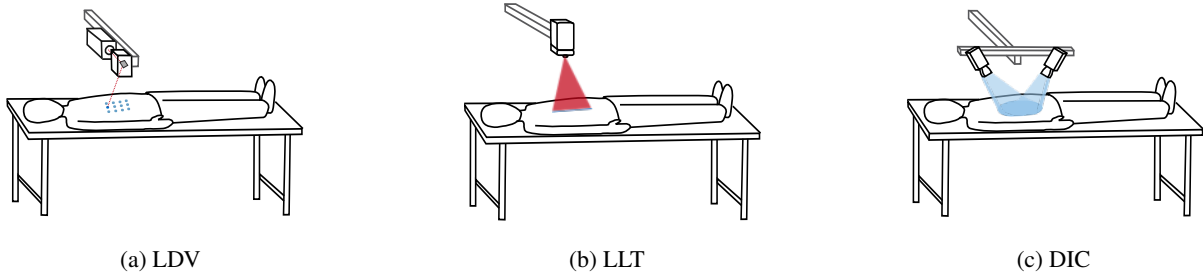


Fig. 3 Overview of the three optical measurement modalities considered for abdominal wall motion sensing: (a) laser Doppler vibrometry, (b) laser line triangulation, and (c) stereo digital image correlation.

BioScanRack

The BioScanRack is a mobile test station carrying the different optical sensors under test. It was developed at the Chair of Applied Mechanics, TUM, to conduct an in-vivo study at the clinic.

Requirements

The design of the BioScanRack had to satisfy numerous requirements from different perspectives. First, the packaging needs to be compatible with ambulance door widths and at the same time transportable with standard vehicles to commute between lab and clinic. All measurement equipment has to be fully contained in the rack to enable a fast and complete retraction from the examination room in case of an emergency. Second, the distance between the surface of the patient's abdomen and the sensors needs to be adjustable to account for different examination table heights and body sizes. Finally, the ScanRack should be modular to serve as a platform for various different sensors.

Mechanical Setup

The core frame of the BioScanRack is built from aluminum profiles on a wheeled platform. An electro-mechanical lift column with manual control carries a horizontal aluminum beam which is connected to the top of the column via a hydraulically damped hinge. This allows the system to safely fold-in for transportation. On the beam, different quick-release clamps are mounted which the individual sensors are attached to. The wiring harness connects the sensors to their control units which are stored in the box. Thin plates around the frame prevent objects from falling out.

Measurement Devices

To acquire useful data, an infrastructure needs to be set up that takes care of fully synchronized data with unique timestamps. The signal infrastructure is built around the open-source data acquisition board OASIS which is a long-term development of the chair.

open-source Data Acquisition - OASIS

The OASIS board serves as the central unit for synchronized data acquisition within the BioScanRack. It is an open-source hardware and firmware platform, originally designed for IEPE sensors but equally suited for direct analog and trigger signal sampling, as required in this setup. A Python API provides a straightforward interface for configuration, control, and data access, making integration into custom measurement pipelines efficient. The open-source nature of the firmware allows individual adaptations for special requirements, ensuring long-term flexibility of the system. All acquired raw data are streamed to the host computer while at the same time being redundantly stored on the internal SD-card, providing a reliable fallback option in case of transmission loss, [12].

Continuous arterial pressure - Finapres, Nova Scan

The Finapres Nova Scan device provides continuous non-invasive arterial blood pressure measurements based on the volume-clamp method originally introduced by Peñáz in 1973 [9]. The measurement principle relies on a finger cuff equipped with an



Fig. 4 BioScanRack - A mobile measurement platform with a vertical column that carries the different measurement devices.

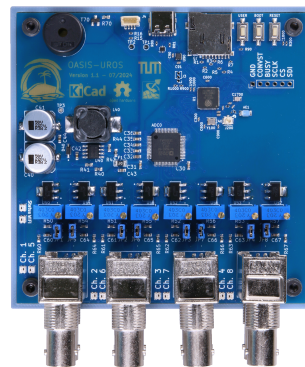


Fig. 5 Modified open-source OASIS system as core data acquisition system for the BioScanRack. All analog signals are gathered and synchronously sampled

inflatable bladder and an optical plethysmograph consisting of an LED and a light detector. The pulsatile changes of arterial diameter during each heartbeat modulate the detected light intensity. A servo-controller compares this plethysmographic signal, representing the unloaded arterial diameter, with a defined setpoint. The difference is amplified to generate an error signal which drives a fast pneumatic proportional valve in the NANO CORE® module. By dynamically adjusting the cuff pressure, the arterial wall is unloaded, such that the instantaneous cuff pressure follows the intra-arterial pressure waveform. This pressure signal thus provides an indirect but continuous measure of finger arterial pressure.

The Nova Scan additionally records ECG and respiration signals, enabling precise temporal alignment of cardiovascular parameters. For integration into the BioScanRack setup, the system provides an analog output channel that can be directly sampled by the OASIS board, ensuring full synchronization with other measurement devices. The raw physiological data are also redundantly stored on the internal memory of the Nova Scan device, guaranteeing a secure backup independent of the acquisition infrastructure.

Infrared Laser Doppler Vibrometer - Polytec RSV-150

The Polytec RSV-150 is a compact infrared laser Doppler vibrometer designed for precise, non-contact vibration measurements. It operates by emitting a laser beam onto the target surface and detecting the frequency shift of the backscattered light caused by the Doppler effect of surface motion. From this frequency shift, the instantaneous velocity of the surface can be

derived with high temporal and amplitude resolution. The use of infrared wavelength ensures compatibility with biological tissue while minimizing interference from ambient light. The vibrometer provides the analog output of the measured velocity and integrated displacement signal, which are directly sampled by the OASIS board to achieve full synchronization with other sensor modalities.

To position the laser measurement point, a Maul-Theet optical scanhead is employed, which allows dynamic redirection of the laser beam to different points on the measurement surface. The scanhead is operated using custom-developed firmware tailored to the requirements of the BioScanRack. Our firmware integrates seamlessly with Python, enabling straightforward configuration and scripted scan protocols directly from the measurement pipeline. This allows automated point-to-point scanning as well as complex predefined trajectories to be executed in synchronization with the other devices. As with all analog sensors in the setup, the scanhead signals can be routed to and synchronously sampled by the OASIS board, ensuring precise temporal alignment of all recorded data streams.

Line Laser Triangulation - Micro-Epsilon scanControl 3002

The Micro-Epsilon scanControl 3002 is a high-resolution laser line triangulation sensor that projects a laser line onto the measurement surface and records the reflected profile using a CMOS array. From the geometrical relation between projection angle, camera optics, and detected line position, the surface contour is reconstructed in real time. This enables the acquisition of dense profile data across a defined measurement width, making the sensor particularly suitable for capturing dynamic shape changes of the abdominal wall.

Within the BioScanRack setup, the scanControl 3002 transmits its profile data via Ethernet directly into the Python-based measurement pipeline, allowing efficient data handling and integration into automated processing workflows. To ensure precise temporal synchronization with the other devices, the scanControl additionally provides a hardware synchronization output. This sync-signal is fed as an analog input to the OASIS board, where it is sampled alongside all other channels. During post-processing, this allows accurate timestamp alignment between the profile data stream and the continuously recorded physiological signals.

Stereo Camera - Zeiss GOM Aramis

The Zeiss GOM Aramis system is a stereo digital image correlation (DIC) setup that consists of two synchronized cameras to capture surface displacements and strains over a defined region of interest. In contrast to full-field projection-based systems, the setup in the BioScanRack does not employ a projector, relying solely on natural or external illumination. The two-camera configuration allows the reconstruction of three-dimensional surface motion by tracking the displacement of speckle patterns applied to the measurement area.

Data acquisition is performed on a dedicated external computer, and thus runs independently from the OASIS-based infrastructure. Synchronization with the other devices is achieved by means of a cross-reference signal, which provides a temporal anchor point for aligning the recorded sequences. After acquisition, specific points of interest on the surface can be interactively selected in the post-processing software, and the corresponding displacement time series are exported. These displacement data are then incorporated into the unified measurement pipeline and can be precisely synchronized with the physiological signals of the other devices.

Data analysis

As a starting point, we adopt a systems-analysis view in which the cardiac cycle provides a periodic excitation and all measured observables are treated as responses to that input. For each spatial measurement point we acquire ≈ 10 s of data (> 10 heartbeats), including the ECG (excitation), noninvasive blood pressure, and optical skin-motion signals from our vibrometry/triangulation/DIC modalities. The ECG was treated as the excitation signal, while blood pressure and optical skin-motion signals were considered as responses. The ECG peak detection procedure is described below. All signals were temporally aligned such that corresponding heartbeats, pressure curves, and displacement patterns coincided across modalities.

After alignment, the signals were mapped onto a normalized cardiac phase using global least-squares beat alignment, allowing beat-wise averaging. In this way, mean values and standard deviations acquired a physiological meaning, as they represented variations of the cardiac response rather than timing differences or sensor latency. This procedure enabled a robust comparison of measurement points within an individual and, importantly, allowed consistent comparisons across different subjects.

The Wiggers diagram (fig. 6) illustrates the temporal relationships between pressure, volume, flow, and electrical stimulation during the cardiac cycle. In the reduced version, it relates the electrocardiogram (ECG) with the aortic pressure, allowing a comprehensive visualization of systole and diastole. By aligning these parameters on a common time axis, the diagram highlights the interplay between electrical excitation, mechanical contraction, valve motion, and blood flow.

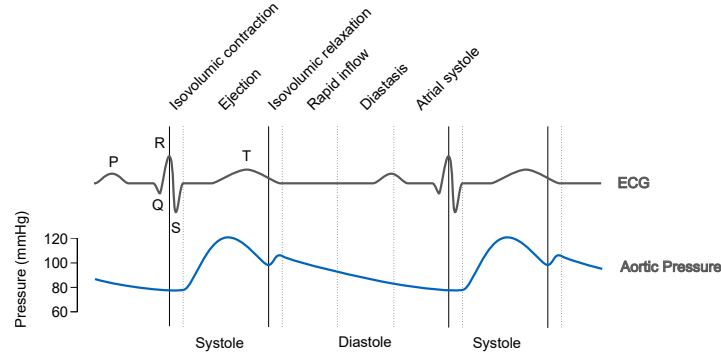


Fig. 6 Wiggers diagram

Electrocardiography (ECG)

A person's heartbeat is triggered by electrical impulses, which are generated by the SA node (sinoatrial node), a small mass of specialized tissue located in the right upper chamber (atria) of the heart. The electrical activity of the heart spreads through the walls of the atria and causes them to contract, forcing blood into the ventricles. Each contraction of the ventricles represents one heartbeat.

The electrical activity of the heart describes the controlled spread of electrical depolarization through the heart mass from one cardiomyocyte (heart cell) to its neighbors, as well as via a specialized electrical conduction system.

The mechanical activity of the heart is a synchronized muscular contraction that is produced as a result of the heart's electrical activity. This contraction of the ventricles forces open and shut specific heart valves, thereby allowing the pumping of blood towards the periphery.

Three-lead ECG refers to the basic configuration of electrocardiographic monitoring, typically involving one positive, one negative, and one ground electrode placed on the body surface. The most common variants are:

Lead I potential difference between the right arm (-) and left arm (+).

Lead II potential difference between the right arm (-) and left leg (+).

Lead III potential difference between the left arm (-) and left leg (+).

These bipolar limb leads form the classical Einthoven triangle, providing orthogonal projections of the cardiac electrical vector in the frontal plane. They allow continuous monitoring of heart rhythm and basic morphology of the P wave, QRS complex, and T wave. We decided to use the Lead II signal for the QRS complex detection.

Pan-Tompkins QRS Detection Algorithm

The Pan-Tompkins algorithm detects QRS complexes by enhancing high-frequency components, extracting energy, and applying adaptive thresholds [11]. Given an ECG signal $x[n]$ sampled at frequency f_s , a band-pass filter (h_{bp} , (≈ 1 Hz to 100 Hz)) is applied to emphasize the QRS energy:

$$x_{bp}[n] = (x * h_{bp})[n],$$

the asterisk $*$ denotes the discrete convolution of the signal with the band-pass filter impulse response. To highlight the slopes, the signal is differentiated:

$$x_d[n] = x_{bp}[n] - x_{bp}[n - 1]$$

Squaring makes all values positive and accentuates steep slopes:

$$x_s[n] = (x_d[n])^2$$

To obtain waveform features, a moving-window integration is applied over $N = T \cdot f_s$ samples (typically $T \approx 120$ ms to 150 ms):

$$x_i[n] = \frac{1}{N} \sum_{k=0}^{N-1} x_s[n-k]$$

The peak detection can be determined from the rising edge of the integration waveform. For irregular heartbeats, an adaptive threshold is implemented. Additionally, a physiological refractory period of $\Delta t \geq 200$ ms is enforced.

The detected indices correspond to QRS candidates; the final R-peak is localized within a short window of the original $x[n]$ by maximal absolute amplitude.

For the next step, the time-series data are cut into pieces around the detected R-peak with a length of 1 s. The alignment of these pieces is then further improved by applying a small time shift in such a sense that the cross-correlation between all pieces is maximized.

Global Least-Squares Beat Alignment

We refer to this approach as **Global LS Alignment (GLSA)**. To align individual beats without assuming a predetermined wave form, we estimated relative time shifts between beats using cross-correlation. For two beat segments $x_i(t)$ and $x_j(t)$, the lag Δt_{ij} is defined as

$$\Delta t_{ij} = \arg \max_{\tau} \int x_i(t) x_j(t + \tau) dt$$

which corresponds to the shift of beat j relative to beat i . Collecting these relations yields an overdetermined system

$$s_j - s_i \approx \Delta t_{ij}$$

where s_k denotes the unknown global shift of beat k . The set of shifts $\{s_k\}$ is obtained by minimizing the least-squares cost

$$\min_{\{s_k\}} \sum_{(i,j)} \left((s_j - s_i) - \Delta t_{ij} \right)^2.$$

This procedure yields consistent global alignment across all beats, with one beat fixed as reference ($s_0 = 0$) to remove the gauge freedom.

Preliminary In-Vivo Measurements

Pilot Study Design

A monocentric pilot study (ongoing at the time of writing) evaluates contact-free optical monitoring of abdominal aortic aneurysms (AAA) using a mobile measurement platform (“BioScanRack”).

The study comprises a transversal phase and a longitudinal phase. In the transversal phase, 10 adults (5 patients with AAA prior to treatment and 5 healthy controls; BMI < 35) will attend one study visit in the vascular surgery outpatient clinic.

During this visit, abdominal wall motion will be recorded with Laser Doppler Vibrometry, Laser Line Triangulation sensor, and (at a later stage) stereoscopic motion capture, synchronized to ECG; continuous noninvasive blood pressure and respiration (CNAP) are acquired in parallel.

In the longitudinal phase, 15 adults with AAA are examined twice: (i) at the pre-admission visit before planned EVAR and (ii) at routine follow-up one month post-procedure.

The primary objective is to assess feasibility and data quality of optical measurements in AAA patients; secondary analyses compare AAA patients with healthy controls and correlate optical metrics with clinical imaging findings.

Exemplary Measurements

As a first exemplary evaluation, measurements with the LDV were processed for one central abdominal point in a control subject and in a patient with an abdominal aortic aneurysm, fig. 7. For each case, individual heartbeats are displayed as transparent lines, while the thick line represents the mean and the surrounding colored band the corresponding standard deviation.

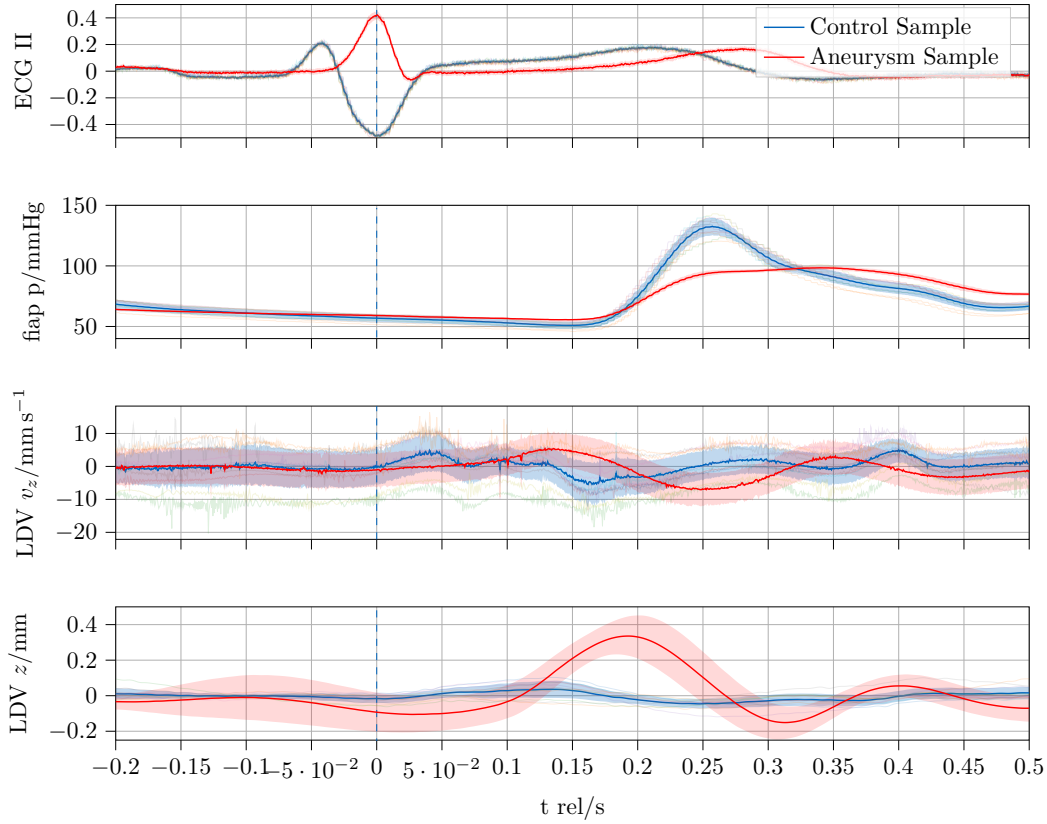


Fig. 7 A single measurement was randomly sampled from one subject of the control group and one subject of the aneurysm group.

Note: The polarity of the QRS complex is not invariant. In slender, tall individuals with a more vertical orientation of the heart, the main depolarization vector may align away from certain leads, resulting in a predominantly negative QRS complex even in physiologically normal cases.

The ECG waveforms appear highly repeatable in both subjects, confirming the robustness of the alignment procedure. The finger arterial blood pressure (fiap) trace of the aneurysm patient, however, shows a visibly less prominent peak compared to the control subject. In addition, the displacement signal, after removal of the respiratory component (by low-pass filtering), is markedly more pronounced in the aneurysm patient. These findings illustrate the potential sensitivity of the approach, yet no conclusions can be drawn at this stage. The observations are based on single individuals, and further measurements on a larger cohort will be required to enable a reliable interpretation.

Conclusion

This contribution shows the first steps in the direction of a non-invasive, non-contact and non-harmful examination method for the monitoring of AAA patients.

The three investigated optical measurement systems differ considerably in terms of spatial coverage, temporal resolution, and practical feasibility. This assessment is independent of the exact sensor type or manufacturer and instead focuses on the general measurement principles.

The point-wise laser Doppler vibrometer (LDV) offers excellent precision in the axial (z-) direction and can achieve very high sampling rates, but it is limited to single-point measurements and entails relatively high cost. Laser line triangulation (LLT) provides a compromise: it captures spatially resolved data along an entire line simultaneously, at moderate sampling rates, and with comparatively low system costs. Digital image correlation (DIC), by contrast, delivers full-field information but at the expense of extremely high data volumes, the need for artificial surface patterns (speckle), and substantial system cost. Considering these trade-offs, LLT emerges as the most favorable method for the current application, with the perspective of extending to three parallel LLT lines as the most promising future option.

However, many future steps remain to achieve a practical system and a robust data extraction which can be utilized in medical practice. As a next step, the full approved pilot study needs to be conducted and evaluated to provide evidence for the relationship between abdominal skin motion and aneurysm size.

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Chapter 3

Facial Tracking and Motion Amplification for Capturing Heartbeats in Video

Yuri Alencar, Camilly Costa, Gustavo Castro, Silvana Oliveira, Victor Cardoso, Moisés Silva, and João Weyl

Abstract Motion magnification is a technique that amplifies imperceptible variations in videos, highlighting subtle movements and even color changes associated with physiological signals. This approach has proven effective for non-invasive monitoring of physiological signals, such as heart rate and respiratory rate, by amplifying subtle motion oscillations linked to blood flow. The technique typically integrates Principal Component Analysis (PCA) and Blind Source Separation (BSS) to remove redundant information and isolate meaningful physiological signals. On the other hand, the Hilbert motion magnification technique offers greater precision in capturing high-frequency, low-amplitude oscillations. However, directly extracting signals from raw video can be compromised by background noise, lighting variations, and unwanted global motion. The implementation of MediaPipe's technique addresses these challenges by accurately isolating and tracking the anatomical regions of interest on a person's face. Continuous tracking of these facial points stabilizes head position, removing unwanted rigid movements before motion decomposition. Dynamically selecting this sub-image significantly reduces the volume of processed data and improves the signal-to-noise ratio, as motion extraction is applied only to relevant pixels, avoiding artifacts from camera movement or vibrations. This preprocessing further enhances the performance of the motion magnification framework, enabling it to more accurately isolate the cardiac component from other independent sources in the amplified signal. The proposed method shows promise for applications in telemedicine, neonatal monitoring, and clinical studies, paving the way for remote, scalable, and contactless healthcare systems due to its automation and low computational cost.

Keywords Motion magnification · Facial tracking · Heart-rate monitoring · Non-contact monitoring · Blind source separation

Introduction

Heart rate (HR) monitoring is a fundamental procedure for analyzing cardiovascular condition and tracking patients in different health contexts. Its relevance is notable in fields ranging from physical training to stress management. HR acts as a thermometer for the body's reactions to various stimuli, such as physical exertion, psychological stress, or illnesses [1]. Current research reinforces that specific metrics, such as resting heart rate (RHR), act as a screening tool for cardiometabolic risk, with chronically high values potentially indicating an autonomic imbalance [1]. Similarly, the speed at which the heart slows down after an activity is a strong indicator of physical fitness and heart health [2]. In this context, heart rate variability (HRV) stands out as a highly relevant marker, as it mirrors the autonomic nervous system's ability to modulate the heart rhythm in different scenarios. High HRV is often interpreted as a sign of good cardiovascular health and an efficient capacity to adapt to stress. Conversely, decreased HRV may suggest autonomic dysfunction, serving as an objective biomarker for stress states and being associated with various pathologies, especially those of cardiovascular origin [3]. Despite its vast utility, HR

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measurement methods, especially the most modern and accessible ones, are not without challenges. Photoplethysmography (PPG) technology, the basis of most wearable devices, although very practical, can have its accuracy affected by several factors. The main source of errors is motion artifacts, which occur during exercise and can generate incorrect readings [4]. Additionally, individual characteristics such as the presence of tattoos at the sensor site or low blood perfusion (common in cold environments) can weaken the signal and compromise data accuracy. These technical limitations show that for medical diagnoses or for athletes seeking maximum precision, data from wearables must be critically analyzed [4].

The acceleration in the adoption of digital health solutions was a direct response to global health crises, consolidating remote care as a pillar for maintaining medical assistance. Large-scale studies, such as systematic reviews, confirm that the patient experience with telehealth is largely positive, reaching levels of satisfaction that often equal or even surpass those of in-person interactions. This high approval is driven by remarkable convenience, savings in time and travel costs, and, especially, expanded access for geographically isolated populations or those with restricted mobility [5]. HR verification via video represents an expanding research field, valued for its convenient and contactless approach. The core methodology behind this technology, known as remote photoplethysmography (rPPG), is based on an optical principle: hemoglobin in the blood absorbs light distinctly. With each heartbeat, the volume of blood in the facial vessels subtly increases, altering the amount of light that is reflected back to the camera. Although these fluctuations in skin coloration are imperceptible to the naked eye, a conventional camera can register them. Image processing algorithms are then applied to isolate this pulsatile signal from the video data, extracting the heart rate. The accuracy of this technique, however, is highly dependent on factors such as the stability of ambient lighting, the individual's skin tone, and, especially, the absence of movement, which can corrupt the signal and hinder measurement [6].

A fundamental approach for contactless video analysis is Eulerian Video Magnification (EVM). This computational technique is designed to reveal phenomena that are imperceptible to the naked eye by selectively amplifying the minute temporal variations that occur in a sequence of images. Instead of tracking objects, the method analyzes the changes in color or intensity of each pixel over time. For HR measurement, EVM can isolate and amplify the frequency band corresponding to human heartbeats. This makes the blood flow under the skin visible as a subtle, pulsating color change. The main limitation of the technique is that it indiscriminately amplifies any signal—including noise—that falls within the selected frequency band. Consequently, the accuracy of the measurement can be compromised by patient movements or fluctuations in ambient lighting if these disturbances occur at a rhythm that overlaps with the cardiac signal [7].

An evolution in video magnification for motion analysis is the phase-based approach, which overcomes limitations found in previous methods, such as the Eulerian one. Unlike techniques that amplify the amplitude of color or brightness signals, this method uses the Hilbert Transform (or its 2D generalization) to isolate and amplify variations in the local phase of the video [8]. As phase is directly correlated with the position of a feature in the image, amplifying its changes over time translates into a more direct and cleaner magnification of motion. The result of this strategy is a superior ability to amplify larger magnitude movements without introducing the noise and visual artifacts that limited the previous technique. This allows for the visualization of motion phenomena on a broader scale and with greater fidelity, making the method more robust for applications in less controlled scenarios [9]. Another tool that can be introduced in the heart rate measurement scenario is MediaPipe, as a component for signal stabilization. Its ability to accurately detect and track facial landmarks in real-time allows for the definition of stable Regions of Interest (ROIs). It can provide up to 468 facial landmarks in the image, which represent the facial geometry in detail, accurately capturing the contours of the eyes, eyebrows, nose, lips, and the overall shape of the face.

The main contribution of this study is the presentation of a simplified and accessible pipeline for contactless HR monitoring. The key advancement lies in using the MediaPipe framework to automate the extraction of facial features, an approach that surpasses traditional methods in efficiency, which demand more intensive data preprocessing and rigorous capture setups. At the core of the methodology, the system analyzes the time series of the 468 three-dimensional facial landmarks provided in real-time by MediaPipe. This raw motion data is then subjected to a sequence of transformations: the Hilbert transform generates a complex analytic signal, PCA reduces its dimensionality, and finally, BSS precisely isolates the pulsatile signal of interest from other noise sources. The effectiveness of this process was rigorously validated, achieving results with high agreement when compared to a pulse oximeter used as a reference. The application potential of this technology is vast, as its optimized architecture shows promise for integration into embedded systems, such as those in smartphones and smart cameras, paving the way for the democratization of continuous health monitoring in everyday scenarios.

Theory

The measurement of HR from video motion analysis is based on the principle of remote ballistocardiography (rBCG). This phenomenon is a direct consequence of Newton's third law (action and reaction) applied to the hemodynamics of the human

body. With each systolic contraction, the left ventricle ejects a volume of blood at high velocity into the ascending aorta; the “action” of this mass of blood being accelerated upwards generates an opposite “reaction”: a subtle recoil impulse throughout the body. Although this recoil is minuscule, it propagates through the skeleton and, due to the biomechanics of the neck, manifests as a periodic and almost imperceptible oscillation of the head. It is this rhythmic movement that computer vision algorithms aim to detect [9].

The capture of the cardiac signal, in this method, does not originate from a global image analysis, but rather from the precise tracking of micro-variations in movement at specific facial points. First, the MediaPipe Face Mesh framework extracts the 3D coordinates of 468 landmarks over time, generating a rich time series of motion data [10]. This coordinate matrix then feeds into a sequence of three processing stages to isolate the heart rate: the Hilbert transform, PCA and BSS.

The methodology begins with the use of MediaPipe face mesh as a robust tool for facial feature extraction. Instead of analyzing raw video pixels, the system employs a sophisticated machine learning pipeline to detect and track 468 three-dimensional (3D) facial landmarks in real time. This process transforms the video into a high-density time series of coordinates, which provides a structured representation of the detailed movement of the facial surface. This data matrix serves as a rich and stable input for the subsequent signal processing steps, overcoming many of the instabilities associated with direct pixel analysis [10].

The analysis of subtle movements in video, such as those associated with physiological signals, often employs a combination of advanced signal processing techniques. The starting point is the application of the Hilbert Transform, which converts a real signal, such as the time series of a pixel’s intensity, into its complex analytic counterpart. The main property of this analytic signal is that its imaginary part has a phase shift of exactly 90 degrees relative to the real part, a fundamental mathematical relationship for phase and envelope analysis [11] Figure 1 provides an overview of the proposed method.

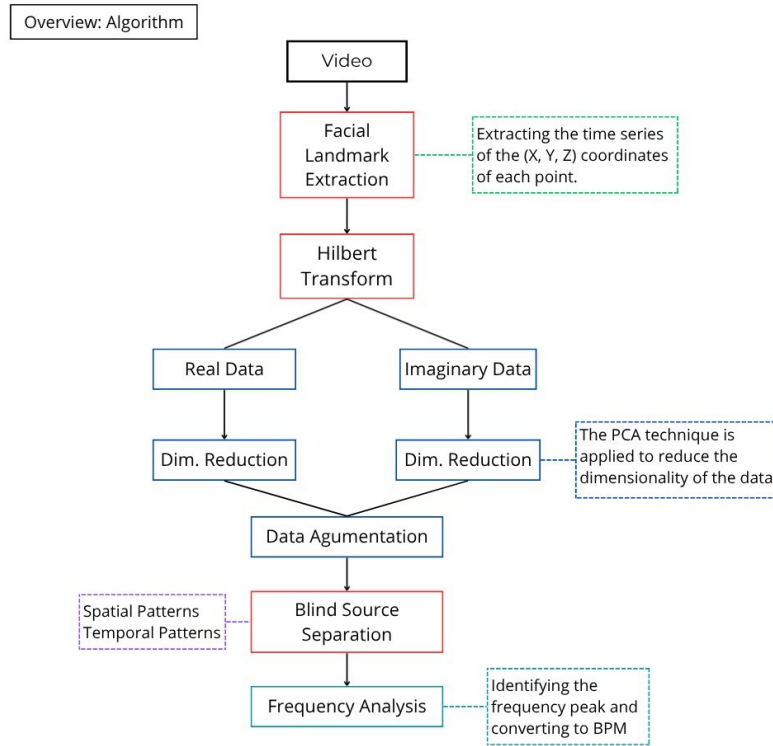


Fig. 1 Flowchart of the proposed approach

After applying the Hilbert Transform to the time series of facial landmarks, the resulting real and imaginary parts are concatenated, forming a single high-dimensional feature matrix, which will be called X . It is upon this matrix that PCA is applied with the goal of drastically reducing its complexity.

PCA is a linear transformation technique that projects data onto a new coordinate system such that the majority of the variance is captured in the first few coordinates, known as principal components. Mathematically, the process begins by calculating the covariance matrix C of the data, which has already been centered (by subtracting the mean from each

column). The covariance matrix is given by:

$$C = \frac{1}{n} X^T X \quad (1)$$

where n is the number of samples (video frames) [12]. Next, the core of PCA consists of solving the eigenvalue and eigenvector problem for this matrix:

$$Cv = \lambda v \quad (2)$$

The eigenvectors v represent the principal components, which are the directions of maximum variance in the data, while the scalar eigenvalues λ indicate the magnitude of this variance. By ordering the eigenvectors according to their corresponding eigenvalues in descending order and selecting the first 10 components, as defined in the code, a reduced-dimensionality representation is obtained that preserves the most significant information of the facial movement. This compressed representation is fundamental to the pipeline's success, as it eliminates redundancies and noise. In doing so, PCA provides the subsequent BSS step with a cleaner and more focused dataset, optimizing the task of isolating the subtle physiological signal of the cardiac pulse. The eigenvectors v represent the principal components, which are the directions of maximum variance in the data, while the scalar eigenvalues λ indicate the magnitude of this variance. By ordering the eigenvectors according to their corresponding eigenvalues in descending order and selecting the first 10 components, as defined in the code, a reduced-dimensionality representation is obtained that preserves the most significant information of the facial movement. This compressed representation is fundamental to the pipeline's success, as it eliminates redundancies and noise. In doing so, PCA provides the subsequent BSS step with a cleaner and more focused dataset, optimizing the task of isolating the subtle physiological signal of the cardiac pulse.

After dimensionality reduction by PCA, the principal components are processed by the final stage of BSS. In the context of this work, BSS is used to separate the different facial movement patterns (captured by the PCA components) into their constituent sources, such as head movement, respiration, and, most importantly, the subtle pulsatile ballistocardiographic signal. The BSS problem is mathematically modeled by assuming that the observed signals $x(t)$ (the PCA components) are a linear mixture of unknown independent sources $s(t)$:

$$x(t) = As(t) \quad (3)$$

The relationship involves A as the unknown mixing matrix and $s(t)$ as the independent source signals. BSS aims to solve for both A and $s(t)$ when only the observations $x(t)$ are known. The process to recover the sources is completed by inverting the mixing matrix A after a reliable estimate for it has been found.

$$s(t) = A^{-1}x(t) \quad (4)$$

This work implements a specific BSS algorithm called complexity pursuit. The premise of this method is that the physiological signal of interest (the cardiac pulse) is more regular and, therefore, temporally more "predictable" (less complex) than other noises or random movements. The algorithm operates iteratively, adjusting the matrix W to minimize a cost function based on the unpredictability of each estimated source. The complexity function $F(w_i, x)$ to be minimized is:

$$F(w_i, x) = \frac{V(w_i, x)}{U(w_i, x)} \quad (5)$$

where $V(w_i, x)$ represents the long-term variance of the extracted signal, and $U(w_i, x)$ represents the short-term variance. The separation matrix W is found by maximizing the temporal predictability of the separated signals. The resulting signal sources correspond to the temporal patterns of each mode of movement in the scene.

Experimental results

The entire computational pipeline was developed in the Python language. For the experiments, video recordings were captured by smartphone cameras at a sampling frequency of 60 Hz, in everyday scenarios with uncontrolled lighting conditions. The method's accuracy was validated by comparing the results with measurements from a pulse oximeter, which served as the ground truth. The final analysis consists of calculating the power spectral density (PSD) of the isolated signal to identify the frequency with the highest energy. Subsequently, this peak in the frequency domain (f) is converted to the unit of beats per minute (BPM) through the following relationship:

$$\text{BPM} = 60 \times f. \quad (6)$$

During all trials, the participants were positioned in a frontal orientation to the camera. The comparative analysis revealed a minimal discrepancy between the heart rate estimated by the algorithm and the reference values provided by the oximeter, a result that robustly corroborates the effectiveness of the proposed methodology.

The initial validation of the methodology yielded encouraging results, prompting supplementary investigations to assess the system's robustness under varied conditions. To isolate the impact of eye movement artifacts, a trial was conducted in which the participant kept their eyes closed, making it possible to verify the influence of blinking. With the material used, it was possible to simultaneously test MediaPipe's facial landmark detection capability and the algorithm's sensitivity in capturing the pulsatile signal with spatial attenuation. Satisfactory results can be observed in Figures 2 and 3, and their comparisons with the ground truth in Table 1.

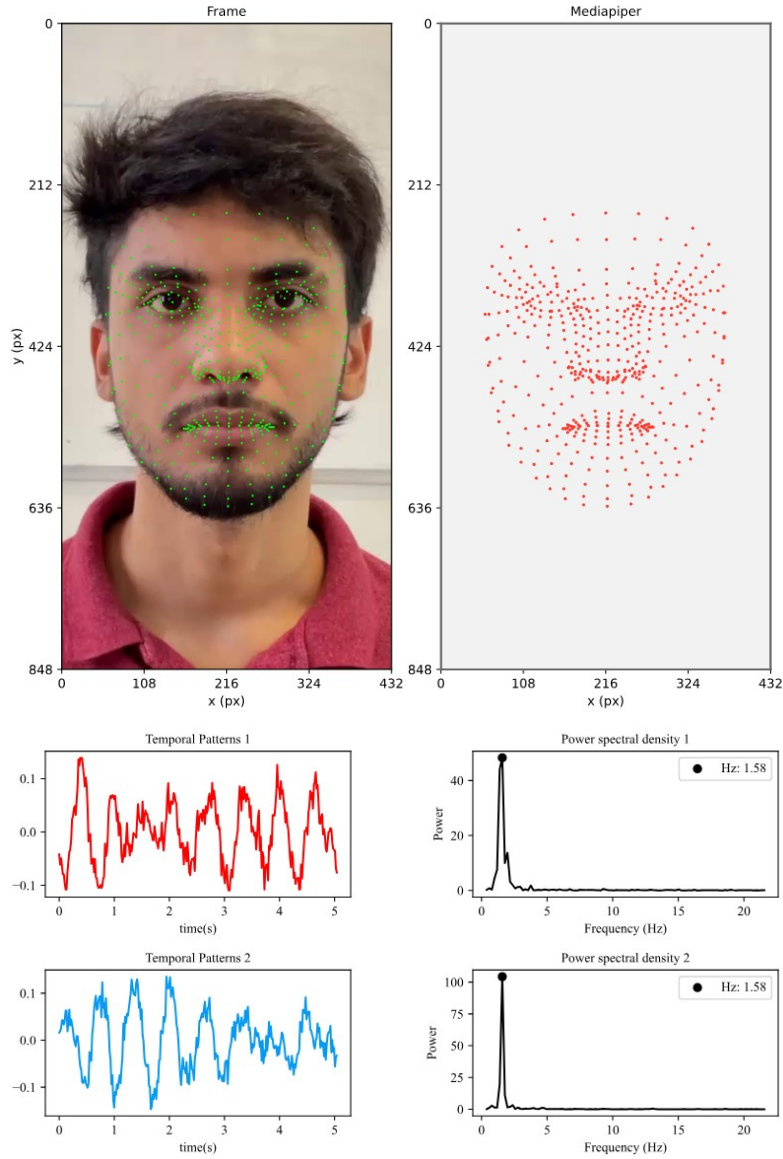


Fig. 2 Map of landmarks, modal coordinates and power spectral density from the test with

In all experiments, the error was insignificant, and variations in head angle did not significantly impact the results. Each setup successfully captured the subtle oscillations of the head caused by the pulsation of the aorta artery, and the heart rate was consistently detected with accuracy. MediaPipe facilitates the identification of points of interest, thereby reducing the amount of data. In this way, the processing did not utilize every pixel; a reshape was required to visualize the intensity of each pixel, and two pixels were added to form a complete 20x24 matrix.

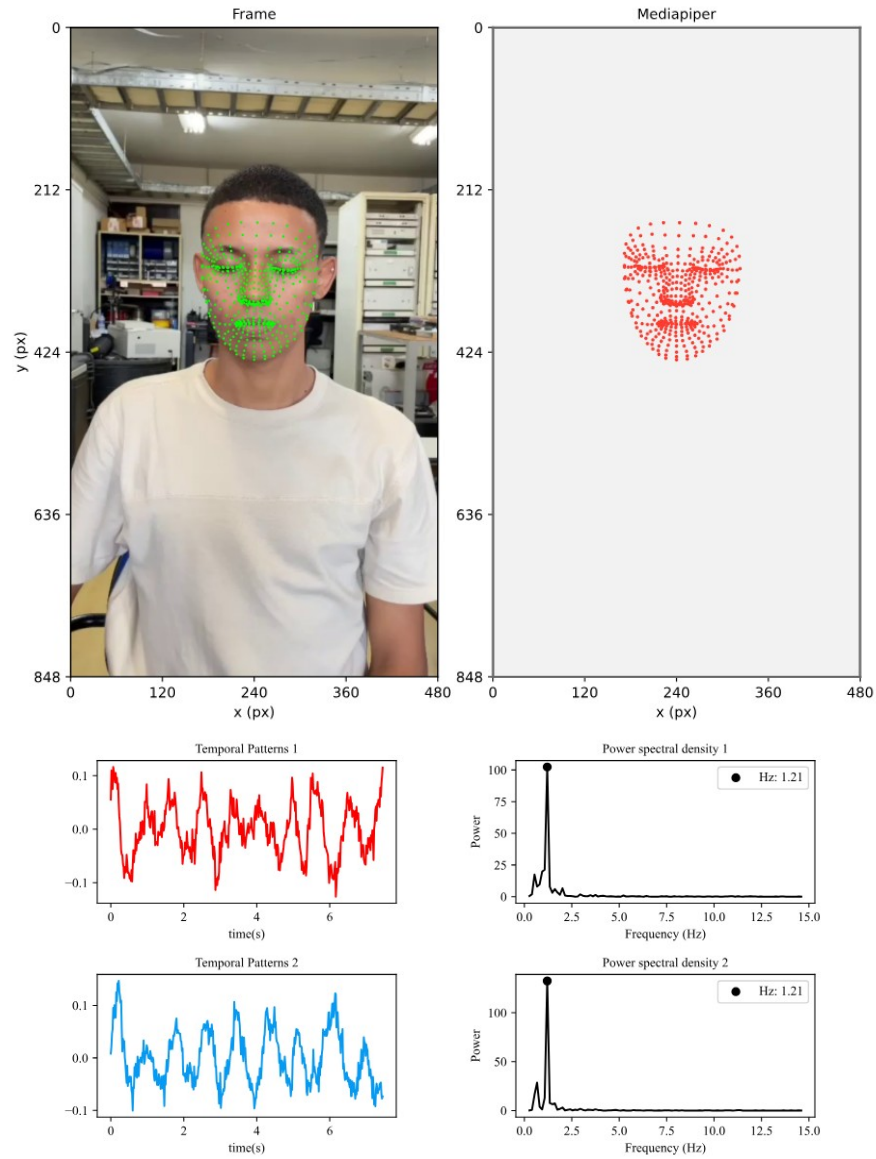


Fig. 3 Map of landmarks, modal coordinates and power spectral density from the test with

Table 1 Comparison of test results with oximeter readings in BPM

	Subject 1	Subject 2
Oximeter	100	73
Hilbert Magnification	94.8	72.6
Error (%)	5.2	0.4

Conclusion

This work presented and validated an innovative methodology for remote heart rate measurement, utilizing a computational pipeline that integrates facial landmark tracking via MediaPipe with advanced signal processing techniques, such as the Hilbert transform, Principal component analysis and blind source separation. The experimental results demonstrated the high effectiveness of the approach, achieving a remarkable agreement with measurements from a reference pulse oximeter.

The system's robustness was confirmed in tests under different conditions, including distance variations and eye occlusion, with the error margin remaining consistently minimal.

Therefore, it is concluded that the proposed approach is not only feasible, but also represents a significant advancement over previous methods, due to its simplicity and automation. The promising performance and efficient architecture pave the way for future optimizations and for the integration of this system into embedded devices, such as smartphones and smart cameras, aiming for the democratization of continuous health monitoring.

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Chapter 4

Mathematical Model and Prototype of Fish-Swimming Motion in a Wheeled Robotic Platform

E. S. Calderón, N. E. Carusetta, M. R. Deverell, J. D. Blessinger, M. J. Ward, J. Ardister, and B. F. Feeny

Abstract Mimicking oscillatory fish swimming dynamics on land platforms, such as autonomous vehicles or domestic robots, can enable more efficient movement. This project explores the development of a “landfish” robot: a wheeled platform that produces fish-like motion on a flat, dry surface through oscillatory motion of the tail relative to the head. A reduced-order model of fish swimming dynamics, which simplifies the fin-water interaction, has been developed in previous work [1]. In this study, a reduced-order model based on nonholonomic constraints in carangiform swimming will be developed and used to inform hardware changes for a wheeled robotic platform. We will then operate the landfish at a range of modulating tail frequencies and amplitudes, measure and characterize the steady-state dynamic response, and compare experimental results with simulations from the reduced-order model.

Keywords Biomimetic robots · Carangiform swimming · Nonholonomic constraints · Fish-inspired locomotion · Reduced-order modeling

Introduction

While human ingenuity has developed machines that excel at their tasks on paved roads or charted waters, these designs are limited in their performance when exposed to unique and difficult environments. By contrast, animals have developed exceptional abilities to survive in inhospitable or niche habitats around the world. These observations have led to a growing field of bio-inspired robots, which are better suited to performing in natural environments compared to their traditional robotic counterparts.

Fish, the most diverse group of vertebrates, are a popular source of inspiration for these designs [2]. Bio-inspired fish robots (e.g. [3], [4], and [5]) have the advantages of high efficiency, advanced maneuverability, low noise emission, and autonomous control capabilities. Beyond the creation of more efficient machines, robot development has additionally allowed scientists to model animal behavior in controlled environments [6]. Understanding how animals move and interact with their surroundings is critical to improving pre-existing engineering designs of underwater and hybrid terrain vehicles and developing new insights about the natural world.

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While there are several common undulatory swimming motions, carangiform [7, 8, 9] is especially popular to research for its balanced high speed and maneuverability. In carangiform motion, the anterior section of the body is stiff relative to the posterior; the last third undulates rapidly to produce propulsion [10]. At sufficiently high Strouhal numbers ($St > 0.22$), the wave speed of the fish exceeds its forward swimming speed, thereby accelerating the flow at the posterior region of the body and preventing flow separation from occurring, which significantly reduces drag [11]. Due to this increased efficiency, as well as an overall balance between speed and maneuverability, carangiform motion has been robustly explored over the past two decades, with notable research including biomimetic tuna [12], wire-driven flapping propulsors [13], and flexible caudal fins [14].

Our work endeavors to explore the movement of carangiform swimming by approximating a fish's motion using non-holonomic constraints imposed by its fins cutting through water [15, 16, 17, 18, 19, 1]. Through exploring the motion of a "landfish" with similar constraints imposed by wheels without slipping, we aim to better understand the motion of fish without the added fluid dynamic complexities like unsteady flow effects or vortex shedding. This research builds on previous Chaplygin sleigh designs that used casters on some wheels instead of nonholonomic constraints on all wheels [20]. While our design cannot perfectly describe the motion of a fish in water due to the lack of continuous body deformation, fluid deformation, and other simplifying assumptions, this research establishes a critical benchmark against which more complex "finned" models can be compared and validated, and can pave the way for faster optimization workflows for future biomimetic robots.

This paper presents a reduced-order model of fish-inspired locomotion, beginning with a mathematical formulation using three distinct approaches, followed by the experimental implementation of a wheeled landfish prototype, comparison of simulation and experimental results across varying tail oscillation parameters, and concluding with insights on the efficacy of our approach for modeling biomimetic locomotion. By eliminating the complexities of fluid dynamics, we can more easily develop and analyze a computationally inexpensive reduced-order model that still captures the essential dynamics of carangiform swimming.

Methodology

Mathematical Model

We used three different mathematical approaches for deriving equations of motion: Lagrange multipliers, nonholonomic Lagrange, and the principle of virtual power. These are common methods for solving nonholonomically-constrained systems [21, 22]. For all three methods, q_k represents a generalized coordinate. After implementing the holonomic constraints, our system has three generalized coordinates to keep track of: x , y , and θ . These will also be referred to as q_x , q_y , and q_θ . A superscript of u or c signifies that the quantity is nonholonomically unconstrained or constrained, respectively.

Figure 1 shows a diagram of the landfish system, which comprises two rigid bodies: the head and the tail. The head position is denoted by coordinates $x_1 = x$, $y_1 = y$, while the tail position is represented by x_2 , y_2 . The angle θ defines the

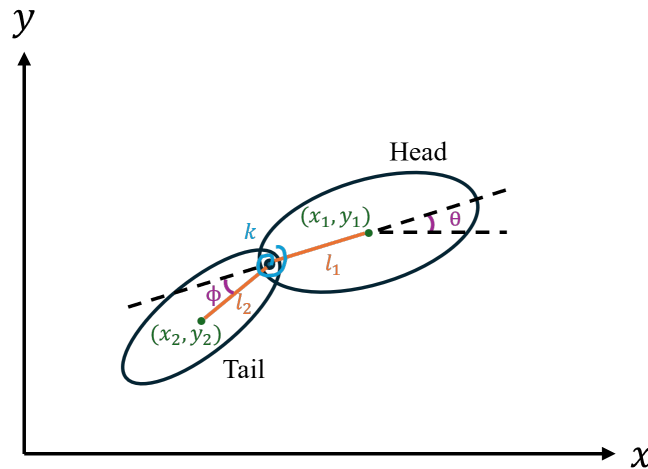


Fig. 1 System diagram of the landfish model showing the two-body configuration with the head and tail, from which the equations of motion for our mathematical model are derived

head's rotation relative to the horizontal x axis, while ϕ represents the tail's rotation relative to the head, which is imposed independently as

$$\phi = a \sin(\omega t + \beta). \quad (1)$$

Table 1 provides definitions for these parameters which include body components and physical coefficients. All parameters are referenced from the center of mass of their respective body.

Table 1 Physical and kinematic parameters of the landfish model

Parameter of Interest	Definition	Units
(x_1, y_1)	Cartesian position of head	m
(x_2, y_2)	Cartesian position of tail	m
θ	Rotational angle of head	deg
ϕ	Rotational angle of tail	deg
a	Amplitude of ϕ	rad
ω	Frequency of ϕ	$\frac{\text{rad}}{\text{s}}$
l_1	Length from head to joint	m
l_2	Length from tail to joint	m
m_1	Mass of head	kg
m_2	Mass of tail	kg
I_1	Moment of inertia of head	$\text{kg} \cdot \text{m}^2$
I_2	Moment of inertia of tail	$\text{kg} \cdot \text{m}^2$
k	Torsional stiffness	$\frac{\text{N}}{\text{rad}}$
C_d	Quadratic drag coefficient	$\frac{\text{kg}}{\text{m}}$
μ_k	Coulomb friction coefficient	[-]

Applying three holonomic constraints and two nonholonomic constraints to $x_1, y_1, x_2, y_2, \theta$, and ϕ reduces our six coordinates to a single degree of freedom system ($n = 1$). This process is described below. Because we are controlling the motion of the tail, there is a constraint on ϕ as seen in Equation 1. There are also two holonomic constraints associated with describing the position of the tail with respect to that of the head through trigonometry. Thus, the position vectors for the head and tail, respectively, can be written in terms of x, y, θ , and ϕ , as

$$\vec{r}_1^u = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \quad (2a)$$

$$\vec{r}_2^u = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x - l_1 \cos(\theta) - l_2 \cos(\theta + \phi) \\ y - l_1 \sin(\theta) - l_2 \sin(\theta + \phi) \end{bmatrix}. \quad (2b)$$

The velocity of each body acts along its respective orientation, creating nonholonomic constraints that can be written as

$$\dot{y}_1 = \dot{x}_1 \tan(\theta) \quad (3a)$$

$$\dot{y}_2 = \dot{x}_2 \tan(\theta + \phi), \quad (3b)$$

which can be cast in Pfaffian (differential) form, or solved for $\dot{\theta}$ and y , generally as

$$a_{jx}dx + a_{jy}dy + a_{j\theta}d\theta + a_{j\phi}d\phi = 0 \quad \text{for } j = 1, 2 \quad (4a)$$

$$\dot{\theta} = \alpha_{\theta x}\dot{x} + \alpha_{\theta\phi}\dot{\phi} \quad (4b)$$

$$\dot{y} = \alpha_{yx}\dot{x} + \alpha_{y\phi}\dot{\phi}. \quad (4c)$$

Note that Equations (4c) and (3a) match given $\dot{y}_1 = \dot{y}$ and $\dot{x}_1 = \dot{x}$. Implementing these constraints to solve for $\dot{\theta}$ results in

$$\dot{\theta} = \frac{-l_2\dot{\phi} - \dot{x} \sin(\phi) \sec(\theta)}{l_1 \cos(\phi) + l_2}. \quad (5)$$

Because our system has a single degree of freedom, we can define an independent coordinate, x , and constrained coordinates, y and θ (whose time derivatives are defined in Equations (3a) and (5)).

Two of the employed methods, Lagrange multipliers and nonholonomic Lagrange, are based on Euler-Lagrangian mechanics and thus use the unconstrained Lagrangian,

$$L^u = T^u - V, \quad (6)$$

where the kinetic energy, T^u , and potential energy, V , are defined as

$$T^u = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}I_1\dot{\theta}^2 + \frac{1}{2}I_2(\dot{\theta} + \dot{\phi})^2$$

$$V = -\frac{1}{2}k_1\theta^2 - \frac{1}{2}k_2\phi^2.$$

The only losses accounted for in our system are Coulomb friction and air drag in the Cartesian plane. For all three methods, we define these forces as

$$\vec{F}^u = \begin{bmatrix} -g \sin(\theta)(C_d(\dot{y}^2 + \dot{x}^2) + \mu_k(m_1 + m_2)) \\ -g \cos(\theta)(C_d(\dot{y}^2 + \dot{x}^2) + \mu_k(m_1 + m_2)) \end{bmatrix}. \quad (7)$$

The **Lagrange multipliers (LM)** method uses

$$\frac{d}{dt} \frac{\partial L^u}{\partial \dot{q}_k} - \frac{\partial L^u}{\partial q_k} = Q_k + \sum_{j=1}^p a_{jk} \lambda_j, \quad (8)$$

where λ_j are introduced as Lagrange multipliers, which in this case represent the magnitude of the generalized force needed to enforce the nonholonomic constraints present in our system. For LM, because there are two nonholonomic constraints ($p = 2$) and one degree of freedom, there are three k indices, which correspond to x , y , and θ . The a_{jk} -values come from the Pfaffian form shown in Equation (4a).

Given that ϕ is an imposed parameter rather than a generalized parameter, the $a_{j\phi}$ coefficient is not included in the constraint formulation. The non-conservative force term, Q_k , is defined as

$$Q_k = \vec{F}^u \cdot \frac{\partial \vec{r}^u}{\partial q_k} + \vec{M} \cdot \frac{\partial \vec{\omega}^u}{\partial \dot{q}_k}, \quad (9)$$

where $\vec{M}$ represents the external moments on the head and tail. Consequently, the generalized forces Q_x and Q_y are equivalent to the corresponding components of $\vec{F}^u$.

The **nonholonomic Lagrange (NHL)** method [21] applies

$$\frac{d}{dt} \frac{\partial L^u}{\partial \dot{q}_k} - \frac{\partial L^u}{\partial q_k} + c_k = Q_k \quad (10)$$

where, because we choose x to be our independent coordinate, $k = x$ and we define

$$c_k = \left(\frac{d}{dt} \frac{\partial L^u}{\partial \dot{y}} - \frac{\partial L^u}{\partial y} \right) \alpha_{xy} + \left(\frac{d}{dt} \frac{\partial L^u}{\partial \dot{\theta}} - \frac{\partial L^u}{\partial \theta} \right) \alpha_{x\theta}.$$

Similar to LM, $p = 2$ continues to represent the number of nonholonomic constraints. The two α -terms are defined in Equations (4b, 4c). They result from solving the nonholonomic constraint Equations (3a, 3b) for our two dependent velocities, $\dot{y}$ and $\dot{\theta}$, only in terms of our independent velocity, $\dot{x}$. The losses term, Q_k , is defined as

$$Q_k = \vec{F}^c \cdot \frac{\partial \vec{r}^c}{\partial \dot{q}_k} + \vec{M} \cdot \frac{\partial \vec{\omega}^c}{\partial \dot{q}_k}, \quad (11)$$

where constrained derivatives of $\vec{r}_1$ in Equation (2a), and the friction force defined in Equation (7) on the head, define $\vec{r}^c$ and $\vec{F}^c$. Note that, beyond the constraint equations, there is only one equation and thus one Q_k term.

The **principle of virtual power (PVP)** method is defined by [22],

$$\sum_{j=1}^N m_i \ddot{\vec{r}}_i \cdot \frac{\partial \dot{\vec{r}}_i^c}{\partial \dot{q}_k} + \sum_{j=1}^N I_i \ddot{\theta}_i \frac{\partial \omega^c}{\partial \dot{q}_k} = Q_k, \quad (12)$$

where $N = 2$ is the number of bodies, k indexes the independent coordinates (only x in this case), and θ_i is the angular displacement of body i with respect to the inertial frame. That is, $\theta_1 = \theta$, and $\theta_2 = \theta + \phi$. Thus Equation (12) can also be written as

$$\sum_{j=1}^N m_i \ddot{\vec{r}}_i \cdot \frac{\partial \dot{\vec{r}}_i^c}{\partial \dot{q}_k} + I_1 \ddot{\theta} \alpha_{x\theta} + I_2 (\ddot{\theta} + \ddot{\phi}) \alpha_{x\theta} = Q_k,$$

where the $\alpha_{x\theta}$ term is derived using the same method as that used in NHL, and the Q_k is given by Equation (11).

All three methods were processed using the symbolic computing software Maple to formulate the equation of motion for $\ddot{x}$. That equation was simplified to be written entirely in terms of $\dot{y}$ and $\dot{\theta}$, as defined in Equations (3a) and (5). Once all three methods produced identical equations of motion, the results were exported to MATLAB, and the built-in MATLAB numerical solver `ode45` was used to solve for the state x , y , θ , and $\dot{x}$. Using trigonometry, the displacement, velocity, and acceleration at positions along the length of the fish could be calculated. Simulations could be run to model position, velocity, and acceleration of any coordinate on the fish (angular and cartesian). Parameters, as defined in Table 1, could be defined according to prototype values and changed as needed for a given run. In simulations, ground perturbations were ignored (assuming a flat and level ground), and a Coulomb damping parameter was chosen to best match the experimental data.

Experimental Methods

As we developed our mathematical model of the landfish, we constructed a prototype for testing. Our experimental platform for implementing fish-like motion consisted of a custom-designed wheeled robot with an elongated chassis of two rigid bodies connected by a gear-driven joint. The fish's total body measured 0.59 m in length, 0.038 m in width, and 0.060 m in height, with a total weight of 3.813 kg. The total coefficient of drag was estimated to be $0.0042 \frac{\text{kg}}{\text{m}}$ by modeling the prototype as a plate with a rectangular frontal area.

Our biomimetic design was inspired by carangiform swimming observed in species such as mackerel. This locomotion pattern is characterized by undulations primarily in the posterior half of the body, with amplitude increasing toward the caudal fin. As shown in Figure 2, we positioned the gear-driven joint at approximately $\frac{2}{3}$ of the landfish's total length, connecting the rigid anterior body to the flexible posterior section. This configuration allows the posterior section to generate

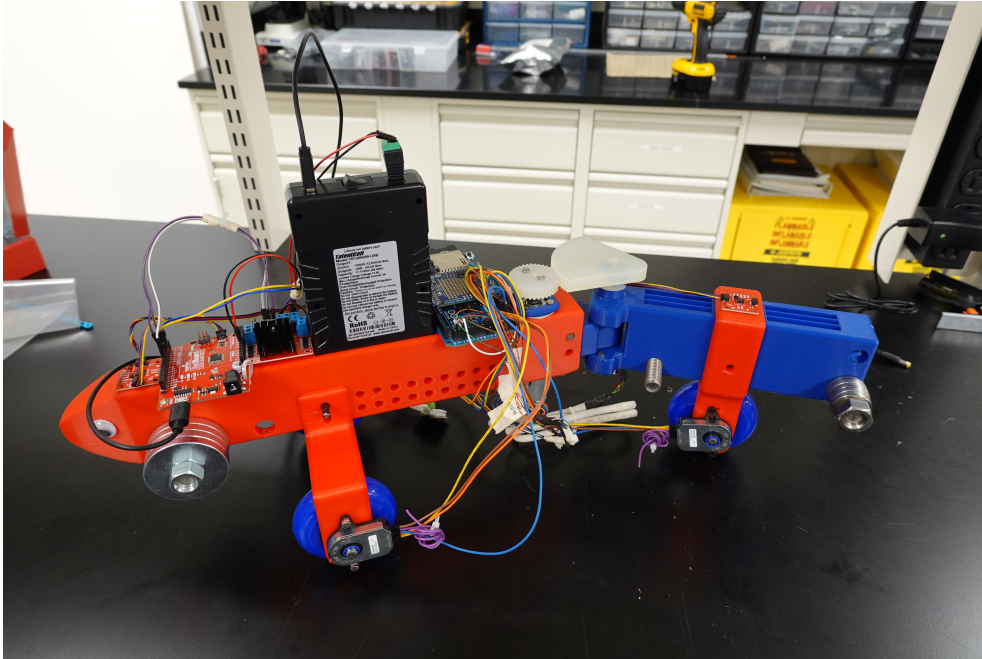


Fig. 2 Detailed view of the landfish prototype showing the two-body design connected by a gear-driven joint

the characteristic undulatory motion of a carangiform fish, with the gear mechanism enabling control of the tail's movement. A three-wheel configuration was chosen for stability, where the two front wheels together enforced the nonholonomic constraint on the head and the rear wheel enforced the nonholonomic constraint on the tail.

The chassis was primarily constructed of 3D-printed polylactic acid (PLA) with circular pin holes for wheel assembly placements. Figure 2 shows a detailed close-up of our prototype, highlighting the key mechanical components and joint configuration. $\frac{1}{2}$ " stainless steel threaded rods, washers of various sizes, and $\frac{1}{2}$ " nuts were added for two purposes: (1) to increase the overall weight of the robot and thereby enforce the nonholonomic constraints on the wheels through friction, preventing the wheels from slipping laterally during tail undulations, and (2) to ensure that the center of mass of each body was located over its respective wheel assembly to prevent tipping and simplify moment of inertia calculations.

For tracking wheel rotations, we integrated incremental encoders with each wheel assembly. To ensure a proper setup, we removed the two bearings from each of the three 70 mm polyurethane roller blade wheels and replaced them with printed PLA hubs. We then installed these bearings inside the 3D-printed wheel assemblies, such that the external surface of each bearing would remain stationary relative to the chassis, while the inner race spun with the axle. This configuration mirrored the encoder mounting, where each encoder body was rigidly attached to the outside of its respective wheel assembly housing. This design ensured that the encoder remained fixed to the non-rotating frame of reference while its sensing element detected the rotation of the spinning axle.

A sinusoidal control signal with adjustable amplitude and frequency parameters drove a 12 V gearmotor to create the undulating movements characteristic of fish locomotion. The gearmotor was housed within the head body's chassis and connected to a 2-gear system with a ratio of 35:95. The input gear was mounted on the motor shaft and the output gear was rigidly connected to the tail section. A long dowel pin ran through the output gear, the tail body, and the joint that connected the head and tail sections, serving as the axis of rotation between the two body segments. An absolute encoder was attached to the bottom of this pin to provide precise angular position feedback of the joint.

The electrical setup was implemented in two interconnected subsystems. The first subsystem consisted of a SparkFun RedBoard microcontroller connected to a motor driver that controlled the gearmotor. This drive system was responsible for generating the fish-like oscillatory motion, with the RedBoard processing the position feedback from the motor's built-in encoder and outputting the appropriate control signals to the motor driver. The second subsystem featured an Arduino Mega as the central controller, equipped with an Adalogger shield for data recording capabilities. This main board interfaced with four separate encoders: three incremental encoders attached to each wheel and one absolute encoder on the joint measuring ϕ . Additionally, two 9-axis inertial measurement units (IMUs) were integrated into this subsystem to capture motion dynamics on each body. The Arduino Mega coordinated the collection of sensor data from all these components, processed the information, and stored it on an SD card in the Adalogger for later analysis. Both the drive subsystem and the data acquisition subsystem were powered by a 12 V battery.

The data we collected primarily served to enable a comparison between our simulation and experimental results. From the comparison, we aimed to inform calibration of model parameters and validate simulation results. We performed standard calibration procedures for our rotational encoders and IMU sensors (linear acceleration and gyroscopic measurements). For our parameter study, we held multiple trials where we varied both amplitude and frequency of the prototype's tail motion and collected all relevant data from our sensors so that we could reconstruct and analyze its motion to compare with our simulations run with similar frequency and amplitude parameters. At the bottom of our range, we tested at a tail frequency of 1 Hz and a tail amplitude of 15 deg. At the top of our range, which was constrained by the limits of our prototype, we tested at 2 Hz and 35 deg. We tested in increments of 0.5 Hz and 5 deg ($\frac{\pi}{36}$ rad), totaling 15 trials in our parameter study.

To prepare our testing space for our experiment, we marked off a region of relatively flat hallway with a length of 20 m. Between trials, we updated and uploaded the motor control code to test a new set of parameters, and saved the data file from the previous trial. We then returned the prototype to the starting position (marked with tape) and aligned the front wheels so that they pointed down the hallway. Before the start of every run, the motor controller calibration routine rotated the tail to its limits in both directions in order to calculate and return to the center. This process occurred after the prototype was placed at the starting line. Thus the motion typically pushed the prototype off of the starting line and resulted in varied initial trajectory for each run. We determined that a trial was over if any of the following criteria were met: the prototype successfully reached the opposite end of the testing space, the prototype stopped moving forward, the prototype began veering severely, or if it was on a collision course with something else in the testing space.

Data Processing

After collecting encoder, accelerometer, and gyroscopic data from our experiments, we developed a standard data processing workflow that consisted of two main steps: cleaning and analysis. The cleaning step involved extracting data from the collected files followed by applying simple calibration tests to align our axes. We then spatially aligned accelerometer

and gyroscopic data, positively aligned encoder data, and zeroed joint encoder data at the center of rotation. During this processing, we observed that our accelerometers were recording non-zero acceleration values in all three axes even when the system was completely at rest, making this data unreliable. Because of this, we decided to extract our position history solely from the encoder data. With the encoder data, we employed odometry to compute the position of each body at each time step. The cleaning process consisted of windowing the raw data to look at time periods we believed to be close to steady-state. To eliminate any influence on the data caused by veering and different starting trajectories, we then detrended it by calculating a linear fit and removing the slope of that line to center the data around $y = 0$ to spatially align it more closely with our model. To reduce noise acquired through numerical processes, we cleaned our simulation results in a similar manner.

Results & Discussion

In this section, we analyze and compare the results of our mathematical model predictions with experimental data from the wheeled landfish prototype, examining both motion patterns and performance across varying tail oscillation parameters.

Numerical solutions of the model with parameter values that matched the prototype's parameters demonstrated carangiform-like oscillatory locomotion which reached a steady-state mean velocity. Figure 3 shows an example of such motion from rest. The red and blue curves indicate the trajectory of the head and tail, respectively, where the initial position and orientation of the head was $(x, y, \theta) = (0, 0, 0)$. The sinusoidal path demonstrates how oscillatory tail motion generates forward propulsion while producing the characteristic undulatory motion observed in carangiform swimming.

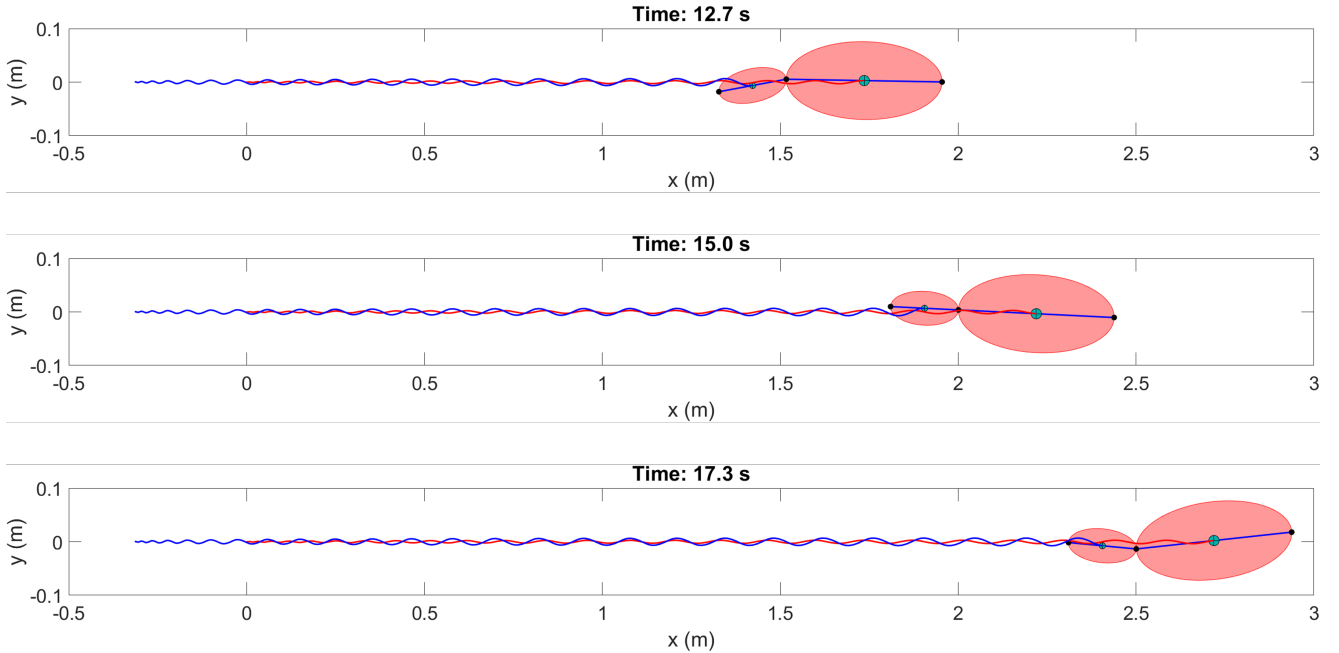


Fig. 3 The figure illustrates the trajectory of the landfish over time, with the head (blue) and tail (red) segments represented at sequential time steps

Our calibration trials successfully aligned coordinate systems and validated sensor functionality. Through linear and gyroscopic tests, we determined the orientation of the IMU axes and performed coordinate transformations. Encoder calibrations also confirmed the manufacturer's specification of 20,480 counts per revolution. However, from our stationary test, we observed unreliable readings from the IMUs. Over a 10-second period where both the IMUs and prototype remained still, the acceleration data indicated that the prototype experienced significant acceleration in all directions, which was deemed unreasonable. Therefore, we relied completely on the encoder data for information about the displacement, velocity, and acceleration. Initial experimental trials produced trajectory data that exhibited both the desired fish-like locomotion patterns and some expected deviations due to environmental factors. Figure 4 presents the unprocessed overhead trajectory of the wheeled landfish prototype over one trial. These paths demonstrate the characteristic undulating motion pattern but also show the veering tendency that necessitated the detrending step of our data cleaning process for comparative analysis.

The varying path curvatures observed in these trials reflect the sensitivity of the real system to slight asymmetries in the mechanical setup and ground irregularities.

After implementing the cleaning process of windowing and detrending, we were able to more directly compare the results of our experiment with our model. Figure 5 shows sample detrended trajectories over an 8 m path. The red and blue curves

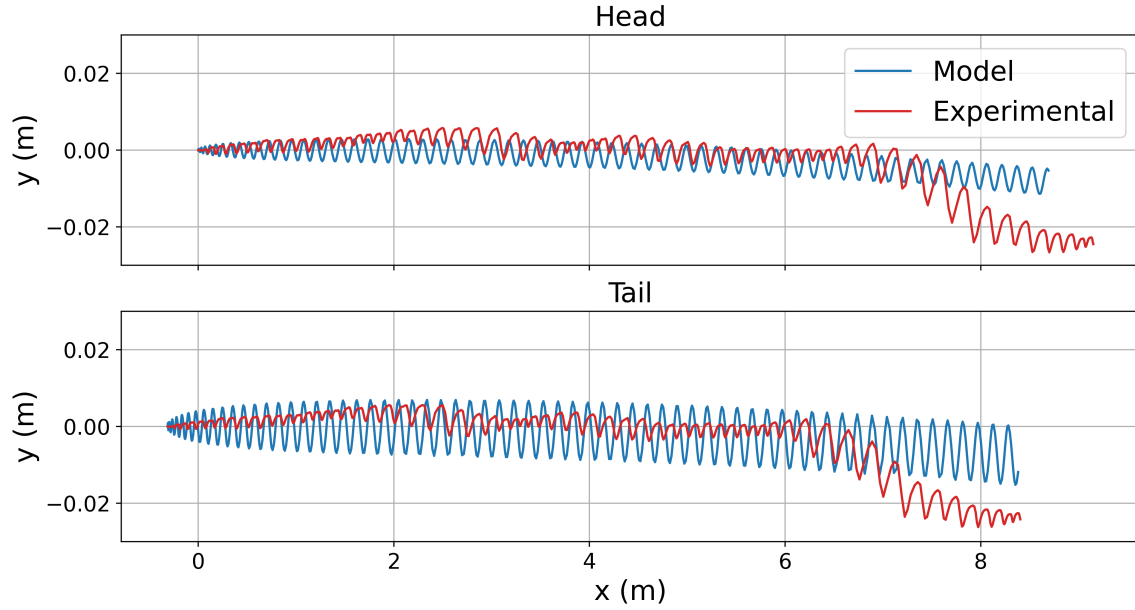


Fig. 4 The two plots compare the simulated/commanded (blue) and experimental/realized (red) overhead trajectories for each body over a 45-second interval with identical commanded tail oscillation parameters (25 deg amplitude, 1.5 Hz frequency)

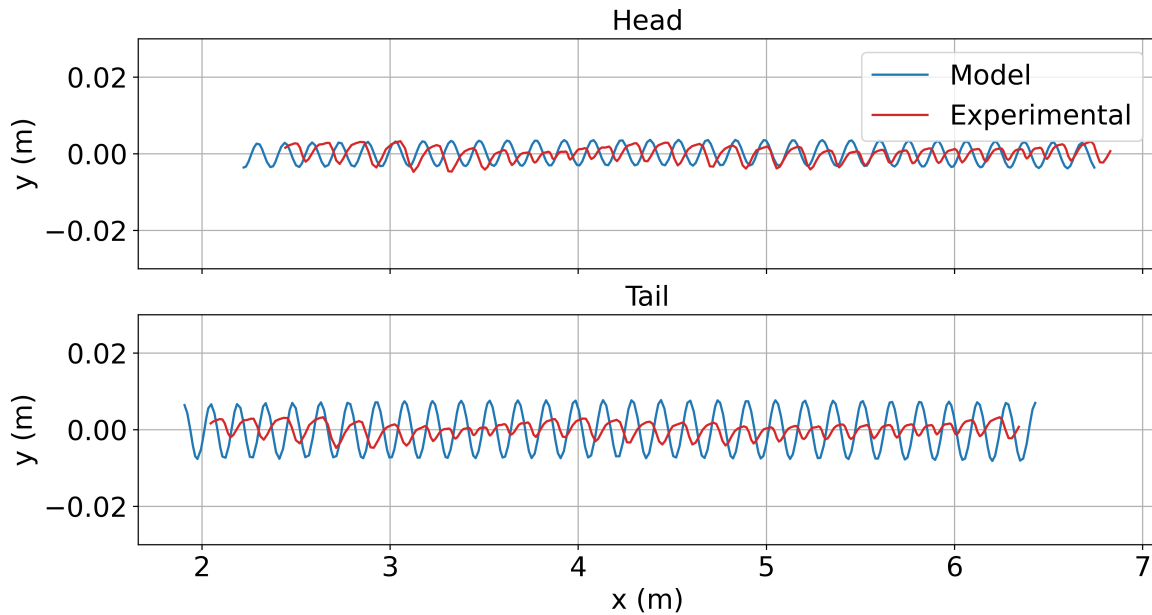


Fig. 5 These plots extend the analysis presented in Figure 4, with data windowed to the 15-35 second interval to exclude transient responses and trajectory deviations likely resulting from surface irregularities

show the experimental trajectory and simulated trajectory, respectively, where simulation parameters were chosen to match the experiment. Our results are encouraging; qualitatively, we see that the experimental data and simulation results produce similar overhead motion plots.

In Figure 6, we compare data from the absolute encoder placed on the joint and the imposed angle ϕ from our most successful run. We see that the frequency was imposed very well, however the amplitude appears to differ from that of the simulation. The experimental data shows a slight shift upward, which likely results from the fact that the friction in the gearmotor was observed to be asymmetrical, causing suspected controller saturation at high ϕ amplitude and frequency. Because of this, instead of producing a smooth sinusoidal curve, the motor oscillated with flattened troughs. At the peaks, the motor appeared to switch direction as desired. However, at the troughs, when the motor switched to rotate in the opposite direction, it appears to struggle (see Figure 6). We hypothesize that this could be one of the causes of the veering we observed in the prototype's overhead motion. These results show that the angle of rotation of the tail, ϕ , is not imposed as a perfect sine wave.

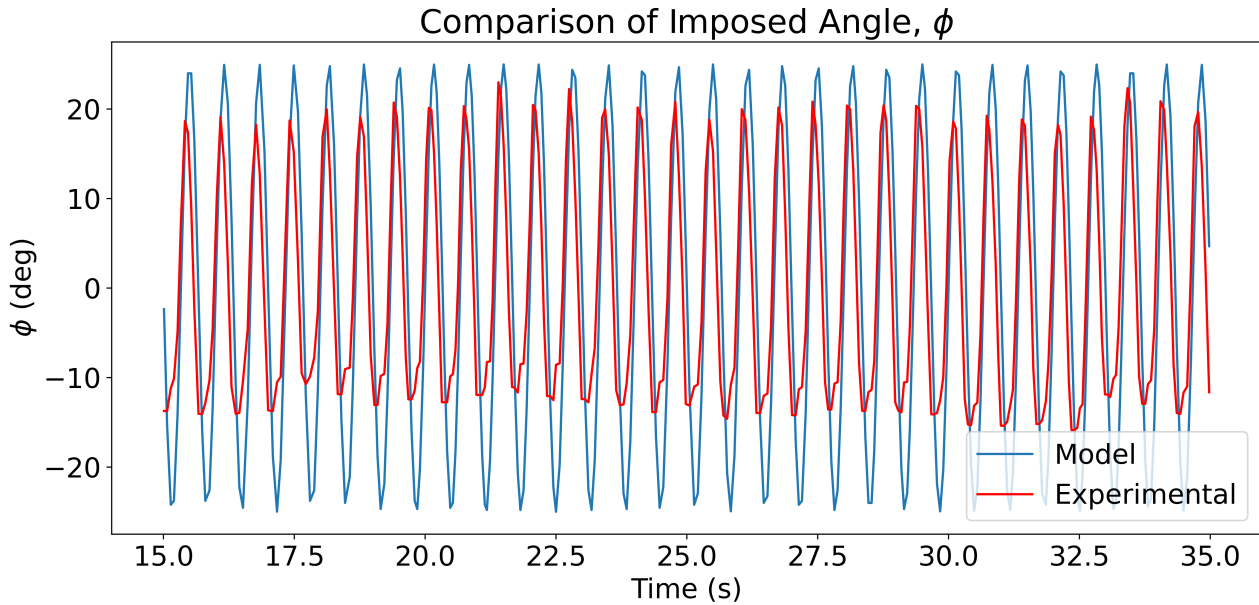


Fig. 6 The plot presents an alignment of simulated/commanded (blue) and experimental/realized (red) imposed tail angles, ϕ , throughout a trial period of 20 seconds with identical commanded tail oscillation parameters (25 deg amplitude, 1.5 Hz frequency)

To understand how changes in tail oscillation parameters affect locomotion, we conducted a parameter study on the mathematical model, which was bounded by the capabilities of our prototype according to the physical limits of the motor. Figure 7(a) shows how the simulation's horizontal steady-state velocity varies with the frequency and amplitude of the imposed tail angle, ϕ . The results from this study confirm our expectations: steady-state velocity increases with both the magnitude and frequency of tail oscillations, with the highest velocities occurring at maximum frequency and amplitude values within our tested range.

The experimental parameter study, shown in Figure 7(b), reveals a similar trend to our model predictions but with notable differences. Due to some of the difficulties with the experimental setup, this trend is only preserved for some ranges, notably where the angle between the head and the tail ϕ is imposed. This suggests the imposition of ϕ is essential to accurate prediction of motion and when ϕ is not accurately imposed there is a dramatic decrease in performance. In general, we found that our prototype behaved much more erratically than the model for several reasons, including ground perturbations in the test environment, asymmetric gearmotor performance in the prototype, and imperfect estimations of friction and drag. These are all sources of experimental uncertainty and potential causes of disagreement with the idealized simulation results.

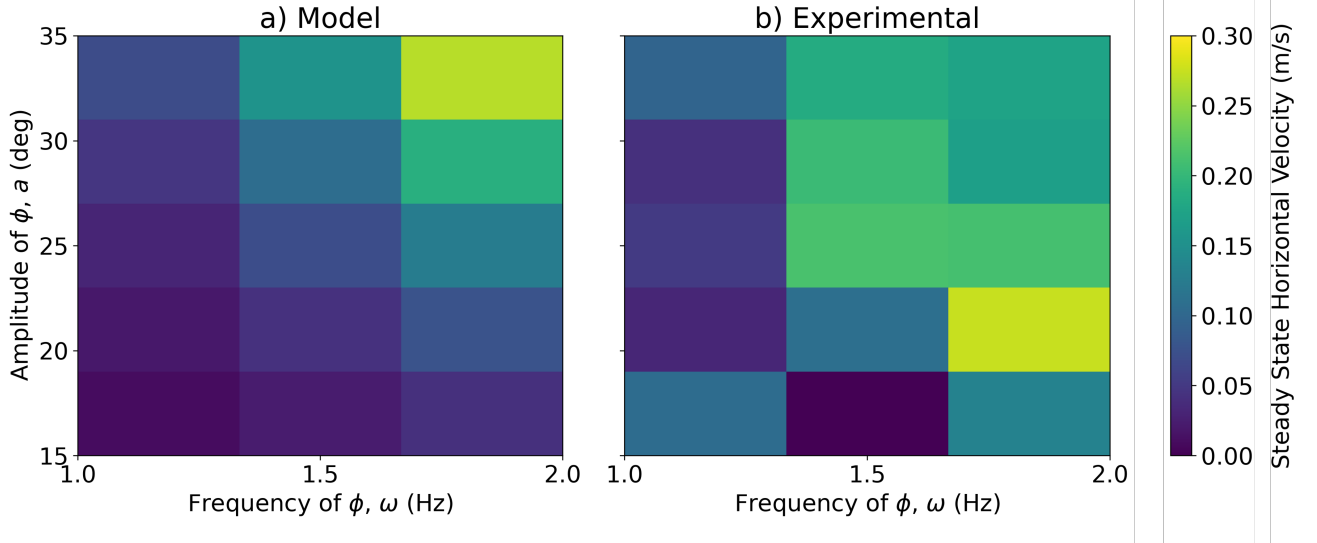


Fig. 7 Plots **a)** and **b)** detail the results of our simulated and experimental parameter studied, respectively. Both plots examine steady-state horizontal velocity as a function of the commanded frequency (x -axis) and amplitude (y -axis) of the imposed tail angle, ϕ . Each box corresponds to a trial at a particular ϕ frequency and amplitude and the color corresponds to the steady-state velocity found in each trial. The parameter space is bounded by our chosen experimental range of values

Conclusion

This study produced a reduced-order model of fish-inspired locomotion on land using nonholonomic constraints. By developing a landfish robot with a two-body system connected by a gear-driven joint, we demonstrated how oscillatory tail motion can generate forward propulsion on a wheeled platform through principles similar to carangiform swimming, without the added complexities of fluid dynamics. This work establishes a simplified framework that bridges biological principles with engineering implementation to rapidly design and optimize biomimetic robotic systems without exhaustive computational resources.

Our mathematical model, which employed three distinct methods (Lagrange multipliers, nonholonomic Lagrange, and the principle of virtual power) to derive the equations of motion, generated identical results that matched our expectations of the system's behavior. The model also predicted a relationship between tail oscillation parameters (frequency and amplitude) and the resulting locomotion patterns, showing that higher frequencies and amplitudes generally produced greater forward velocities within the operational limits of our prototype.

While the prototype successfully demonstrated fish-like propulsion in some runs, several challenges were encountered, which produced significant uncertainty in the experimental results. Ground irregularities prevented achieving true steady-state motion observed in simulations and caused the robot to veer during extended trials, highlighting both the sensitivity of nonholonomic systems to initial conditions and the limitations of our model's perfectly flat surface assumption, which is difficult to recreate in real-world testing environments. The robot's discrete segments further limited our ability to replicate the continuous and smooth undulation of fish locomotion.

The prototype could improve from future developments, such as a flexible body as well as smoother power transmission between bodies. For practical applications beyond laboratory settings, future designs would likely require robust and adaptive control systems for autonomous operation. Additionally, where our design positions centers of mass above wheel assemblies to simplify our model, investigation into varied mass distributions along the body would yield valuable insights into stability optimization, energy efficiency, and expanded maneuverability capabilities that could significantly enhance real-world performance in varied experimental conditions. Future work may address these limitations through the integration of additional articulated segments, improved surface interaction dynamics, enhanced control strategies for straight-line stability, and examination of varying surface conditions on operational performance. Along with these prototype modifications, the mathematical model would undergo equivalent changes. These would include new system-defining equations as bodies are added, an in depth parameter study to determine more accurate loss coefficients, a nonzero rotational stiffness coefficient of the head to ensure head oscillation about $y = 0$ and minimize veering, and introducing a third dimension to account for ground perturbations. With these changes, we hope the model and prototype will produce results that match even

more accurately. Because of time constraints, we were not able to perform very in-depth or rigorous comparisons. Therefore, employing other analysis approaches that produce quantitative insights as well as qualitative would be beneficial. For example, beyond comparing overhead motion and ϕ profiles, proper and complex orthogonal decomposition could provide further insight into the motion of the landfish [23, 24].

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Chapter 5

Tuneable Dynamic Behaviour in Beams with Phase-Changing Materials

T. de Ruiter, E. Habtour, and D. Di Maio

Abstract This research investigates the dynamic properties of phase change materials (PCMs), specifically their damping and eigenfrequency, which are directly related to stiffness. The goal of this study is to demonstrate that the composition of PCMs can be tailored to achieve desired material properties. In this work, the PCM composition was modified by combining hard paraffin wax with silicone rubber. The temperature was adjusted to exploit the high thermal expansion coefficient of the wax, thereby altering the material properties. The silicone rubber served as an encapsulating material to contain the wax when it melts. To test this hypothesis, two carbon fibre beams incorporating wax and silicone were manufactured and experimental modal analysis (EMA) was performed to determine their modal properties. The results indicate that adding wax to the system increases the eigenfrequency and locally enhances damping by up to 250%. Furthermore, the study identifies two competing mechanisms that determine the material's final damping and stiffness.

Keywords Adaptive materials · Phase changing materials · Paraffin wax · Silicone rubber · FRF · Loss factor

Introduction

In the aerospace and space industry, there is a continuous drive to develop innovative materials that are lighter, stronger and more cost-efficient. In the current design process, the evaluation of a structure's dynamic properties is typically conducted at the final stage. As a result, implementing design changes at that point often becomes impractical and costly.

Phase-change materials (PCMs) offer a potential solution. These materials can transition between phases, for example, from solid to liquid, when exposed to external stimuli such as temperature or pressure. This phase transition alters the material properties, which provides a unique advantage: instead of being restricted to one material with fixed characteristics, the properties can be adapted after manufacturing. This flexibility brings two major benefits. First, it increases design freedom, since properties can be tuned even after fabrication. Second, it facilitates optimisation by enabling the structure to be adjusted to specific load cases. Third, PCMs reduce the need for worst-case design by allowing properties to change dynamically during operation, thereby avoiding over-engineering and resulting in lighter, more cost-effective structures.

Phase Changing Materials (PCMs) can be divided into three categories: organic, inorganic and eutectic compounds, as shown in Figure 1 [1]. The scope of this study is limited to organic paraffin compounds. Paraffin compounds are the result of refined de-waxed petroleum and are divided into soft and hard wax. The difference is the length of the hydrocarbon chains (C_nH_{2n+2}), where n typically ranges from 20 (for soft wax) to 60 (for hard wax) carbon atoms. This results in a lower melting point for soft paraffins than for hard paraffins, at 45 °C and 68 °C, respectively [2]. Hard paraffin wax is a tasteless, odourless, white translucent material. They are safe to use, inexpensive, non-corrosive, chemically inert and have a great Coefficient of Thermal Expansion (CTE). The CTE is the greatest when the wax changes phase and can reach up to 1500 $\frac{ppm}{K}$ [3].

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However, using hard paraffin wax as PCM, poses some drawbacks. Firstly, hard paraffin waxes exhibit a relatively low thermal conductivity of approximately $0.2 \frac{W}{mK}$ [4]. Consequently, when exploiting the thermal expansion characteristics, it is important that the material efficiently absorbs heat to raise its temperature. Secondly, waxes are used in flammable candles, meaning they should be handled with care [5]. Thirdly, the Young's modulus is in the order of 50 to 200 MPa [6], but drops when the material reaches its melting temperature, and it is also very brittle. Lastly, when the wax transitions from the solid to the liquid phase it becomes fluid; as a result, it will leak and lose its material properties. To prevent this, a second material is added, which acts as an encapsulation of the wax. This material is silicone rubber and has a higher melting point than wax.

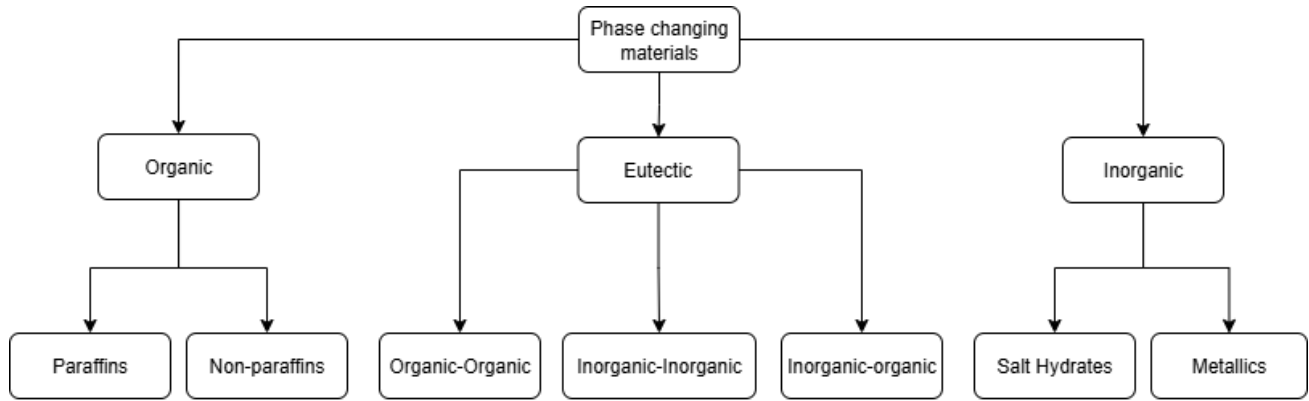


Fig. 1 Classification of Phase Changing materials

Silicone rubber is an elastomer consisting of multiple elements. It is a polymer composed of silicon and oxygen atoms in the backbone and side branches consisting of methyl groups; it is also referred to as polydimethylsiloxane. Silicone rubbers can be one- or two-part polymers. In the case of a two-part polymer, there is an A component, which is the base polymer and provides the silicone with its bulk properties, and there is a B component, which acts as the catalyst. When the A and B components are mixed, crosslinking is activated, and the material starts to cure. A special type is room-temperature-vulcanised (RTV) silicone, which cures at room temperature. This type comes in different curing times, ranging from 10 minutes to 24 hours. It also comes in different hardnesses, ranging from 10A (soft, comparable to a rubber band) to 100A (hard, comparable to shopping cart wheels). Silicone rubber can be used across a wide temperature range, from -50°C to 230 [7]. The CTE of silicone rubber ranges from $200\text{--}300 \frac{\mu\text{m}}{\text{K}}$, which slightly contributes to the total thermal expansion [8]. Additionally, the thermal conductivity of silicone rubber is similar to that of paraffin wax [9], which means that a sufficient dwell time is critical to achieve complete melting of the wax at elevated temperatures. Lastly, the Young's modulus of silicone is a factor of 10 lower than paraffin wax, ranging from 1 to 10 MPa [10]. However, the modulus remains relatively constant across the temperature service range, and it is also very ductile. All these properties make it an excellent encapsulating material.

Method

An implementation of PCMs was presented in the Master Thesis of Flint [11]. In his study, the goal was to demonstrate how the eigenfrequency and damping ratio of beams could be tuned as a function of temperature by embedding materials with differing thermal and mechanical properties. The present work builds upon and extends his findings.

To this end, two hollow carbon fibre beams with a symmetric layout of $[-45/45/0/45/-45]$ were manufactured. Both beams had identical geometry but were filled with different core materials. The beam dimensions and an example of the geometry are shown in Figures 2 and 3, respectively. The first beam served as a reference and was filled with silicone rubber, while the second beam was filled with a multiphase composition consisting of 85% silicone and 15% paraffin wax by volume. Both beams were equipped with Kapton foil heating elements to control the internal temperature, as well as thermocouples to monitor it. To prevent leakage of molten wax and maintain a closed system, the beams were sealed with plastic end caps. Additionally, the weight and theoretical eigenfrequencies were calculated, as summarised in Table 1.

The dynamic properties were characterised using Experimental Modal Analysis (EMA). In this procedure, the beam is excited using an electrodynamic shaker, while an accelerometer measures the response. The input is the force from the shaker, and the output is the acceleration of the beam. For spatial resolution, the beam was divided into 13 equal segments

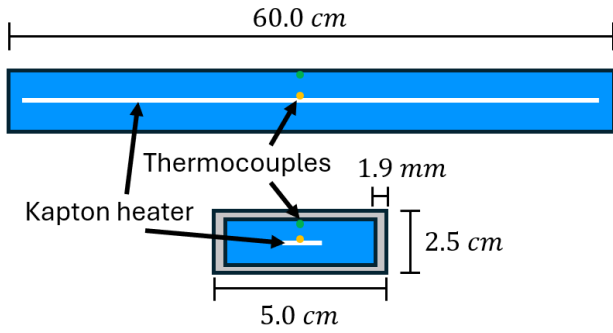


Fig. 2 Schematic representation of the beam dimensions and content

Table 1 Composition and weight of the beams and analytically calculated first eigenfrequency

Sample	Si/wax [%]	Weight [g]	ω_1 [Hz]
Beam 1	100/0	1024	370
Beam 2	85/15	978	379

with 14 nodes. A preliminary test was conducted to identify the most suitable excitation and response points. Node 12 was chosen as the excitation point, and node five was selected as the response point, as shown in Figure 4. This figure also shows that the beam is freely suspended at two points by a small string. It also displays the force sensor, which measures the input force from the shaker, and the thermocouple cables.

The excitation was applied as a frequency sweep, ranging from 50 to 500 Hz, with the force level kept constant. Since the first bending eigenfrequency of the beam was expected between 370 and 380 Hz. During post-processing in Matlab, the frequency response function (FRF) was computed by dividing the Fast Fourier transform of the output by that of the input. From the FRF, the first eigenfrequency and its corresponding damping ratio were extracted using the peak-picking method.



Fig. 3 Close up of the outside of the carbon fibre beam

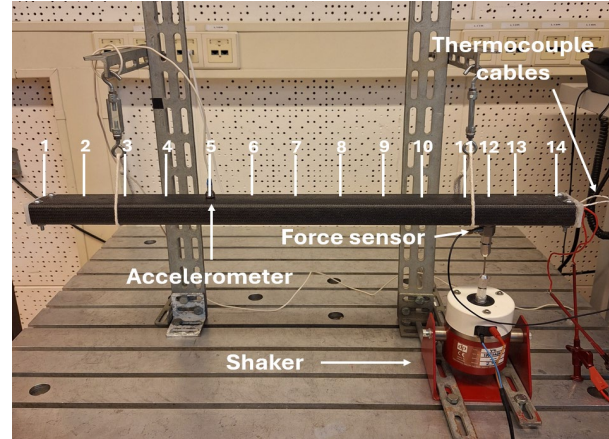


Fig. 4 Experimental test setup

The measurements were repeated at multiple temperatures to study the thermo-mechanical effect of the PCM. Starting from room temperature (20 °C), the temperature was increased stepwise up to 50 °C, which is slightly above the melting point of the paraffin wax used. The ramp rate is 0.2 °C/min. At each step, the heating elements were allowed to dwell for 5 min before the modal test was repeated.

Results

Figure 5 (eigenfrequency) and Figure 6 (loss factor) plot the modal response of the two beam compositions as a function of mean beam temperature (x-axis). The eigenfrequency and loss factor are shown normalised to their first measurement. For Beam 1 (100% silicone), the normalised first-mode frequency rises rapidly to a maximum near 32 °C ($\approx 108\%$, approximately +12 Hz in absolute terms), after which it declines approximately linearly with a slope of about -0.36 Hz/°C to roughly +3.9% at 50 °C. Beam 2 (85% silicone / 15% wax) exhibits the same trend, but with a lower peak near 29 °C

($\approx +105\%$, approximately 9 Hz) and a steeper post-peak decline of about $-0.49 \text{ Hz}/^\circ\text{C}$, which brings the frequency below its initial value at higher temperatures. These numerical trends are reproduced in the duplicate runs. Before the peak, minor deviations are observed between the data points; however, beyond the peak, the measurements converge and follow a single consistent trend.

The behaviour of the frequency response can be understood as the result of two competing thermo-mechanical effects. First, increasing temperature increases polymer chain mobility and reduces the stiffness of the matrix material, which is directly related to a reduction in the eigenfrequency. Second, near the wax melting range, the paraffin undergoes a large constrained volumetric expansion (up to 15%), which increases internal pressure and acts like a stiffening preload on the beam walls, thereby raising stiffness. This effect explains the sharp rise in frequency around the melting interval. Once the wax is fully molten, the expansion-driven stiffening vanishes and the temperature-dependent softening of the polymer dominates, causing the frequency to fall again.

The results also indicate that beam two consistently exhibits a higher eigenfrequency than beam one. Since a higher eigenfrequency is directly associated with increased stiffness, it can be argued that the addition of wax enhanced the stiffness of the system not only at the peak but across the entire temperature range. There are two likely explanations for this increase: first, the Young's modulus of wax is approximately an order of magnitude greater than that of silicone rubber, and second, the density decreased. Because eigenfrequency is inversely related to density, and wax possesses a lower density than silicone rubber, the eigenfrequency consequently increased.

The loss factor curves follow a distinct pattern. Beam 1 begins near 0.118 and increases with $8.6\text{E-}3 \text{ } 1/^\circ\text{C}$, peaks around 29°C ($\approx 165\%$), then falls rapidly to 48 % at 39°C before it stays constant. Beam 2 starts lower at 0.063, but reaches a larger relative peak at 25°C (250 %) and then falls below the initial value, after which it remains constant. This behaviour indicates that the added wax shifts the peak loss factor by a few degrees. When paraffin wax is embedded into a silicone matrix, its phase transition introduces an additional dissipation mechanism that is absent in the single-phase system. Paraffin undergoes a solid-to-liquid transition over a narrow temperature range, during which it exists in both solid and liquid states. Under dynamic loading, these domains continuously rearrange, and the solid-liquid interface then creates internal friction. This internal friction results in higher hysteresis losses during each vibration cycle, thereby increasing the effective loss factor.

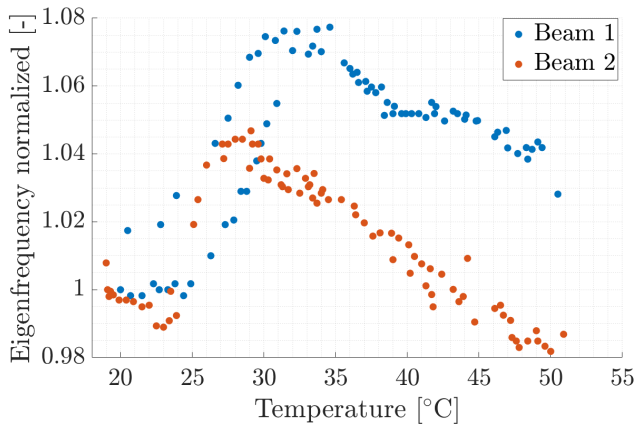


Fig. 5 The eigenfrequency results of the EMA for different temperatures

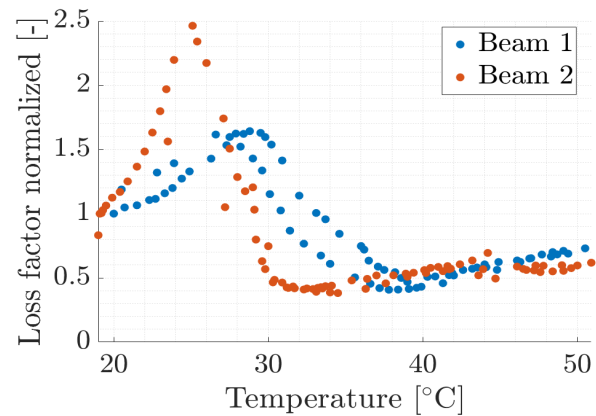


Fig. 6 The loss factor results of the EMA for different temperatures

Conclusions

This study demonstrated that combining silicone rubber with hard paraffin wax enables temperature-dependent tuning of dynamic properties in composite beams. Two competing mechanisms were identified: stiffness reduction due to increased polymer chain mobility and stiffness enhancement resulting from wax expansion during the phase transition. The results showed that eigenfrequency can be shifted and local damping improved by up to a factor of 2.5. These findings provide a proof of concept for integrating phase-change materials into structural components to achieve adaptive behaviour. While this work provides a proof of concept, future studies should address higher wax compositions and more in-depth research about the behaviour of PCMs.

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Chapter 6

Wave-Smart Absorption: A Parametric Study of Dashpot Location in Beam Structures

Samikhshak Gupta, Abigail Smith, and Vijaya V.N. Sriram Malladi

Abstract Flexural waveguides carry vibration as traveling waves that are partly reflected and absorbed at local impedance changes. Characterizing these interactions through frequency-dependent power reflection and absorption coefficients, $\rho(\omega)$ and $\alpha(\omega)$, is critical for structurally efficient vibration control. We present a wave-energy framework for optimally placing a compact passive absorber—a spring-damper (dashpot)—to maximize $\alpha(\omega)$. Broadband excitation is introduced with a piezoelectric actuator, and spatiotemporal responses are captured using scanning laser Doppler vibrometry (SLDV). Sensor spacing is selected to suppress evanescent content, ensuring the analysis isolates propagating waves that dominate in large, lightweight structures. An undamped host-beam baseline isolates the absorber's effect, while wave-amplitude decomposition provides $\rho(\omega)$ and $\alpha(\omega)$. Parametric sweeps map α versus absorber location and frequency. A repeatable pattern emerges: peak absorption occurs at multiple placements repeating at odd quarter-wavelength intervals—velocity antinodes—along the waveguide. With increasing frequency (shorter wavelength), the number of absorption peaks grows, reflecting the harmonic nature of the response. These results yield a practical design rule: placing the absorber at $(2n + 1)\lambda/4$ achieves strong absorption without the mass cost of blanket damping. The resulting location–frequency map operationalizes this rule and highlights sensitivities near node–antinode transitions. Together with sensor-placement guidelines that suppress evanescent effects, the framework provides concise, wave-informed rules for passive vibration reduction and sets the stage for extensions to transmission modeling, multi-absorber layouts, and broadband absorption.

Keywords Wave-decomposition · Passive-absorber · Power-absorption · Sensor-layout · Dashpot-location

Introduction

Many structural elements, such as beams, plates, pipes, and stiffeners, act as waveguides that carry vibration as traveling waves [1]. Irregularities [2]—joints [3, 4], fasteners, material transitions [5], cracks [6], and added hardware—scatter energy and alter global response, yet their effective properties remain difficult to characterize. Uncertain boundary data limit the applicability of analytical and numerical models, while finite-element simulations often require highly accurate inputs. Conventional damping layers can reduce vibration but impose prohibitive weight costs in transportation [7] and aerospace applications [7]. These challenges motivate experimental, wave-based strategies that act locally at interfaces. Within this framework, the absorption coefficient, derived from reflection and transmission, provides an energy-based and experimentally accessible measure to characterize discontinuities and guide lightweight vibration control [8, 9].

Dynamic vibration absorbers (DVAs) provide a practical means of tailoring impedance. Passive devices are reliable and straightforward, but they are inherently narrowband. Semi-active and active devices [10–21, 21–26] introduce adaptability through actuators and control but at the cost of added complexity, expense, and potential stability issues [27]. Hybrid approaches often pair a passive baseline with active or semi-active augmentation. Although DVAs are widely used in machinery, vehicles, and buildings, the role of absorber placement in governing wave absorption remains underexplored.

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This study adopts a purely passive approach [28–33], employing a compact spring–damper (Airpot 2KS160) mounted on a beam, with spanwise location as the design variable. Relocating the dashpot modifies local mobility and interface impedance, thereby influencing reflection and absorption. Wave decomposition is used to extract reflection and absorption coefficients, and parametric studies establish the relationship between dashpot placement and absorption. The results demonstrate how location-dependent impedance can be leveraged to maximize absorption at target frequencies, providing a lightweight, wave-informed strategy for vibration reduction.

Numerical Model & Wave Decomposition

A beam made of Al 6061-T6 with a rectangular cross-section of 19.05 mm × 1.5875 mm and a length of 1.113 m is considered in this study. Using the Timoshenko beam formulation, the governing equations are described as

$$\begin{aligned} GA\kappa \left(\frac{\partial \psi(x,t)}{\partial x} - \frac{\partial^2 y(x,t)}{\partial x^2} \right) + \rho A \frac{\partial^2 y(x,t)}{\partial t^2} &= q(x,t), \\ GA\kappa \left(\frac{\partial y(x,t)}{\partial x} - \psi(x,t) \right) + EI \frac{\partial^2 \psi(x,t)}{\partial x^2} &= \rho I \frac{\partial^2 y(x,t)}{\partial t^2}, \end{aligned} \quad (1)$$

where G is the shear modulus, A is the area of cross-section, $y(x,t)$ and $\psi(x,t)$ are the transverse displacement and the bending rotation angle at location x and time t , q is the load, E is the Young's Modulus, ρ is the density, and I is the moment of inertia. This section provides a concise overview of the modeling approach used to analyze a beam incorporating a spring–damper (SD) system that functions as a passive absorber (PA). The schematic representation of the clamped–clamped beam, incorporating a discrete spring–damper with stiffness denoted by the stiffness is denoted by k [N m^{−1}] and the damping coefficient by c [N − s/m]. The beam is driven by a harmonic point load $F(t) = F_0 e^{i\omega t}$ applied near the right clamp ($x \approx 0.88 \times L$). Clamped boundaries at $x = 0$ and $x = L$ impose $y(0,t) = y(L,t) = 0$ and $\psi(0,t) = \psi(L,t) = 0$. A MATLAB-based FE-model comprising 400 quadratic shape functions has been thoroughly tested and validated in previous research endeavors [34–37] using the equation of motion as,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{B}_f F(t) \quad (2)$$

where the mass, damping, and stiffness matrices are denoted as $\mathbf{M}, \mathbf{C}, \mathbf{K} \in \mathbb{R}^{1598 \times 1598}$. A proportional (Rayleigh) damping model is adopted, allowing $\mathbf{C}$ to be expressed as a linear combination of $\mathbf{M}$ and $\mathbf{K}$ [35, 38–41], as

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \quad (3)$$

where $\alpha = 0.7099\text{s}^{-1}$ and $\beta = 8.8166e^{-8}\text{s}$ are finite element constants determined experimentally. In particular, a thorough analysis shows that the natural frequencies converged within the applicable frequency range. The spring–damper assembly is positioned at $L_0 = 0.0988L$, and the corresponding modeshapes are shown in Figure 1.

MATLAB is utilized to solve the equation of motion Equation (4) employing the inverse matrix technique;

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{C} + cB_{sd})\dot{\mathbf{x}} + (\mathbf{K} + kB_{sd})\mathbf{x} = \mathbf{B}_f F(t) \quad (4)$$

where $\mathbf{x}$ is the displacement vector; $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the assembled mass, damping, and stiffness matrices; k and c parameterize the SD stiffness and damping; $F(t)$ denotes an excitation applied near the far end; B_f distributes the external force; and B_{sd} distributes the SD interaction [42]. The parameters k and c are optimized, subject to specified bounds, to maximize the absorption coefficient at a selected frequency. Consider the same beam model with a spring–damper system—akin to an acoustic black hole—attached at $L_0 = 0.0988L$. For a given frequency, the Timoshenko-beam wavenumber follows from the dispersion relation as

$$\frac{EI}{\rho A} k_f^4 - \frac{I}{A} \left(1 + \frac{E}{G\kappa} \right) k_f^2 \omega^2 - \omega^2 + \frac{\rho I}{GA\kappa} \omega^4 = 0. \quad (5)$$

Here, κ is the Timoshenko correction factor; ω the angular frequency; ρ the density; E the Young's modulus; G the shear modulus; A the cross-sectional area; and I the second moment of area. Because the solution for k_f depends quadratically on ω , the beam exhibits dispersion. The resulting response by superposition [43, 44] is written as,

$$\mathbf{W}(x,t) = \left(\underbrace{\tilde{A} e^{-ik_f x}}_{\text{forward propagating}} + \underbrace{\tilde{B} e^{+ik_f x}}_{\text{backward propagating}} + \underbrace{\tilde{C} e^{-k_f x}}_{\text{forward evanescent}} + \underbrace{\tilde{D} e^{+k_f x}}_{\text{backward evanescent}} + \underbrace{\ddots}_{\text{higher-order modes}} \right) e^{i\omega t}. \quad (6)$$

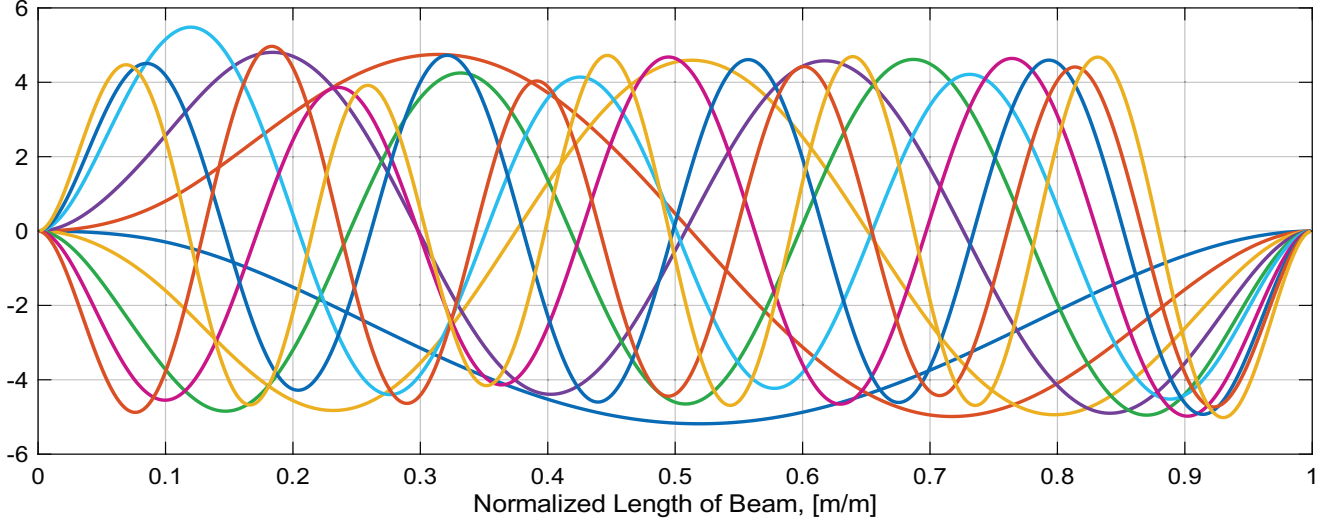


Fig. 1 Modeshapes of the host beam with a dashpot attached at the end

In the above expression, k_f is the spatial frequency obtained from Equation (5) at the chosen ω . Although a solution exists for each ω_n , the explicit summation over frequencies is suppressed for clarity [44]; see [45] for a spectral treatment of wave propagation. Neglecting noise and nonlinear higher-order terms, the sensor measurement $\mathbf{W}(x_i, t)$ at position x_i is a local instance of the field:

$$\mathbf{W}(x_i, t) = \left(\tilde{A} e^{-\iota k_f x_i} + \tilde{B} e^{+\iota k_f x_i} + \tilde{C} e^{k_f x_i} + \tilde{D} e^{k_f x_i} \right) e^{i\omega t}.$$

Applying a Fast Fourier Transform (FFT) at x_i yields, for each frequency,

$$\mathbf{W}(x_i) = \tilde{A}(\omega) e^{-\iota k_f x_i} + \tilde{B}(\omega) e^{+\iota k_f x_i} + \tilde{C}(\omega) e^{k_f x_i} + \tilde{D}(\omega) e^{k_f x_i}. \quad (7)$$

In Equation (7), the wavenumber k_f is obtained from the Timoshenko model using known material and geometric properties, and the sensor position x_i is assumed measurable (constraints discussed later). Each measurement thus involves four unknown complex amplitudes, $\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D} \in \mathbb{C}$, which describe the modal content of the field. To streamline wave amplitude estimation, we neglect evanescent terms by adopting a far-field placement of sensors—evanescent contributions are strongest near boundaries/discontinuities and the sources of excitation—and by selecting sensor locations accordingly. The ensuing constraints specify dashpot placement and sensor spacing to suppress evanescent effects, enforce wavelength-based separation between adjacent sensors, and respect practical experimental limitations for future tests.

Table 1 Constraints for Sensor Placement

	Constraint Condition	Result
<i>Far-Field Assumption</i>	$e^{-k_f l} = 0.1, \forall \omega \in [0.775, 20] \text{ kHz}$	$l = 0.05 \text{ m}$
<i>Wavelength-Based Spacing</i>	$\Delta_{\max} < \frac{\lambda_{\min}}{2}$	$\Delta_{\max} = 0.012 \text{ m}$
<i>Experimental Spacing</i>	$\Delta_{\min} > \text{SLDV spatial resolution}$	$\Delta_{\min} = 0.009 \text{ m}$

Table 1 summarizes the placement rules used to suppress evanescent components. To treat evanescent terms as negligible, the response is required to decay by 90% over the propagation distance l from the source [46]; accordingly, sensors are placed at least $l = 0.05 \text{ m}$ from boundaries, the dashpot, and the excitation point. To avoid spatial aliasing, inter-sensor spacing follows the half-wavelength criterion across the bandwidth of interest [4]. With $\lambda_{\min}$ attained at $f_{\max} = 20 \text{ kHz}$, the allowable spacing is $\Delta_{\max} = 0.012 \text{ m}$. Finally, a practical resolution limit for future SLDV measurements sets a minimum spacing of $\Delta_{\min} = 0.009 \text{ m}$. Also, to avoid high condition numbers while doing the inverse problem, a non-uniform sensor layout is adopted [47, 48].

For the end-mounted dashpot case in Figure 2, we assume no transmission and only reflection to simplify the analysis. At $x = x_0$, the response is expressed through a decomposition that isolates positive- and negative-going wave amplitudes.

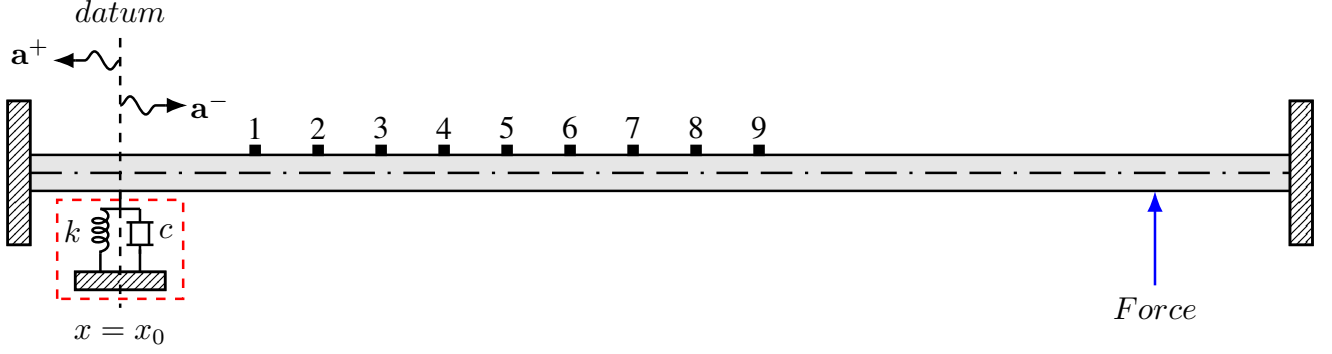


Fig. 2 Schematic Diagram of a clamped–clamped beam with a spring-damper at an end

The responses measured by the n sensors (displacement, velocity, or acceleration) are represented as $\mathbf{W} = \Lambda \mathbf{A}$, where

$$\mathbf{W} = \begin{Bmatrix} \mathbf{W}(x_1) \\ \mathbf{W}(x_2) \\ \vdots \\ \mathbf{W}(x_n) \end{Bmatrix}, \Lambda = \begin{bmatrix} e^{-\iota k_f x_1} & e^{\iota k_f x_1} \\ e^{-\iota k_f x_2} & e^{\iota k_f x_2} \\ \vdots & \vdots \\ e^{-\iota k_f x_n} & e^{\iota k_f x_n} \end{bmatrix}, \mathbf{A} = \begin{Bmatrix} a_p^+ \\ a_p^- \end{Bmatrix}. \quad (8)$$

Because the sensors are positioned to suppress evanescent/near-field terms, only the propagating amplitudes $a_p^\pm$ remain, with $+$ and $-$ signs denoting positive and negative propagation. $\mathbf{W}(x_i)$ is the measured output at $x = x_i$ for $i = 1, 2, \dots, n$. Although two sensors suffice, here $n = 9$ are used so the same beam model accommodates future cases with multiple dashpots. For implementation, we express $\mathbf{W}(x_i)$ as the acceleration-to-force transfer function, rather than in terms of spatial displacement; the resulting wave amplitudes are accelerations per unit force, which does not alter the reflection or transmission coefficients. Wave amplitudes are reconstructed from measurements of $\mathbf{W}$ via

$$\mathbf{A} = \Lambda^{-1} \mathbf{W}, \quad \mathbf{A} = (\Lambda^H \Lambda)^{-1} \Lambda^H \mathbf{W},$$

for square and overdetermined systems, respectively. The reflection coefficient is then defined by,

$$r_p = \frac{a_p^-}{a_p^+}. \quad (9)$$

Because the phase terms in Equation (9) are sensitive to errors in k_f and the datum–sensor distances, accordingly, this study focuses on the power coefficients in Equations (10) and (11). The power reflection coefficient is defined as,

$$\rho = |r_p|^2, \quad (10)$$

and the corresponding power absorption coefficient is shown as.

$$\alpha = 1 - |r_p|^2, \quad \text{or} \quad \alpha = 1 - \rho. \quad (11)$$

Figures 3a and 3b show the reflection and power-reflection coefficients, respectively. When propagation coefficients are included in the damping model, the plots reveal appreciable structural damping: even without a passive damper, the power absorption coefficient $\alpha(\omega)$ in Figure 3c remains nonzero across the band. To avoid conflating intrinsic material damping with added damping, we therefore adopt an undamped host-beam model in which $\rho(\omega) = 1$ and $\alpha(\omega) = 0$, as indicated by the black dotted curves in Figures 4b and 4c. These figures also show that introducing a dashpot alters both ρ and α , confirming that a spring–damper modifies the beam’s absorption characteristics. Building on this result, the next section examines how relocating the dashpot along the beam affects the power absorption coefficient.

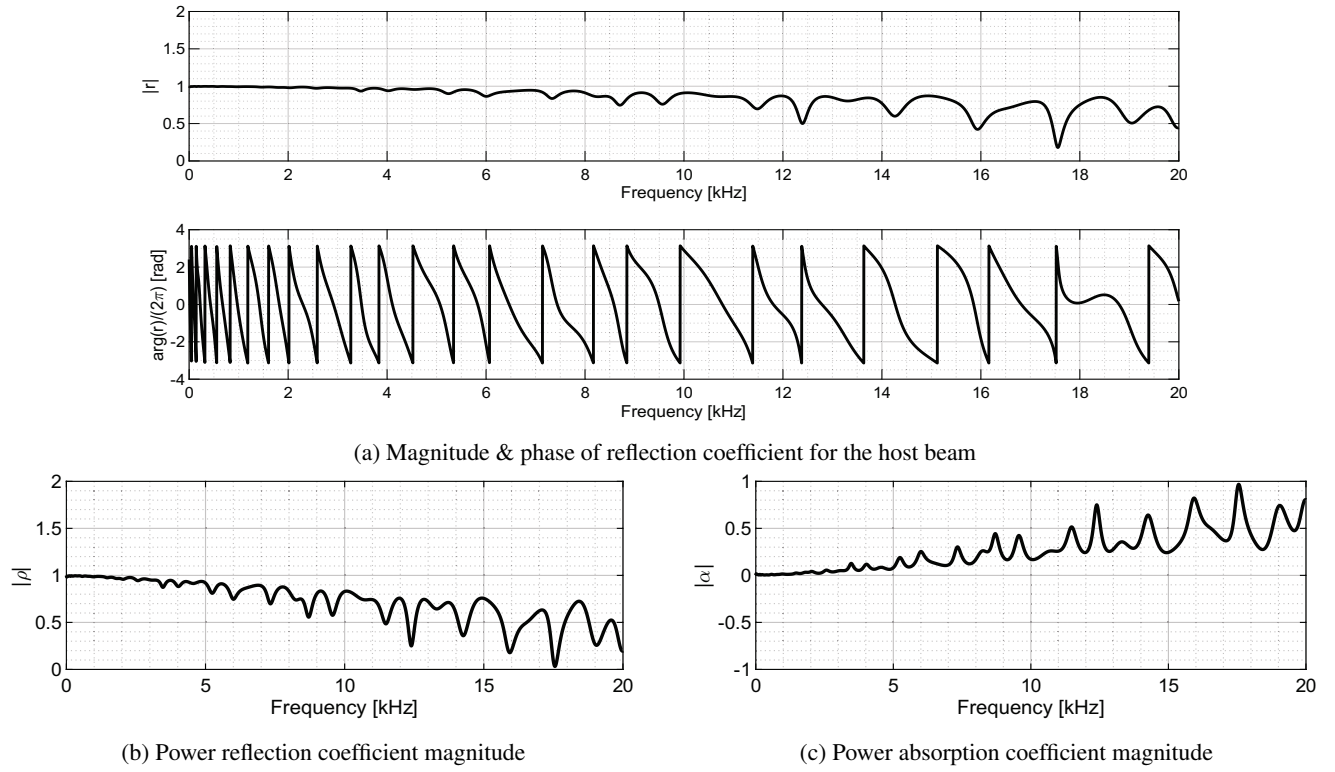


Fig. 3 Wave-amplitude coefficients for the host beam with a proportional damping model

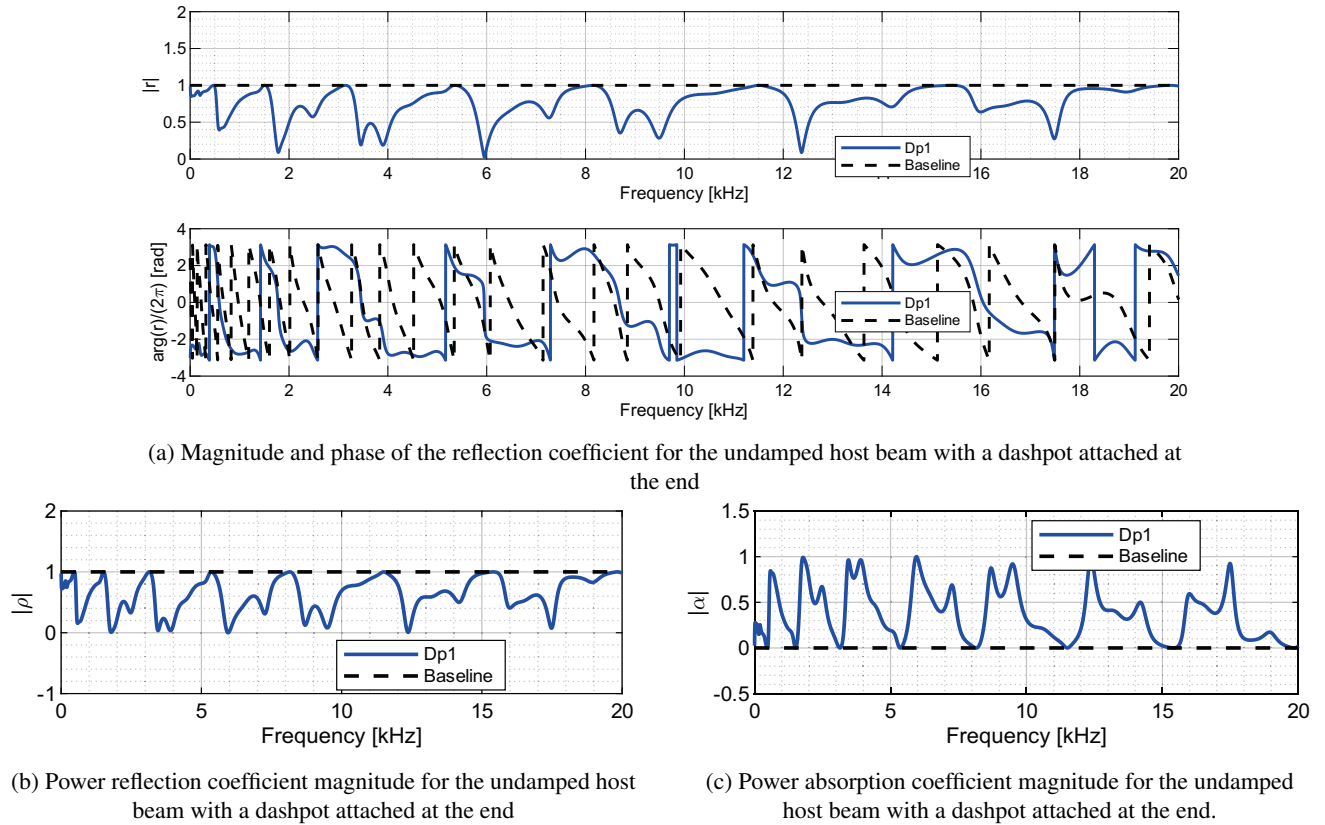


Fig. 4 Wave amplitude coefficients for the undamped host beam with a dashpot attached at the end

Results

The results in Figures 4b and 4c indicate that adding a spring–damper alters both ρ and α , highlighting its impact on the beam’s absorption behavior. We next assess how relocating the dashpot along the span affects the power absorption coefficient. At a fixed frequency, $\alpha(\omega)$ depends on both the damping coefficient and the passive absorber (PA) location; to isolate location effects, the damping coefficient is set to the value that maximizes α at that frequency. The PA is then swept from 0.017 m to 0.149 m measured from a clamped end, under the maintained assumption of no transmission. The spring–damper properties used in this sweep are held constant and reported in Table 2. It is evident from Figure 5a that, at the

Table 2 Spring-Damper Properties

Frequency	Damping Coefficient	Wavelength	Number of Peaks
(a) 2940 Hz	29.06 N – s/m	$\lambda_1 = 0.07$ m	4
(b) 11623 Hz	41.02 N – s/m	$\lambda_2 = 0.035$ m	7

single tuning frequency 2940 Hz, multiple dashpot locations deliver peak absorption for the same damping coefficient—i.e., optimal placement is not unique. At 2940 Hz the wavelength is $\lambda_1 = 0.07$ m. Halving it to $\lambda_2 = 0.035$ m (so $\lambda_2 \simeq \lambda_1/2$) and applying the dispersion relation yields a corresponding frequency of 11623 Hz, as listed in Table 2. At this higher frequency, the number of absorption peaks increases from 4 to 7 (see Figure 5b), consistent with the harmonic character of the power absorption coefficient: when the wavelength is halved, the spatial peak pattern approximately doubles, and the principal peaks of the power absorption coefficient align with harmonic wavelength scaling.

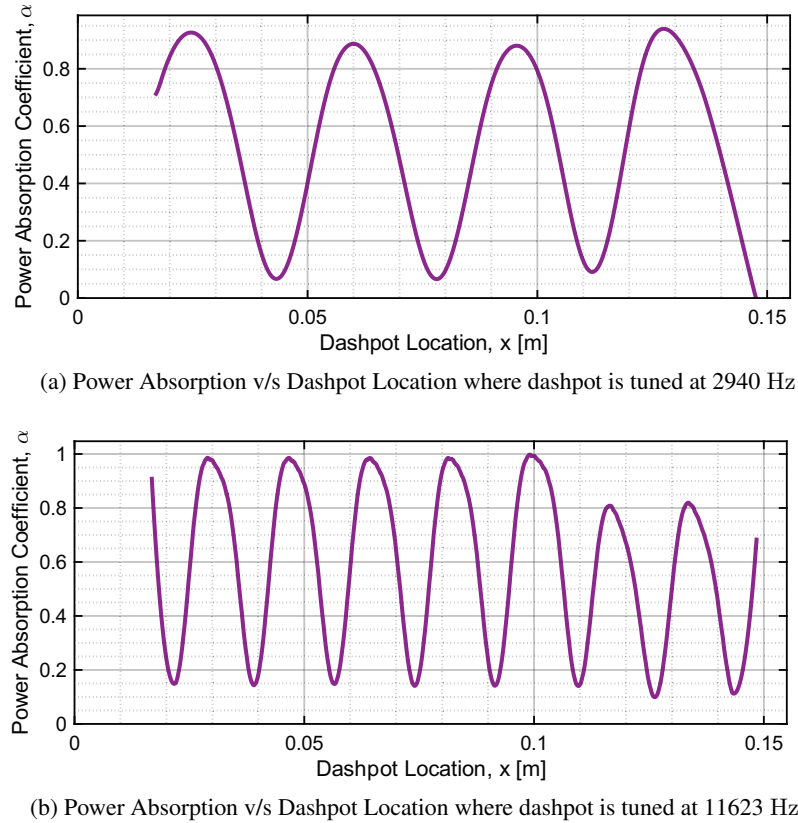


Fig. 5 Power Absorption v/s Dashpot Location

Conclusion

This work introduced a wave-based strategy for positioning a compact spring–damper on a beam to maximize power absorption. Establishing an undamped baseline— $\rho(\omega) = 1$, $\alpha(\omega) = 0$ —isolated the absorber’s influence and showed that adding a dashpot markedly reshapes both the power reflection and absorption coefficients. Parametric sweeps then quantified how α varies with spanwise location. A simple, repeatable pattern emerges: optimal placements are not unique but repeat at approximately half-wavelength intervals, and halving the wavelength (i.e., moving to higher frequency via the dispersion relation) increases the number of absorption peaks, consistent with the harmonic nature of the response.

In practice, these results lead to a clear placement rule: maximum absorption occurs when the dashpot is positioned at the vibration antinodes, which lie at odd quarter-wavelengths from the clamped end.

$$\text{Dashpot Location} = \frac{(2n + 1)\lambda}{4}, \quad n = 0, 1, 2, \dots$$

Placing the absorber at velocity antinodes maximizes vibration absorption while avoiding the mass penalty of blanket damping. This rule yields broad regions of effective placement but also heightened sensitivity near node–antinode transitions, where small shifts in position can markedly change α . Paired with sensor-placement guidelines that suppress evanescent content, the method offers an experimentally grounded, wave-informed route to lightweight vibration reduction. A natural next step is to include transmission to generalize the coefficients and capture through-energy paths. Coordinating multiple absorbers—passive, semi-active, or active—can broaden impedance control, and SLDV hardware tests will assess robustness to modeling and boundary uncertainties. Overall, the results provide concise, wave-informed placement rules and a path to scalable, mass-efficient vibration control in structural waveguides.

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Declaration of generative AI and AI-assisted technologies in the writing process

While preparing this work, the author(s) used ChatGPT to edit the language. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the publication’s content.

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Chapter 7

Modelling Homogeneous Populations of Structures using the Statistical Finite Element Method (statFEM)

Brandon J. O'Connell, Keith Worden, and Timothy J. Rogers

Abstract Population-based structural health monitoring (SHM) promises to leverage diagnostic information from multiple systems for a shared benefit. To move towards population-based prognosis, physical models are required. Incorporation of uncertainty across population models will be a key technology to achieve this goal. The Statistical Finite Element Method or statFEM has recently emerged as a new method of combining stochastic finite element (FE) models with noisy observed data in a Bayesian framework, to obtain a posterior over possible solutions. Despite its promising capabilities, case studies exploring the application of this technology, particularly to structural applications, remain limited. Turning to the population-level approach, the current work will demonstrate a population-based application of statFEM to simulated data from multiple structures. In this study, a statFEM model of a simple structure – a clamped-clamped beam – will be demonstrated where a homogeneous population of beams can leverage a shared underlying statistical representation despite having unique variations in properties. Via an implementation of statFEM, it will be shown how one can recover posterior densities over the model solutions, using a shared prior FE, given data from multiple structures.

Keywords Finite element · Population-based SHM · statFEM

Introduction

Population-based approaches to data-driven learning have arguably brought about a new paradigm in the field of structural health monitoring (SHM), with several proposed methods aiming to address the problem of data scarcity. The resulting field of *population-based SHM* (PBSHM) has thus seen considerable growth in the last few years [1, 2]. Despite significant work on advancing data-driven approaches for PBSHM, little research has been conducted on population-based modelling for SHM, and by extension model updating, particularly under uncertainty.

As practitioners increasingly seek new and more informative ways to make decisions, combining physics-based models with measured data under uncertainty [3], population-based modelling via finite element (FE) methods are of interest, particularly where structures are often considered nominally the same *i.e.* a homogeneous population [1]. Of the available approaches to uncertainty quantification in FE modelling (FEM), the *statistical finite element method* (statFEM) [4] has emerged as a new promising solution to this problem and one that provides the necessary Bayesian framework to consider a population-level approach.

Although statFEM has been applied to a selection of simple PDEs in the original and related works [4, 5, 6], the application of statFEM to problems in structural mechanics remain limited. Few examples are present in the literature, notably the work on the Staffordshire bridge [7], and that applying statFEM to buckling of railway sleepers [8]; both of which apply simplified forms of the full statFEM approach. In both cases only the force or the model parameter are modelled as being uncertain, not both simultaneously. However, in the setting of PBSHM one expects to see unknown variability in both the model parameters and the loading conditions. Therefore, consideration of uncertainty from both sources is desired.

In this paper, application of statFEM to the static elasticity problem in structural mechanics is demonstrated and applied to a population of simulated clamped-clamped beams. A shared prior FE model is generated, assuming that the forcing

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and a model parameter are stochastic. This FE prior is then combined with (simulated) observation data to obtain posterior densities over the possible FE solutions. Two different health states are shown, with the observation data for these systems generated using two FE models with known parameters, respectively.

Background

The incorporation of uncertainty in FE and model updating procedures is not a new discipline but has seen continuous and growing attention. Particular areas of interest include computational efficiency [9], Bayesian approaches [10], and the scalability of such methods [5]. One research challenge has been the coalescence of the two fields of stochastic FE and statistical FE; where one aims to unify stochastic models and noisy data in a statistical framework. In stochastic FE, model parameters (*e.g.* material properties, loads) are considered uncertain and treated probabilistically. These uncertainties are then propagated through an FE model to obtain probabilistic outputs [11, 12]. Alternatively, in statistical FE traditional models are combined with noisy data using statistical inference to obtain updated and improved estimates of the system under investigation [13]. Often such an approach is attempted within a Bayesian framework [13, 14].

The *statistical finite element method* or statFEM, formally introduced by Girolami in 2021 *et al.* [4], provides a unifying approach to these challenges under a fully Bayesian framework; allowing synthesis of observed noisy data and stochastic FE models to obtain a posterior density over the possible set of solutions. In the formulation of statFEM, the forward (stochastic FE) problem considers the solution to the PDE as random field (see [11]) and is solved as a probabilistic FE. This is achieved via the introduction of Gaussian processes over the uncertain model parameters and by propagation of the uncertainty through the classic FE problem. Following discretisation of the relevant domain, the prior probability density over the FE solution is obtained using a first-order perturbation (Taylor series approximation). The resulting density has a mean, given by the standard FE solution, and an associated covariance (Eq. (27) in [4]); thus approximating the density as Gaussian. It is noted by Girolami *et al.* that this approximation is only valid for small variances in the model parameters.

Statistical inference is then performed by decomposing the observed data y into three random components (see [15]); a finite element component u , a model mis-specification component d , and a measurement noise component e , to give

$$y = \rho Pu + d + e, \quad (1)$$

where ρ is a random hyperparameter and P is a projection operator. Given access to the full joint density, the posterior densities over each of the random variables can then be obtained in the usual way, by employing Bayes’ rule. In practice, the misspecification and measurement components are considered to be Gaussian processes with some chosen kernel and controlled by a set of learnable hyperparameters.

Although initially shown for Poisson’s equation in the original work, statFEM can be adapted to the problem of static deflection because the elasticity PDE is also elliptic by definition [16]. Using the weak form of the general elasticity PDE, one obtains the following discrete system of linear equations,

$$K(\theta)u = f, \quad (2)$$

where $K(\theta) \in \mathbb{R}^{n_u \times n_u}$ is the stiffness matrix, $\theta \in \mathbb{R}^{n_e}$ is the vector of uncertain model parameters, $u \in \mathbb{R}^{n_u}$ is the vector of nodal solutions, and $f \in \mathbb{R}^{n_u}$ is the uncertain nodal load vector.

Method

Using a custom-built FE package in Python, a 200 node Timoshenko beam element mesh was generated for a rectangular cross-section (prismatic) aluminium beam using the parameters defined in Table 1. A mesh convergence study was used to find the appropriate number of elements to ensure verification against the theoretical result. Using the same base mesh, several alternate health states were simulated by varying the thickness along the beam, equivalent to a scaling on the Young’s modulus. The functions of thickness used for each state are given in Table 2. Given that traditional Timoshenko beam elements assume prismatic beams *i.e.* a constant cross-section [16], the mean thickness in each element was calculated using the corresponding values of thickness function at the element nodes.

To incorporate variation in the loading, the amount of force applied to the beam was changed in each scenario from that of the prior. The values of force are also provided in Table 2. In this study, the applied load was distributed evenly between the two nodes either side of the mid-span ($x = 0.25$), replicating a simple point load. In all examples, the beam was subjected to Dirichlet boundary conditions at either end, ensuring a clamped-clamped setup.

Table 1 Geometric and Material Properties

Parameter	Value	Units
Length (l)	0.5	m
Height (h)	0.02	m
Thickness (t)	0.02	m
Cross-Sectional Area (A)	th	m^2
Second Moment of Area (I)	$th^3/12$	m^4
Young's Modulus (E)	70	GPa
Shear modulus (G)	26	GPa
Poisson's Ratio (ν)	0.33	-
Shear Correction Factor (κ)	5/6	-

Table 2 Health scenarios and their associated thickness and applied load

Scenario	Thickness Function $t(x)$ (m)	Applied Load P (N)
1 (Healthy - Prior)	0.020	3500
2 (Damaged)	0.017	3550
3 (Damaged)	$0.002(1-x)\cos(2\pi x) + 0.018$	3200

Prior Finite Element

To obtain the prior FE solution, Gaussian processes (GPs) were introduced over the thickness t (*i.e.* the uncertain model parameter) and the applied force f , both modelled by squared-exponential kernels $K_t(x, x')$ and $K_f(x, x')$, such that

$$t \sim GP(\bar{t}, K_t(x, x')), \quad (3)$$

and

$$f \sim GP(\bar{f}, K_f(x, x')) \quad (4)$$

where (see [17]),

$$K_{SE}(x, x') = \sigma^2 \exp\left(-\frac{(x - x')^2}{2\ell^2}\right) \quad (5)$$

In line with [4], the prior mean is equivalent to the classical FE solution which can be easily obtained using traditional FE approaches. The mean is computed using the prior mean force $\bar{f}$ and prior mean thickness $\bar{t}$ (Scenario 1 in Table 2), such that

$$\bar{u} = K(\bar{t})^{-1} \bar{f}. \quad (6)$$

Using a custom Python implementation of statFEM developed by the authors, the prior covariance over the FE solution was also computed using Eq.(27) from [4]. Given the element stiffness matrix for a Timoshenko beam element [18], the derivative of the element stiffness with respect to the thickness t , could be easily obtained. The hyperparameters of the two GPs were constrained to fixed values, as detailed in Table 3. These parameter values were chosen sensibly, given insight of the problem but could be optimised if required. The calculated prior density over the FE solution is shown in Figure 1.

Table 3 Hyperparameters

Hyperparameter	Value
σ_t	0.04
ℓ_t	100
σ_f	10
ℓ_f	10

In each axis, the mean solution is given by the blue dot-dashed line, and variance by the incremented shaded regions. The top axis in Figure 1 displays the y (transverse) displacement and the corresponding variance, the middle axis displays

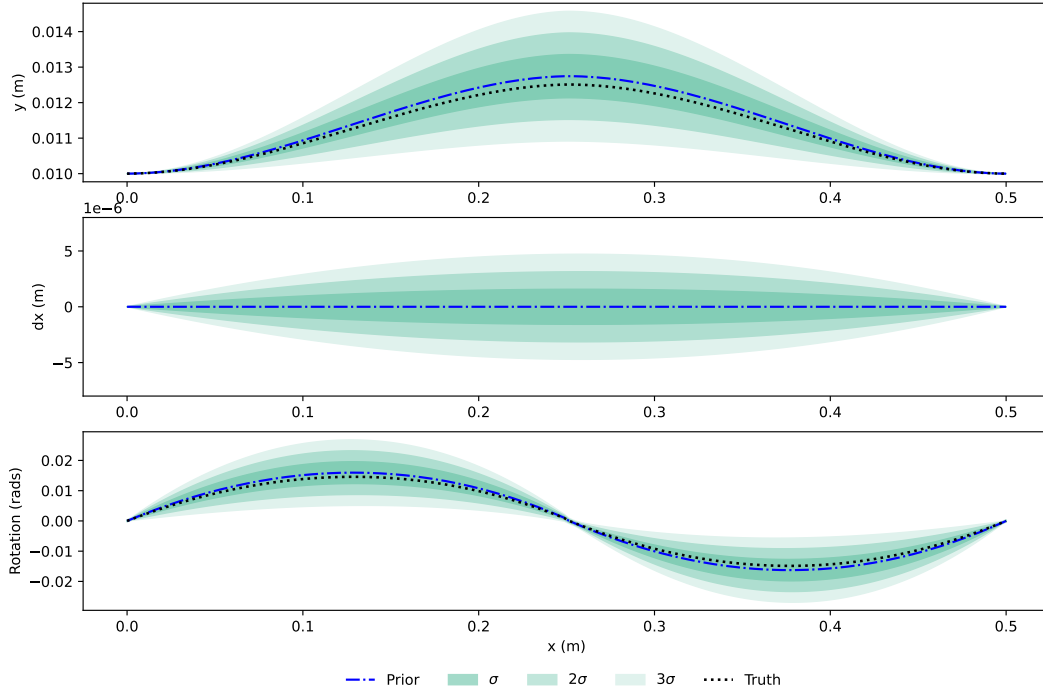


Fig. 1 Prior density over the finite element solution computed using statFEM. Top: transverse displacement. Middle: difference in axial displacement. Bottom: shear rotation angle

the difference in the axial displacement, and finally the bottom axis shows the shear rotation angle. The axial displacement and covariance of beams is difficult to interpret in any case, but is included here as the difference in displacement between the mesh and the prior for completeness.

Results and Discussion

In this section, the results from the application of statFEM using data generated for two health scenarios are shown. Recall that the 'damage' in this study corresponds to a variation in the beam thickness along the length, as defined in Table 2. Ten measurements of the vertical displacement were generated at each of the ten observation locations, spaced evenly along the beam, directly from other FE models constructed with the true parameters. The posterior density over the FE solution was then obtained using statFEM, combining the prior FE with the observed data. The hyperparameters chosen for the discrepancy d and the noise variance e are given in Table 4.

Table 4 Hyperparameters

Hyperparameter	Value
σ_d	10^{-7}
ℓ_d	100
σ_e	10^{-6}

Figures 2 and 3 show the posterior densities over the FE solution for Damage Case One and Damage Case Two, respectively. As evidenced by the posterior, These results demonstrate that statFEM can be successfully used to update a (shared) FE prior density and converge toward the true underlying system, given noisy observations of a system.

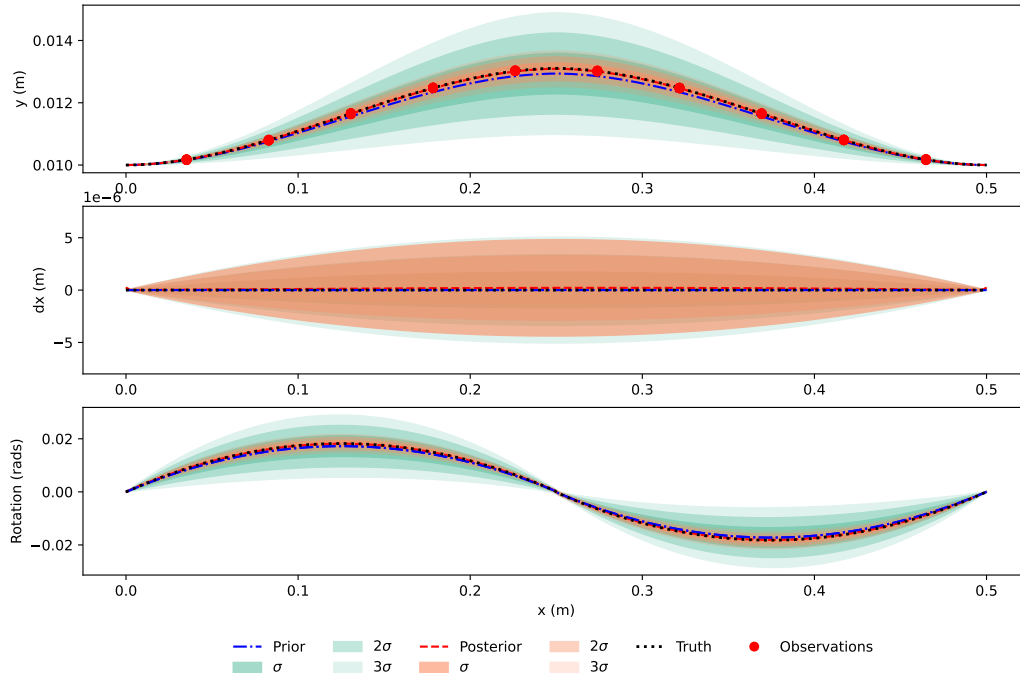


Fig. 2 StatFEM Posterior FE solution for Damage Case 1

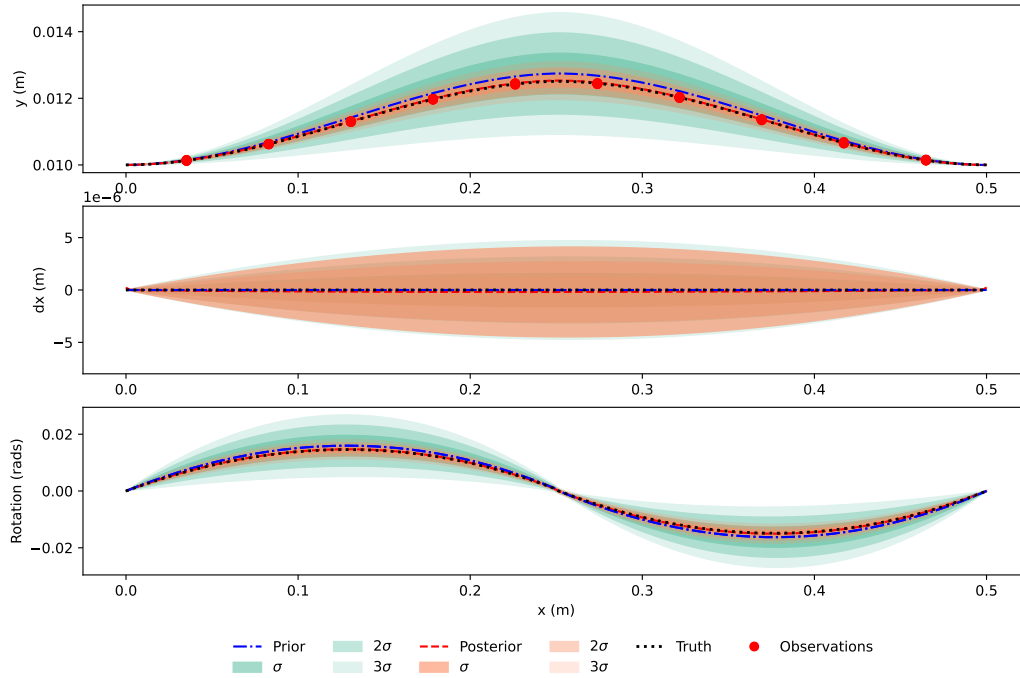


Fig. 3 StatFEM Posterior FE solution for Damage Case 2

Interestingly, the uncertain models arising from statFEM (considering both the prior and posterior densities) can be considered to constitute what is known as a *Form*; in the context of PBSHM [1]. A Form is that which encompasses both the essential nature of an object of interest, *i.e.* the mean FE solution of the cantilever beam, together with a measure of the expected variation from the ideal across a population, *i.e.* the probability density over possible solutions. Note that a Form need not be an object, but could also exist as a feature. In this case, the prior distribution provides an initial Form, based on pre-existing knowledge of the uncertainty in the parameters for a population, while the posterior gives an updated-Form, on observation of the real system in the physical world. Given access to the uncertainty over possible solutions, the prior outputs of statFEM (a Form) could be considered a general representation of a homogeneous population, which could be used to assist diagnostic tasks in PBHSM.

Conclusion

This work has illustrated how the statistical finite-element method (statFEM) can be adapted to the problem of static mechanics and used to model the prior uncertainty over the static deflection of a structure assuming both uncertainty in the force and the stiffness, given uncertainty in a model parameter (*e.g.* the thickness or Young’s Modulus). Assuming uncertainty in the thickness and the applied load, the prior density over the finite element (FE) solution was obtained for the forward problem using the statFEM approach. Simulated data from two alternate health states were then used to obtain the posterior densities over the two beams. In each example, it was shown how statFEM accurately recovers a posterior mean corresponding to the true underlying system whilst also providing an updated estimate for the variance.

The statFEM approach to stochastic and statistical FE has the potential to provide a cohesive and unified approach to Bayesian model updating and digital twinning of structural systems under static loads. Future work aims to apply this technology to physical, real-world structures through an experimental campaign designed to validate its applicability to measured data. Moreover, the authors aim to further explore the methods applications to population-based learning and population-based SHM.

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Chapter 8

OMA-based Approach for Structural Health Monitoring: the Case of Palazzo Lombardia

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Abstract Palazzo Lombardia in Milan is one of Italy's tallest buildings due to its 161 m height. Since 2015 the building has been equipped with a monitoring system that continuously tracks key parameters such as acceleration and inclination across its structure. In recent years, various Structural Health Monitoring (SHM) techniques have been developed to assess the building's condition throughout its lifespan. One promising approach is Operational Modal Analysis (OMA), which employs variations in modal parameters to monitor structural conditions. By continuously evaluating OMA outcomes, SHM strategies can be refined, enabling more effective tracking of the building's condition over time. This study focuses on an OMA-based strategy for Palazzo Lombardia, using long-term data collected from its monitoring system since 2015. Due to a gap in the monitoring data between 2018 and 2021, the data from before the interruption were used for training, while the data from 2021 onward served for validation. The results from the validation set were compared with those from the training set. Since the building remains in a healthy condition, the analysis of both sets is expected to yield similar results.

Keywords Operational modal analysis · Structural health monitoring · Principal component analysis

Introduction

The ongoing aging of infrastructure has led to a rising need for effective maintenance. This requires not only regular inspections but also a continuous monitoring approach. The field that aims to develop techniques for improving the monitoring systems is known as Structural Health Monitoring (SHM). Among the numerous SHM techniques developed in recent years, those based on vibration measurement are promising. As part of the global approach field, these techniques can determine the general health condition of the structure through measurements performed by transducers that are easy to install, even in already existing structures. The vibration features usually employed in these techniques are the modal parameters outcomes such as natural frequencies, damping ratios, mode shapes and modal curvature. The basic concept of modal-based SHM strategies is that the variation in mass, stiffness or energy dissipation properties of the structure influences its overall vibration response. Therefore, the continuous observation of the modal parameters in long-term monitoring may help to develop effective SHM strategies. In [1] the authors present a non-destruction damage detection technique based on outcomes from an Experimental Modal Analysis (EMA) performed on both reinforced concrete beam and slab. The damage detection is conducted by evaluating the variations in natural frequencies and mode shapes in different conditions of the systems under analysis. However, in operating conditions, the excitation term is often unknown, especially in large structures such as bridges or skyscrapers. In this field, Operational Modal Analysis (OMA) is particularly beneficial. Assuming that environmental excitation can be modeled as a white-noise term, OMA can determine modal parameters by measuring only the system's output. Several SHM techniques have been developed for OMA parameter extraction. In [2] a SHM strategy is proposed by evaluating modal parameters on a long-term monitoring of an offshore wind turbine. In [3] the authors developed an SHM approach using modal parameters collected from an iron arch bridge.

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In this work, an automated OMA is employed to collect modal parameters from the long-term monitoring of Palazzo Lombardia. The outcomes are used to develop an SHM strategy to assess the building's structural conditions. The monitoring system has been installed since 2015, but it was suspended between 2018 and 2021. Therefore, the data collected before the interruption has been used as the training set for the development of a SHM strategy. The data collected after the interruption have been employed as a test set. Since the structure is considered healthy, the modal parameters should not vary significantly between the test and the baseline sets. However, it is known from the state of the art that these parameters are particularly susceptible to environmental variations [4]. Therefore, in this paper, a strategy using the Principal Component Analysis (PCA) is proposed to mitigate the influence of ambient variations as much as possible.

Monitoring System Layout

Thanks to its 161 m height, Palazzo Lombardia is one of Italy's tallest skyscrapers. Since 2015 it has been equipped with a monitoring system that constantly measures useful information related to the structure conditions. Moreover, the state of the building can be assessed in case of exceptional events (e.g., earthquakes, strong gusts of wind). The overall system is briefly summarized below, while its complete details are reported in [5].

The building consists of 45 floors (three underground) and structure features such as accelerations and tilts are constantly measured through appropriate devices (accelerometers and tiltmeters). The sensors have been installed to cover the entire geometry of the skyscraper, comprehending floors -3, 9, 30, 37 and the rooftop. All floors are provided with three accelerometers and two clinometers installed on the concrete core to measure accelerations in the three principal directions and the rotation in the horizontal plane. Moreover, floors 9, 30 and 37 are provided with additional 3 accelerometers placed at a point horizontally farthest from the principal core.

The sensors adopted in the monitoring system are listed in Table 1. The monitoring system continuously collects data at a sampling frequency of 250 Hz through a DAQ-based device with an antialiasing filter.

The modal parameters extraction is deepened in the following section.

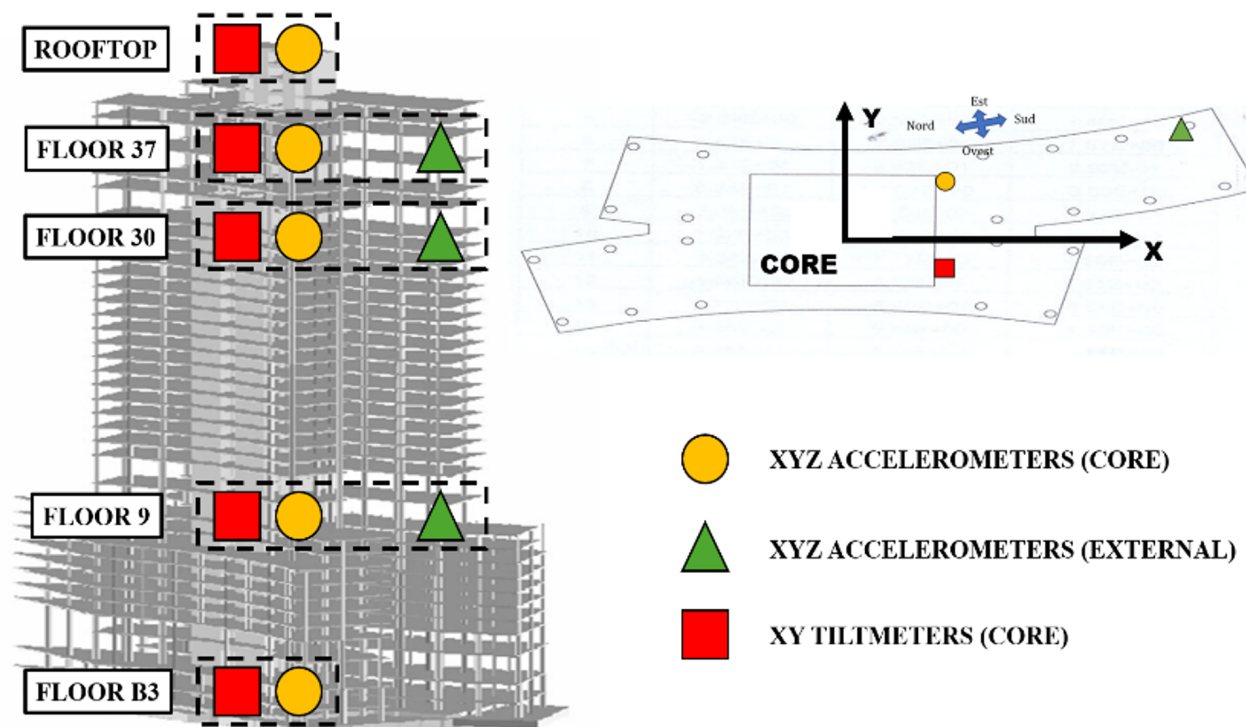


Fig. 1 Layout of the monitoring system and sensor positions

Table 1 List of sensors installed and their characteristics

Sensor	Manufacturer	Model	Full-scale	Bandwidth
Tiltmeter	Singer	Ts	5 [°]	≤ 3 [Hz]
Accelerometer	PCB	393B31	4.9 [m/s ²]	0.1-200 [Hz]

Modal Parameters Extraction

In this work, modal parameter extraction is performed using an automated Frequency Domain Decomposition (FDD) algorithm developed in Matlab. In this technique, assuming the natural excitation represented by a white-noise signal [6], the power spectra of the responses are processed in the frequency domain through the Singular Value Decomposition (SVD). The whole process can be summarized by the following

$$G_{xx}(\omega) = U(\omega) S(\omega) U(\omega)^H \quad (1)$$

where S is a diagonal matrix that contains the scalar singular values and is used to detect the structure modes. Natural frequency and damping are obtained using the Peak Picking (PP) method on $S(\omega)$, while mode shapes are estimated through $U(\omega)$ [7].

The greatest drawback of this method is the frequency resolution due to the cross-spectra processing. Therefore, the peaks in the singular value function have been modelled through a minimization employing the Lorentzian Peak-Shape function [8] by means of the Non-Linear Least-Squares (NLLS) method [9].

Modal parameters extraction for each observation has been performed employing acceleration data covering a period of 2 hours filtered by a Butterworth band-pass filter with a bandwidth of 0.05-3 Hz. Thus, the frequency resolution of the power spectra is $1.38 \cdot 10^{-4}$ Hz. An example of the procedure is shown in Figure 2.

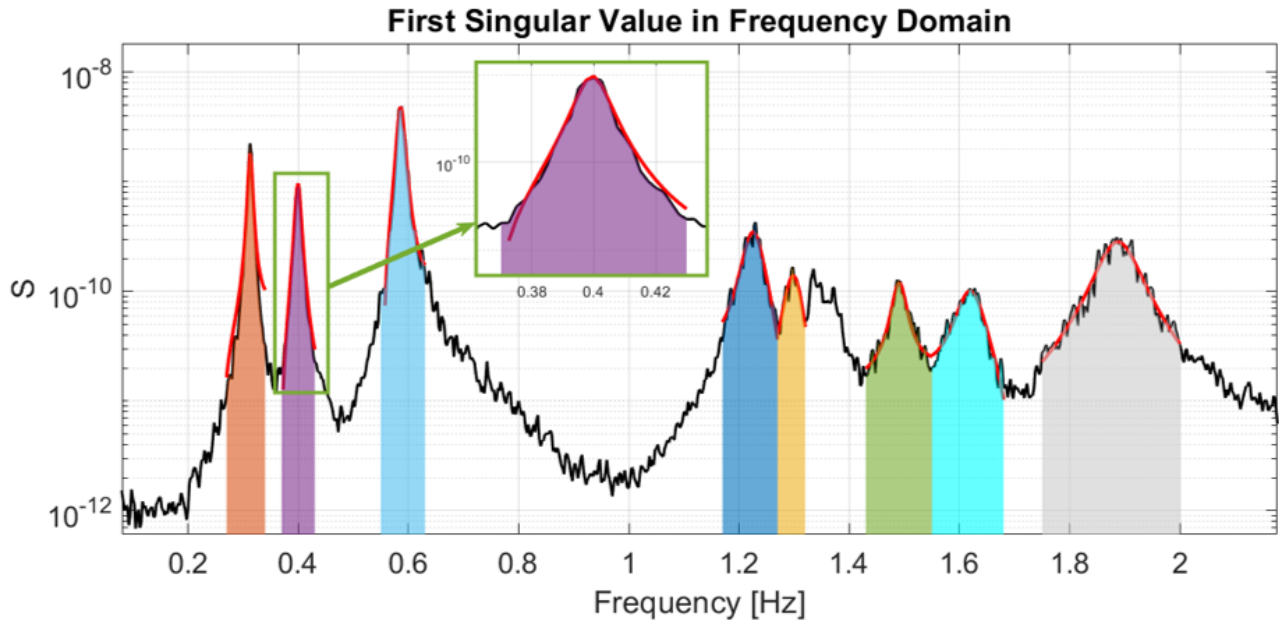


Fig. 2 FDD performed on Palazzo Lombardia data with bands of interest analyzed and reconstruction of the resonance peaks (red)

Since modes at approximately 1.3 and 1.8 Hz did not show consistent reliability during the monitored period, presenting peaks split or attenuation, they have been excluded from the presented analysis. The remaining modes with their characteristics are shown in Table 2.

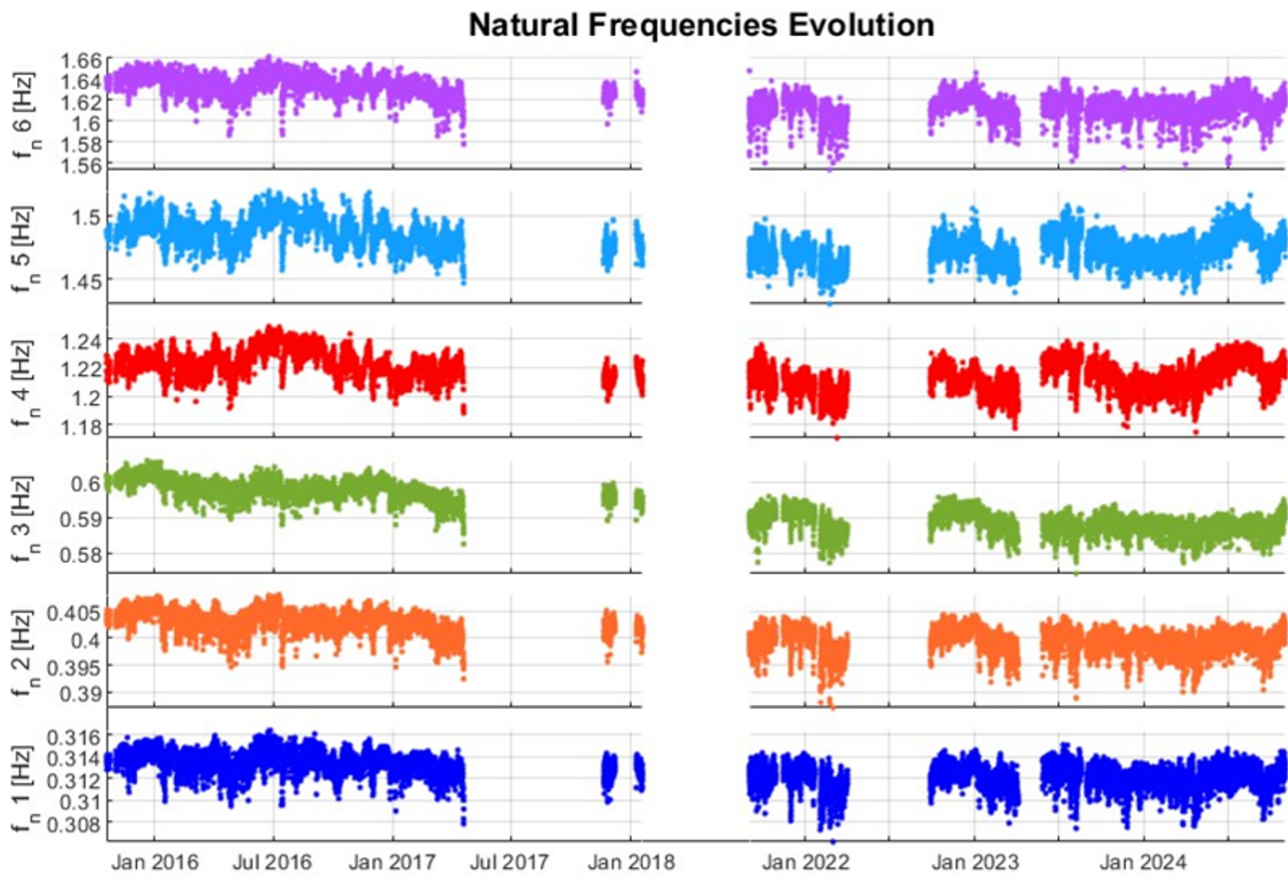
Results obtained during the monitored period and their processing are presented in the following section.

Table 2 Palazzo Lombardia modes detected

Mode	Type	Natural Frequency [Hz]
1	1st Bending Mode Y-axis	0.31
2	1st Bending Mode X-axis	0.4
3	1st Torsional Mode	0.59
4	2nd Bending Mode Y-axis	1.22
5	2nd Bending Mode X-axis	1.49
6	2nd Torsional Mode	1.62

Modal Analysis Outcomes

Since automated results can be affected by processing or measurement failures, files considered unreliable have been removed using statistical tools applied to raw data. Furthermore, modal parameter outcomes have been filtered with a strategy proposed and deepened in [10]. Natural frequency trends evaluated during the monitored period are shown in Figure 3.

**Fig. 3** Palazzo Lombardia natural frequencies evaluated during the monitored period

The natural frequencies obtained through the analysis exhibit a behaviour that is caused by the influence of environmental conditions as shown in [11] for the first three natural frequencies. This hypothesis is extended in this work by comparing the six natural frequencies considered in the baseline set (obtained before the monitoring interruption occurred in January 2018) with environmental measurements.

Principal Component Analysis and Environmental Influence

PCA is a technique derived from multivariate analysis that uses orthogonal projection on a given dataset to reduce its dimensionality. This process maximizes the variance of the projected data onto a lower-dimensional linear space. The new variables, or components, are uncorrelated by definition and are arranged such that the first few components capture most of the variation in the original dataset. The full theory behind PCA is detailed in [12], while the main steps of the calculation are outlined below, based on the natural frequencies set.

Natural frequencies are arranged as columns of the matrix X with dimensions $n \times m$, where n is the number of observations and m is the number of the modes considered. Considering the training set length of 6200 samples, in the proposed analysis, the dimensions of X are 6200×6 . After the natural frequencies standardization, the first step of PCA is performed by computing the covariance matrix Σ as follows

$$\Sigma = \frac{1}{n-1} X^T X \quad (2)$$

and to diagonalize that

$$\Sigma = \Lambda A A^T \quad (3)$$

In Eq. (3), Λ is a diagonal matrix with eigenvalues arranged in decreasing order along the diagonal, and A is a matrix of eigenvectors (the singular value decomposition is commonly used). The eigenvectors represent the principal axes or principal directions of the data. The projections of the data onto the principal axes are referred to as principal component (PC) scores, which are the columns of the matrix Z , obtained through the following expression

$$Z = X A \quad (4)$$

PC scores from the baseline set are shown in Figure 4.

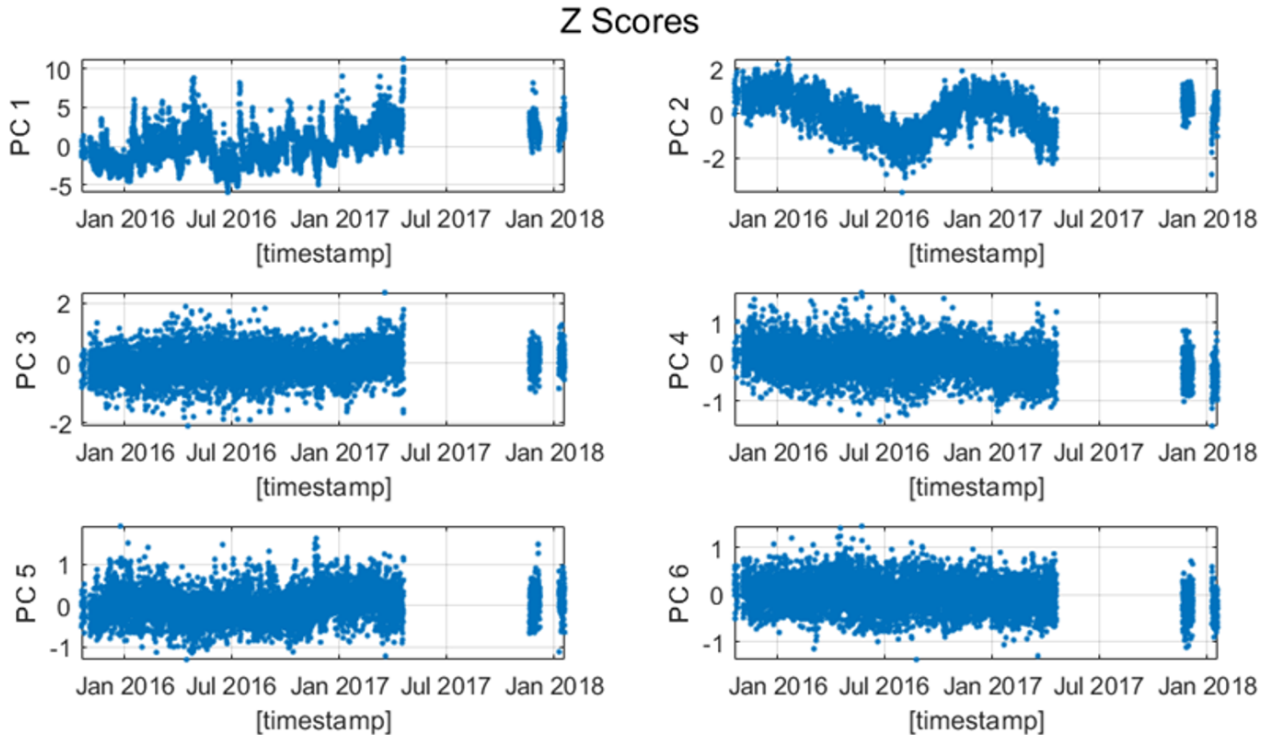


Fig. 4 Z scores evaluated from the baseline set

Observing the PC scores behaviour, the first two PCs seem to present a dependency on external conditions. In order to verify that, environmental data taken from the Linate weather station have been analyzed [13]. Temperature and wind speed measurements are shown in Figure 5.

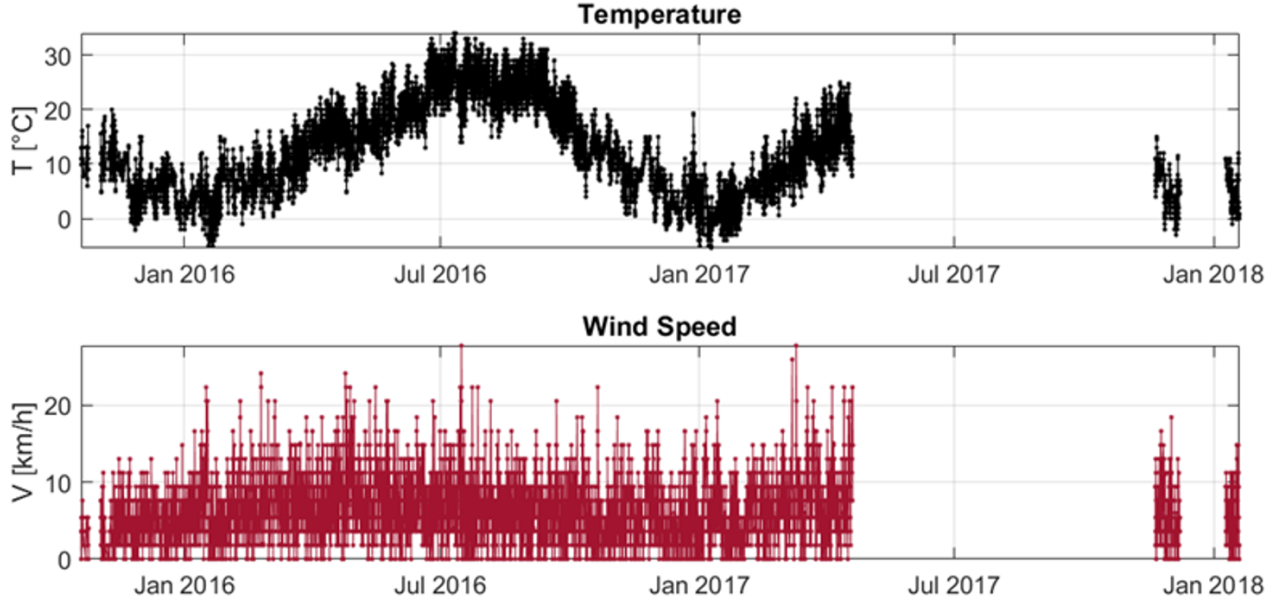


Fig. 5 Environmental data from Linate weather station

The correlation between PC scores and environmental conditions has been investigated through the evaluation of the Pearson's Coefficient, described in the following equation

$$\rho_{xy} = \frac{Cov(x, y)}{\sigma_x \sigma_y} \quad (5)$$

This index demonstrates a high correlation between two variables when its absolute value is greater than 0.7, a good correlation between 0.7 and 0.3, and a weak correlation below 0.3. Concerning the wind speed, only values greater than 7 km/h have been considered. Pearson's coefficient evaluation is reported in Table 3.

Table 3 Pearson's Coefficients evaluated between PC scores and environmental conditions (temperature and wind speed)

	Pearson Coefficient ρ_{xy}	
	Wind Speed	Temperature
PC 1	0.37	-0.19
PC 2	-0.22	-0.78
PC 3	-0.13	0.02
PC 4	0.10	0.01
PC 5	0.05	-0.15
PC 6	0.06	0.05

Results show a good correlation between the first PC and the wind speed, and a strong correlation between the second PC and the temperature. Thus, the first 2 PCs are removed from the structural condition evaluation.

The evaluation of the structure condition is performed by exploiting the Mahalanobis Squared Distance (MSD). MSD is a measure used to quantify the distance between a point and a distribution. It accounts for correlations of the data and normalizes the scale of the features [14]. The synthetic index proposed in this work is evaluated as follows

$$D_i = [Z_i^* - \bar{Z}_0^*]^T \sum^{-1} [Z_i^* - \bar{Z}_0^*] \quad (6)$$

where Z_i^* is a new observation of natural frequencies standardized with respect to the baseline averages and standard deviations, projected to the baseline PCs and with the first 2 PC scores removed, $\bar{Z}_0^*$ is the baseline average of the PC scores with the first 2 removed and $\sum^{-1}$ is the baseline covariance matrix. The evolution of the index is shown in Figure 6.

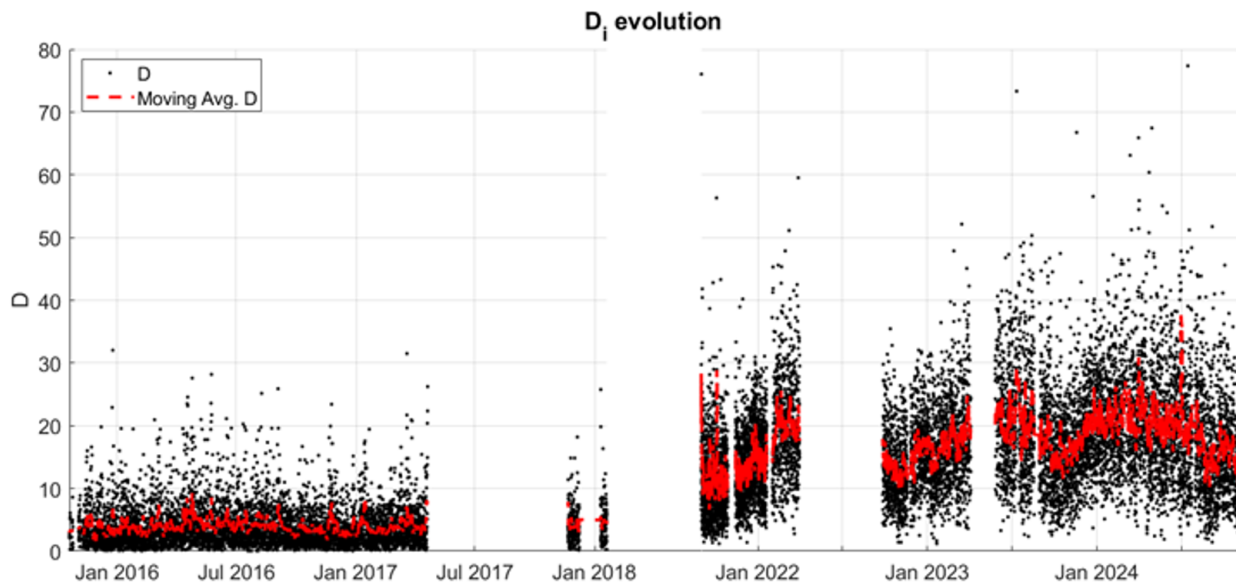


Fig. 6 Synthetic index evolution during the monitored period

The index proposed shows a strong immunity to environmental conditions during the training set since variations due to the wind speed and seasonal temperature seem to be removed. However, a certain discrepancy between the validation and training sets is noticeable. Furthermore, the proposed index in the validation set seems to be influenced by a seasonal variation. The causes behind these discrepancies are still being investigated and are yet to be fully understood.

Conclusions

This article presents a SHM strategy based on OMA outcomes. Acceleration measurements from the monitoring system of Palazzo Lombardia (Milan, Italy) have been employed to develop this technique. Results from OMA demonstrate a consistent variation caused by environmental factors. Therefore, the PCA has been performed to detect and remove the ambient effects. Finally, a synthetic index based on the MSD is evaluated to verify the structure condition without the environmental impact. Results show a strong immunity to ambient variations. However, a certain discrepancy between the validation set and the training set is evident. The deviation of the proposed index is still under further investigation.

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Chapter 9

Shock Response of LPBF Beams with Pockets of Trapped Powder

R Thayne Wagner, Mason Rollans, Matthew S. Allen, and Jeffrey R. Hill

Abstract Laser powder bed fusion (LPBF) is a widely used additive manufacturing technique for producing a variety of components. A common challenge in this process is designing parts that allow unfused powder to be evacuated from internal cavities. However, LPBF can also be leveraged to enhance damping in structures subjected to harsh mechanical environments by intentionally retaining unfused powder within internal cavities. This retained powder enables particle damping, which can reduce resonant vibrations and mechanical shock—offering protection to sensitive electronic components. While several recent papers have explored the frequency response or modal behavior of powder-filled components, there is comparatively less literature on their transient shock response. Understanding the behavior of these components in shock environments can provide insights into damage mitigation that frequency-domain analysis alone cannot offer. The industry standard for transient shock analysis involves comparing shock response spectra (SRS) curves. In this study, we compare the shock responses of powder-filled and monolithic beams using SRS analysis. We investigated their behavior under various impact conditions, measuring input forces with force sensors and structural responses with accelerometers. Results show that, compared to a monolithic beam, beams with unfused powder reduce maximum absolute acceleration by up to 28%.

Keywords Laser powder bed fusion · Impact · Transient response · SRS · Particle damping

Introduction

Additive manufacturing has become an increasingly prominent method for producing complex components. Unlike traditional processes, it enables single-piece designs with intricate geometries, eliminating the need for assemblies of smaller parts [1]. Laser powder bed fusion (LPBF) is a widely used metal additive manufacturing process that fuses fine metal powders layer by layer. However, one challenge is that cavities in a part inevitably trap unfused powder unless provisions are made for draining or post-processing. A second drawback is reduced structural damping. In most assemblies, damping is dominated by joints [2]; eliminating these joints reduces the system's inherent damping. Because damping mitigates vibration-induced damage and noise [3], its reduction can compromise structural performance. Interestingly, these two issues—powder entrapment and reduced damping—can counteract one another when exploited strategically [4, 5, 6, 7, 8, 9, 10].

By embedding unfused powder within LPBF beams, damping can be substantially increased. These pockets function similarly to particle dampers: they dissipate vibrational energy through friction and wall interactions. Prior work has primarily examined the modal behavior of such beams, demonstrating their effectiveness. However, there is limited research on their transient shock response. While particle damping has shown promise in impact-excited beams [11], most prior studies used external filler materials such as plastic beads or tiles [12]. Although effective, such methods introduce additional manufacturing steps and complexity. By instead utilizing the unfused powder inherent to the LPBF process, manufacturing is simplified. The objective of this work is to compare the shock response of a monolithic LPBF beam with that of an LPBF beam containing unfused powder.

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The SRS is a fundamental tool in shock analysis [13]. Typically, acceleration data from a test environment are transformed into an SRS curve. A non-destructive test article can then be excited with inputs of similar frequency content, and its acceleration data converted into an SRS for comparison. If the test article withstands shocks with an SRS equal to or greater than that of the environment, it is deemed suitable. In this study, rather than rating the beams against a specific environment, we subjected a monolithic beam to impact, and then subjected the beam with powder to similar impacts and compared its SRS to the monolithic beam to evaluate peak accelerations.

Methods

Two rectangular 316L stainless steel beams were used, shown in Figure 1. Fabrication details are provided in [4, 5]. One beam was monolithic with no unfused powder, while the other contained a 140mmx16mmx500 μ m cavity filled with particles with a mean size of 25 μ m. Both beams were identical in overall dimensions. The specimens were subjected to multiple impacts from two modal hammers and a portable pneumatic projectile launcher. Acceleration data were collected and processed into SRS curves for comparison.

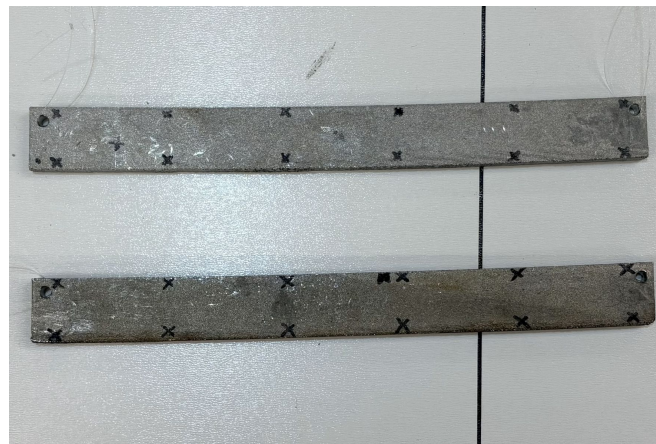


Fig. 1 316L stainless steel beams fabricated by LPBF: powder-filled beam (top) and monolithic beam (bottom). Impact location markers are indicated with black X's.

Hammer Hits

Two hammers were used: a PCB 086D05 modal hammer and an NV-TECH Scalable Automatic Modal Hammer (SAM1). Both were fitted with metal tips to produce short impulses and higher frequency excitation. A PCB 352A21 accelerometer was mounted on the back, left edge of the beam. The beams were suspended by fishing line and bungee cords to approximate free-free boundary conditions, as shown in Figure 2. Each hammer impacted the second marked location from the right end of the beam on the same side as the accelerometer. Accelerometer data were collected at a sampling frequency of 1280 kHz with 20,480 total samples. Five tests were performed for each beam and hammer type. Acceleration was averaged and used to construct SRS curves. Force data were collected and plotted, but used only for reference and not to calculate the SRS.

Portable Pneumatic Projectile Launcher

High-frequency excitation was achieved using a Daisy Powerline 880 portable pneumatic projectile launcher with 4.5 mm diameter (.177 cal) zinc-plated steel shot. The test configuration is shown in Figure 3. The launcher can be seen on the right in each picture and the sample on the far left, with the striker resting on it. Beams were again suspended for free-free conditions. Because direct impacts risked damaging the beams, a striker plate was used. The striker plate measured $4\frac{1}{8} \times 4 \times \frac{1}{8}$ in, and was fabricated from steel. A 1/4 in diameter, 1/8 in thick cylindrical brass bar stock was brazed to the rear of the plate to distribute pressure and soften the impact. The striker plate was suspended in front of the beam so that the brass element just contacted the second marked location from the right end of the beam. A DYTRAN 3224A4 accelerometer was mounted opposite the impact site.

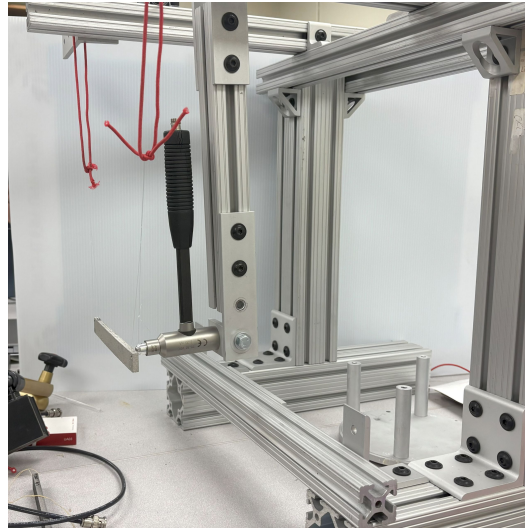
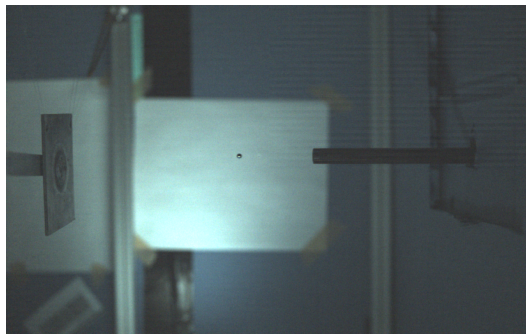


Fig. 2 Experimental setup with beam suspended by fishing line and bungee cords to approximate free-free boundary conditions. A pendulum-mounted modal hammer provided consistent impacts.



(a) Pre-impact



(b) Post-impact ricochet

Fig. 3 High-speed video frames of projectile impact on striker plate: (a) initial contact and (b) ricochet. Frame analysis was used to determine projectile velocity before and after impact.

Projectile velocity was measured using a Phantom v1610 high-speed camera. Videos were recorded at 10,000 fps. From these, pre- and post-impact velocities were determined. An additional video at 130,000 fps captured the impact duration. SRS curves were then generated for both monolithic and unfused powder beams.

Results and Discussion

SRS curves were generated using a modified Matlab SRS program. The original program was found at [14]. A damping ratio of 0.002 was used. Acceleration data were captured for both beam cases and used to generate the SRS. Average acceleration data for the SAM1 hammer test are shown in Figure 4 for reference. The acceleration data for the hammers were averaged over the 5 tests, then inputted into the SRS program.

Hammer Tests

Figures 5 and 6 present the SRS results for the SAM1 and PCB hammers, respectively. Five acceleration samples for each case were averaged and normalized by impact force. The trigger for the data collection allowed the data to be synchronized and no correction was needed. This normalization allowed direct comparison of maximum absolute accelerations at the 900 Hz peak. The overall shapes of the SRS curves matched closely between beams, indicating consistent impacts.

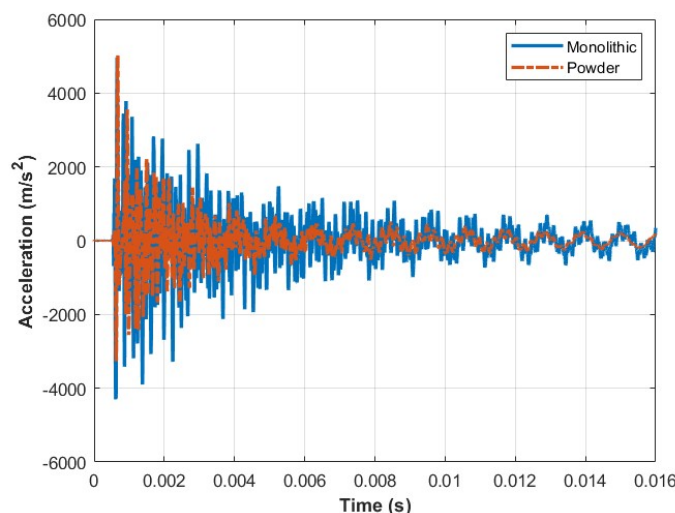
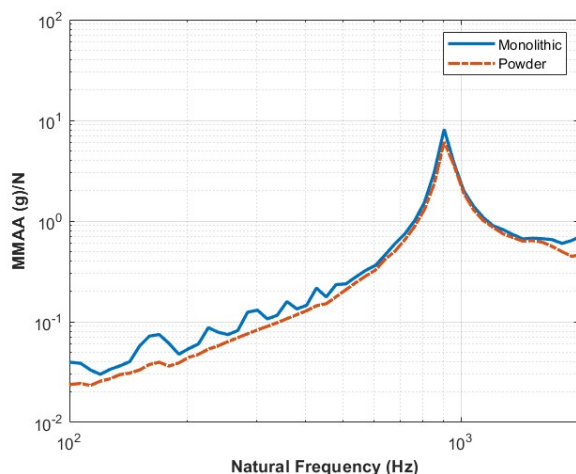
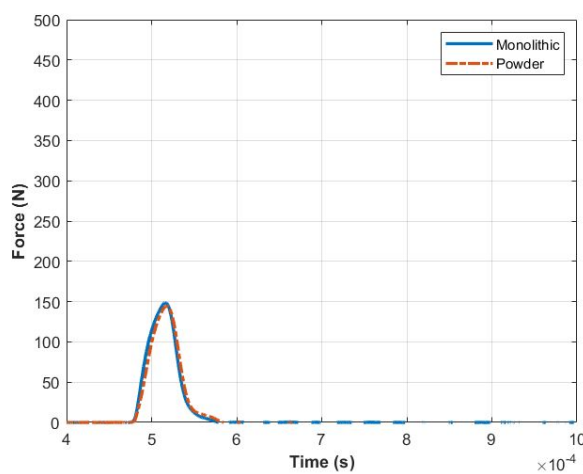


Fig. 4 Average acceleration for the SAM1 hammer of the monolithic beam and the unfused powder beam.

For the SAM1 hammer, the monolithic beam produced a normalized peak of 8.15 g/N, compared to 6.10 g/N for the powder beam—a 28.8% reduction. For the PCB hammer, the monolithic beam produced 14.0 g/N, compared to 13.1 g/N for the powder beam—a 6.6% reduction. In both cases, unfused powder effectively reduced peak acceleration.



(a) SAM1 SRS



(b) SAM1 Force

Fig. 5 SRS of NV-TECH SAM1 hammer tests. (a) Maximum absolute acceleration normalized by average force. Monolithic peak: 8.15 g/N; powder peak: 6.10 g/N. (b) Averaged force values for monolithic (148 N) and powder (144 N) beams. Data was collected over 16 ms, but was trimmed for the force plot to highlight its peak force.

Portable Pneumatic Projectile Launcher Tests

Projectile velocities averaged 190 m/s (pre-impact) and 12 m/s (post-impact). Variation across trials was less than 5%, justifying use of average values.

SRS curves for steel shot impacts are shown in Figure 7. Unlike the hammer tests, results were inconsistent. This discrepancy likely arose from test variability. Since the portable pneumatic projectile launcher required manual repositioning between shots, small changes in firing trajectory and striker plate alignment produced non-uniform impacts. The pendulum-mounted striker plate further contributed to variability.

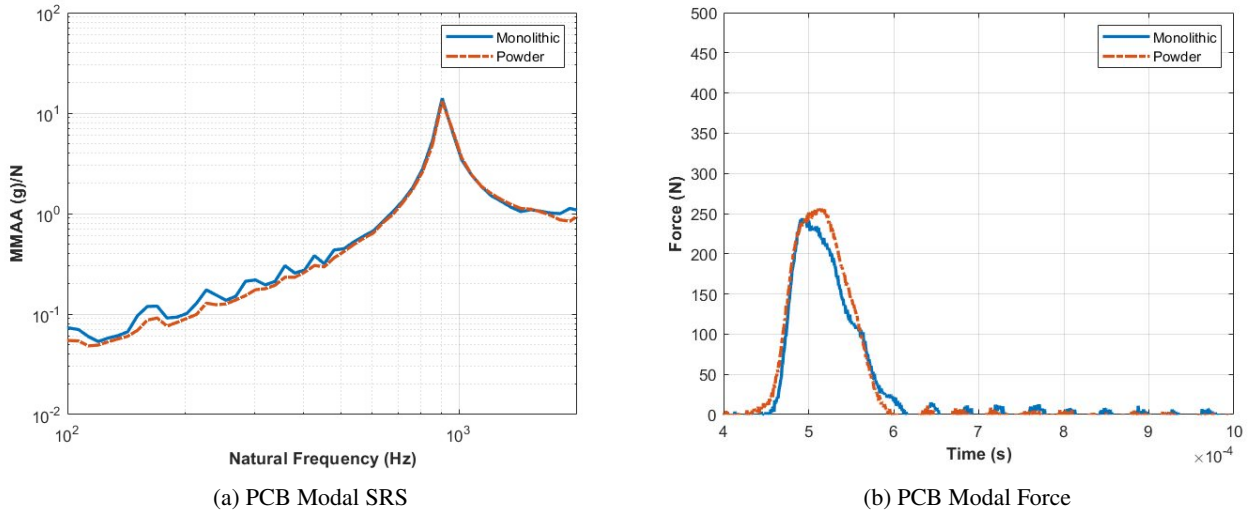


Fig. 6 SRS of PCB 086D05 hammer tests. (a) Maximum absolute acceleration normalized by average force. Monolithic peak: 14.0 g/N; powder peak: 13.1 g/N. (b) Averaged force values for monolithic (244 N) and powder (251 N) beams. Data was collected over 16 ms, but was trimmed for the force plot to highlight it's peak force.

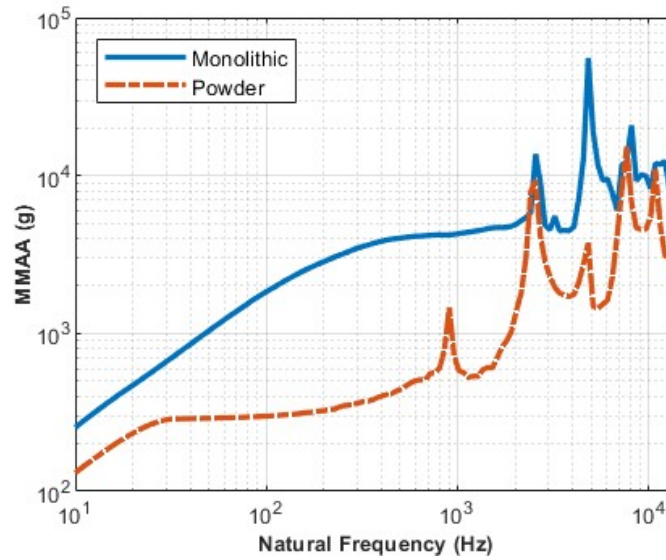


Fig. 7 SRS of beams impacted by steel shot. Monolithic and powder SRS shapes do not match, indicating inconsistent impacts and limiting comparability.

To improve consistency, future tests should employ a gas gun with a tuned resonator plate, as developed in [15]. This method provides a repeatable projectile impact with controlled frequency input, yielding more reliable high-frequency response data.

Conclusions

Unfused powder in LPBF beams effectively increased damping, as evidenced by reduced maximum absolute accelerations compared to monolithic beams under hammer excitation. These results suggest that incorporating unfused powder can reduce structural damage or protect mounted components in dynamic environments.

However, high-frequency tests using a portable pneumatic projectile launcher yielded inconsistent results, highlighting the need for improved impact methods. Future work will employ a gas gun with a resonant plate to provide controlled, repeatable excitation. Additionally, optimization of unfused powder pocket geometries and internal infill structures could further enhance damping performance for specific applications and impact conditions.

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Chapter 10

Interplay of Light, Mechanics, and Material in Light-powered Actuation of Fibers

Alireza Ahmadi and Neda Maghsoodi

Abstract We investigate the self-sustained oscillatory behavior of azo-dye-doped liquid crystal elastomer (azo-LCE) cantilevers under constant illumination. A coupled photomechanical model is developed to capture photo-isomerization dynamics, light absorption, and the resulting nonuniform strain distribution driving bending and flapping motion. Our simulations reveal that the system evolves into a steady-state oscillation at frequencies close to the natural resonance of the structure. The results highlight the interplay of light absorption, self-shadowing, and mechanical response in producing periodic motion, offering insights into designing light-powered actuators and soft oscillators.

Keywords Photomechanical materials · Light-induced bending · Periodic motion

Introduction

Self-sustained oscillation occurs when periodic motion is generated and maintained by a power source that is not inherently periodic. Self-sustained oscillations are non-equilibrium phenomena that interconvert between two or more kinetically stable and metastable states of soft actuators [1]; the actuators directly sense a constant stimulus, deform to a position away from the stimulus source, and autonomously return to their initial state based on an embedded negative feedback loop to obtain regular energy to maintain their periodic motion [2, 3]. Interestingly, the frequency of oscillations depends only on the inherent parameters of the structure [4, 5]. Recent developments in active materials have introduced new possibilities for designing self-sustained oscillators. Liquid crystal elastomers (LCE) are one example of such materials that can respond to incident light when proper photo-responsive moieties (like azobenzene molecules) are incorporated into their structure (Fig. 1). The mechanism by which azo-LCEs respond to light depends on illumination wavelength. Exposure to UV radiation at 365 nm prompts the azobenzene linkage to transition from its *trans* to *cis* form, leading to macroscopic deformation (Fig. 1). The goal of this work is to investigate the photo-induced self-sustained oscillations of high modulus, glassy, monodomain azo-LCE cantilevers via a self-shadow-induced negative feedback loop.

Methods

Consider an inextensible and unshearable monodomain long and thin LCE strip with a rectangular cross-section subjected to the planar incident light intensity I_0 (Fig. 1). Let s denote the coordinate along the centerline of the strip. The balance of linear and angular momenta can be expressed as [6, 7]

$$m \frac{\partial^2 \mathbf{R}(s, t)}{\partial t^2} = \frac{\partial \mathbf{f}(s, t)}{\partial s} + \mathbf{f}_{\text{ext}}(s, t) \quad (1)$$

$$\Pi \frac{\partial^2 \theta(s, t)}{\partial t^2} = \frac{\partial q(s, t)}{\partial s} + (\hat{\mathbf{t}}(s, t) \times \mathbf{f}(s, t)) \cdot \hat{\mathbf{e}}_3 + q_{\text{ext}}(s, t) \quad (2)$$

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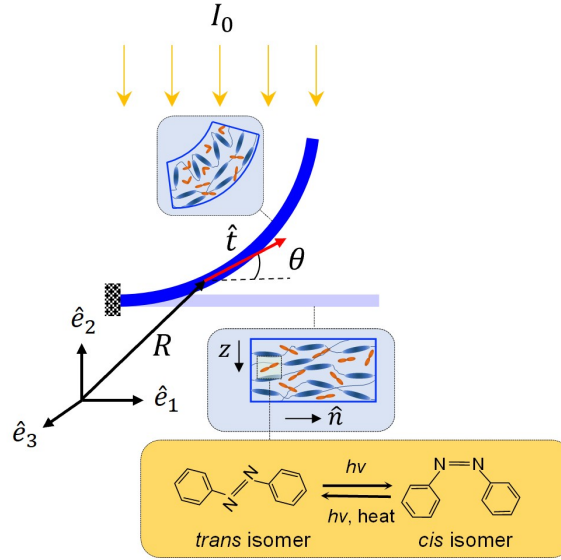


Fig. 1 Azo-LCE deformation under light.

where $\mathbf{f}$ and q are the internal force vector and moment, respectively, $\mathbf{f}_{\text{ext}}$ and q_{ext} are any external force vector and moment per unit length applied to the strip, respectively, m and Π denote the mass and rotational moment of inertia per unit length, respectively. Let n_c denote the fraction of azo molecules in the *cis* state and n_t denote the fraction of azo molecules in the *trans* state. At a depth z at time t , the rate of change of the *cis* volume fraction is [8]

$$\frac{\partial n_c(s, z, t)}{\partial t} = A_t I(s, z, t) n_t(s, z, t) - A_c I(s, z, t) n_c(s, z, t) - \frac{n_c(s, z, t)}{\tau} \quad (3)$$

where $n_t + n_c = 1$ and I is the absorbed light intensity at depth z at time t and point s . At any point s , the light intensity diminishes with depth as it is absorbed by the azo molecules

$$\frac{\partial I(z, t)}{\partial z} = \frac{-I(z, t)}{d_0} \quad (4)$$

where d_0 is the penetration depth. For the definition of utilized parameters, refer to [7, 9]. The photo-isomerization of the azo molecules leads to a light-induced spontaneous axial strain $\varepsilon_0(z, t) = \lambda n_c(z, t)$ in the material where λ is a material constant. The non-uniform distribution of light-induced axial strain at each cross-section causes the strip to bend. Note that any part of the strip may block light from reaching another part, a phenomenon known as *self-shadowing*. In the self-shadowed region, $I = 0$; therefore, the *cis* molecules relax to the *trans* state, causing the strip to thermally relax in the shadowed area. The initial-boundary-value problem above for a photomechanical strip is solved via a finite difference method by discretizing the equations in both space and time.

Results and Discussion

Figure 2 shows the simulation results for a monodomain LCE cantilever illuminated perpendicularly from above with a light intensity of $I_0 = 4000 \text{ W/m}^2$. Other simulation parameters are listed in Table 1. Figure 2A shows that, beyond a critical illumination intensity, the rod reaches a steady-state flapping motion with a frequency of 19 Hz. Interestingly, this value is very close to the natural frequency of a clamped-free rod, which in this case is 18 Hz. Our analysis shows that the strip largely bends up and makes shadow on the illuminated section near the clamped end, so the strip unbends to relax the bending energy. Indeed, self-shadowing plays the role of negative feedback loop in self-sustained oscillation. As expected, the oscillation frequency is independent of external stimulus (illumination intensity). Figures 2B, C also show that the steady-state response, once reached, is symmetric, but as the velocity curve clearly shows, the symmetry point is not exactly at $x = 0$. This is because the rod is initially placed in the positive x direction, and since it is clamped at one end, it cannot bend completely to the other side, leading to a small asymmetry. Figure 2D shows the evolution of photo-induced

spontaneous strain. As can be seen in this figure, the steady state photo-strain at the centerline is very small and can be neglected, showing that the assumption of the rod's inextensibility is correct. There are multiple parameters that can affect the motion of the rod and its time scale. A more detailed analysis can lead to interesting results, paving the way towards a self-sustained motion using light.

Table 1 Simulation parameters

Parameter (symbol)	Value
Young's modulus (E)	4 GPa
Poisson's ratio (ν)	0.3
Mass density (ρ)	1400 kg/m ³
Strip thickness (H)	15 μ m
Strip width (W)	1 mm
Strip length (L)	15 mm
<i>trans</i> absorption coefficient (A_t)	5×10^{-3} m ² /J
<i>cis</i> absorption coefficient (A_c)	5×10^{-4} m ² /J
Thermal relaxation time (τ)	0.2 s
<i>cis</i> -strain proportionality (λ)	-0.05 [10]

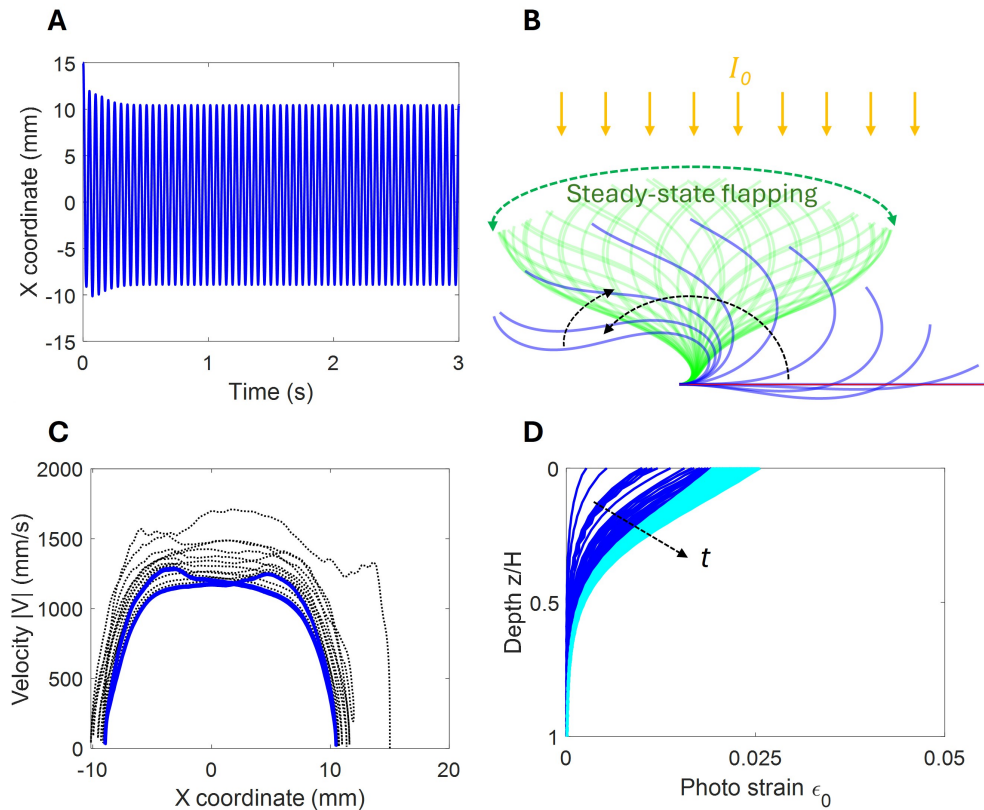


Fig. 2 Self-sustained flapping of the Azo-LCE under constant light illumination using typical parameters for Azo-LCE. (A) The x coordinate of the end-point vs. time, (B) the transient and steady-state flapping of the rod. The red line indicates the initial configuration, blue lines indicate the transient response, and green lines show the self-sustained steady-state flapping of the rod. (C) The velocity of the end-point vs. the x coordinate. The dotted lines show the response in the transient state and the solid blue lines show the steady-state response, (D) the evolution of the light-induced spontaneous strain ϵ_0 in depth and time, showing a very small strain at the centerline of the rod ($z/H = 0.5$, where H is the thickness of the rod). The light blue lines indicate the steady-state response.

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Chapter 11

Seal Whisker-Inspired Sensor for Amplifying Wake-Induced Vibrations in Underwater Marine Animal Monitoring

Yuyan Wu, Sanjay Giridharan, Leixin Ma, and Hae Young Noh

Abstract Underwater marine animal monitoring is essential for assessing biodiversity, evaluating ecosystem health, and understanding the effects of offshore structures. Traditional approaches such as tagging, sonar, and camera systems are often invasive, energy-intensive, or limited by poor visibility and water turbidity. Inspired by the hydrodynamic sensing of seal whiskers, wavy whisker vibration sensors have been developed for flow velocity and angle-of-attack detection. However, most prior work has focused on sensor characterization and only forward modeling, with limited exploration of the inverse problem of inferring animal movement. Moreover, current sensor sensitivity to vortex street wakes generated by swimming animals remains insufficient for practical monitoring. To address this gap, we develop a whisker-inspired sensor with a spiral-perforated base that amplifies vibrations within frequency ranges relevant to animal-induced wakes. We further characterize the influence of spiral parameters on the sensitive frequency band, enabling adaptation of the design to specific species. We evaluated the amplification effect of the spiral-perforated design using frequency response simulations of the whisker-base structure under harmonic water pressure. Results show up to 51× enhancement in root mean squared displacement at the target sensor location within frequency bands associated with animal-induced wakes compared to the baseline design, confirming the effectiveness of the amplification.

Keywords Underwater marine animal monitoring · Bio-inspired sensing · Seal whisker · Vibration amplification structure · Wake-induced vibrations

Introduction

Underwater marine animal monitoring, including tracking their populations and behaviors, is essential for understanding biodiversity, assessing ecosystem health, and evaluating the ecological impacts of offshore infrastructure. Keystone species [1] such as whales and sharks play critical roles in maintaining the underwater food web balance [2, 3, 4]. Monitoring their populations is important for detecting early signs of ecological imbalance. In addition, many marine animals, such as corals, fish, and mammals, are sensitive to temperature shifts, ocean acidification, and oxygen depletion [5, 6, 7]. Monitoring changes in their distribution, migration, and behavior provides critical insight into the effects of climate change on marine ecosystems. Furthermore, marine animal monitoring is crucial for understanding how offshore infrastructure, such as wind farms and oil rigs, affects marine fauna. Construction and operational noise can disrupt communication, navigation, and foraging [8, 9]. Infrastructure may also create artificial reefs that attract certain species or disrupt migration routes of sea turtles, fish, and marine mammals [10, 11]. Animal monitoring identifies ecological changes and reveals how animals adapt to offshore infrastructures, enabling governments to implement adaptive management strategies that minimize harm.

Existing methods for monitoring marine animals include tagging, camera-based systems, acoustic sensing, and artificial lateral lines (ALL). Tagging involves attaching archival biologgers or transmitting biotelemetry to record individual movement and behavior [12, 13]. This method provides detailed, long-term data but is invasive and impractical for large populations. Cameras offer a non-invasive alternative, capturing images to monitor species population and behavior [14, 15, 16].

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However, its effectiveness is constrained by underwater lighting conditions, making it unsuitable in dark or turbid environments. Acoustic sensing, mainly including sonar, transmits sound pulses and analyzes returning echoes to track the population and movement of marine animals and identify species [17, 18, 19]. It is non-invasive, effective in turbid or dark waters, and covers large ranges. However, its active sensing mechanism is energy-intensive. ALL systems, inspired by the sensory organs of fish and amphibians, detect flow velocity and pressure gradients through arrays of pressure or flow sensors (e.g., MEMS-based or piezoelectric) mounted on underwater robots [20, 21, 22]. They are non-invasive, energy-efficient, and robust in dark or turbid waters. However, ALL sensors exhibit limited sensitivity to wakes generated by marine animals and are highly susceptible to turbulence and background currents in complex underwater environments, which reduces their effectiveness for long-distance detection [23, 24].

To overcome these limitations, we introduce seal whisker-inspired sensors for underwater marine animal monitoring. These sensors emulate the mechanism by which seals detect prey using their whiskers. Swimming marine animals generate vortex street wakes through body movement, and these wakes induce vibrations in seal whiskers at their characteristic natural frequency. Seals then detect these vibrations to localize and track prey [25]. Seal whiskers have a wavy morphology to suppress self-induced vortices and thereby enhance the signal-to-noise ratio for long-distance wake detection [26, 27]. Compared to other methods, whisker-inspired sensing is non-invasive, passive, energy-efficient, and independent of lighting conditions. By reducing self-motion noise, it enables more reliable detection of animal-induced vortex street wakes over long distances. Seal whisker-inspired sensors have recently been used to measure hydrodynamic parameters, such as flow velocity, angle of attack, and pressure [28, 29, 30, 31]. However, existing whisker-inspired sensor works primarily focus on characterization and only forward modeling, using wavy morphologies to suppress self-induced vortices [32, 29, 30, 31]. The inverse problem of inferring marine animal movement remains challenging, as animal-induced wakes are small in amplitude and difficult to detect. Swimming marine animals typically induce velocity fluctuations on the order of several centimeters per second [33, 34], corresponding to dynamic pressure variations of roughly 1 Pa (according to the Bernoulli relation). This magnitude falls below the sensitivity threshold of current underwater pressure-sensing arrays [35]. Furthermore, these vortices dissipate rapidly [36], making detection even more challenging.

We address this challenge through a novel sensor hardware design of a spiral perforated base that amplifies whisker vibrations within targeted frequency ranges. By adjusting the base geometry, the design can be tuned to amplify whiskers with different natural frequencies, enabling customization for specific target frequency bands. This amplification enhances the signal-to-noise ratio for underwater marine animal monitoring by amplifying the desired vibration response while suppressing noise from self-motion and background underwater disturbances with a wavy shape whisker geometry. Furthermore, we characterize how geometric parameters of the whisker base influence amplification rates and frequency-band shifts. This characterization provides guidelines for tuning geometry parameters to optimize amplification for different target frequency bands.

We evaluated the effectiveness of the whisker base design by conducting frequency-response simulations and comparing the results with a conventional whisker sensor design. The simulations demonstrate that our whisker base design amplifies whisker vibration amplitude by up to 51 times.

The main contributions of this paper are as follows:

- We develop a seal-whisker-inspired sensor for underwater marine animal monitoring through whisker vibrations induced by animal-generated wakes.
- To overcome the challenge of inherently small amplitudes of animal-induced wakes, we design a novel sensor hardware with a spiral-perforated whisker base that amplifies wake-induced vibrations within the target frequency range, thereby enhancing the sensitivity of the whisker-inspired sensor. We further characterize how whisker geometry parameters influence amplification rates and frequency bands.
- We evaluated the proposed design through frequency-response simulations of the seal whisker-base system, demonstrating the effectiveness of amplification.

In the remainder of this paper, we first introduce the background on the mechanisms by which seals detect prey through their whiskers and the geometric characteristics of seal whiskers. We then present the design of the whisker base and evaluate its effectiveness through frequency-response simulations. Finally, we characterize how the geometric parameters of the whisker base affect amplification and the associated frequency ranges.

Background: Mechanisms of Animal Detection by Seal Whiskers

Seals can detect prey over 180 meters away using their whiskers [37]. As marine animals move underwater, they generate trailing vortices known as reverse von Kármán vortex streets [38]. When these vortex street wakes pass over seal whiskers,

the whiskers vibrate in synchrony with the wakes (see Figure 1a). Mechanoreceptors at the whisker base transduce these vibrations into neural signals that provide information about prey location and movement. Moreover, because different marine animals produce distinct vortex patterns [39, 40], seals can also identify prey species from the wake-induced whisker vibrations [41]. Seal whiskers exhibit a characteristic wavy geometry with elliptical cross-sections (see Figure 1b). This geometry can be parameterized by $(a, b, k, l, M, \alpha, \beta)$ as defined in Figure 1b, where (a, b) and (k, l) the lengths of the semi-major and semi-minor axes of the maximum and minimum elliptical cross-sections, respectively; M denotes the distance between the adjacent cross-sections; and (α, β) specify the orientation angles of these cross-sections. The whisker tapers from a thick base to a gradually thinner tip. Furthermore, seals possess whiskers with diverse natural frequencies, enabling detection across a broad frequency band (Figure 1c) [42].

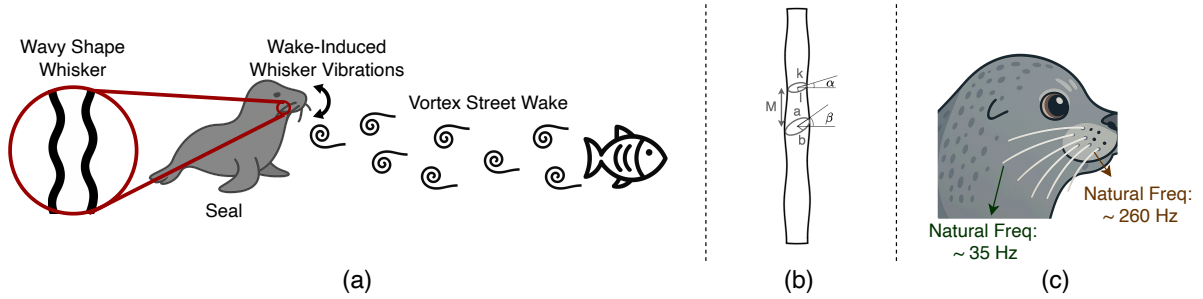


Fig. 1 (a) Seals track prey using wavy shape whiskers that detect vibrations induced by vortex street wakes generated by swimming animals. (b) Enlarged view of the wavy whisker cross-section, illustrating elliptical geometry with major and minor axes (a, b, k, l) and orientation angles (α, β) . (c) Seals have an array of whiskers with various natural frequencies, facilitating broadband hydrodynamic sensing across multiple frequency ranges.

Seal whisker-inspired sensors [29, 30, 31, 28] replicate the wavy geometry of seal whiskers, which suppress self-induced vibrations during swimming and filter out background noise [26, 27]. When a straight, cylindrical whisker moves through water, it creates strong von Kármán vortex streets, which are the alternating vortices that are shed off the whisker in a regular pattern [43, 44]. This vortex shedding produces vibrations (self-noise) that mask the tiny water movements caused by prey. The wavy geometry disrupts these regular vortex streets, breaking them into smaller, less coherent vortices [26, 27]. This reduces the strength of vibrations transmitted to the whisker base, making them less sensitive to noise induced by self-motion. These wavy shape whiskers enable seals and whisker-inspired sensors to detect and track hydrodynamic wakes left by animals over a long distance. However, current studies on whisker-inspired sensors have primarily focused on mitigating self-induced vortices [32, 29, 30, 31], with limited exploration of mechanisms to enhance sensitivity to animal-induced vortices.

Whisker-Inspired Sensor Design for Wake-Induced Vibration Amplification

To enhance sensitivity to wake-induced vibrations within the frequency range of the animal-induced wakes, we designed a spiral-perforated whisker base that increases structural flexibility and amplifies vibration amplitude in the target band. The whisker is mounted at the base center (Figure 2a), with the bottom view shown in Figure 2b. The sensing point, located on the opposite surface directly beneath the whisker attachment, is intended for embedding a vibration sensor. This spiral-perforated design amplifies displacement at the sensing point under harmonic pressures generated by animal-induced wakes.

The spiral perforations reduce the effective stiffness of the whisker base by extended deformation paths, thereby amplifying the vibrations of the whiskers. Modeling the whisker base system as a single degree-of-freedom (SDOF) translational oscillator, its displacement $x(t)$ under vortex forcing $F_0 e^{j\omega t}$ is governed by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_0 e^{j\omega t}, \quad (1)$$

where m is the combined effective mass of the whisker and base, c is the viscous damping coefficient, and k is the effective stiffness determined by the spiral-perforated base and whisker structure. The steady-state harmonic response is

$$X(\omega) = \frac{F_0}{k - m\omega^2 + jc\omega}, \quad |X(\omega)| = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad (2)$$

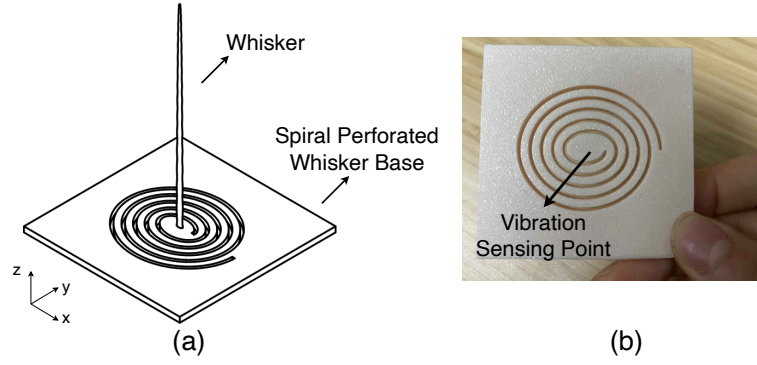


Fig. 2 (a) Design of the whisker-inspired sensing unit and spiral perforated base. The whisker is mounted on a spiral perforated base, which amplifies vortex-induced vibrations. (b) Photo of the spiral perforated base. The vibration sensing point is at the middle of the base, where a vibration sensor will be mounted to capture vortex-induced whisker vibrations.

with $X(\omega)$ denoting the Fourier-domain displacement amplitude. Modeling the solid segments within the spiral region of the base as beam elements, the effective stiffness $k \propto \frac{EI}{L^3}$, where E is the Young's modulus, $I = \frac{bh^3}{12}$ is the second moment of area with b representing the width of the spiral region and h the thickness of the whisker base, and L is the effective load-path length, positively correlated to the total spiral beam length. The spiral geometry increases L and reduces b , thereby decreasing k and enhancing the vibration amplitude $|X(\omega)|$.

Vibration amplification occurs through resonance coupling when the whisker and base exhibit closely matched natural frequencies in a coupled vibration mode. In the case where the exciter frequency lies below the natural frequency of the coupled system, the whisker and base primarily respond in phase. Under this same-phase condition, resonance coupling reinforces their motion and amplifies vibration, enabling efficient energy transfer [45, 46]. At resonance, the whisker and base interact strongly, oscillate coherently, and enhance the efficiency of energy transmission.

The spiral geometry provides multi-directional flexibility, enabling torsional deformation in all three principal directions (x , y , z). As shown in Figure 3a–c, the whisker base exhibits resonant modes in all three directions, confirming its capability for lateral, vertical, and rotational deformation. For in-plane lateral flexing in the x - and y -directions, the spiral perforations create space within the whisker base to compress and extend, thereby increasing flexibility in response to horizontal forces and amplifying vibration. In the z -direction, compared to conventional solid bases, which restrict vertical motion due to their resistance to out-of-plane deformation, the spiral-perforated base behaves as a compliant spring, allowing greater vertical displacement of the whisker. This multi-directional flexibility is crucial for capturing animal wake dynamics, which are inherently three-dimensional [47]. Alternating vortices shed from the animal's tail generate forces in both horizontal directions, while body undulations and vortex interactions produce vertical and torsional components. The spiral-perforated base acts as a multi-axis amplifier, converting these complex 3D vortex forces into measurable vibrations across all directions (x , y , z), thereby enhancing the system's sensitivity to multi-directional vortex-induced oscillations.

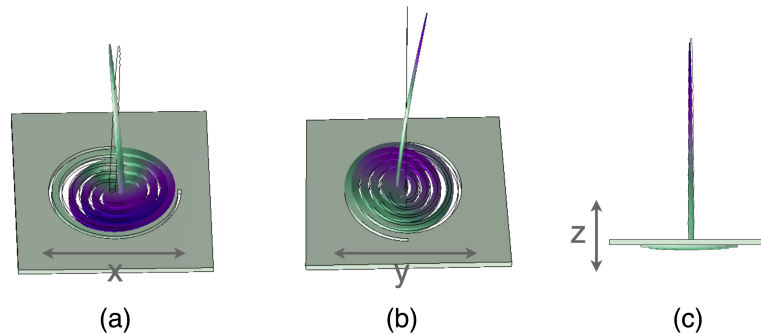


Fig. 3 Mode shapes of the spiral-perforated whisker base showing flexible vibrations along (a) the x -axis, (b) the y -axis, and (c) the z -axis.

Characterization of Spiral Base Parameters on Whisker Vibration Amplification

We characterize the influence of spiral-shaped whisker base geometry on vibration amplification and frequency response through frequency-response simulations in COMSOL Multiphysics. The vibration amplification is represented by the root mean square (RMS) displacement measured at the sensing point located at the center of the whisker base (Figure 2b) under harmonic water-pressure excitation at varying frequencies. The geometric parameters investigated include base thickness, number of spiral turns, spiral growth rate, aspect ratio (minor-to-major axis ratio), and perforation slot width. In addition, we analyze the frequency response when vortex forces are applied along different directions (x and y axes).

The simulation parameters were configured as follows: the rectangular spiral-perforated whisker base was fixed along all four lateral surfaces, and a harmonic water pressure of 15 Pa was applied in the x-direction, aligned with the whisker's major cross-sectional axis. The whisker geometry for characterization consisted of 33 sections, corresponding to lengths of 73.7 mm. The other whisker parameters were $M = 1.3$ mm, $(a, b, k, l) = (0.7, 0.29, 0.62, 0.33)$ mm, and $\alpha = 8.7^\circ$, $\beta = 13.24^\circ$, based on the grey seal whisker geometry reported in previous work [37]. The taper factor followed $taper\ factor = (1 - n/N)^{0.5}$, where N is the total number of sections and n is the section index ($n = 1$ at the whisker base). For the n -th cross-section, the elliptical dimensions (a, b, k, l) and inter-section spacing $2M$ were scaled by the corresponding taper factor. The whisker and base material were selected as PLA with density $\rho = 1240\text{ kg/m}^3$, Young's modulus $E = 3\text{ GPa}$, and Poisson's ratio $\nu = 0.35$. PLA was selected because it is the primary 3D printing material for fabricating the whisker and base structures. An extra fine mesh was employed in the COMSOL simulations to ensure numerical accuracy. The default whisker base configuration had a thickness of 2 mm, 5 spiral turns, a growth rate of 0.5, an aspect ratio of 0.6, a slot width of 1 mm, with harmonic pressure applied along the x-axis.

The whisker base thickness influences both the amplification frequency range and magnitude. Increasing the base thickness shifts the amplification effect to higher frequencies while reducing its magnitude. As shown in Figure 4a, when the base thickness increases from 1 mm to 3 mm, the peak frequency of the RMS displacement response shifts from 20 Hz to 60 Hz. This occurs because greater thickness increases the structural stiffness of the whisker base, thereby raising the natural frequency. However, the amplification magnitude at 3 mm thickness is notably smaller than at 1 mm and 2 mm. According to Equation 2, increased stiffness k reduces the displacement amplitude under constant force, resulting in lower amplification.

Increasing the number of spiral turns in the whisker base shifts the large displacement frequency range to lower frequencies and enhances the amplification magnitude (see Figure 4b). This occurs because additional turns increase the effective length of the supporting beam, thereby reducing the structural stiffness. According to Equation 2, lower stiffness results in

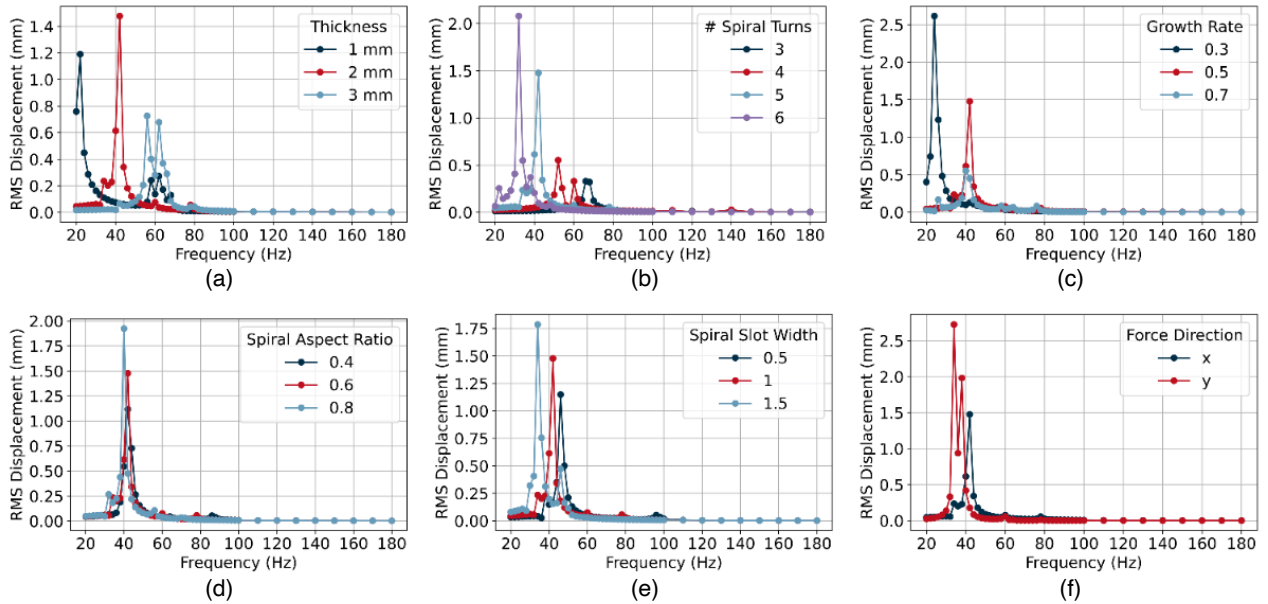


Fig. 4 Frequency response of RMS displacement at the sensing location under an applied pressure of 15 Pa. Subplots compare the effects of different whisker base parameters: (a) whisker base thickness, (b) number of spiral turns, (c) spiral growth rate, (d) spiral aspect ratio, (e) spiral slot width, and (f) force direction (x vs. y).

greater amplification at lower frequency ranges.

Increasing the spiral growth rate shifts the sensitive frequency range to higher frequencies while reducing the amplification magnitude. As shown in Figure 4c, increasing the growth rate from 0.3 to 0.7 raises the peak frequency from 24 Hz to 40 Hz while decreasing the RMS displacement, indicating reduced vibration amplification. This occurs because a higher growth rate increases the effective width of the supporting beam, thereby increasing the structural stiffness and resulting in higher resonant frequencies with lower displacement amplitudes.

The aspect ratio of the spiral has a relatively minor effect on both amplification and frequency range. The aspect ratio is defined as the ratio of the short axis to the long axis of the elliptical spiral. As shown in Figure 4d, increasing this ratio from 0.4 to 0.8 does not significantly affect the peak frequency or the amplification magnitude. This occurs because the aspect ratio does not substantially alter the dimensions of the supporting beam and therefore has minimal impact on the structural stiffness.

Increasing the spiral slot width shifts the amplification frequency range to lower frequencies while enhancing the amplification magnitude. As shown in Figure 4e, increasing the slot width from 0.5 mm to 1.5 mm shifts the resonant peak from 42 Hz to 36 Hz, with a corresponding increase in RMS displacement. This occurs because wider slots reduce the effective width of the supporting beam, thereby decreasing the structural stiffness k and resulting in lower resonant frequencies and larger displacements under the same applied force.

Harmonic water pressure applied in the y-direction induces larger vibration displacement at lower frequencies. As shown in Figure 4f, when the same pressure amplitude (15 Pa) is applied to the whisker surface, the y-direction exhibits greater displacement amplitude. This occurs because the y-axis is perpendicular to the long axis of the elliptical whisker cross-section, resulting in a larger contact area and consequently a greater effective force on the whisker structure. Additionally, due to the elliptical geometry of the whisker base, vibration along the y-direction (corresponding to oscillation along the short axis) represents a lower-order mode with a lower natural frequency. Therefore, the resonant peak occurs at a lower frequency than for the x-direction pressure.

Evaluation of Amplification Effectiveness

We evaluate the amplification effect of the developed whisker base design through frequency-response simulations in COMSOL Multiphysics. Similar to simulations in characterization, root mean square (RMS) displacement at the sensing point (whisker base center) is used to quantify amplification. We compare the RMS displacement of our design against a baseline whisker structure for three whisker morphologies of different lengths, evaluating its effectiveness across various whisker geometries. Compared to the baseline rectangular-plate support, our design successfully amplifies vortex-induced vibrations for all three whisker morphologies, achieving amplification factors of up to 51.

We compared the RMS displacement of our whisker base design with the baseline design [30], in which the whisker was mounted on a single rectangular plate clamped along one short edge, with displacement measured at the opposite free edge. We evaluated three whisker morphologies with 33, 28, and 23 sections, corresponding to lengths of 73.7, 54.2, and 38.3 mm, respectively (referred to as Whisker 1, 2, and 3). These configurations were selected to represent the diverse whisker morphologies and natural frequencies observed in seal whisker arrays [42]. The simulation parameter configurations are identical to those in the characterization section.

To achieve resonance coupling, the whisker base geometry was adjusted so that its natural frequency matched the whisker's natural frequency in the coupled torsional mode. The optimized parameters were: for Whisker 1, thickness = 2 mm and number of turns = 5; for Whisker 2, thickness = 4 mm and number of turns = 5; for Whisker 3, thickness = 2 mm and number of turns = 3. All other geometric parameters remained constant: slot cut width = 1 mm, ellipse major axis = 5 mm, and ellipse minor axis = 3 mm. These configurations were selected to match the whisker's natural frequency with the natural frequency of the whisker base corresponding to the torsional mode shape in the x-direction. Figure 5a–c illustrates the frequency matching for Whisker 1, showing the base torsional mode, the whisker vibration mode, and the coupled whisker–base system. Figure 5d summarizes the matching of whisker and base natural frequencies across all three whisker geometries.

Our spiral-perforated base design achieves significantly larger RMS displacement amplitudes for all three whisker geometries compared to the baseline, demonstrating the effectiveness of the amplification. Figure 6 shows the frequency response for both our design and the baseline across all three whisker geometries. Whisker 1 exhibits a sensitive frequency range of 30–40 Hz with an amplification factor of 51.13. Whisker 2 is responsive to frequencies of 40–100 Hz with an amplification factor of 3.46, while Whisker 3 demonstrates sensitivity in the 100–150 Hz range with an amplification factor of 4.23. The amplification factor is calculated as the ratio between the maximum RMS displacement among all the frequency values

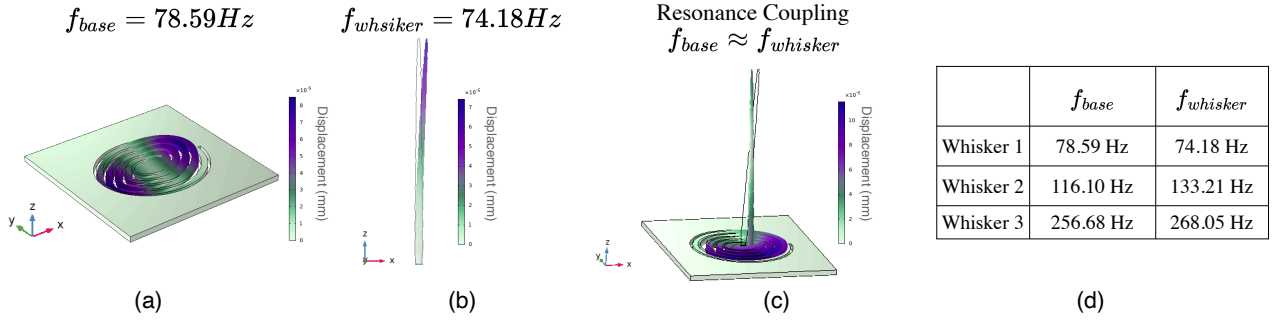


Fig. 5 Resonance coupling between whisker base and whisker modes. (a) Base torsional mode shape about the x-axis for Whisker 1 with the natural frequency of 78.59 Hz. (b) Whisker bending mode shape along the x-axis with natural frequency of 74.18 Hz. (c) Coupled system visualization demonstrating near-resonant interaction between base torsion and whisker bending modes. (d) Modal frequency comparison for three whisker geometries showing base (f_{base}) and whisker ($f_{whisker}$) natural frequencies in Hz.

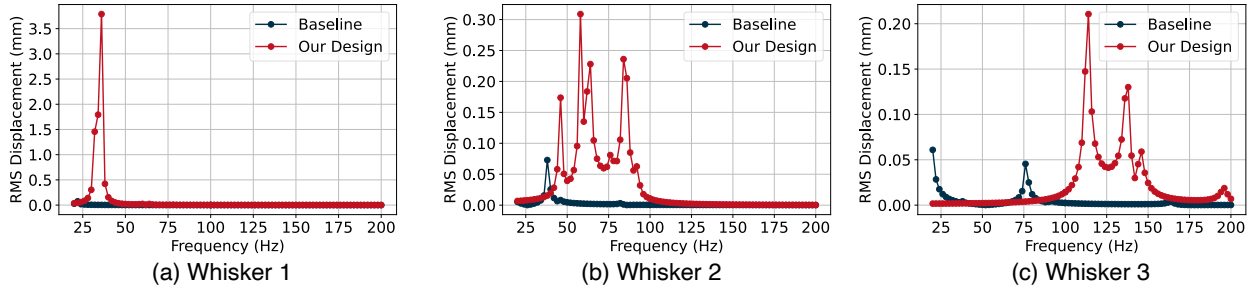


Fig. 6 Frequency response of RMS displacement at the center sensing point for (a) Whisker 1 (33 sections), (b) Whisker 2 (28 sections), and (c) Whisker 3 (23 sections), comparing the baseline design and our spiral-perforated whisker base design.

for each design. These sensitive frequency ranges align with the natural frequencies of biological seal whiskers [42, 30], which primarily span 30–200 Hz. These results confirm that the spiral-perforated whisker base design effectively amplifies vibrations across the diverse frequency ranges characteristic of natural seal whisker arrays.

We observe that the peak frequency of vibration amplification for each whisker is smaller than the natural frequency of the uncoupled whisker or base (f_{base} and $f_{whisker}$). This shift arises because the two systems are fixed together and dynamically coupled. Once coupled, the combined system no longer resonates at the original single-body frequencies. Instead, its modal frequencies split into two: one lower than the lowest original natural frequency and the other higher than the highest [45]. The lower mode corresponds to the in-phase response, which reinforces motion and produces the amplification peaks we observe [45, 46]. Consequently, the dominant amplification appears at a frequency below either of the original natural frequencies.

Conclusions and Future Works

In this work, we introduce a seal whisker-inspired sensor for underwater marine animal monitoring and develop a spiral-perforated whisker base design that amplifies weak vibration amplitudes induced by animal-generated wakes. The spiral perforation pattern increases the structural flexibility of the whisker base and reduces effective stiffness, thereby enhancing whisker sensitivity within frequency bands corresponding to target animals. We characterize the influence of geometric parameters on both amplification magnitude and frequency range, providing guidelines for adapting whisker base designs to target frequency bands. To evaluate the amplification effect, we conducted frequency-response simulations on three whisker morphologies targeting 30–40 Hz, 40–100 Hz, and 100–150 Hz, respectively. Our whisker base design achieved up to a 51× increase in RMS displacement compared to the baseline whisker base. The amplification frequency range can be tuned to

specific detection targets by adjusting whisker morphology and base geometry.

In the future, we will extend our work on whisker-inspired sensing for underwater marine animal monitoring through real-world experiments. First, we will evaluate the sensor sensitivity and the effectiveness of whisker-inspired sensor design in detecting animal-induced wakes in natural underwater environments. Additionally, we will develop algorithms for detecting, classifying, and tracking marine species to enable long-term, non-invasive monitoring. Furthermore, we will design whisker arrays comprising sensors tuned to different frequency ranges. By optimizing the cooperative behavior of these arrays, we aim to enhance robustness and adaptability across diverse species and hydrodynamic conditions.

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Chapter 12

From Short-Term Monitoring to Long-Term Insight: VAE Driven Synthetic Data for SHM Applications



M. Conti, G. Battista, S. Pavoni, and M. Vanali

Abstract Structural Health Monitoring (SHM) is essential for ensuring the safety and durability of structures, buildings, and industrial components. In recent years, data-driven SHM techniques have gained increasing attention due to their versatility for different structures and their ability to detect damage and assess structural conditions. However, their main limitations lies in the need for large dataset to effectively train the algorithms. Acquiring such data requires continuous monitoring over extended periods (sometimes years) making the process costly, time-consuming, and often impractical.

To overcome these challenges, recent studies have explored synthetic data generation to supplement real-world datasets. Among the most promising approaches is the use of Variational Autoencoders (VAEs), a class of generative models capable of learning compact representations of input data and producing realistic new samples based on learned distributions. VAEs are probabilistic generative models that combine encoder-decoder networks to learn the characteristics of input data and generate novel coherent data samples.

In this study, VAEs are employed to generate synthetic structural response parameters using only a limited set of real-world measurements from a short monitoring period. The benchmark is a laboratory truss girder subjected to environmental variations (temperature drift and thermal gradient evolution). The goal is the augmentation of the available data and the improvement of effective training of SHM algorithms, even when long-term monitoring data is unavailable.

Keywords Structural health monitoring · Variational autoEncoders · Machine learning · Data augmentation

Introduction

Ensuring the safety and durability of civil and mechanical structures is a major challenge in modern engineering. Structural Health Monitoring (SHM) has emerged as a key discipline to address this challenge, providing diverse methods to evaluate the structure's condition. These strategies involve the continuous observation of the structural characteristics and the extraction of damage-sensitive features used for the evaluation of the system's state [1]. In recent years, strategies based on vibration measurements have gained a central role, due to their easy implementation in sensor installation and their non-destructive nature [2]. Several studies have demonstrated that modal parameters are found to be particularly effective in the detection of the condition variations of the structures investigated [3]. However, these strategies present a critical drawback, since the employed damage-sensitive features can be strongly influenced by environmental and operational factors too [4]. For this reason, especially in the data-driven methods, the collection of a large number of observations is necessary to both remove the external impact [5] and train machine learning algorithms [6]. In fact, data-driven algorithms usually require the collection of very large datasets to capture all variations caused by daily and seasonal environmental variations. In many cases, acquiring such datasets requires continuous monitoring over extended periods (sometimes spanning years), making the process costly, time-consuming, and often impractical [7].

To address these challenges, recent research has progressively focused on developing synthetic data generation strategies in order to supplement existing real-world datasets. These generative models (GMs) typically learn the distribution of a specific given dataset and generate new data coherent with the original ones [8]. Common architectures of GMs employed in this field are the Generative Adversarial Networks (GANs) [9], the Diffusion Models (DMs) [10], and the AutoEncoders

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(AEs) [11]. Among the three techniques mentioned, AEs provide a simpler and more stable training process through direct reconstruction loss, avoiding the instability of adversarial training. Additionally, their feed-forward decoders allow for rapid data generation [12]. Particularly useful in this context are the Variational AutoEncoders (VAEs), which, unlike the classic AEs, map the input image in a probabilistic distribution (usually a Gaussian) instead of a single point in the latent space.

In this work, Convolutional Neural Network (CNN) [13] based VAEs are employed to generate new data to implement a pre-existing dataset collected to develop novel SHM strategies. The research aims to increase the observations of the modal parameters affected by environmental influence, used to improve the characterisation of the ambient variability in the baseline of an already employed SHM strategy. The vibration measurements are collected from a laboratory truss girder, realised ad hoc in the noise and vibration laboratory at the University of Parma, for SHM purposes [14]. As the structure is subjected to a white-noise excitation, the generation of vibration measurements directly is impractical, because VAEs are not able to detect patterns in such a case. Thus, the data generated are the power and cross-spectra evaluated between the acceleration measurements, which present a more consistent and perceptible pattern. In the following section, a brief introduction to the benchmark is proposed.

Benchmark and Experimental Setup

The benchmark structure is a laboratory Pratt truss girder. It is a 2D truss anchored to the floor with rigid supports and Teflon roller connections, spanning 4.2 m in length and 0.8 m in height. The truss consists of aluminium diagonal and vertical rods, and two horizontal steel H-beams. Stability is preserved by seven 15 kg masses at the base and four struts with roller cylinders, allowing only vertical movement.

Since ambient excitation is insufficient, the structure is vertically forced using a PCB 2007E electrodynamic shaker with a band-limited white-noise (10-250 Hz). The response is measured with six MEMS accelerometers (TE4030, ± 2 g), while four thermocouples monitor ambient conditions. Tests are repeated every 30 minutes to track changes in modal and environmental parameters. In Figure 1 the experimental setup layout is presented.

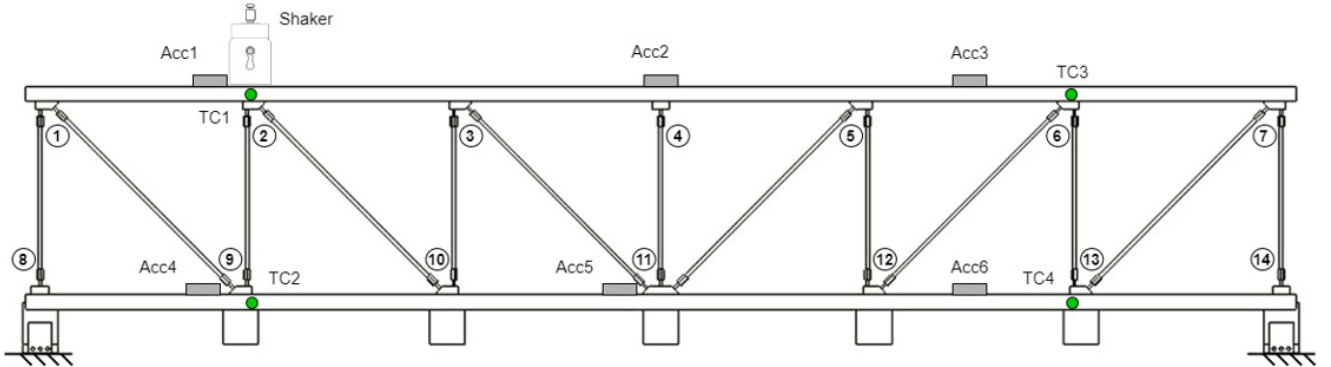


Fig. 1 Truss sketch with node numeration and data acquisition system layout with shaker, accelerometers (grey rectangles) and thermocouples (green circles) location

Damage-Sensitive Features Extraction

The damage-sensitive parameters employed in already improved SHM strategies are the modal parameters, in particular, natural frequency and mode shapes. However, in this work, only the enhancement of the natural frequencies dataset is investigated. Therefore, natural frequencies are extracted using the power and cross-spectra by exploiting an enhanced Frequency Domain Decomposition (FDD) method. In more depth, the response power spectra are processed using Singular Value Decomposition (SVD) as follows

$$G_{xx}(\omega) = U(\omega) S(\omega) U(\omega)^H \quad (1)$$

where $S(\omega)$ is a diagonal matrix containing the singular values, which are employed to identify the resonances. Natural frequencies are extracted by applying the Peak Picking (PP) method to $S(\omega)$ [15]. To reduce the effects of the frequency

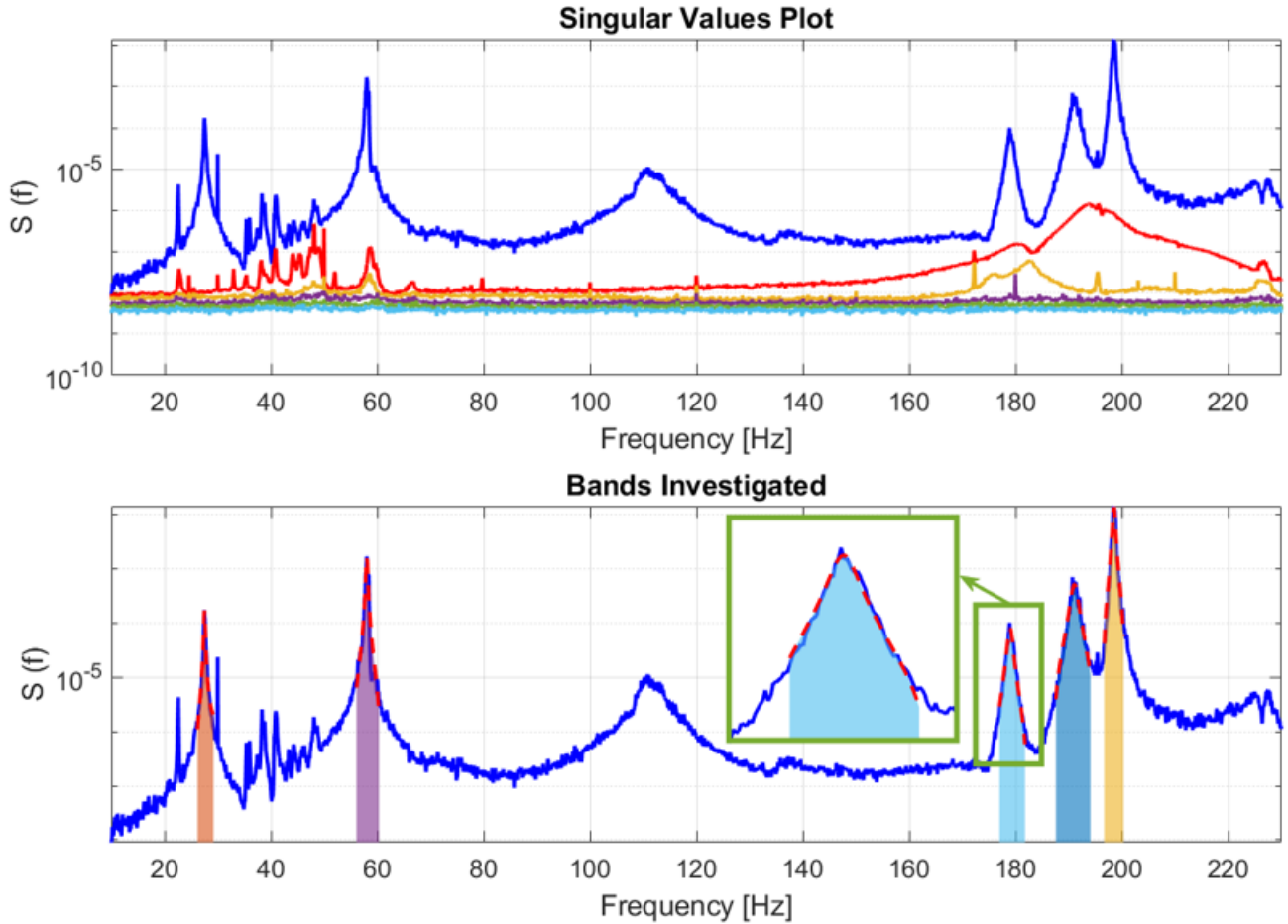


Fig. 2 Enhanced FDD performed on the truss girder data with bands analysed and reconstruction of resonance peaks

resolution, the singular value is minimised using a Lorentzian Peak-Shape function [16]. An example of the procedure is shown in Figure 2.

The natural frequencies investigated are listed in Table 1.

Table 1 Natural frequencies of the truss girder

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Natural Frequency [Hz]	27	58	179	191	198

Natural frequencies collected while the structure was in a healthy condition and used as the baseline set, encompass the period from 6th April to 22nd July and consist of 3014 observations. In Figure 3, the baseline relationship between natural frequencies is shown.

These observations are employed to train a generative model for enhancing the baseline set. Details of that process are introduced in the following section.

Generative Model and Generation of New Samples

Variational Autoencoders (VAEs) Introduction

VAEs are generative models that learn how to represent the features of a dataset in a structured latent space. In contrast to classic AEs, which compress inputs into fixed vectors, VAEs encode data into probability distributions (usually Gaussian).

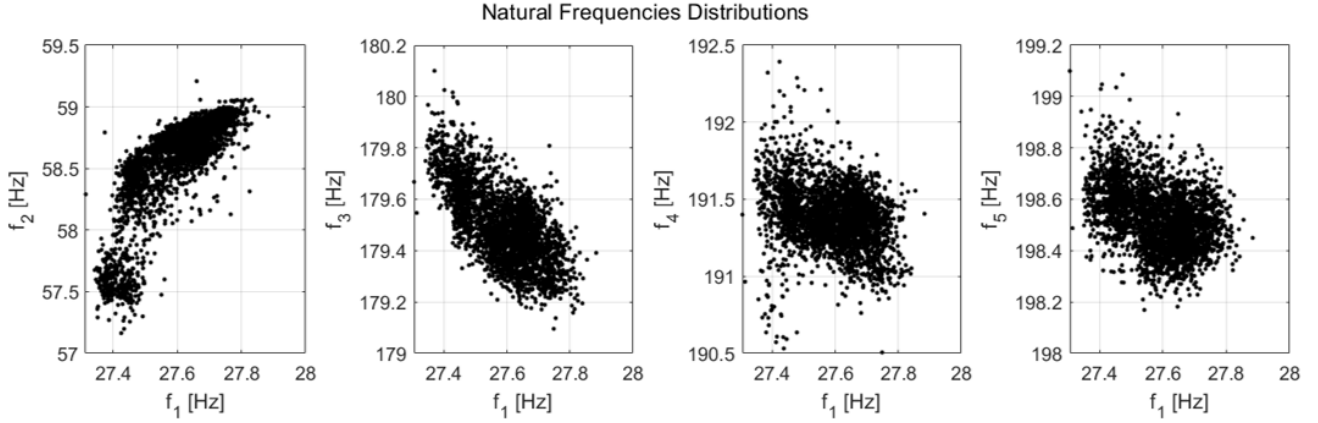


Fig. 3 Relationship between natural frequencies collected during the baseline period

Indeed, VAEs consist of an encoder that maps inputs into a probability distribution using a latent space, and a decoder that reconstructs the initial data. VAE's generic architecture is shown in Figure 4.

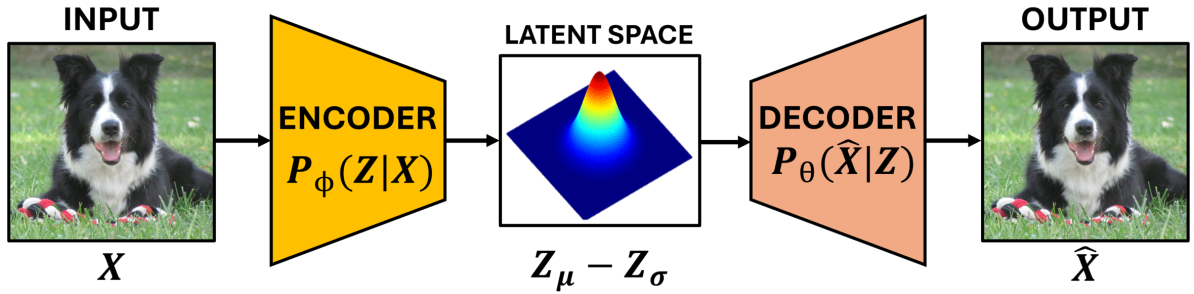


Fig. 4 VAEs architecture

Both the encoder and decoder can be formed by either Fully Connected Neural Networks (FCNN) or Convolutional Neural Networks (CNN). This enables the network to learn specific and characteristic patterns in the images used as input data. Particularly effective are the CNNs, which employ filters (called Kernels) to recognise image characteristics.

Through this architecture, it is possible to train the network with target images and then generate new coherent samples by using the decoder structure fed with a random Gaussian distribution.

VAEs Input Data

VAEs are able to detect determined patterns in a set of images. For this reason, in the present work, acceleration measurements are not used due to their random nature. Thus, the functions employed for data generation are the power and cross-spectra for two reasons:

- spectra converted into images present recognisable specific patterns.
- spectra are used for modal parameters extraction.

However, some consideration must be given regarding the power and cross-spectra employed as input:

- Spectra need to be processed in terms of real and imaginary parts, as the magnitude and phase differ by several orders of magnitude.
- Spectra must be normalised to allow a conversion into images. In this work, the normalisation was performed taking into account the maximum and minimum values of all spectra.
- Spectra need to be raised to a power (in this study, -5) to enhance characteristic patterns, making them more identifiable by the network (see Figure 6).

- Spectra have to be rearranged to obtain a squared image. This is made by neglecting the complementary functions, eliminating the imaginary part of the power spectra (zero), and dividing the functions into strips. With six accelerometers, 36 functions are used to construct the image, as shown in Figure 5, where the power and cross-spectra exploited are deepened.

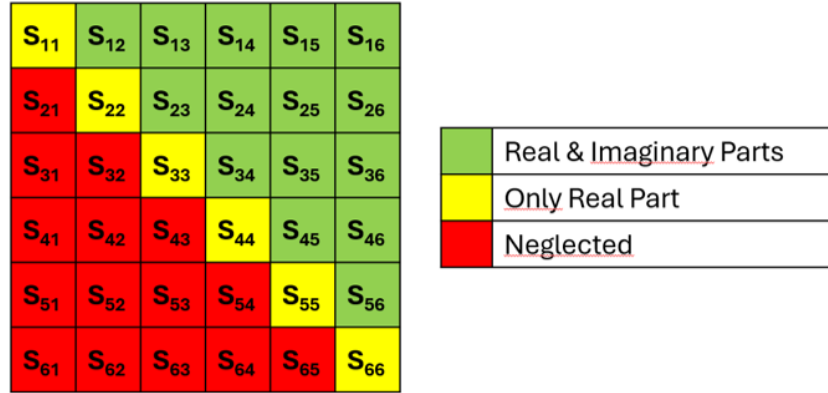


Fig. 5 Power and cross-spectra employed

Since the image generated by the process is square (a better format for CNN), it has been arranged to have a resolution of 288 x 288 pixels, covering the final frequency range of 10-240.3 Hz.

In Figure 6, the image conversion process is shown in more detail.

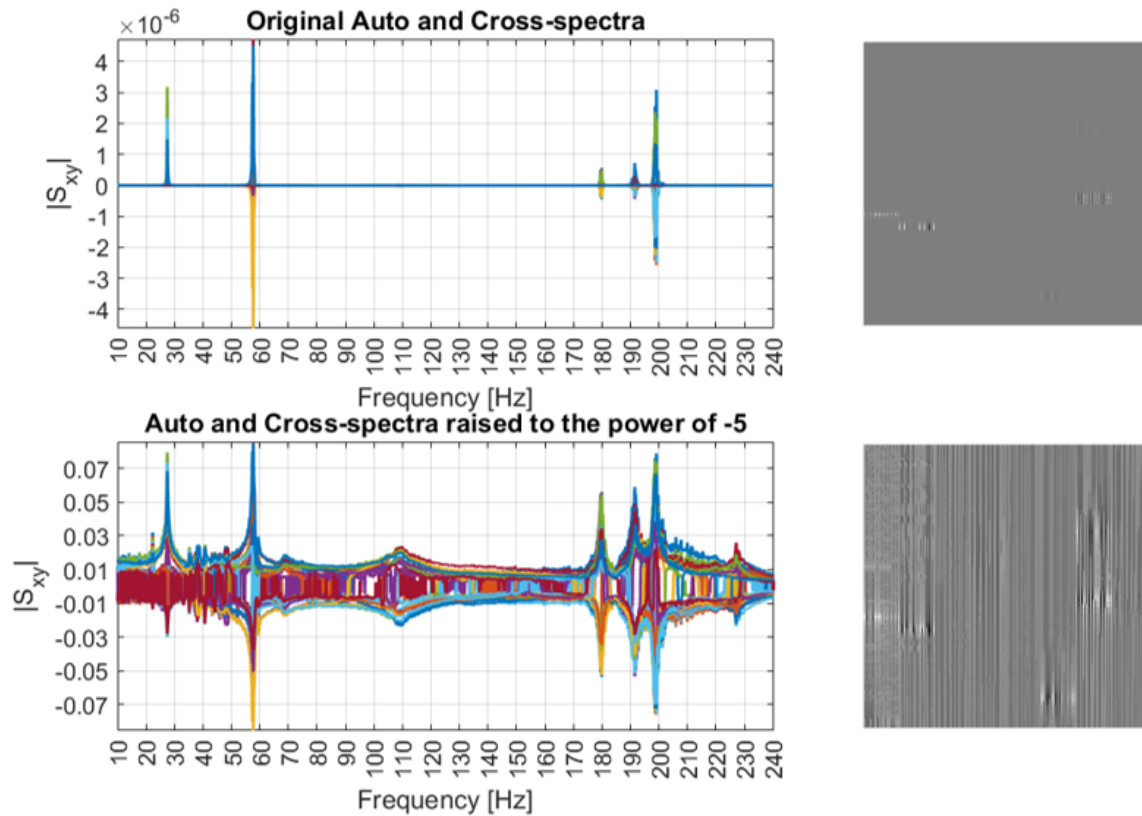


Fig. 6 Power and cross-spectra processed and converted into images. Above the original spectra, below the processed spectra

VAEs Architecture

Several layouts were tested to obtain a network with generated data as coherent as possible with the training test. The network's performance was, first of all, evaluated through the ELBO Loss function (Evidence Lower Bound), which indicates simultaneously the good reconstruction of the input data and the preservation of a well-structured latent space. The following formula represents the ELBO function

$$\mathcal{L}_{ELBO}(x) = \mathbb{E}_{q(z|x)} [\log p(x|z)] - D_{KL}(q(z|x)||p(z)) \quad (2)$$

where the first term $\mathbb{E}_{q(z|x)} [\log p(x|z)]$ measures the reconstruction fidelity, while the second one $D_{KL}(q(z|x)||p(z))$ is the Kullback-Leibler divergence and serves to approximate posterior $q(z|x)$ to match the prior $p(z)$, typically a Gaussian [17].

Finally, the networks were evaluated based on damage-related features by analysing the generated natural frequency distributions and assessing how closely they matched the training set.

The final VAE architecture consists of:

- 4 convolutional layers with 6 kernels (3x3) with 2-element stride
- 1 fully connected layer with 1944 elements
- a latent space dimension of 50
- a latent vector (82944x1) converted into a feature map (18x18x256)
- 4 deconvolutional layers with 6 kernels (3x3) with 2-element stride.

The final architecture of the VAE employed is shown in Figure 7.

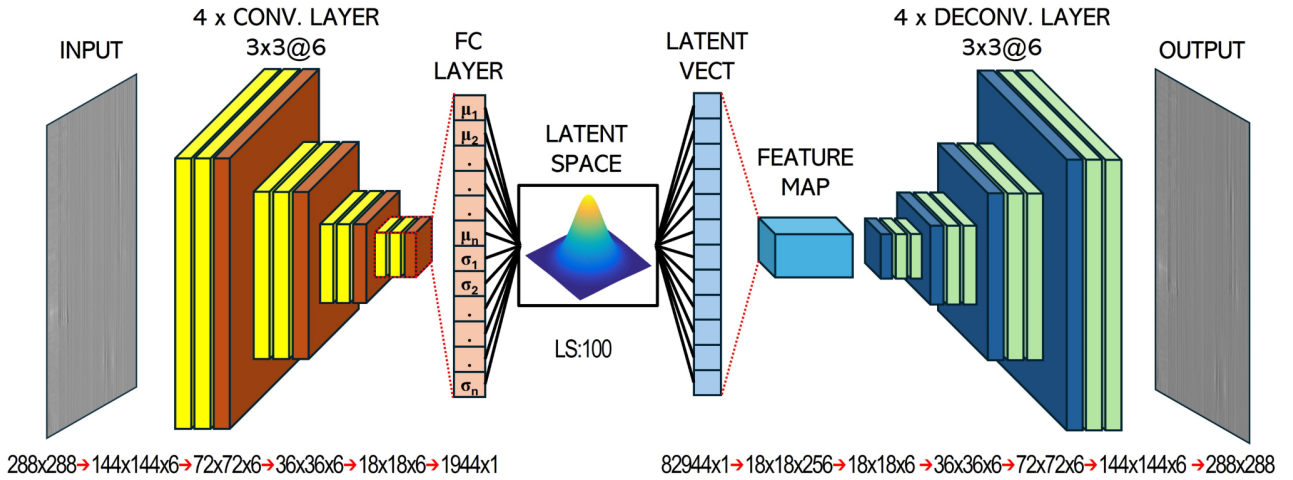


Fig. 7 VAE architecture with layers and dimensions below

Results

The entire process of conversion between the images generated and the spectra functions (and vice versa) has been deepened in order to assess the reliability of the procedure.

An example of spectral functions and the relative image generated with a trained VAE is shown in Figure 8.

First preliminary approach

An initial attempt was carried out using either the entire baseline set or portions of it (30, 50 and 70 %) to train the VAE network. The reconstruction of the spectral functions appeared to be consistent with the experimental ones. However, by

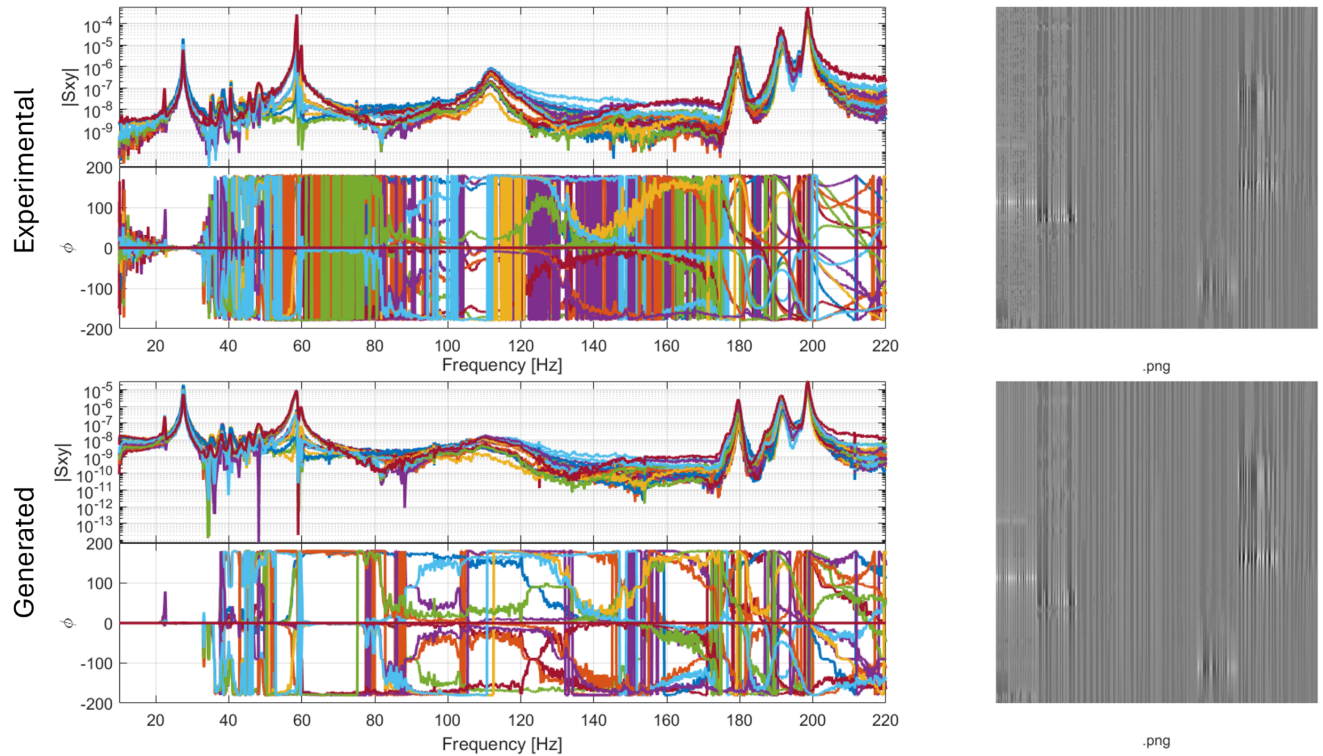


Fig. 8 Spectra from an experimental test (above) and generated with VAE (below) with the corresponding images on the right

observing the distributions of the natural frequencies extracted from the generated data, the variability of the baseline observations seemed not to be properly represented. As shown in Figure 9, natural frequencies from generated data likely tend to capture the average dynamic behaviour of the structure.

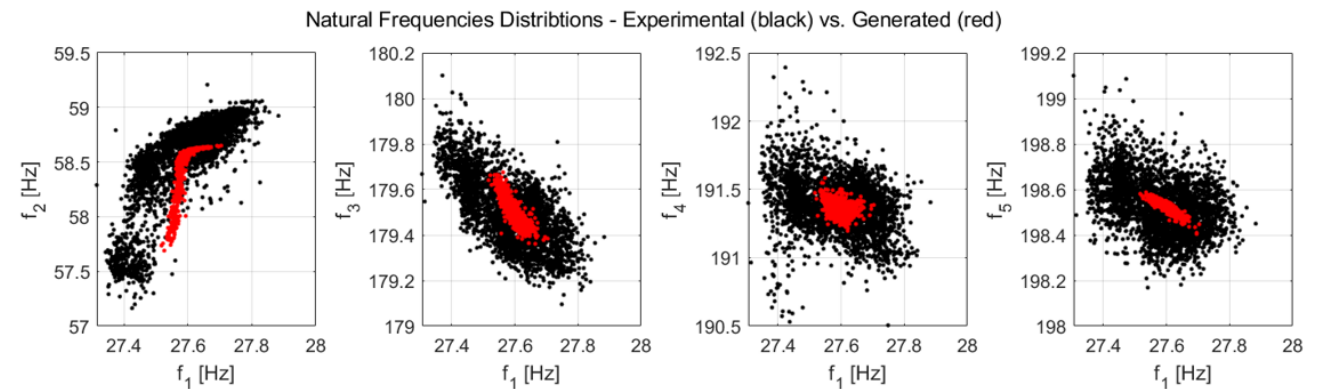


Fig. 9 Comparison between experimental and generated natural frequencies distributions

Such a result cannot be considered reliable, since the variability of the structural dynamic behaviour does not seem to be adequately represented by the generated data.

Final approach

A different approach was considered to obtain more reliable results. In order to obtain newly generated observations coherent with the experimental variability, the baseline set was split into subsets according to a weekly grouping. Then, a series of individual VAEs was trained using the data as input on a week-by-week basis. Unfortunately, the number of observations for each group was not enough to train a VAE in a proper way, which led to underfitting. Therefore, to address this problem, a pilot VAE was pre-trained using the entire baseline set. Subsequently, the network parameters were optimised by retraining the principal VAE with the weekly observations. The final procedure is shown in Figure 10.

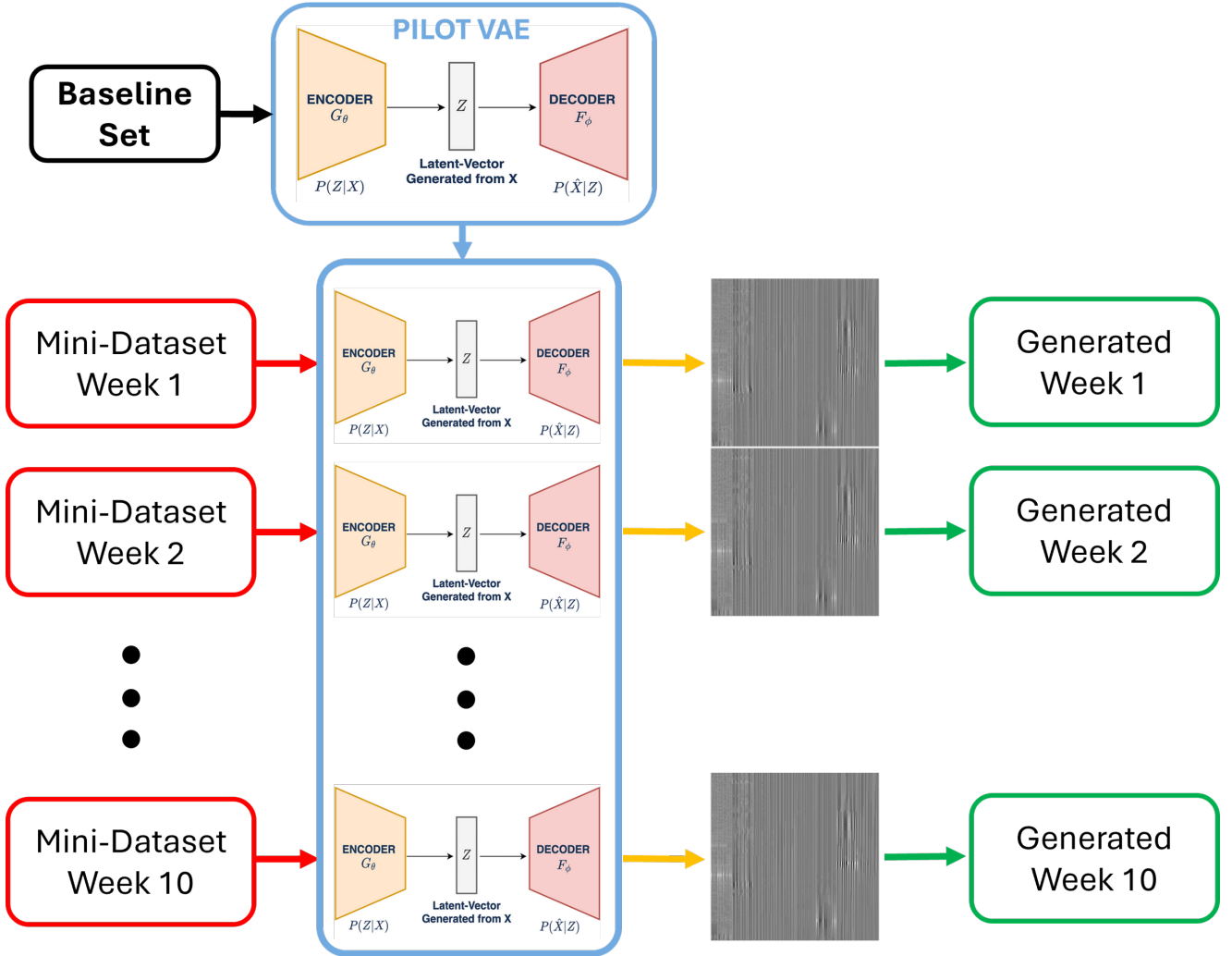


Fig. 10 Definitive procedure for new data generation

This approach led to a better representation of the variability of the dynamic behaviour of the structure, as shown in Figure 11.

A total of 10000 new observations were generated through the trained decoders. To avoid possible generation failures, the outcomes were filtered using a probabilistic strategy. Only a few dozen observations were removed.

Damage detection enhancement

To illustrate the potential benefits of the proposed strategy, a damage detection strategy based on the Principal Component Analysis (PCA) and Mahalanobis Squared Distance (MSD) exploiting the natural frequencies is evaluated using both original and enhanced baseline sets. This method is further deepened in [18]. Results in Figure 12 demonstrate the improvement in the damage detection (1 kg placed in several points of the structure) using the generated data for baseline augmentation.

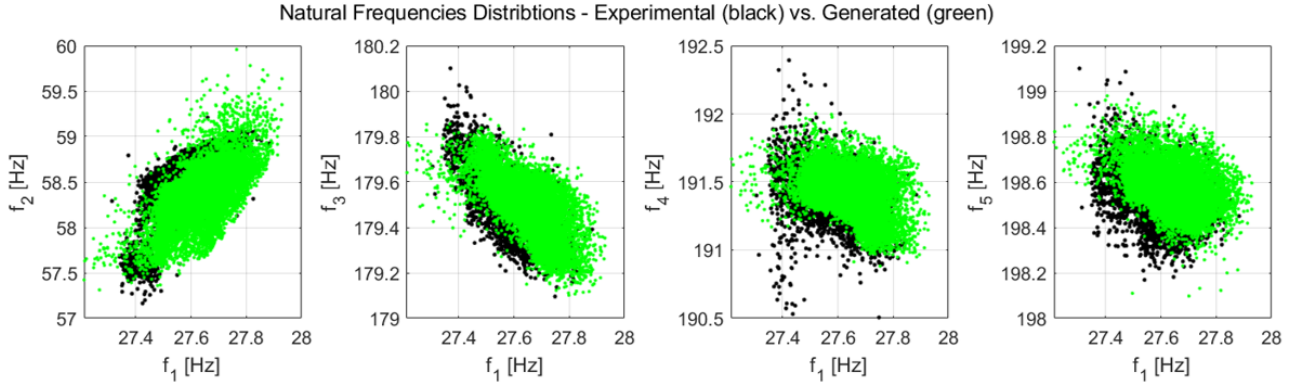


Fig. 11 Comparison between experimental and generated natural frequencies distributions with the final approach

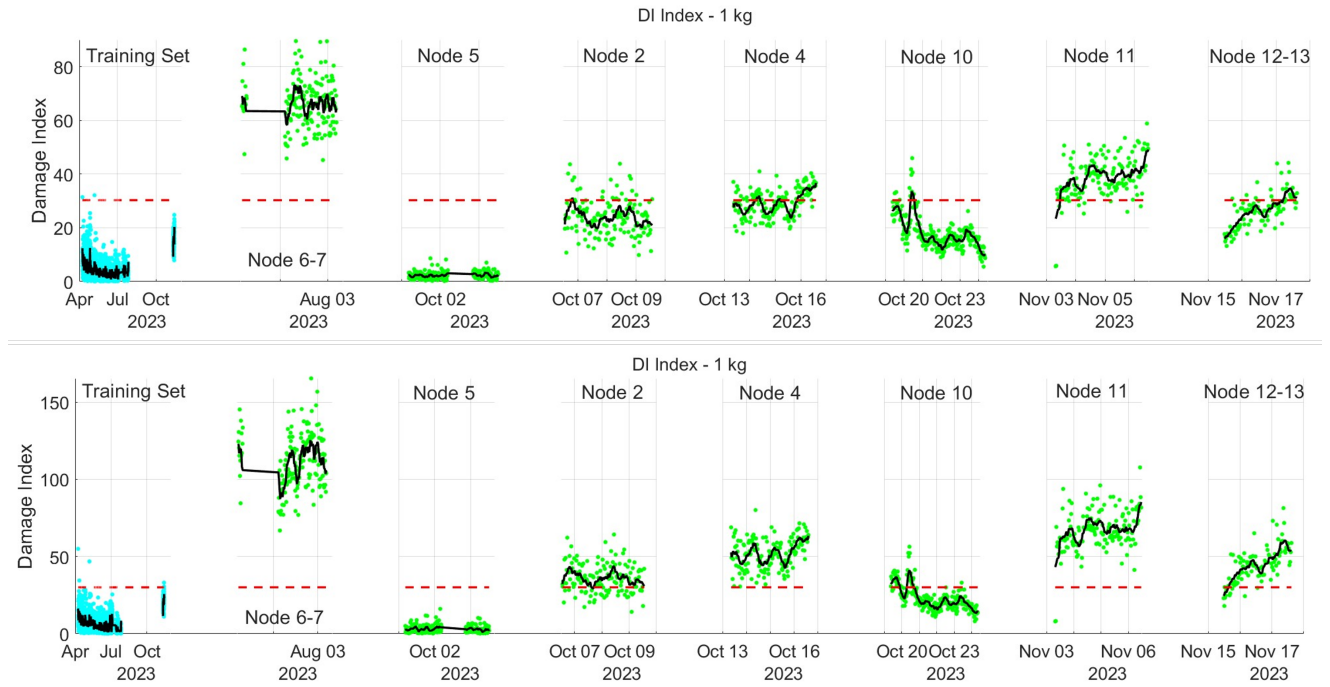


Fig. 12 Damage detection strategy performed with the original baseline (above) and the enhanced baseline (below)

Conclusion

This work demonstrates the improvement of a damage detection technique by enhancing a limited baseline dataset using generative models. The data used for the generative network consisted of the experimental power and cross-spectra from a laboratory truss girder subjected to environmental variations. The damage-sensitive features adopted in this work were the natural frequencies extracted from the spectral functions.

A series of VAEs was trained by weekly observations to properly capture the variability of the baseline set due to different ambient conditions. Subsequently, the trained decoders were employed to generate new spectra and, consequently, new natural frequencies consistent with the experimental ones. These synthetic observations were employed to augment the limited baseline set, with the aim of improving an already developed damage detection technique.

The results demonstrate a benefit in the detection of induced damage to the laboratory structure. Possible future developments may involve the employment of different modal parameters, such as mode shapes, to explore alternative damage detection strategies.

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Chapter 13

Bridge Health Monitoring Using Vehicle-Mounted Sensors and Machine Learning Classification

Burak Duran, Saeed Eftekhari Azam, and Daniel G. Linzell

Abstract This work presents a novel framework for bridge structural health monitoring (SHM) by integrating indirect drive-by sensing with a 1D Convolutional Neural Network (CNN) model. In a full-scale field study, a reinforced concrete (RC) bridge in Nebraska was subjected to a series of controlled, progressive damage scenarios: an initial healthy state (D0), a localized parapet saw-cut (D1), simulated scour (D2), and deck keyway damage (D3). A U-Haul truck equipped with five accelerometers gathered vibration data while traversing the bridge at varying speeds (0-30 mph), replicating authentic traffic conditions. The collected time-series data were preprocessed and fed into a custom-designed 1D CNN for damage detection and classification. The model achieved a training accuracy of 99.9%, a validation accuracy of 94.44%, and a testing accuracy of 93%. A detailed examination of the confusion matrix indicated high diagnostic precision across all four damage states, with the healthy (D0) and most severe damage (D3) conditions classified with over 95% accuracy, scour (D2) with over 92% accuracy, and the subtle parapet damage (D1) with a robust performance above 85%. This research marks a significant step forward by successfully applying a 1D CNN to raw drive-by data from a full-scale, in-situ bridge, providing a strong proof-of-concept for a non-intrusive, scalable alternative to traditional SHM techniques.

Keywords Drive-by sensing · Convolutional neural networks · Full-scale bridge testing · Intelligent transportation systems · Damage detection

Introduction

Bridges are essential infrastructure, but their aging condition poses a growing global concern. In the U.S. alone, a substantial portion of the bridge inventory is approaching or has exceeded its design life, necessitating more proactive maintenance and monitoring strategies [1, 2]. Traditional SHM methods, such as visual inspections and periodic non-destructive testing (NDT), are often labor-intensive, time-consuming, and difficult to scale across a large number of bridges. Although vibration-based methodologies have shown potential in detecting stiffness change and structural degradation, they typically require extensive sensor deployments and introduce challenges associated with operational and environmental variabilities and measurement uncertainties.

Drive-by sensing has emerged as an innovative alternative, using instrumented vehicles as mobile platforms to indirectly assess bridge health [3, 4]. This approach offers significant advantages in cost-effectiveness, safety, and scalability. However, its practical implementation faces a primary challenge: the vehicle's dynamic response, coupled with road roughness and ambient environmental noise, can heavily contaminate the measured signals, masking the subtle signatures of structural damage.

Deep learning techniques, particularly Convolutional Neural Networks (CNNs), are well-suited to address this challenge due to their capacity for automated feature extraction from raw data. While CNNs have been applied to SHM, most of these studies have been confined to numerical models or small-scale laboratory experiments [5]. This work stands apart

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by demonstrating the efficacy of a 1D CNN framework using drive-by data from a full-scale, operational bridge under controlled, realistic cumulative damage scenarios. This foundational study aims to bridge the gap between idealized experimental validations and the complexities of practical large-scale deployments, thereby laying the groundwork for a truly scalable SHM solution.

Methodology

Experimental Setup

The study was conducted on an operational bridge, Bridge Q110, located in the southeast of Lincoln, Nebraska, US, a single-span prestressed and pre-tensioned concrete double-tee bridge. The Q110 bridge had two lanes with a width of 30 feet and a span length of 44 feet. Bridge views from the site are provided in (Figure 1)



(a) Overview of the bridge including parapets



(b) Bottom view

Fig. 1 Bridge Q110 views

A prior state of the bridge was assumed to be healthy. Then, a sequence of systematically imposed three damage states was introduced: D0 (healthy), D1 (a localized saw-cut to the north parapet), D2 (excavation of soil around abutment piles to simulate scour), and D3 (saw-cuts to the keyways between the double-tee beams) (Figure 2). These scenarios were designed to replicate potential failures and deterioration that could arise under real-world operating conditions, thereby providing meaningful challenges for evaluating the proposed CNN-based SHM approach. This progressive approach created a comprehensive dataset reflecting progressive structural degradation.



(a) Parapet after saw-cut (D1)



(b) Exposed pile scour detailed view (D2)



(c) Shear key-way being saw-cut (D3)

Fig. 2 Visual comparison of damage states showing D1, D2, and D3

A 26-foot U-Haul truck served as the sensing platform, equipped with five accelerometers placed at the front axle, rear axle, front suspension, rear suspension, and center of gravity (Figure 3). The vehicle's total weight was approximately

21,300 lbs, and it traversed the bridge at speeds of 10, 20, and 30 mph. This extensive test matrix, consisting of over 1,000 live-load runs, generated a robust dataset of acceleration time-series signals sampled at 500 Hz.

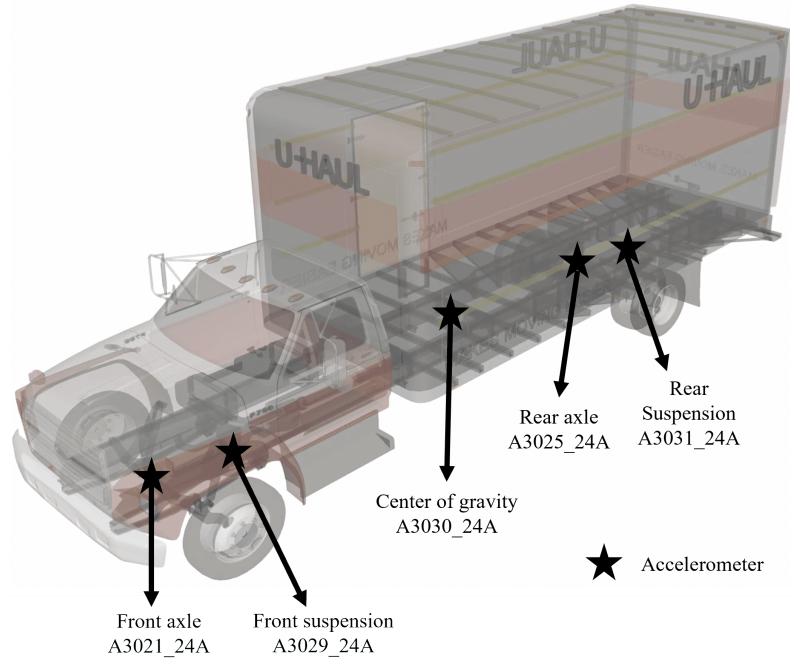


Fig. 3 Truck instrumentation plan and Accelerometer IDs

Data Preprocessing and Model Architecture

The raw acceleration signals, which varied in length, were resampled to a uniform length of 1500 data points using cubic interpolation. This method was chosen to provide a smooth approximation of the original signals while preserving key characteristics. The data were then normalized using a z-score standardization approach to feed the proposed CNN model.

The core of our framework is a custom 1D CNN model. The architecture was finalized through an iterative design and a grid search for hyperparameter optimization. The final network comprises four convolutional layers with increasing filter counts (64, 128, 256, 512), interspersed with max-pooling layers for dimensionality reduction. The network concludes with four fully-connected layers and a final SoftMax output layer for multiclass classification. The training was conducted over 200 epochs using the Adam optimizer on a dataset split into 70% training, 15% validation, and 15% testing subsets.

Results and Discussion

The 1D CNN model demonstrated an overall testing accuracy of 93%. A detailed analysis of the confusion matrix revealed robust per-class performance (**Figure 4**). The confusion matrix analysis of the 1D CNN model for Bridge Q110 demonstrates its overall effectiveness in classifying structural health states with high accuracy. For the healthy condition (D0), the model achieved a true positive rate of 95.3%, indicating strong capability in correctly identifying undamaged states and minimizing false alarms in structural health monitoring applications. In the case of north barrier damage (D1), the true positive rate was lower at 85.8%, suggesting that this damage type shares overlapping features with other states, which leads to a higher rate of misclassification and highlights the need for model refinement. For scour damage (D2), the model maintained a strong performance with a true positive rate of 92.3%, reflecting its effectiveness in detecting this critical form of deterioration that can significantly compromise bridge stability. The highest performance was observed for shear keyway damage (D3), with a true positive rate of 98.5%, demonstrating that this damage state has highly distinguishable characteristics that the model can reliably detect. Overall, the results confirm the model's strong capability in identifying both healthy and damaged conditions, with particularly high precision for scour and shear keyway damages, while also emphasizing the necessity of further improvements in differentiating north barrier damage from other states.

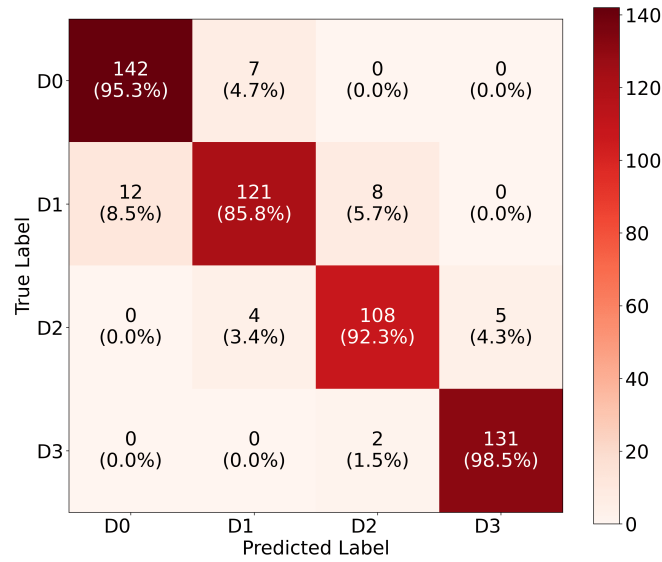


Fig. 4 Confusion matrix

Conclusion

This study offers a pioneering validation of a 1D CNN framework for drive-by sensing on a full-scale, operational bridge, accounting for diverse operational conditions, environmental variations, and measurement uncertainties. The findings demonstrate that the proposed methodology can reliably and accurately classify multiple cumulative damage states directly from raw, noise-contaminated acceleration data. Although the results are derived from a single bridge subjected to specific progressive damage scenarios and loading conditions, they highlight the potential of the 1D CNN framework to significantly advance bridge SHM by reducing dependence on costly, labor-intensive inspections.

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Chapter 14

Data-Driven Aerodynamic Modeling for Flapping-Wing Robots



Costanza Speciale, Silvia Milana, Antonio Concilio, and Antonio Carcaterra

Abstract Flapping-Wing Aerial Vehicles (FWAVs), also known as ornithopters, draw inspiration from nature and use rapidly oscillating wings to generate sufficient lift and thrust to move forward while sustaining their weight. Their similarities to birds and compliance with the natural environment, combined with their lower noise levels compared to propeller-driven aerial vehicles, make them ideal tools for monitoring sensitive ecosystems with minimal disturbance.

Predicting the dynamic behavior of these bio-inspired robots is particularly challenging due to the difficulty in estimating the aerodynamic forces resulting from the complex, unsteady aerodynamic phenomena generated by the oscillating motion of the wing surfaces under variable wind conditions. This challenge is further compounded by the flexibility of their membrane wings, which typically undergo large shape changes under aerodynamic loads depending on factors such as flight speed, attitude and maneuvering.

A promising solution lies in data-driven reduced-order models. In this study, Volterra theory is explored as a framework to develop such a model for the unsteady aerodynamics of a flapping wing. Trained on experimental measurements or high-fidelity numerical simulations of an ornithopter wing operating under varying conditions, the resulting model can provide accurate and computationally efficient aerodynamic force predictions.

Keywords Flapping-wing aerial vehicles · Biomimetics · Aerodynamic modeling · Volterra theory · Robotics

Introduction

Researchers have long been fascinated by the unique flight capabilities of birds, as attested by the studies of Leonardo Da Vinci in the sixteenth century [1], who sketched machines intended to allow humans to fly inspired by avian flight (Figure 1a).

Despite pioneering attempts such as those of Otto Lilienthal, who designed and realized multiple manned, engine-powered flapping-wing machines other than gliders, the development of ornithopters was hindered for decades by their inherent complexity and by the success of fixed-wing aircraft [2].

With technological advancements, however, interest in natural fliers was renewed in the early 2000s, this time focusing on the development of Unmanned Aerial Vehicles (UAVs). Since then, several flapping-wing aerial vehicle (FWAV) prototypes have been realized, including Festo's SmartBird [3], the Dove [4], and the more recent RoboFalcon [5].

The growing interest in these platforms stems from their unique characteristics, which allow them to minimize disturbance to the surroundings, both acoustically and visually. These features makes FWAVs more suitable than other UAV types for a wide range of operations, such as wildlife monitoring in protected areas, forest surveillance, vehicle tracking and infrastructures inspection in urban environments with minimal impact on the local population.

Despite these advantages, several limitations still prevent their use in practical applications. These include strict payload restrictions, limited take-off and landing capabilities, and challenges in modeling and control, arising from the unsteady three-dimensional flow generated by the flapping-wing motion. The focus of this work lies in addressing this latter challenge.

There exist different approaches for unsteady aerodynamic modeling, which can be grouped into three main categories: theoretical modeling, computational modeling and data-driven modeling [6].

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Theoretical models, such as the classical analytical models of Wagner [7] and Theodorsen [8], remain valuable for certain applications but rely on the small perturbation assumption inherent in linear potential theory, which limits their accuracy when dealing with the large-amplitude motion typical of ornithopter wings.

Computational Fluid Dynamics (CFD) simulations, on the other hand, can predict aerodynamic loads with high accuracy, but their high computational cost makes them unsuitable for scenarios where fast predictions are required.

For real-time applications, rough estimates based on a flapping frequency of 1 Hz suggest a desired update rate around 20 Hz, which is incompatible with the computational demands of such simulations.

A promising alternative is the use of data-driven approaches, which leverage data from experiments or high-fidelity numerical simulations to construct Reduced Order Models (ROMs). These can provide both accurate and computationally efficient aerodynamic force estimations, which are fundamental, for example, for enabling real-time model-based control of ornithopters. This work focuses on a particular class of data-driven aerodynamic models based on system identification (i.e. the process of deriving mathematical models from observed system input-output data signals [9]) and specifically on those based on the Volterra theory [10]. Volterra-based models have been widely applied in both aerodynamic and aeroelastic modeling. The first to do so was Silva [11], who employed a second order Volterra series to model the transonic unsteady aerodynamic response of a pitching wing. Marzocca et al. [12] developed a Volterra-based aeroelastic ROM for a two-dimensional airfoil and used it to predict flutter instability. De Paula et al. [13] proposed a method to identify Volterra kernels for multiple-input, multiple-output systems using time-delay neural networks and applied it to the unsteady aerodynamic modeling of a heaving and pitching airfoil. This study is positioned within the context of ROMs for flapping wings and builds on the works of Liu et al. [14] and Ruiz et al. [15] by focusing on Volterra-series-based aerodynamic modeling.



Fig. 1 (a) A full-sized replica of Leonardo da Vinci's ornithopter at the Museum of Aviation in Košice, Slovakia. Photo by ZemplinTemplar, licensed under CC BY-SA 4.0. (b) Festo's SmartBird [3]. Photo by Matti Blume, licensed under CC BY-SA 4.0

Volterra Theory

The Volterra series [10] is a mathematical tool widely used in many engineering applications to describe the behavior of nonlinear, time-invariant systems with fading memory (i.e. systems in which the output depends not only on the current input but also on its past values, with the influence of past states gradually decreasing over time [16]).

Considering a single-input, single-output system, its output $y(t)$ can be expressed using a Volterra series expansion as an infinite sum of multidimensional convolution integrals:

$$y(t) = k_0 + \sum_{n=1}^{\infty} \int_0^t \dots \int_0^t k_n(\tau_1, \tau_2, \dots, \tau_n) \prod_{i=1}^n u(t - \tau_i) d\tau_1 d\tau_2 \dots d\tau_n \quad (1)$$

where $u(t)$ is the system input, k_0 is the zero-input response and $k_n(\tau_1, \tau_2, \dots, \tau_n)$ is the n^{th} -order Volterra kernel, which represents the combined effect of n past input states on the present output.

The convolution structure of the series introduces memory effects into the system response, making it particularly suitable for applications such as unsteady aerodynamic modeling, where it can capture effects like the influence of a wing's vortex wake on the aerodynamic loads, which becomes significant when the characteristic timescale of the phenomenon is comparable to the convective timescale associated with the inflow speed.

The Volterra series can be also extended to the general case of multiple-input, multiple-output systems. The j^{th} output $q_j(t)$ can be expressed as:

$$\begin{aligned} q_j(t) = & k_j^0 + \sum_{i=1}^p \int_0^\infty k_{j;i}^{(1)}(\tau_1) u_i(t - \tau_1) d\tau_1 \\ & + \sum_{i_1=1}^p \sum_{i_2=1}^p \int_0^\infty \int_0^\infty k_{j;i_1,i_2}^{(2)}(\tau_1, \tau_2) u_{i_1}(t - \tau_1) u_{i_2}(t - \tau_2) d\tau_1 d\tau_2 \\ & + \dots \\ & + \sum_{i_1=1}^p \dots \sum_{i_n=1}^p \int_0^\infty \dots \int_0^\infty k_{j;i_1,\dots,i_n}^{(n)}(\tau_1, \dots, \tau_n) \prod_{m=1}^n u_{i_m}(t - \tau_m) d\tau_1 \dots d\tau_n + \dots \end{aligned} \quad (2)$$

Here, p is the number of inputs, $u_i(\cdot)$ are the system inputs ($i = 1, \dots, p$) and $k_{j;i_1,\dots,i_n}^{(n)}(\tau_1, \dots, \tau_n)$ denotes the n^{th} -order multi-input Volterra kernel, where the subscript j identifies the system output and the indices following the semicolon indicate the inputs to which the kernel corresponds.

Methodology

The use of Volterra theory is explored to develop a model capable of accurately and efficiently estimating the aerodynamic forces of a flapping-wing.

Consider a flapping airfoil, representing a section of the ornithopter's wing, undergoing combined heaving and pitching motions, described by the heave displacement h and pitch angle θ , respectively. Focusing on the modeling of the lift force L (a similar procedure can be applied to drag and aerodynamic moment), it can be approximated by a Volterra series expansion truncated at the first order:

$$L(t) = k_l^0 + \int_0^t [k_{l;1}(\tau)u_1(t - \tau) + k_{l;2}(\tau)u_2(t - \tau) + \dots k_{l;N}(\tau)u_N(t - \tau)] d\tau = k_l^0 + \mathbf{k} * \mathbf{u} \quad (3)$$

Where $\mathbf{k} = [k_{l;1} \ k_{l;2} \ \dots \ k_{l;N}]$ is the vector of the first-order kernels, while $\mathbf{u} = [u_1 \ u_2 \ \dots \ u_N]^T$ is the input vector containing N variables that can be selected, using an appropriate criterion, from an initial set $\mathbf{u}_0$, that may include, for example, $h, \theta, \dot{h}$ and $\dot{\theta}$ [17].

To identify the unknown kernels $k_{l;1} \ k_{l;2} \ \dots \ k_{l;N}$, a numerical or experimental training dataset containing inputs and output time histories obtained under varying conditions, such as different airspeeds, flapping frequencies and angles of attack, can be employed.

A possible technique for kernel identification from such datasets, which has shown promising results in flapping-wing applications [14, 15], is direct identification, which consists in identifying the Volterra kernels directly from input-output data without relying on intermediate transformations. For clarity, consider the single-input case, where the input may be, for instance, the time derivative of the effective angle of attack as in [15].

In discrete time, and assuming $k_l^0 = 0$, the first-order Volterra model for the lift force can be written as:

$$L[\tau] = \sum_{n=0}^M k_l[n]u[\tau - n]\Delta t \quad (4)$$

where τ and n are the time indices, M is the memory depth in time steps and Δt is the time step. The terms $k_l[n] = k_l(t)|_{t=n\Delta t}$ and $u[n] = u(t)|_{t=n\Delta t}$, are the discrete kernel and input values, respectively. Equation 4 can then be rearranged into matrix form, enabling the kernel values to be identified using least-squares or regularization techniques. In addition to direct identification, other methods for Volterra kernel identification can also be considered, these include the cross-correlation method [18], orthonormal basis functions [19], and time-delay neural networks [20].

Finally, once the model has been identified, its generalizability can be evaluated by validating it against data obtained under conditions not used during training.

Conclusions and further developments

Ornithopters are complex aerial vehicles that present several significant challenges. Among these, particularly critical are modeling and control, which are difficult due to the unsteady aerodynamics associated with their wing motion.

To address the challenges related to aerodynamic modeling, a promising approach is to develop a ROM based on Volterra theory. Such a model can combine computational efficiency with adequate accuracy, making it a particularly valuable tool for applications such as real-time control, where fast and reliable aerodynamic force prediction is a fundamental requirement. The resulting aerodynamic ROM can be indeed integrated into a dynamic model of the ornithopter, which, due to the structure of the Volterra series, would take the form of integro-differential equations. Advanced control strategies specifically developed for integro-differential systems, such as those proposed by Paifelman et al. [21], could then be applied to enable effective control of the ornithopter's flight.

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Chapter 15

Footfall Forensics: Real-Time Reactive Balance Classification from Sparse Floor-Vibration Sensing

Samikhshak Gupta, Rachel Cole, Wil Reehl, Emily Dowdy, Elizabeth Hoy, Vijaya V.N. Sriram Malladi, and Carolyn Duncan

Abstract Falls remain a major source of injury and mortality among older adults, motivating the development of unobtrusive methods that can detect early signs of instability. This study presents a building-integrated, contact-free approach that uses floor vibrations as diagnostic indicators of balance control, capable of distinguishing *reactive-balance* responses from nominal gait. Unlike wearable or camera-based systems, this framework treats the floor as a distributed sensing medium and leverages vibration-based system identification to extract gait signatures directly from structural responses. Experiments were performed in Michigan Tech's H-STEM Functional Mobility Laboratory on a reinforced-concrete floor bay instrumented with eight seismic accelerometers arranged in a single-bay grid. The array included five PCB 393B04 and three PCB 393B31 sensors to capture broadband vertical responses during low-amplitude excitation. A pilot study with one participant evaluated performance across two surface conditions—an anti-slip mat and a low-pile carpet. On each surface, two tests were conducted: a nominal gait trial, where the participant walked at a self-selected pace to establish baseline footfall patterns, and a reactive-balance trial using a lean-and-release apparatus that induced forward perturbations at 10° and 18° lean angles. The resulting vibration data revealed distinct spatiotemporal features associated with balance recovery dynamics. These signatures demonstrate that reactive-balance events produce measurable deviations in modal content and force distribution compared to steady walking. The proposed vibration-centric framework thus enables scalable, privacy-preserving, and continuous monitoring for early fall-risk detection in real architectural environments.

Keywords Gait analysis · Reactive balance · Seismic accelerometers · Fall detection · Mobility

Introduction

Falls are a leading source of nonfatal injury and a major contributor to injury-related mortality among older adults in the United States [1, 2, 3, 4]. Incidence rises sharply with age—about one in three adults over 65 falls each year [5, 6], with risk increasing further beyond 80 [7, 8]—and annual rates span $\sim 25\%$ for community-dwelling seniors to as high as 75% in nursing homes [9, 10, 11]. In 2021, falls led to nearly three million emergency-department visits, over 800,000 hospitalizations, and more than 38,000 deaths among those aged 65+ [12]; prior falls substantially elevate recurrence risk (likelihood ratio 2.3–2.8) [13, 14]. Costs for nonfatal falls have grown from \$23.3 billion (2008) to $\sim$ \$80 billion (2020) [15, 16]. Risk is multifactorial: intrinsic contributors (age-related declines in strength, neuromuscular control, proprioception, reaction time; gait/balance impairments) interact with extrinsic hazards (poor lighting, slippery or uneven surfaces, unexpected perturbations) [17, 18]; deconditioning and fear of falling further reduce stability [19, 20]. These trends motivate the development of unobtrusive indicators that detect incipient balance loss before a fall occurs.

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Structural dynamics provides such an avenue: instrumented floors can serve as distributed sensors that convert footfalls into measurable responses. Building on the foundations of experimental modal analysis for estimating natural frequencies and mode shapes in bridges and buildings [21, 22, 23, 24, 25], recent “smart building” deployments use floor responses for operational assessments. Source-identification techniques, analogous to epicenter localization, infer loading and human-structure interaction from ambient data [26, 27, 28, 29, 30]; underfloor accelerometer networks resolve footstep activity [31, 32]; and full-scale arrays (e.g., Goodwin Hall) demonstrate in situ localization with seismic-grade sensors [33, 34, 35, 36, 37, 38, 39, 40]. In this context, our premise is to exploit floor-vibration signatures to separate nominal gait from *reactive-balance* events and to treat the emergence of reactive signatures as an early indicator of instability—aligning modal analysis with proactive fall-risk detection in operational environments [41, 42, 43, 44, 45, 46].

Reactive balance denotes the rapid, unplanned stepping response that stabilizes the center of mass (COM) relative to the base of support (BOS) following an external perturbation, and it is dynamically distinct from volitional (pre-planned) gait that executes an intended COM-BOS trajectory [47, 48]. In reactive events, the control objective switches from trajectory tracking to immediate recovery: step initiation latency shortens, swing-leg unloading accelerates, and step velocity increases, yielding spatiotemporal patterns (step time, stance/swing durations, double support, stride/step length, and step speed) that diverge from steady walking [49, 50, 51, 52]. From a modal-analysis perspective, these control shifts imprint measurable differences in floor-response signatures: reactive steps tend to excite broader-band content, altered mode participation (including antisymmetric/torsional components), and higher peak ground-reaction impulses that modify the amplitude and phase of the operating-deflection shape relative to nominal gait. Because such compensatory postural reactions are a primary line of defense against balance loss and strongly tied to fall risk in older adults [53, 54], their early detection is clinically meaningful. This work therefore exploits contact-free floor-vibration sensing as a system-identification channel to separate reactive balance from nominal gait in situ, leveraging the building’s dynamics as the sensing medium and avoiding the deployment burdens of cameras or wearables [55, 56, 57, 58]. We treat the presence of reactive-balance signatures as a precursor for fall detection and a trigger for targeted monitoring, enabling a pathway to scalable, privacy-preserving, building-integrated surveillance of instability [59, 60, 61].

Current solutions for gait tracking span clinic-and home-based modalities with varied fidelity and scalability: structured monitoring and early intervention improve balance, reduce fall risk, enhance mobility, and support quality of life [50, 54, 62, 63, 64, 65, 66], yet population aging (~ 1.5 billion ≥ 65 by 2050) [67] and access barriers in rural/underserved settings constrain clinic-centric care [68, 69]. Affordable modalities include wearables, marker-based/markerless 3D motion capture, video analytics, and remote supervised monitoring [70, 71, 72], but each exhibits domain-specific limitations: augmented-reality devices are sensitive to lighting/sensor interference and narrow fields of view that under-sample whole-body kinematics; wearables suffer from placement variability, motion artifacts, partial-state observability, and user-compliance drift in older adults [73]; and video systems pose persistent privacy/security risks and observer effects that bias natural gait [74, 75].

To address these limitations, this study introduces a contact-free framework that treats the floor itself as a distributed sensing medium. By analyzing vibration responses induced by footfalls, the system identifies distinct *reactive-balance* signatures that differentiate them from nominal gait. Within this framework, floor vibrations are modeled as structure-borne excitations whose spatiotemporal features reveal balance recovery dynamics. The approach enables continuous, in situ monitoring capable of detecting early signs of instability—without the need for wearable devices or cameras. Through cost-effective signal processing and vibration-based system identification, the proposed method offers a scalable, privacy-preserving pathway for telehealth integration and fall-risk assessment in real architectural environments [76, 77, 78].

Methods

Experiments were conducted in Michigan Tech’s H-STEM Functional Mobility Laboratory on a reinforced-concrete floor bay with a clear span of approximately 9 m. Eight seismic accelerometers were surface-mounted within a single-bay grid and oriented to capture the vertical vibration response. The array included five PCB 393B04 sensors (sensitivity 10^3 mV g^{-1}) for broadband measurements and three high-sensitivity PCB 393B31 units (10^4 mV g^{-1}) positioned near midspan and antinodal regions to enhance the signal-to-noise ratio during low-amplitude footfall excitation. All sensors were bonded to the slab using a low-creep adhesive, gravity-leveled, and powered as IEPE channels. Data acquisition was performed using Siemens Testlab, equipped with per-channel anti-aliasing filters, at a sampling rate of 10.24 kHz. Each trial consisted of a 20 s time record initiated by a common hardware trigger to ensure phase coherence across all channels. A pilot study was conducted with a single participant to evaluate structural and human response under two surface conditions: one covered with an anti-slip mat and the other with a low-pile carpet. On each surface, both nominal gait and reactive-balance tests were performed to examine differences in foot-floor interaction, traction behavior, and vibration response characteristics.

In the first phase, corresponding to the nominal gait condition, the participant walked across the instrumented span at a self-selected, comfortable pace, generating steady and periodic footfall excitations representative of natural walking dynamics. This dataset provided a consistent baseline for gait-related loading patterns and served as a reference for comparison with reactive-balance trials.

Reactive balance performance was evaluated using a lean-and-release (tether-release) apparatus [78, 79]. At the start of each trial, the participant adopted a standardized stance for balance assessment [80]. While secured to a posterior wall-mounted tether, the participant leaned forward by rotating about the ankle joints in a controlled, plank-like posture until the tether became taut Figure 1. Tether length was adjusted to achieve the desired shank inclination, measured as the angle between the shank and the vertical using a handheld goniometer aligned with the lateral malleolus. Two lean conditions, 10° and 18° from vertical, were tested. Shank orientation was determined using anatomical landmarks at the lateral malleolus and lateral femoral epicondyle, with all measurements collected by a trained researcher experienced in landmark identification and goniometric assessment.

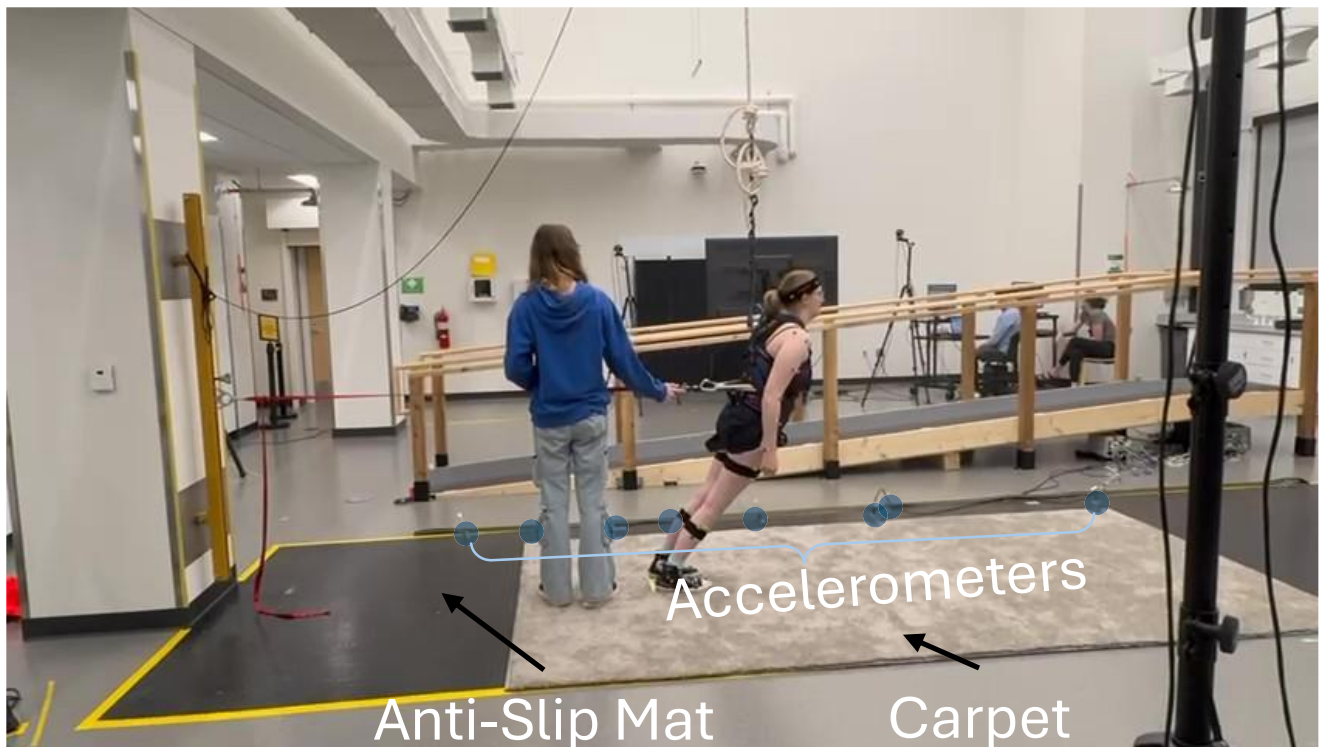
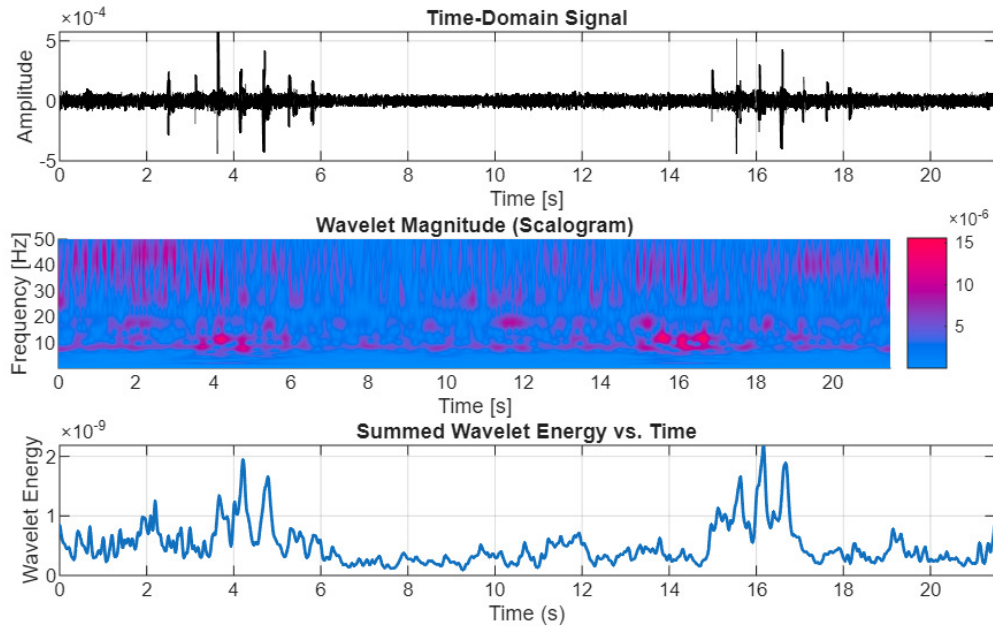


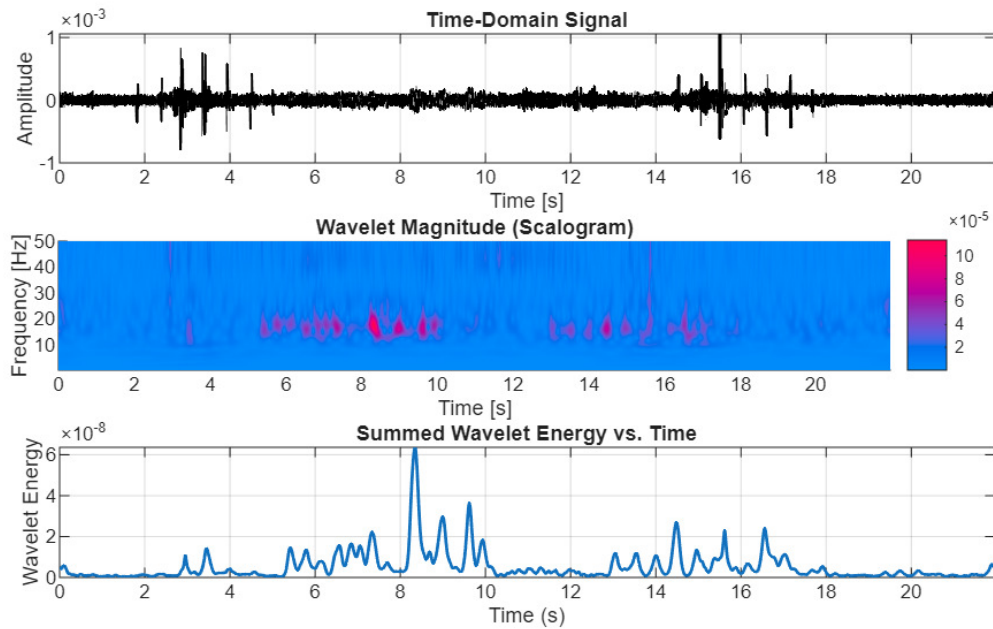
Fig. 1 Experimental setup showing the instrumentation and test configuration

Preliminary Results

Individual footfalls were successfully detected across all trials using the accelerometer array, with each event producing distinct vibration impulses consistent with heel-strike loading. Time–frequency analysis via continuous wavelet transformation (CWT) was then applied to the vertical acceleration signals to capture transient energy variations within the 0–50 Hz band Figure 2. The resulting wavelet scalograms will be used to get differences between nominal gait and reactive-balance events. Footstep segmentation will allow for the extraction of step-wise features, including dominant frequency content, temporal energy envelopes, and inter-sensor phase shifts. This classification pipeline will demonstrate a robust separation between nominal and reactive events, forming the basis for a real-time fall-risk indicator within building-integrated monitoring frameworks.



(a) Time Domain Analysis for the Anti-Slip Mat



(b) Time Domain Analysis for the Carpet

Fig. 2 Time Domain Analysis for the two surfaces

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Declaration of generative AI and AI-assisted technologies in the writing process

While preparing this work, the author(s) used ChatGPT to edit the language. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the publication's content.

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Chapter 16

Mapping Propulsive Efficiency of Flexible Bio-Inspired Fins

E. Paifelman, G. Costantino Muniz, S. Milana, A. Carcaterra, and E. Ciappi

Abstract This work presents a detailed parametric study of the propulsive performance of bio-inspired fish-like elastic fins through combined numerical modeling and experimental validation. The objective is to generate propulsion maps and identify the key parameters influencing propulsive efficiency, with direct implications for the design of efficient underwater propulsion systems. A fluid–structure interaction model is employed to predict the dynamic response of flexible fins across a wide range of material properties, stiffness levels, and geometric configurations. Extensive hydrodynamic channel tests have been conducted using a diverse set of fin prototypes, allowing systematic comparison with numerical predictions. The experimental data validate the model’s accuracy and provide a rich dataset to assess thrust generation, elastic deformation, and efficiency under varying actuation conditions. The combined numerical-experimental analysis enables the creation of comprehensive propulsion maps, highlighting the sensitivity of performance to material and geometric parameters. The results offer valuable insights into the design space and establish a foundation for the optimization of future bio-inspired propulsion systems.

Keywords Fish-like · Bionspired robot · FSI model · Modal analysis · Biomimetic propulsion

Introduction

Biomimetic propulsion, inspired by fish locomotion, offers quiet operation, reduced risk in the event of contact with marine fauna, and high maneuverability characteristics that are particularly valuable for prolonged missions and operations in cluttered or sensitive habitats. Designing such systems, however, requires identifying which physical parameters most strongly govern thrust generation and efficiency, and understanding how these parameters interact across materials, geometries, and operating conditions. To address these needs, we combine a fluid–structure interaction (FSI) modeling framework with an experimental campaign aimed at mapping the propulsive response of flexible, fish-like fins representative of carangiform caudal propulsion. The numerical model enables dynamic simulations under controlled variations of stiffness distribution, plate geometry, and actuation, thereby exploring a design space relevant to marine robotics. The experimental activity in a water channel complements the simulations by validating trends and quantifying performance, using fin prototypes of different materials and thicknesses spanning both rigid and biomimetic stiffness ranges. In contrast to many previous studies focused on small propulsors [1, 2], we explicitly consider meter-scale platforms by employing plates of 500×150 [mm], a size compatible with robotic bodies of about one meter in length and missions at significant depths. Methodologically, this work builds on our previous contributions to analytical hydroelastic modeling of oscillatory fish-like propulsion and to control strategies for systems with memory effects [3, 4, 5, 6, 7, 8, 9, 10, 11]. The outcome is a set of propulsion maps that synthesize the role of dimensionless stiffness, inertia ratio, reduced frequency, and Strouhal number, providing practical guidance for selecting materials and geometries and for tuning actuation to achieve efficient, scalable bio-inspired propellers suitable for real-world marine operations.

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Hydroelastic Modeling Framework

The two-way model developed to generate propulsion maps was obtained by coupling the one-dimensional Euler–Bernoulli plate equation with potential flow theory. This model represents the cross-section of the caudal fin and is generalized to handle non-rectangular shapes as well. The development and validation of the model, together with the formulation of the coupling terms (mass, stiffness, added damping, and memory effects) in closed form, are presented in detail in [3, 4]. The closed-form solution was derived using complex analysis, making the model semi-analytical [12]. Moreover, the model is formulated entirely in the time domain to allow integration with control logic [11, 10]. From this formulation, an integro-differential equation of motion was obtained, incorporating convolutional memory effects [8]. In this framework, Wu’s theory is extended by replacing Theodorsen’s frequency-dependent function with Wagner’s time-dependent function. The caudal fin is thus modeled as a pitching/heaving Euler–Bernoulli beam subjected to two main external forces: one arising from pitch and heave accelerations, and the other of hydrodynamic origin. The hydrodynamic contribution accounts for both the non-circulatory and circulatory components, the latter associated with vortex shedding. The final modal equation of motion of the system is written as

$$\widetilde{\underline{M}} \ddot{\underline{q}}(t) + \widetilde{\underline{C}} \dot{\underline{q}}(t) + \widetilde{\underline{K}} \underline{q}(t) + \left[\widetilde{\underline{\Psi}} \ddot{\Phi}(t) + \widetilde{\underline{\Upsilon}} \dot{\Phi}(t) \right] * \underline{q}(t) = \widetilde{\underline{F}} \quad (1)$$

Here, the matrices $\widetilde{\underline{M}}$, $\widetilde{\underline{C}}$, and $\widetilde{\underline{K}}$ represent the effective mass, damping, and stiffness, including both structural and fluid contributions. Vortex shedding generates additional forces on the plate, described by the convolution matrices $\widetilde{\underline{\Upsilon}}$ and $\widetilde{\underline{\Psi}}$, which represent memory effects. Finally, the vector $\widetilde{\underline{F}}$ accounts for the external forces (see [3]).

Parametric Design maps

In order to properly design the propulsion system, it is necessary to consider multiple parameters simultaneously. Parametric maps provide an effective way to achieve this. Here, maps based on dimensionless stiffness $S = \frac{h_b^3 E}{\rho_f U_\infty^2 L^3}$ and solid-to-fluid inertia $R = \frac{\rho_s h_b}{\rho_f L}$ are proposed, both associated with the reduced frequency $\tilde{f} = f_\alpha L / U$. Since the system is intended to operate in an underwater environment, it is important that both R and S fall within the ranges observed in nature. Marine biology studies indicate that the efficiency of fish locomotion occurs for $R \ll 1$ and $10^{-3} < S < 10^3$ [13]. For this reason, our goal is to investigate these intervals, expanding the analysis to large geometries and engineering materials relevant for marine applications. The leading parameter for selecting the optimal configuration is the thrust coefficient

$$C_T = \frac{\bar{T}}{\frac{1}{2} \rho U^2 d L}.$$

However, optimal operation requires meeting two simultaneous conditions [14]. The first is that the actuation frequency f matches the first wet natural frequency of the panel f_{wet}^1 , thereby inducing resonance and amplifying deformation. The second is that the Strouhal number $St = \frac{f_\alpha \chi}{U}$, with χ the peak-to-peak amplitude, falls within the range $[0.25 - 0.35]$. This ensures wake resonance and an increase in thrust production. When both conditions are satisfied, the motion is inherently efficient. A further requirement concerns the trailing edge velocity V_{TE} , which must satisfy $V_{TE} \ll U_\infty$ for the model to remain valid. An example of the maps is reported in Figure 1. All plate materials were experimentally characterized to ensure high fidelity of the numerical results. In the C_T maps, the white lines represent the boundaries of the optimal St range, while the magenta dots indicate the first wet frequencies of the tested plates. Optimal configurations are selected by restricting the candidate plates to those whose natural frequencies lie between the two white lines, thereby meeting the resonance and wake conditions. As shown, the ranges of both S and R fall within biomimetic values thanks to the combination of plate geometries and thicknesses employed. The lower row of Figure 1 also verifies the model validity, confirming that for the identified range of plates $U_\infty \approx 1$ [m/s] and $V_{TE} < 0.6$, thus satisfying $V_{TE} \ll U_\infty$. Among all tested plates, the highest thrust coefficient $C_T = 0.27$ was obtained with a 3 [mm] HDPE plate. This procedure guided the selection of the most promising plate for the experimental campaign and the development of a first robotic prototype, which demonstrated encouraging results and supports further refinement of the proposed methodology.

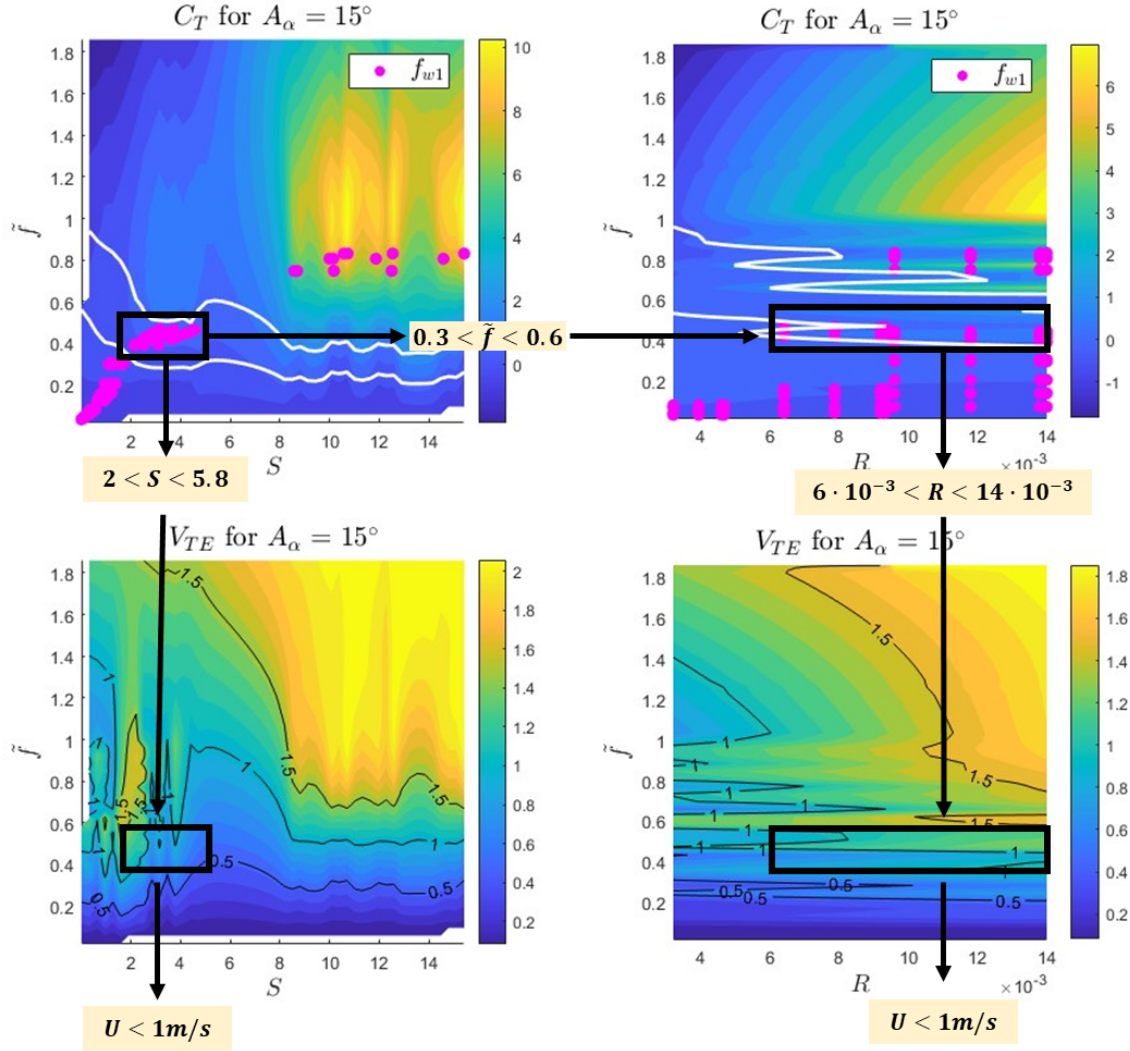


Fig. 1 Extraction of the best fin geometry using design maps. White lines indicate the boundaries of the optimal St range, while magenta dots mark the first wet frequencies of the tested plates

Experimental setup and validation

The experimental campaign is being conducted in the water channel of the Institute of Marine Engineering. The facility features a test section with a width of 536 [mm] and a depth of 650 [mm], allowing flow velocities of up to 1.8 [m/s]. The setup consists, from top to bottom, of a brushless Smartmotor (1.45 [Nm] continuous torque, 3.39 [Nm] peak torque, encoder resolution 8,000 [counts/rev], supporting DeviceNet, PROFIBUS, and CANopen communication protocols), connected to a planetary gearbox, a 6-axis load cell for measuring forces and moments (50 [N], 5 [Nm]), and a rod to which the plates are attached (Figure 2). The system is closed by a flanged bushing at the bottom of the channel, acting as a flat hinge for the lower end of the rod. Because of vibrations caused by the water pumps, the setup is mounted on a structure designed to isolate the load cell and rod as much as possible. This significantly reduced noise in the force measurements, which are recorded using a Dewesoft acquisition system. In the lower part of the test section, beneath the plate, a window provides optical access for a Photron high-speed camera, enabling reconstruction of plate deformation during post-processing. The experimental campaign also investigates the influence of flow velocity. This parameter is controlled by varying the pump revolutions per minute and measuring the actual velocity with a Dantec Dynamics laser Doppler system. Tests were performed at different inflow speeds ($U_\infty = 0.1, 0.2, 0.4$ [m/s]) and pitching angles ($\alpha = 5^\circ, 12.5^\circ, 25^\circ$). The plates tested so far include 2, 3 [mm] HDPE, 2, 3 [mm] PMMA, and 1, 2 [mm] aluminum plates. The actuation frequency was varied between 0.1 and 1.5 [Hz], with each experiment repeated at least three times to ensure repeatability. This frequency range is

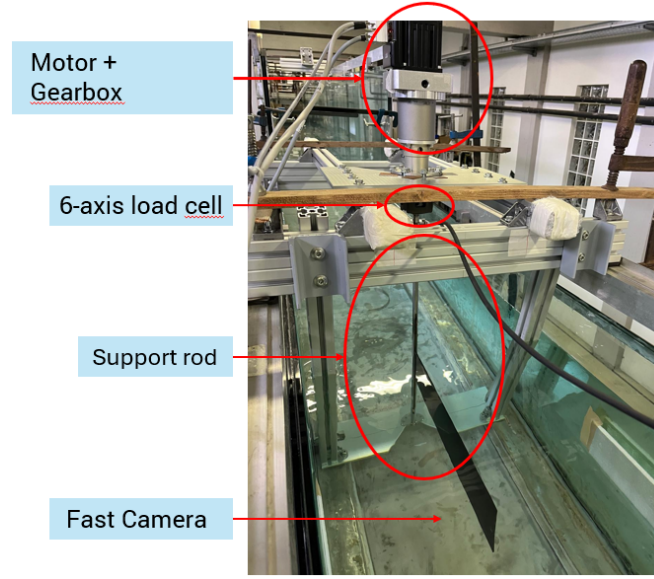
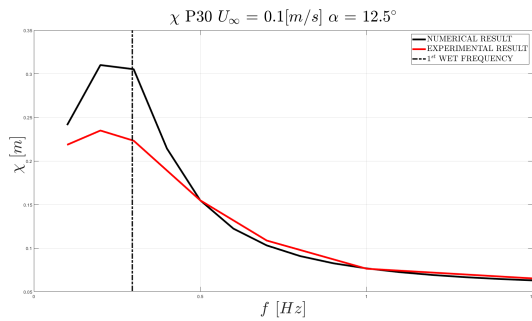
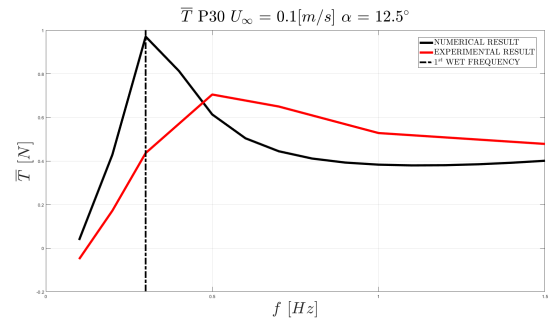


Fig. 2 Experimental setup with the 3 [mm] HDPE plate

lower than that used in the numerical model, as the first wet frequency of all plates lies below 1.5 [Hz], making it unnecessary to extend the range further. The experimental results are compared with the numerical predictions to assess the accuracy of the model. Figures 3a and 3b illustrate the case of the 3 [mm] HDPE plate at $U_\infty = 0.1$ [m/s] and $\alpha = 12.5^\circ$. The model provides an excellent approximation of both the peak-to-peak amplitude and the thrust production. However, both quantities are slightly overestimated at the first wet frequency of the system (dashed line), a discrepancy attributable to the modal nature of the model.



(a) Numerical (black) and experimental (red) comparison of peak-to-peak amplitude



(b) Numerical (black) and experimental (red) comparison of thrust production

Fig. 3 Comparison between numerical and experimental results for the 3 [mm] HDPE plate at $U_\infty = 0.1$ [m/s] and $\alpha = 12.5^\circ$. (a) Peak-to-peak amplitude. (b) Thrust production

Conclusions and Ongoing Work

The purpose of this article is to illustrate the tools we have developed to optimize the design of biomimetic propulsion systems. Starting from the development of a hydroelastic model, systematic simulations were performed and collected into maps to provide a synoptic view of thrust behavior as different parameters vary. This approach made it possible to identify the most efficient configuration in terms of thrust generation, which was then used as the starting point for the experimental campaign. The experimental tests validated the theoretical model, confirming the accuracy of the generated maps. Nevertheless, model updates are required to correct its behavior at low frequencies near the first wet resonance. To this end, additional

experimental tests are currently being carried out to expand the parameter range and isolate the most influential factors. Furthermore, the introduction of corrective coefficients is being studied to compensate for the approximations inherent in the potential flow theory approach. Once updated and refined, the model will also be applied to investigate the effects of caudal fin shape and combined heaving–pitching motions, providing broader guidelines for designing optimal propellers for specific operating scenarios.

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Chapter 17

Analytical Modeling of Flexible Caudal Fins with Arbitrary Biomimetic Shapes

E. Paifelman, S. Milana, G. Costantino Muniz, A. Carcaterra, and E. Ciappi

Abstract This study presents an extension of an analytical model for the dynamic behavior of flexible bio-inspired fins, with the goal of enabling real-time predictions across arbitrary planform geometries. The work addresses the need for fast and versatile modeling tools to support the design of next-generation soft and smart fin technologies. The proposed fully analytical formulation captures the coupled fluid–structure interaction dynamics of elastic fins and is generalized to accommodate non-rectangular planforms, including trapezoidal and biomimetic shapes. This extension allows efficient exploration of geometric effects on propulsion without the computational cost of full numerical simulations. A systematic numerical study is performed to assess the influence of planform shape on key propulsion metrics such as thrust generation and efficiency. The results reveal clear trends in performance as a function of geometric features, providing valuable guidance for selecting optimal shapes in the design of adaptive and soft-actuated fins. The analytical model’s real-time capability and generality make it a powerful tool for future integration with control strategies and smart actuation systems, paving the way for more efficient and versatile underwater robotic propulsion.

Keywords Fish-like · Bionspired robot · FSI model · Modal analysis · Biomimetic propulsion

Introduction

The design of efficient underwater propulsion systems increasingly draws inspiration from flexible bio-inspired fins, whose deformable nature can exploit fluid–structure interactions (FSI) to enhance thrust and propulsive efficiency. Foundational theories of aquatic locomotion established the role of unsteady lift, body–fin kinematics, and wake mechanics in thrust generation [6, 20, 19], while later studies clarified performance maps and optimality ranges (e.g., Strouhal range) for oscillating foils and flexible panels [17, 5]. Comprehensive reviews summarize undulatory and oscillatory swimming strategies and their scaling [18]. From an FSI standpoint, low-order analytical and semi-analytical formulations offer physical insight and computational efficiency, whereas high-fidelity CFD/FSI provide detailed wake resolution at higher computational cost [7]. The influence of flexibility, both distributed and localized, has been linked to resonance phenomena, thrust peaks, and efficiency trade-offs [4]. Geometry is likewise pivotal: trailing-edge shaping and planform effects modulate vortex dynamics and propulsive metrics [21]. In our previous publication, we presented a fluid–structure coupled framework for oscillatory fish-like propulsion in which only the added-mass contribution was derived in closed analytical form [8, 12]. That added-mass formulation was validated both against numerical simulations and via comparison with experimental data from the literature, confirming accuracy while maintaining a markedly low computational cost [12, 3]. Nevertheless, the prior model was limited to fins with a uniform rectangular planform (constant span along the chord), which restricts the exploration of span-varying, biomimetic geometries commonly observed in nature [2, 1]. The present work addresses these limitations by extending the framework to variable-span fins and non-rectangular planforms (e.g., trapezoidal and biomimetic/lunate) and,

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crucially, by promoting the formulation to a fully analytical model: all coupling coefficients that govern the FSI dynamics (including damping, stiffness-like terms, and vorticity-related operators) are now provided in closed form. This generalization preserves real-time or near-real-time capability across a broad geometric design space, enabling systematic interrogation of how planform variation influences thrust and efficiency without resorting to costly high-fidelity simulations for every candidate [3, 5]. The availability of a fully analytical reduced-order description paves the way for geometry-aware control and smart actuation strategies that explicitly exploit real-time prediction. In this context, our prior developments on optimal and feedback control for systems with memory and hydroelastic couplings provide a natural foundation for integrating the present modeling advances into closed-loop architectures [11, 10, 9, 16, 14, 15, 13].

Extended Analytical Model for Variable-Span Fins

The analytical framework previously introduced in [12] is here generalized to fins with non-rectangular planforms, in order to capture span variations typical of biomimetic morphologies. Three representative geometries are considered: a rectangular fin, a trapezoidal fin, and a biomimetic fin featuring a trailing-edge circular cut-out (see Figure 1).

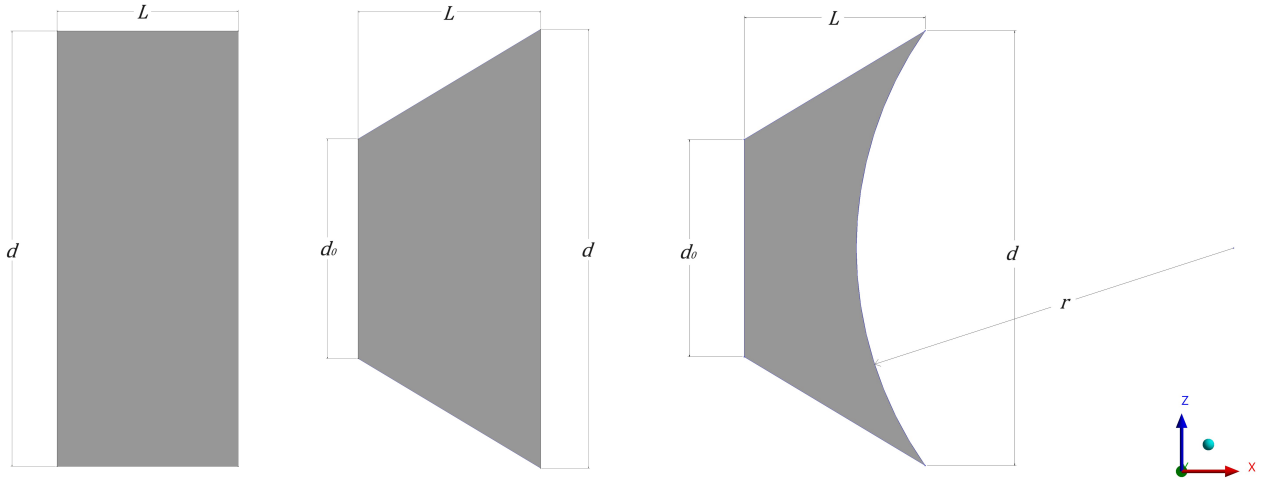


Fig. 1 Caudal fin geometries: rectangular, trapezoidal, and biomimetic/lunate

All configurations share the same material properties, namely polyethylene with Young's modulus $E_Y = 2 \cdot 10^9$ Pa and density $\rho = 1300$ kg/m³, the same thickness $h_b = 0.38$ mm, and the same aspect ratio $AR = 2.4$. In this way, all fins can be inscribed in the same bounding rectangle of chord L and maximum span d , thus ensuring comparability across planforms while maintaining consistency with available experimental data. The spanwise distribution is generalized from the constant value of the rectangular plate to more complex functional laws. In the trapezoidal case, the span varies linearly with the chordwise coordinate x , while in the biomimetic case it decreases nonlinearly due to the trailing-edge cut-out. In analytical terms, the span law is represented as $d(x) = \sum_{k=0}^m d_k x^k$, where the coefficients d_k encode the chosen geometry. The hydrodynamic response of the fin is modeled by coupling a linear potential flow formulation for the surrounding incompressible, inviscid fluid with the Euler–Bernoulli beam representation of the fin's mid-span section. For additional mathematical details the reader is referred to [12]. The dynamics of the fin in modal coordinates $\mathbf{q}(t)$ are governed by

$$\widetilde{\mathbf{M}} \ddot{\mathbf{q}}(t) + \widetilde{\mathbf{C}} \dot{\mathbf{q}}(t) + \widetilde{\mathbf{K}} \mathbf{q}(t) + \left[\widetilde{\Psi} \ddot{\mathbf{q}}(t) + \widetilde{\Upsilon} \dot{\mathbf{q}}(t) \right] * \varphi(t) = \widetilde{\mathbf{F}}(t), \quad (1)$$

where $\widetilde{\mathbf{M}}$ is the generalized mass matrix, $\widetilde{\mathbf{C}}$ the damping matrix, $\widetilde{\mathbf{K}}$ the structural stiffness matrix, $\widetilde{\Psi}$ and $\widetilde{\Upsilon}$ the aerodynamic coupling operators, $\varphi(t)$ the bound circulation, and $\widetilde{\mathbf{F}}(t)$ the forcing vector due to heaving and pitching motions. All operators are expressed in closed form as functions of the span law coefficients d_k [12]. The application of the model requires suitable modal shapes. Two strategies were adopted. In the first, finite element (FE) simulations were employed to compute the natural frequencies and eigenmodes of each geometry, providing a high-fidelity reference. In the second, a Galerkin projection was used: the modal basis of a clamped uniform beam was chosen as reference, and the governing

equations were projected onto this basis to derive approximate modal shapes. The resulting eigenvectors provide the coefficients of the modified modes, expressed as $\Phi_i^*(x) = \sum_{j=1}^N c_j \Phi_j(x)$, which approximate the true shapes with different levels of accuracy depending on the geometry. For trapezoidal fins, the Galerkin approach yields excellent agreement with FE predictions, while for the biomimetic geometry higher-order contributions become significant, leading to a frequency deviation of about 15% compared with FE.

Results and Discussion

The performance of the three fin geometries was evaluated in terms of thrust production and propulsive efficiency. Once the coupled fluid–structure dynamics were solved, the instantaneous hydrodynamic forces were obtained from the unsteady pressure distribution along the fin surface, reconstructed from the bound circulation predicted by the linear potential flow model. The thrust was computed by integrating the streamwise component of the pressure forces over the fin surface, while the propulsive efficiency was defined as the ratio between the useful propulsive power and the input mechanical power associated with the prescribed heave–pitch kinematics. Formally, these two indicators can be written as

$$T = \int_S p(\mathbf{x}, t) \mathbf{n} \cdot \hat{\mathbf{x}} dS, \quad \eta = \frac{\overline{TU}}{\overline{P_{in}}}, \quad (2)$$

where T is the instantaneous thrust, $p(\mathbf{x}, t)$ the unsteady pressure field, $\mathbf{n}$ the outward surface normal, $\hat{\mathbf{x}}$ the streamwise unit vector, U the forward swimming speed, and P_{in} the input mechanical power. In Eq. (2), the overline denotes averaging over one oscillation period.

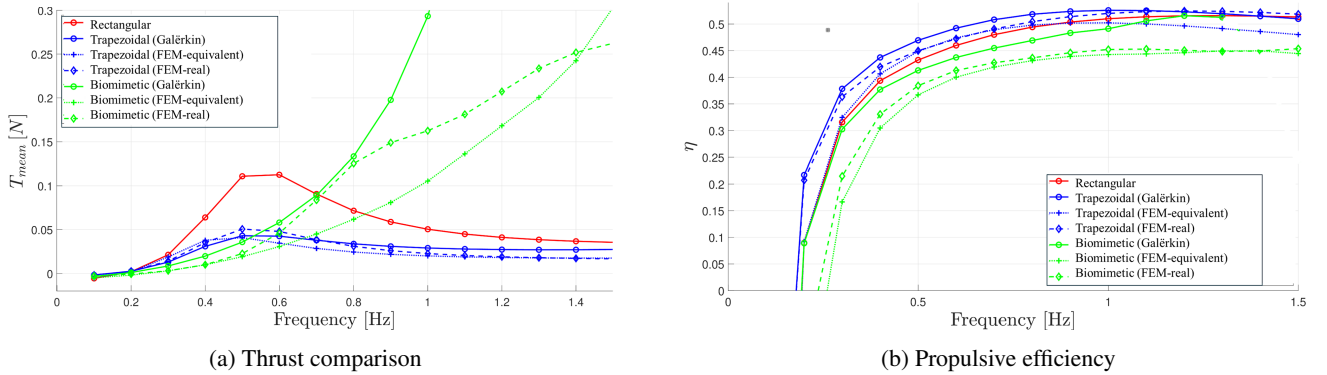


Fig. 2 Comparison across geometries and methods: (a) thrust and (b) efficiency

As shown in Figure 2 (a), the trapezoidal fin exhibits excellent agreement between the different modal identification methods across the full frequency range, confirming the robustness of the extended formulation for this geometry. For the biomimetic fin, the predictions remain consistent up to about 0.8 Hz, beyond which the Galerkin approximation begins to diverge from the finite element reference. Nevertheless, all approaches indicate that the biomimetic geometry is capable of producing larger thrust at higher frequencies, a consequence of the nonlinear span reduction and its effect on the dynamic response. The corresponding efficiencies in Figure 2 (b) show that the trapezoidal fin, although producing the lowest thrust, achieves the highest efficiency across the investigated frequency range, with a nearly perfect match between the different computational approaches. The rectangular fin occupies an intermediate position, balancing thrust and efficiency. The biomimetic fin, in contrast, is the least efficient configuration, even though it delivers the strongest thrust at high frequencies. This confirms that span variation introduces a trade-off between force production and energetic cost, with trapezoidal geometries favoring efficiency and biomimetic ones favoring thrust. Overall, these results demonstrate that the extended analytical model is able to capture the propulsive differences between variable-span fins in a consistent manner, while preserving the real-time capability required for design exploration and for potential integration with control strategies.

Future Remarks

Ongoing work is focused on extending the validation of the present framework through a dedicated experimental campaign aimed at building a comprehensive database of hydrodynamic and structural responses across a broad range of planform shapes. In parallel, an optimization study is being conducted on the basis of the proposed fully analytical model, exploiting its computational efficiency to explore large design spaces systematically. The ultimate objective is to provide designers with a real-time decision-support tool capable of identifying the most suitable fin geometry according to the specific operational requirements of biomimetic underwater vehicles.

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Chapter 18

Integrated Virtual Sensing Method for Real-Time Damage Detection in Mechanical Systems

Rodrigo Salazar Colunga, Nimish Pandiya, Christian Dindorf, and Frank Naets

Abstract The automotive industry seeks to improve the reliability and safety of mechanical systems while reducing maintenance costs. Structural Health Monitoring (SHM) provides a promising solution through real-time integrity assessments and early damage detection, helping to prevent catastrophic failures. The presented work investigates a virtual sensing approach for automotive components, combining parametric (model-based) and non-parametric (signal-based) techniques. Unlike physical sensors, which are costly and require maintenance, virtual sensors estimate difficult-to-measure quantities using data from a limited number of sensors. In this paper, the mechanical system under study is a notched aluminum beam, representative of an overhanging automotive component. Acceleration responses are acquired through vibration testing on a 3D-shaker to simulate a dynamic operational environment. A virtual sensing approach in the form of a tuned Kalman Filter (KF) is utilized to monitor the mechanical system. This work focuses on an initial assessment of the accuracy of the KF virtual sensing methodology when implemented with a generalized base excitation input in mechanical systems.

Keywords State estimation · Kalman filter · Virtual sensing · Damage detection · Structural health monitoring

Introduction

Modern vehicles operate under diverse and often unpredictable load conditions, which makes effective structural integrity monitoring essential [1]. Traditional maintenance practices based on scheduled inspections, may not adequately capture the irregular progression of wear that can lead to premature failures [2]. In this context, SHM provides real-time assessments that enable early damage detection and fatigue life prediction, supporting proactive maintenance strategies and extending component lifespan [3].

Recent literature on SHM addresses both model-based and signal-based approaches [4]. Model-based methods rely on mathematical or Finite Element (FE) models of the structure, but their effectiveness depends on highly accurate models, often requiring complex nonlinear analyses that are computationally expensive [5]. On the other hand, signal-based methods, compare measured responses with previously acquired data to detect anomalies [6]. While being more adaptable, they depend heavily on sensor placement and availability. In many practical cases, direct measurements at critical locations are infeasible due to integration challenges or sensor limitations [7].

To address these challenges, this study presents a virtual sensing approach that combines limited sensor data with a reduced-order FE model to estimate system states. This approach is demonstrated on a notched aluminum beam tested under random base excitation on a 3D shaker. A tuned KF is employed for a full-field response estimation, enabling the

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recovery of “difficult-to-measure” quantities. The estimated states can then be used for a variety of applications, such as predicting of stresses/strains and fatigue loads, real-time structural health monitoring or damage identification, among others [8, 9, 10, 11].

To the best of the author’s knowledge, no existing formulation explicitly addresses state estimation for cro:MIMO Multiple Inputs Multiple Outputs (MIMO) cro:MDOF Multiple Degree of Freedom (MDOF) systems under applied base-excitation conditions, which motivates the approach proposed in this study.

The paper is outlined as follows: Section [FE Model Validation](#) describes the object of study, the system characterization via an cro:EMA Experimental Modal Analysis (EMA), as well as the description and validation of the FE model. Section [System Modeling and Validation](#) introduces the mathematical modeling framework for base-excited systems and its validation. Section [State Estimation with Kalman Filter](#) presents the state estimation algorithm. The results of the full-field response are presented in Section [Full-field response results](#), and conclusions with future perspectives are stated in Section [Conclusions](#).

FE Model Validation

System Description

The structure under study is a aluminum beam with a rectangular cross section of 5×10 mm and a length of 390 mm, cut from *AlMgSi0.5* stocks. As shown in Figure 1, a round notch of depth 1.65 mm is machined on the long edge of the beam acting as a stress concentrator.

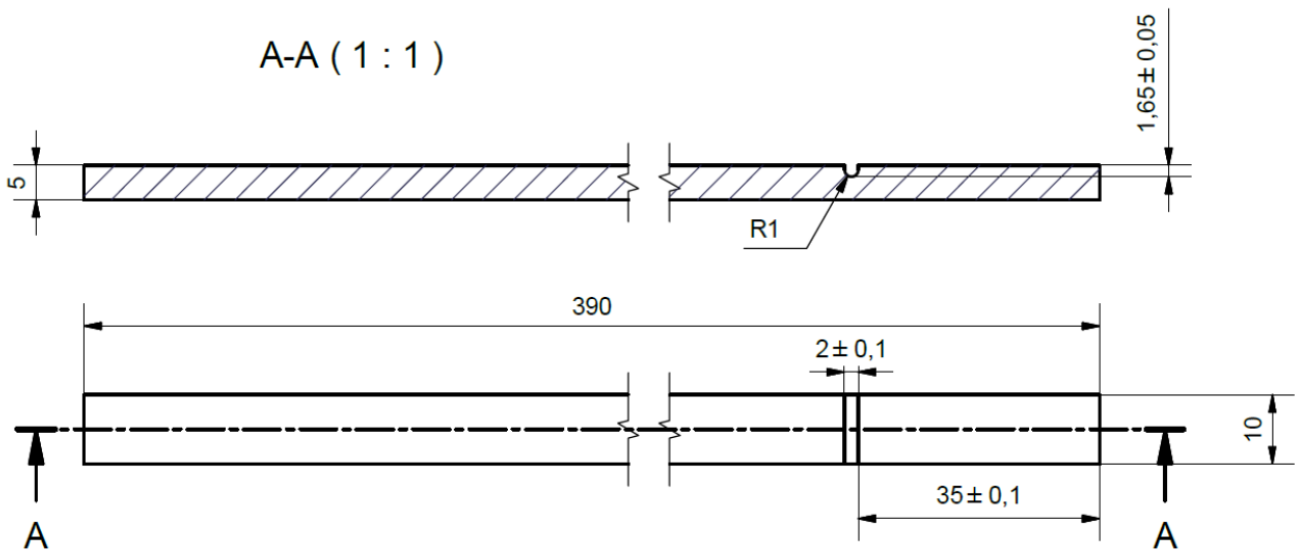


Fig. 1 Technical drawing of specimen [12]

Experimental Modal Analysis

To characterize the dynamic properties of the beam, an in-situ EMA was performed. The beam was mounted in a cantilever configuration using a fixture on the shaker table as shown in Figure 2 (left) to minimize the effect on the modelling of the boundary conditions. Five triaxial accelerometers were distributed along the beam, with Sensor *M21* near the notch and Sensor *M25* at the free tip as shown in Figure 2 (right) as an illustration. It can be seen on Figure 2 (left) other sensors on the fixture besides the already described, these sensors were used later for shaker control. An instrumented impact hammer was used to apply impulse excitation on the beam, and the resulting acceleration responses were recorded. This procedure enabled the identification of cro:MP Modal Parameters (MP), which were later used to calibrate the numerical FE model.

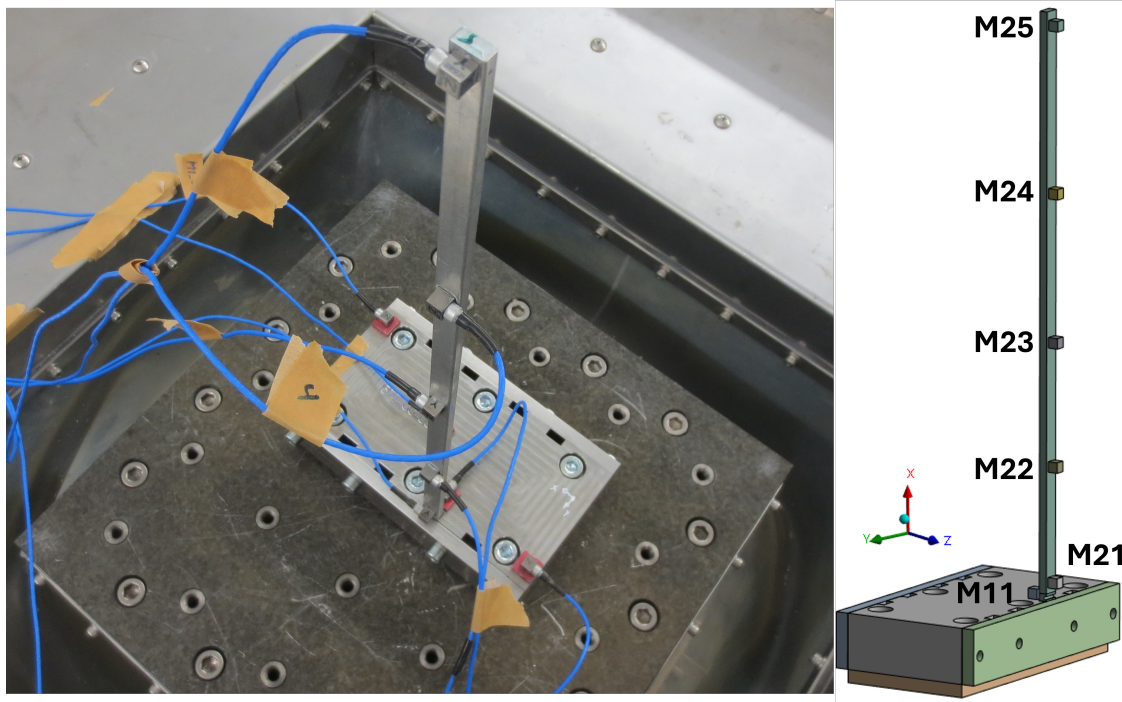


Fig. 2 Experimental (left) and FE illustrated sensor configuration (right)

Numerical Model

A FE model of the notched beam was developed in ANSYS® Mechanical. The beam was meshed with 49980 linear HEX-SOLID185 elements, and the boundary condition at one end was defined as fully fixed to replicate the cantilever setup. Material properties were calibrated using the native parametric optimization tool in ANSYS® Workbench against the experimental modal parameters obtained from the EMA. The natural frequencies obtained experimentally and numerically are compared in Table 1. The discrepancies remain below 4% across the first seven modes, demonstrating strong agreement between the two data-sets. Furthermore, the mode shape correlation was quantified using the cro:MACModal Assurance Criterion (MAC) [13], shown in Figure 3. MAC values close to 1 confirm that the FE model reproduces the system dynamics with good accuracy. The simulated mode shape of mode 6 and 7, does not correlate with the corresponding experimental mode, but appear to be switched. No effort was made to correct this mode order because it was taken into account in the KF algorithm.

Table 1 Comparison of experimental and numerical natural frequencies.

No	FEM $f_{\text{fix-free}}$ (Hz)	Experimental $f_{\text{fix-free}}$ (Hz)	Error (%)
1	28.83	27.797	-3.58
2	55.40	56.4	+1.81
3	176.41	174.55	-1.05
4	345.36	352.47	+2.06
5	493.43	489.24	-0.85
6	963.26	958.92	-0.45
7	972.06	981.88	+1.01

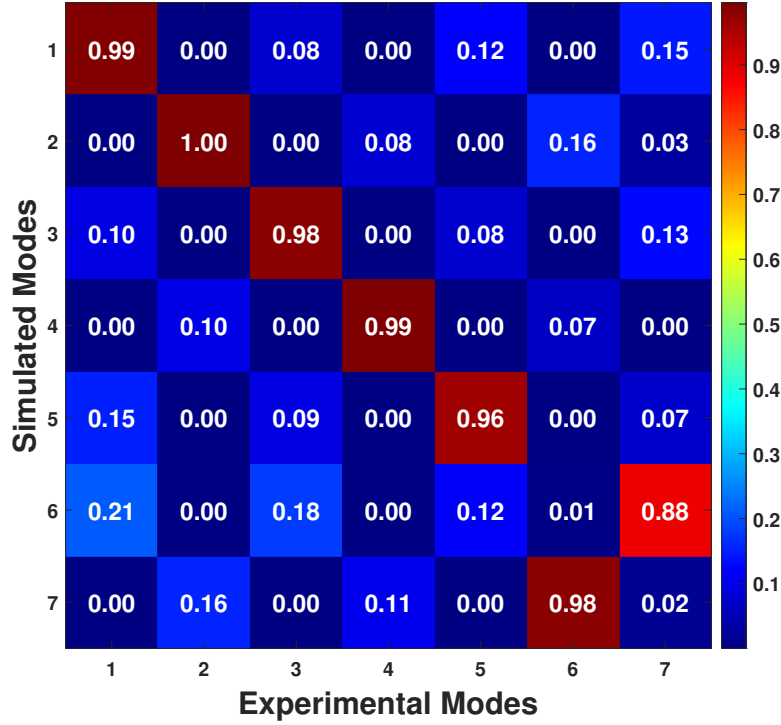


Fig. 3 Experimental and FE Modal Assurance Criterion (MAC) for the first 7 modes.

System Modeling and Validation

Equation of motion

The dynamics of a linear time-invariant mechanical system with n_{DoF} degrees of freedom can be described by the following discretized equation of motion in Eq. (1):

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \quad (1)$$

where $x(t) \in \mathbb{R}^{n_{\text{DoF}} \times 1}$ is the displacement, and $M, C, K \in \mathbb{R}^{n_{\text{DoF}} \times n_{\text{DoF}}}$ denote the mass, damping and stiffness matrices, respectively. The external excitation is given by the force vector $f(t) \in \mathbb{R}^{n_{\text{DoF}}}$.

Eq. (1) can be reformulated for cases in which the excitation is applied as a prescribed base motion rather than a direct force input. While base-excitation formulations have been extensively studied in civil and earthquake engineering, where they are typically restricted to cro:SIMOSingle Input Multiple Outputs (SIMO) systems [14] as shown in Eq. (2):

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -ML_o\ddot{x}_g(t) \quad (2)$$

where L_o is the ground input selection vector, specifying locations of the applied base excitation, typically a column vector of ones indicating that the input is applied equally to all degrees of freedom. Relative displacement of the mass with respect to the ground is represented by $x(t)$ and the imposed ground acceleration by $\ddot{x}_g(t)$.

It is worth emphasizing that the extension of this approach to MIMO MDoF mechanical system remains relatively unexplored. This motivates the present work, which develops and validates a generalized formulation suitable for mechanical structures. Such systems are typically modeled using FE models, where motion is described in terms of coupled displacements in the x , y and z directions. A generalized partition of the system can be performed on the basis of b and f nodes in Eq. (3):

$$\begin{bmatrix} M_{bb} & M_{bf} \\ M_{fb} & M_{ff} \end{bmatrix} \begin{bmatrix} \ddot{x}_b \\ \ddot{x}_f \end{bmatrix} + \begin{bmatrix} C_{bb} & C_{bf} \\ C_{fb} & C_{ff} \end{bmatrix} \begin{bmatrix} \dot{x}_b \\ \dot{x}_f \end{bmatrix} + \begin{bmatrix} K_{bb} & K_{bf} \\ K_{fb} & K_{ff} \end{bmatrix} \begin{bmatrix} x_b \\ x_f \end{bmatrix} = \begin{bmatrix} f_b(t) \\ f_f(t) \end{bmatrix} \quad (3)$$

where the subscripts b and f denote the base and free cro:DoFsDegrees of Freedom (DoFs) of a FE model, respectively. The portion of Eq. (3) corresponding to the free nodes (x_f) constitutes the unknown equation to be solved for the system

response under base acceleration. The base nodes (x_b) are typically prescribed by the imposed ground input, so their values are known.

By introducing the relative displacement of the free nodes with respect to the base, defined in Eq. (4), into Eq. (3), the expression for x_f can be reformulated as shown in Eq. (5)

$$x_r(t) = x_f(t) - x_b(t) \quad (4)$$

$$M_{ff} \ddot{x}_r(t) + C_{ff} \dot{x}_r(t) + K_{ff} x_r(t) = -M_{be} \ddot{x}_b(t) \quad (5)$$

where:

$$M_{be} = M_{fb}L_{ob} + M_{ff}L_{of} \quad (6)$$

L_{ob} and L_{of} are Boolean selection matrices. They project the imposed base motion onto the corresponding DoFs of the system. This ensures consistent dimensions in the matrix products. By neglecting both damping and external base excitation, Eq. (5) reduces to Eq. (7), which is an homogeneous, undamped system.

$$M_{ff} \ddot{x}_r(t) + K_{ff} x_r(t) = 0 \quad (7)$$

assuming a harmonic solution of the form of Eq. (8)

$$x_r(t) = \phi e^{i\omega t} \quad (8)$$

with mode shape vector ϕ and natural frequency ω , substitution into Eq. (7) yields to

$$(K_{ff} - \omega^2 M_{ff}) \phi = 0 \quad (9)$$

which represents the generalized eigenvalue problem of the system. Collecting all eigenvectors ϕ_j into the mode shape matrix $\Phi \in \mathbb{R}^{n_{DoF} \times n_{DoF}}$, and the corresponding eigenvalues ω_j into the diagonal matrix Ω^2 gives the following compact form

$$K_{ff} \Phi = M_{ff} \Phi \Omega^2. \quad (10)$$

Introducing the modal coordinate transformation of Eq. (11)

$$x(t) = \Phi q(t) \quad (11)$$

where $q(t) \in \mathbb{R}^{n_m}$, n_m being the number of modes; and premultiplying Eq. (5) by Φ^T yields the system in modal form as:

$$\Phi^T M_{ff} \Phi \ddot{q}(t) + \Phi^T C_{ff} \Phi \dot{q}(t) + \Phi^T K_{ff} \Phi q(t) = -\Phi^T M_{be} \ddot{x}_b(t) \quad (12)$$

Eq. (12) can be modally decoupled by exploiting the orthogonality conditions associated with a set of mass-normalized eigenvectors, $\Phi^T M \Phi = I$ and $\Phi^T K \Phi = \Omega^2$. Assuming proportional damping, we obtain $\Phi^T C \Phi = \Gamma$, where Γ is a diagonal matrix containing the terms $2\xi_j\omega_j$, and ξ_j denotes the j^{th} modal damping ratio. The resulting decoupled equations of motion from Eq. (12) in modal coordinates are given by Eq. (13):

$$\ddot{q}(t) + \Gamma \dot{q}(t) + \Omega^2 q(t) = -M_{\Phi be} \ddot{x}_b(t) \quad (13)$$

Continuous-time state space model

The dynamics of the beam can be expressed in first-order continuous-time state-space form by introducing the state vector $z(t)$ as shown in Eq.(14):

$$z(t) = \begin{pmatrix} x(t) \\ \dot{x}(t) \end{pmatrix} \in \mathbb{R}^{n_s}, \text{ where } n_s = 2n_{DoF}, \quad (14)$$

The second-order relative displacement equation of motion Eq. (5) can then be reformulated as a first-order state equation as shown in Eq. (15):

$$\dot{z}(t) = A_c z(t) + B_c u(t) \quad (15)$$

where $u(t) \in \mathbb{R}^{n_p}$ is the input vector representing external forces, with n_p the number of independent inputs. The state dimension is $n_s = 2n_{DoF}$, where n_{DoF} denotes the number of DoFs. The matrices $A_c \in \mathbb{R}^{n_s \times n_s}$ and $B_c \in \mathbb{R}^{n_s \times n_p}$ define the system dynamics and the input mapping, respectively, and are constructed as:

$$A_c = \begin{bmatrix} 0 & I \\ -M_{ff}^{-1} K_{ff} & -M_{ff}^{-1} C_{ff} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ -M_{be}^{-1} \end{bmatrix} \quad (16)$$

In addition to the state equation, the system output is defined as:

$$y(t) = Cz(t) + Du(t) \quad (17)$$

This allows for the measurement or observation of specific system responses, such as displacements, velocities, or accelerations, at various DoFs. Here, $y(t) \in \mathbb{R}^{n_d}$ is the output vector, representing the measured quantities, where n_d is the number of measured outputs. The output matrix $C \in \mathbb{R}^{n_d \times n_s}$ and the feed-through matrix $D \in \mathbb{R}^{n_d \times n_p}$ are defined in Eq. (18).

$$C = \begin{bmatrix} I & 0 \\ 0 & I \\ -M_{ff}^{-1}K_{ff} & -M_{ff}^{-1}C_{ff} \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ -M_{be}^{-1} \end{bmatrix} \quad (18)$$

The state-space representation can be formulated in the modal domain by using Eq. (13) and introducing Eq. (19) as the modal state vector

$$\tilde{z}(t) = [q(t)^T, \dot{q}(t)^T]^T \quad (19)$$

Following the same structure as Eqs. (15) and (17), the system can be expressed as:

$$\dot{\tilde{z}}(t) = \tilde{A}_c \tilde{z}(t) + \tilde{B}_c u(t) \quad (20)$$

$$\tilde{y}(t) = \tilde{C} \tilde{z}(t) + \tilde{D} u(t) \quad (21)$$

where $u(t)$ is equal to $\ddot{x}_b(t)$ representing the base acceleration inputs, and the tilde indicates quantities expressed in modal coordinates [15]. The state, input, measurement and output matrices in the modal domain are given by:

$$\tilde{A}_c = \begin{bmatrix} 0 & I \\ -\Omega^2 & -\Gamma \end{bmatrix}, \quad \tilde{B}_c = \begin{bmatrix} 0 \\ -M_{\phi be} \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} I & 0 \\ 0 & I \\ -\Omega^2 & -\Gamma \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} 0 \\ 0 \\ -M_{\phi be} \end{bmatrix}. \quad (22)$$

Discrete-time state space model

In this section the discrete-time formulation is introduced because it is used in the implementation of the KF, which operates naturally in discrete time. Using a sampling rate of $1/\Delta t$, the continuous-time state equation Eq. (15) can be discretized to obtain its discrete-time equivalent:

$$z_{k+1} = A_d z_k + B_d u_k \quad (23)$$

where k denotes the discrete time step. Since the output equation is an algebraic relation between the states and inputs at each time step, discretization does not affect its structure. Therefore, the output equation in discrete time is given by:

$$y_k = C z_k + D u_k \quad (24)$$

where A_d and B_d are the discrete-time system matrices [16, 17] derived from their continuous-time counterparts, defined as:

$$A_d = e^{A_c \Delta t}, \quad B_d = (A_d - I) A_c^{-1} B_c \quad (25)$$

Frequency Response Validation

The system cross-Frequency Response Function (FRF) from Eq. (5), enables the understanding of the mechanical structures reaction to dynamic loading at different excitation frequencies. The FRF expresses the relationship between all inputs and all outputs of the system at certain frequencies.

$$H_{pq} = \frac{X_{r_p}(s)}{X_{b_q}(s)} = -(s^2 M_{ff} + s C_{ff} + K_{ff})^{-1} s^2 M_{be} \quad (26)$$

where s is the Laplace variable, representing complex frequency. The system response can also be expressed in modal coordinates by using the Eq. (13) leading to:

$$H_{pq} = \frac{\tilde{X}_{r_p}(s)}{X_{b_q}(s)} = -\Phi(s^2 I + s \Gamma + \Omega^2)^{-1} s^2 M_{\Phi be} \quad (27)$$

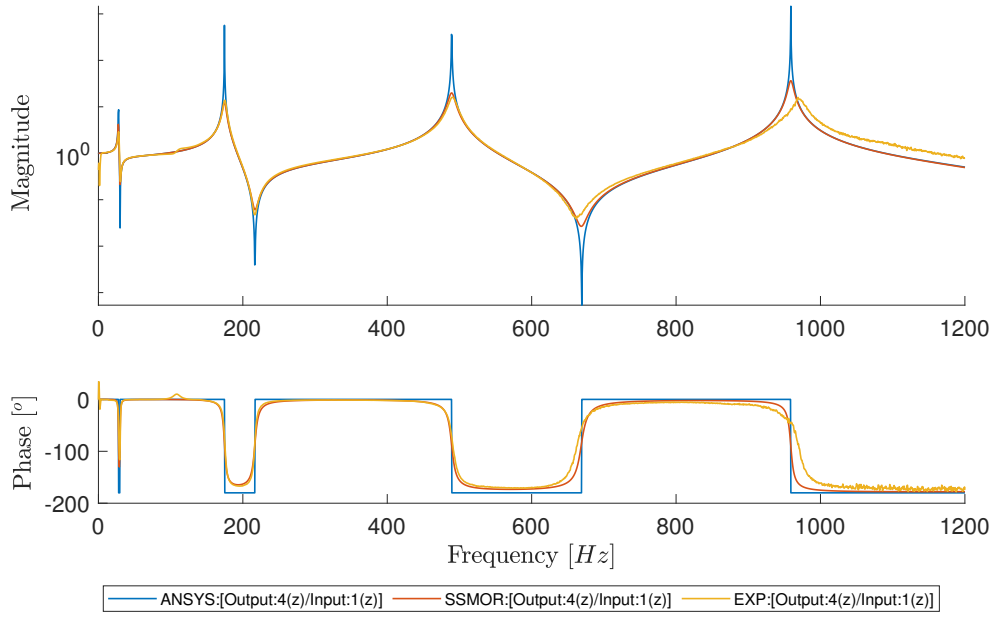


Fig. 4 Transmissibility FRF at Sensor M24 (acc/acc)

To validate the base-excitation formulation, a comparison between the FRF from the analytical reduced-order state-space model, the ANSYS® Mechanical FE harmonic simulation, and experimental measurements was performed. Figure 4 shows the (acceleration/acceleration) transmissibility FRF from sensor *M24*. The close agreement among all three confirms the accuracy of the analytical base-excitation model.

Similar agreement is also observed at the other sensor locations, confirming the consistency of the FRF transmissibility across the system. The slight differences in peak amplitudes between the ANSYS® Mechanical FE model, and both the experimental and analytical FRFs are attributed to the fact that the ANSYS® Mechanical simulation was performed using an undamped model.

State Estimation with Kalman Filter

The Kalman Filter (KF) is a deterministic–stochastic approach that fuses model predictions with measurement data to estimate the states of a dynamic system [18]. A key component of the KF is the cross-covariance Kalman Gain (KG), which enables an optimal balance between model predictions and measurement data, accounting for their respective uncertainties in an interactive way. The KF algorithm is divided into two main steps.

1. Prediction step:

$$x_{k|k-1} = A_d \hat{x}_{k-1|k-1} + B_d u_k \quad (28)$$

$$P_{k|k-1} = A_d P_{k-1|k-1} A_d^T + Q \quad (29)$$

At each step, the KF estimates the current state based on the previous state and the system model. The predicted state $x_{k|k-1}$ is projected forward in time using the system dynamics and the current input u_k , while the predicted state covariance $P_{k|k-1}$ propagates uncertainty from the previous estimate and the process noise covariance Q , which accounts for modeling errors and unmodeled disturbances [18]. The prediction is refined in the correction step.

2. Correction (or update) step:

$$S = C P_{k|k-1} C^T + R \quad (30)$$

$$KG = P_{k|k-1} C^T S^{-1} \quad (31)$$

$$x_{k|k} = x_{k|k-1} + KG(y_k - (Cx_{k|k-1} + Du_k)) \quad (32)$$

$$P_{k|k} = P_{k-1|k-1} - KGCP_{k-1|k-1} \quad (33)$$

where the innovation covariance S combines the predicted state covariance $P_{k|k-1}$ with the measurement noise covariance R , representing uncertainty in the sensor data. The KG determines how much the new measurement influences the state update. The state estimate $x_{k|k}$ is corrected using the measurement residual $y_k - (Cx_{k|k-1} + Du_k)$, and the state covariance $P_{k|k}$ is updated accordingly, typically reducing uncertainty. This procedure is repeated iteratively at each time step, providing an updated and more accurate state estimate over time [18].

Full-field response results

This section presents the full-field estimation using a real data-set. The data was acquired during a five minutes vibration test, where the input was a random cro:PSDPower Spectral Density (PSD) signal applied in the “Z-direction” of the beam, as shown in Figure 2 and indicated in Figure 5 by the green arrow. The acceleration on the fixture was measured by sensor $M11$, shown in Figure 2, and used as the input base acceleration on the beam. To reduce the computational effort of the large FE model, a cro:ROMReduced Order Model (ROM) of the beam was created. The ROM was built in modal coordinates, retaining 14 normal modes after applying modal truncation. For this test, only accelerometers were used. Out of the five measured responses along the beam, only accelerometer $M24$ was taken as the observation in the KF. This approach represents a “best-case” scenario in which a single measurement is used to estimate the responses at the remaining four locations, as might occur when sensor availability is limited or when other locations are inaccessible. The other accelerometer responses were estimated by the KF, and the predicted responses were compared against the measured data to evaluate the performance of the KF. The parameters for the KF implementation were:

- initial state vector set to zero,
- initial covariance P_0 set to ten,
- noise covariance matrix R defined as a diagonal matrix based on static test measurement, and
- process noise covariance Q defined under the assumption that uncertainty in uncorrelated modes is higher than in correlated ones.

Figures 6–9 show the *measured* and *estimated* acceleration responses over the first 50 seconds of the test for each accelerometer along the beam. The left panel of the figures shows the complete time history of the acceleration response, showing the complete dynamic behavior of the beam under base excitation. The right panel provides a detailed view of a segment of the response. Furthermore, the cro:NRMSNormalized Root Mean Square Error (NRMSE) between the *estimated* and *measured* acceleration time histories at accelerometers $M21$ – $M25$ was calculated using Eq. (34) and the results are reported in Table 2.

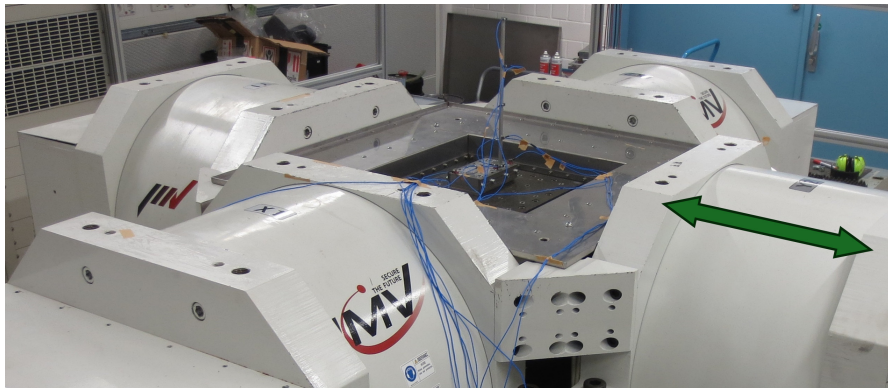


Fig. 5 Random vibration test setup on the electrodynamic 3D shaker. The green arrow indicates the excitation direction.

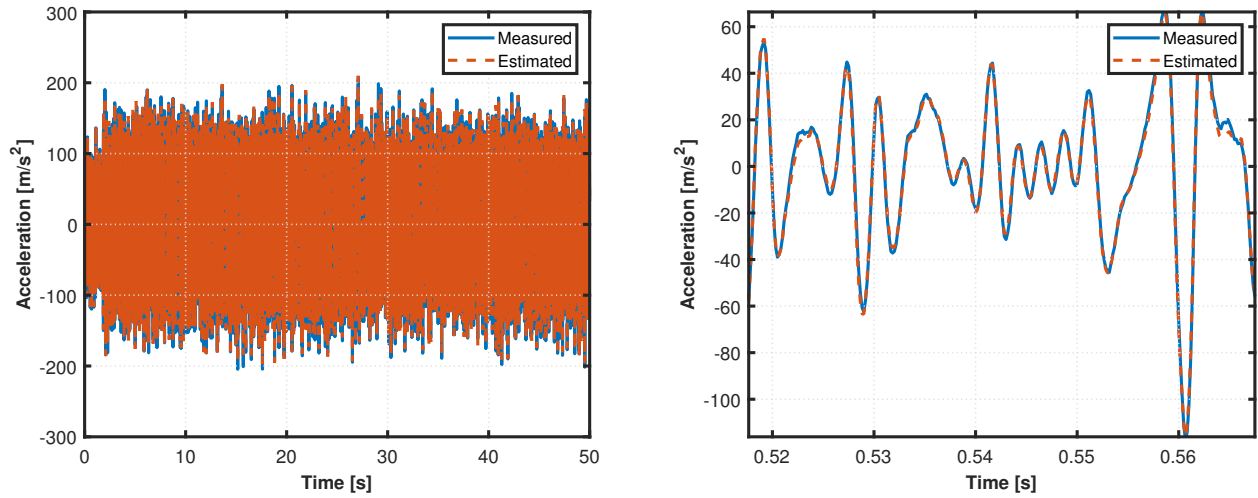


Fig. 6 Response at accelerometer M21: full time history (left) and detail (right).

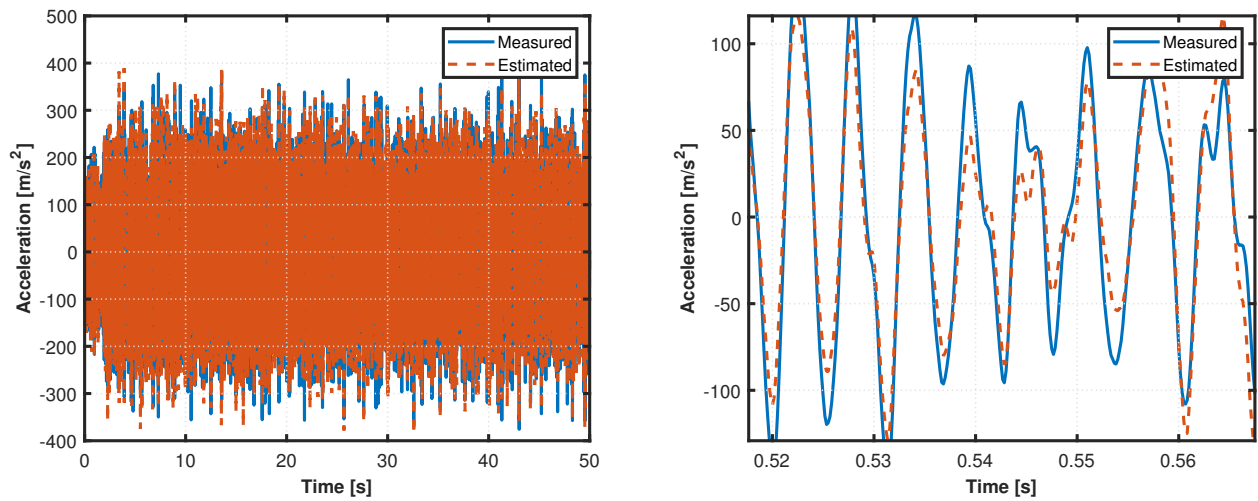


Fig. 7 Response at accelerometer M22: full time history (left) and detail (right)

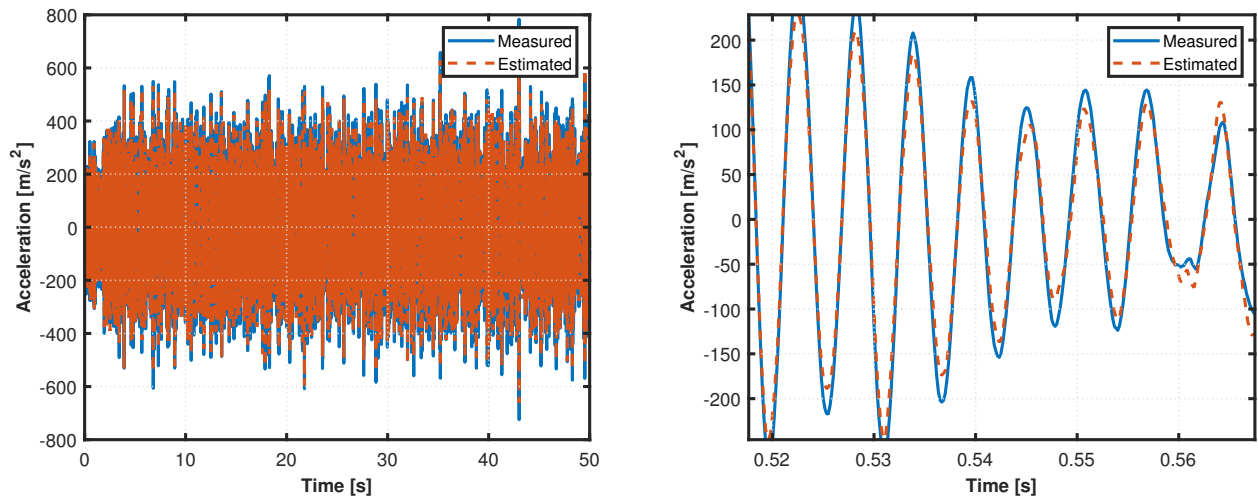


Fig. 8 Response at accelerometer M23: full time history (left) and detail (right)

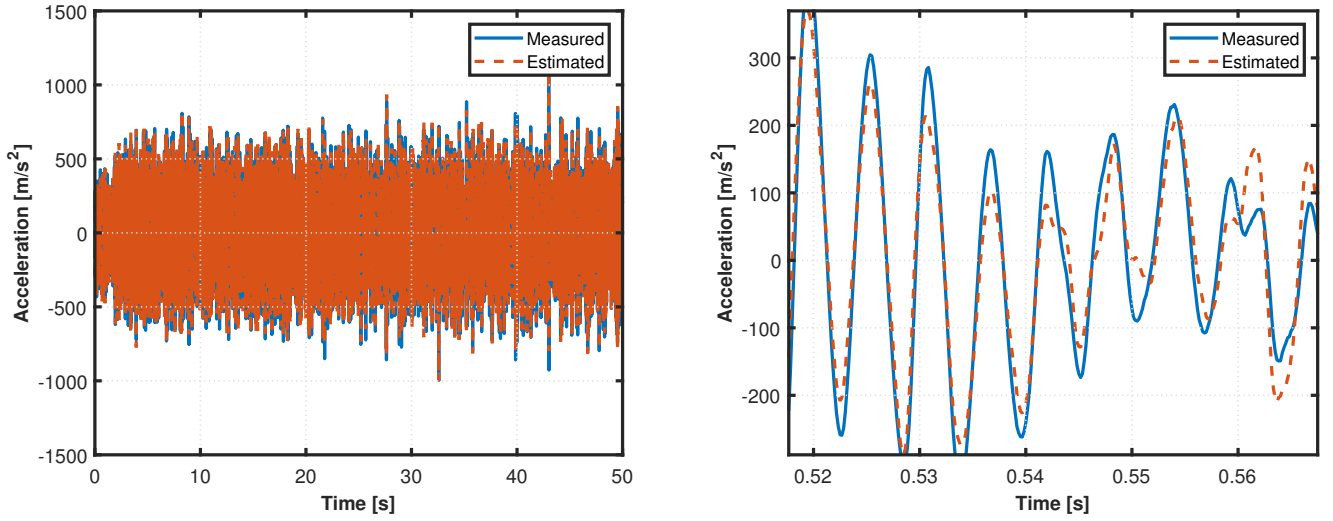


Fig. 9 Response at accelerometer M25: full time history (left) and detail (right)

Table 2 NRMSE between measured and estimated acceleration at sensors M21–M25.

Sensor	NRMSE [%]
M21	2.95
M22	10.28
M23	3.55
M24	8.74×10^{-14}
M25	4.83

$$\text{NRMSE} = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2}}{\max(y) - \min(y)}, \quad (34)$$

where y_i and $\hat{y}_i$ denote the *measured* and *estimated* signals at time step i , respectively, and N is the number of samples. The numerator corresponds to the cro:RMSE Root Mean Square Error (RMSE), while the denominator normalizes the error with respect to a range [19]. For the calculation of the NRMSE value, the 50 seconds of data was used. The lowest NRMSE among the estimated sensors was obtained at sensor *M21* (2.95%), which is the accelerometer closest to the notch region of the beam. Sensor *M23* also showed a low NRMSE value of 3.55%, confirming that the estimator performs reliably at intermediate locations along the structure. Higher NRMSE values appeared at sensor *M22* and *M25*, 10.285% and 4.831%, respectively. However, these locations are not the main analysis goal, and even at sensor *M22*, the NRMSE remains within an acceptable range. Therefore, the estimation accuracy can still be considered satisfactory across the structure. The NRMSE value between *measured* and *estimated* responses at sensor *M24* was essentially negligible, which is expected since *M24* was used as the observation measurement in the KF.

Conclusions

This work presented the application of virtual sensing techniques on a notched aluminum beam subjected to base excitation. The acceleration measured on the fixture was taken as the input base excitation, together with five acceleration responses measured on the beam. A KF was then employed to estimate the responses at *unmeasured* locations from a single sensor observation.

From the results presented on Table 2, it is clear that estimation accuracy varies slightly across the beam. The KF was tuned to achieve the best estimation with the smallest NRMSE value at sensor *M21*. This criteria was chosen due to the proximity of sensor *M21* to the notch of the beam, which can be defined as a zone prone to damages in the studied structure.

Overall, these findings demonstrate that the KF can accurately reconstruct the dynamic response of a beam subjected to a base excitation using limited measurements. Although this study considered a single “best-case” scenario using only one observation, it demonstrates that high estimation accuracy can still be achieved on the other location. Future work will

focus on optimal sensor placement strategies, the application of virtual sensing under multi-directional excitation to predict full-field responses of mechanical systems, and the use of this approach for real-time monitoring and structural damage detection.

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Structural Health Monitoring, Metamaterials, Metastructures and Additive Manufacturing and Dynamics of Biomedical and Bioinspired Systems, Volume 10

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