



## Chapter 6

# Design and Optimization of Shock Absorbers Made of Grade Density Foams

C. Sabbatini, G. Zandri, and M. Sasso

**Abstract** In this paper, the adoption of density graded polymeric foams is evaluated for energy absorption purposes. The use of functionally graded materials is becoming increasingly widespread in the field of personal protection and industrial applications, thanks in part to recent advances that have simplified their production. To properly design an energy absorber, it is first necessary to identify the material used and characterize its behavior at different densities and strain rates. The necessary experimental tests were conducted using a pneumatic compression machine for low and intermediate strain rates, while for dynamic tests, the Hopkinson bar was employed. Tests were initially conducted on uniform samples of different densities; this allowed the analysis and calibration of a visco-elasto-plastic constitutive model borrowed from the literature. Then, the mentioned constitutive models have been implemented in numerical simulations with Abaqus/explicit finite element software, where the energy absorption capability of a density graded absorber was evaluated reproducing a puncture test. The density distribution in the absorber was designed and optimized to simultaneously reduce the peak of acceleration of the impacting mass and the maximum stress experienced by the absorber during the impact event.

**Keywords** Polymeric foams · Ogden hyperfoam model · Viscoelasticity · Functionally graded materials

## Introduction

Polyvinyl chloride (PVC) foam is widely used in various industries due to its lightweight structure, energy-absorbing capacity, and excellent mechanical properties. Understanding its behavior under different loading conditions is essential for optimizing its performance in applications such as impact protection [8], cushioning systems, and structural components [12].

As new technologies make it possible to produce increasingly complex cellular structures at lower costs [5], the use of functionally graded materials whose mechanical and physical properties vary spatially is gaining traction, depending on the specific application requirements [3, 9]. For foamed materials, one way to achieve such structures is to create a graded density distribution. However, for proper design and optimization of these materials, accurate characterization is necessary to enable reliable finite element simulations.

In this study, compression tests were conducted on PVC foam across a range of strain rates, from quasi-static to dynamic, and considering different nominal densities. This approach allowed for the evaluation of both strain rate effects and density-dependent behavior [7].

The experimental results were then used to calibrate the Ogden hyperelastic model [10], the Prony series for viscoelasticity, and the Mullins effect to account for foam damage and stress softening.

The goal of this work is to develop a user-defined constitutive model (VUMAT) capable of implementing all three models, thus accurately describing the behavior of PVC foam. Finally, an impact test simulation will be carried out on an absorber with a variable density distribution to verify the effectiveness of the VUMAT and assess the ability of graded-density structures to enhance energy absorption efficiency compared to an absorber with the same average density.

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## Material and Experimental Tests

In this work, parallelepiped samples of closed cells PVC foam were tested to characterize the constitutive behavior of the material. We investigate the material response of commercial PVC foams with different densities, namely 130, 200 and 250 kg/m<sup>3</sup>, provided by the company Diab and here indicated with their commercial name: Divinycell HP130, HP200 and HP250, respectively. These densities are selected to have foams with a high mechanical response and suitable for high-end applications, as the mechanical properties increase with foam density.

The tests were conducted in a wide range of nominal strain rate, i.e.  $10^{-3}$ ,  $10^{-1}$ ,  $10^1$  and  $10^3$  s<sup>-1</sup>. All PVC foam samples have a nominal 12 mm × 12 mm square cross section. The length of the samples was 15 mm for the tests at  $10^{-3}$  and  $10^{-1}$  s<sup>-1</sup>, while it was 10 mm for the ones tested at  $10^1$  and  $10^3$  s<sup>-1</sup>; samples with different lengths were adopted to match the strain rate over the prescribed range according to our testing facilities.

All samples were tested in compression, with the load acting parallel to the growth (out-of-plane) direction. Three repetitions were carried out for each test condition, obtaining a strong repeatability of results; hence, only one curve for each test condition will be shown hereinafter. In addition, as expected for this class of material, a negligible lateral expansion was observed during compression tests.

We perform the compression tests at low and medium strain rate, from  $10^{-3}$  to  $10^1$  s<sup>-1</sup> on a pneumatic testing machine, model Siplan<sup>®</sup> equipped with a 3 kN load cell. The machine piston can reach a speed of 100 mm/s. The tested samples were obtained from PVC planks of three different densities, i.e. 130, 200 and 250 kg/m<sup>3</sup>, corresponding to 0.09, 0.14, and 0.18 relative density, respectively. In the low strain rate regime tests, i.e.  $10^{-3}$  and  $10^{-1}$  s<sup>-1</sup>, the piston was gently moved downwards, approaching the upper sample surface, and the test started when the preload reached the value of 2 N. In the tests at medium strain rate (i.e.  $10^1$  s<sup>-1</sup>), the piston was moved to the uppermost position and accelerated from 0 to 100 mm/s before going into contact with the sample.

The dynamic tests were performed by means of the Split Hopkinson Pressure Bar (SHPB), whose calibration and adaptation to test soft materials are extensively described in [7, 8]. The SHPB apparatus is made up of three aligned bars named pre-stressed, input and output bars, which are 3.0, 7.5 and 4.0 m long, respectively. All bars have a diameter of 18 mm. A compression wave is produced inducing a sudden release of the elastic energy stored in the pre-stressed bar, which is statically pre-loaded by means of an electro mechanical actuator. This input wave travels through the input bar at the sound speed of the bar material and reaches the sample that is placed between the input and the output bars. While the sample is quickly deformed, the wave is partially transmitted to the output bar and partially reflected into the input bar. The strain induced by the incident, reflected and transmitted waves, respectively, is measured by strain gauge rosettes placed on the input and output bars in full Wheatstone bridge configuration. The signals obtained are used to compute the load at the two edges of the sample that are in contact with the input and output bars. The pre-stressed and input bars are made of aluminum (AA7075 T6) whereas, polyethylene terephthalate (PET) has been used for the output bar with the aim of increasing the sensitivity in the load measurement.

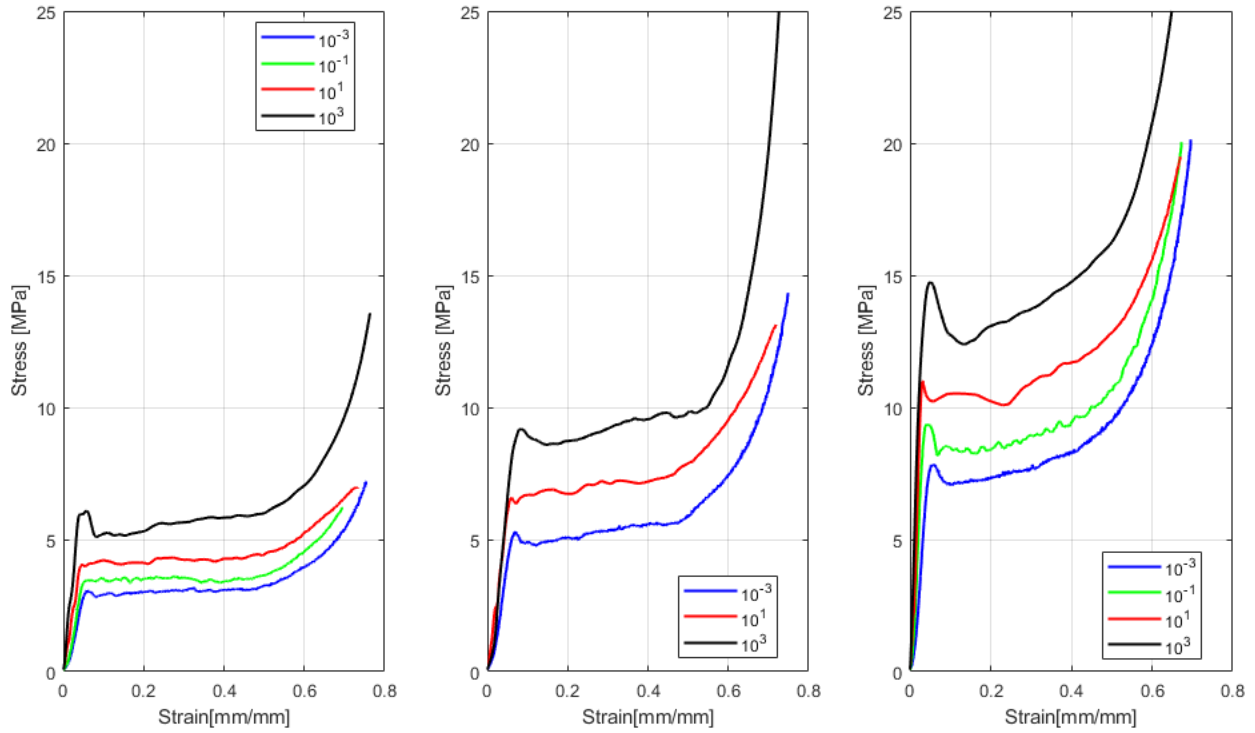
The experimental results, reported in Fig. 1, highlight the characteristic nonlinear behavior of foams, marked by an initial linear elastic region, followed by a stress plateau, and concluding with a densification phase. Furthermore, the mechanical response of the foams is strongly influenced by both density and strain rate: as these parameters increase, the plateau stress rises, and the onset of densification occurs at lower strains.

## Constitutive Material Modelling

In this work, a user-defined constitutive model (VUMAT) was implemented to describe the material behavior, accounting simultaneously for the effects of density and strain rate. The following sections will provide a detailed description of the hyperelastic, viscoelastic models, the Mullins effect, and the variable density representation.

### Hyper-elasticity

The hyperelastic model formulated by Ogden well describe the non-linear behavior of PVC foam. This model is available in the most commonly used finite element analysis software and is suitable for modeling both the static and time-dependent response of polymer foams. The model is based on the strain energy function, which depends on principal stretches, defined as  $\lambda = L_f/L_0 = 1 + \epsilon$ .



**Fig. 1** Experimental compression test results, stress-strain curves

$$W(\lambda_1, \lambda_2, \lambda_3) = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} \left( \lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 + \frac{1}{\beta_i} (J^{-\alpha_i \beta_i - 1}) \right) \quad (1)$$

where  $N$  is the order of the model,  $J$  denotes the volume ratio (determinant of the deformation gradient  $F$ ),  $\mu_i$ ,  $\alpha_i$ ,  $\beta_i$  are model parameters:  $\mu_i$  has the unit of stress. In fact, it is directly correlated to the initial shear modulus  $G$ . Instead,  $\alpha_i$  and  $\beta_i$  are both dimensionless, in particular  $\alpha_i$  represents the hardening or softening exponent of the model and  $\beta_i$  is related to the effective Poisson's ratio  $\nu$ .

$$G = \sum_{i=1}^N \mu_i \quad \nu_i = \frac{\beta_i}{1 + 2\beta_i} \quad (2)$$

Starting from an hyperelastic strain energy potential  $W$  expressed according to the Ogden hyperfoam model, the principal stress of first Piola-Kirchhoff tensor  $P_k$ , can be obtained as:

$$\sigma = \frac{\partial W}{\partial \lambda_1} = \sum_{i=1}^N \frac{2\mu_i}{\alpha_i \lambda_1} (\lambda_1^{\alpha_i} - 1) \quad (3)$$

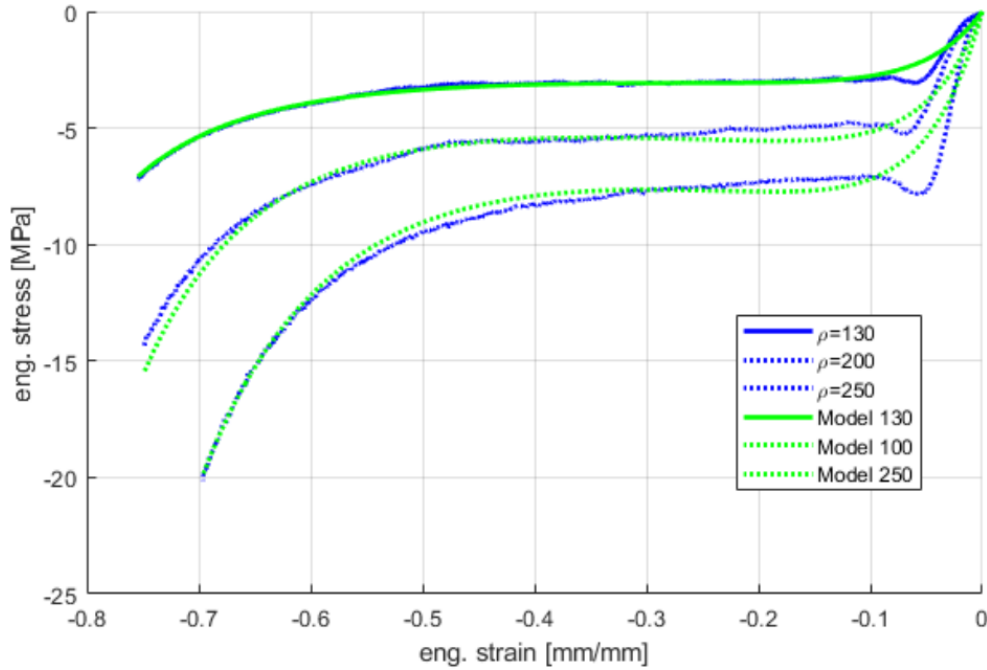
The Eq.(3) is used to fit the best parameters of the model using the quasi-static compression curve [6, 11].

### Effect of density

To simulate the behavior of a functionally graded material, the parameters of the Ogden model were defined as a function of the generic density  $\rho$ , as shown in Eq. 4, based on the parameters calibrated from the mechanical behavior at a density of  $130 \text{ kg/m}^3$ .

$$\begin{aligned}
\mu_{1,\rho} &= \mu_{1,130} \left( \frac{\rho}{\rho_{130}} \right)^{q_1} \\
\mu_{2,\rho} &= \mu_{2,130} \left( \frac{\rho}{\rho_{130}} \right)^{q_2} \\
\mu_{3,\rho} &= \mu_{3,130} \left( \frac{\rho}{\rho_{130}} \right)^{q_3} \\
\alpha_{i,\rho} &= \alpha_{i,130}
\end{aligned} \tag{4}$$

The parameters  $q_i$  were obtained through optimization based on the experimental curves for densities of 200 and 250 kg/m<sup>3</sup>, and the results are shown in Fig. 2. This density-dependent formulation, once implemented in the VUMAT, allows for the simulation of structures with a linearly varying density, rather than being limited to solids with layer-by-layer density variation [4].



**Fig. 2** Results of different density parameters fitting

### Viscoelasticity and relaxation

The PVC foam shows a viscoelastic behavior that emerges in dynamic load conditions [13]. Consequently, the viscoelastic behavior must also be modeled in conjunction with the hyperelastic model. The Prony series (or generalized Maxwell) model was chosen. This model considers the material as a branch with a spring representing the long-term stiffness  $G_\infty$ , in parallel with a series of  $N_G$  Maxwell branches, where a spring and a viscous damper are present. Each Maxwell branch is characterized by a relaxation time  $\tau_i$ . The global relaxation modulus of such a system is given by:

$$G(t) = G_0 \left[ g_\infty + \sum_{i=1}^{N_G} \left( g_i e^{-t/\tau_i} \right) \right] \tag{5}$$

In Eq. (5),  $g_i$  are the relative moduli, so that  $g_\infty + \sum g_i = 1$ ;  $G_0$  represents the short term, or instantaneous, stiffness that is obtained as the sum of all branches' stiffness. The long term and short term stiffnesses are related as  $G_\infty = g_\infty \cdot G_0$ . The stress along time in the  $i$ -th layer of the Generalized Maxwell model is given by [2]:

$$\left( \mathbf{S}_i^{2d} \right)_{n+1} = \exp \left( -\frac{\Delta t}{\tau_i^G} \right) \left( \mathbf{S}_i^{2d} \right)_n + \alpha_i^G \exp \left( -\frac{\Delta t}{2\tau_i^G} \right) \left[ \frac{d\Phi}{d\mathbf{C}_{n+1}} - \frac{d\Phi}{d\mathbf{C}_n} \right] \tag{6}$$

where  $\mathbf{S}_i^{2d}$  is the second Piola-Kirchhoff stress tensor and  $\mathbf{C}$  is the right Cauchy-Green deformation tensor. The true stress can be obtained from:

$$\sigma = \frac{2\mathbf{F}\mathbf{S}_i^{2d}\mathbf{F}^T}{J} \quad (7)$$

where  $\mathbf{F}$  is the deformation gradient.

The total stress can be obtained from the sum of the hyperelastic stress with the  $N_G$  viscoelastic stresses.

In order to correctly model permanent energy dissipation effects and stress softening in PVC foam, Mullins effect is implemented, providing an extension to the elastomeric foam model [1]. Therefore, this damage model is used to include the damage present in elastomeric foams, modeling the energy absorption in foam components subjected to dynamic loading, with high strain rates compared to the characteristic relaxation time of the foam. In this model, energy dissipation effects are considered by introducing an augmented strain energy density function of the shape:

$$W(\lambda_i, \eta) = \eta W(\lambda_i) + \phi(\eta) \quad (8)$$

The function  $W(\lambda_i, \eta)$  is a continuous function of the damage variable,  $\eta$ , and is related to the damage function  $\phi(\eta)$ . The damage variable,  $\eta$ , varies continuously during the course of deformation and always satisfies  $0 < \eta < 1$ , with  $\eta = 1$  at the points of the primary curve (described by the Hyperfoam model). Taking into account the Mullins effect, the stresses are calculated by:

$$\sigma(\lambda_i, \eta) = \eta \sigma(\lambda_i) \quad (9)$$

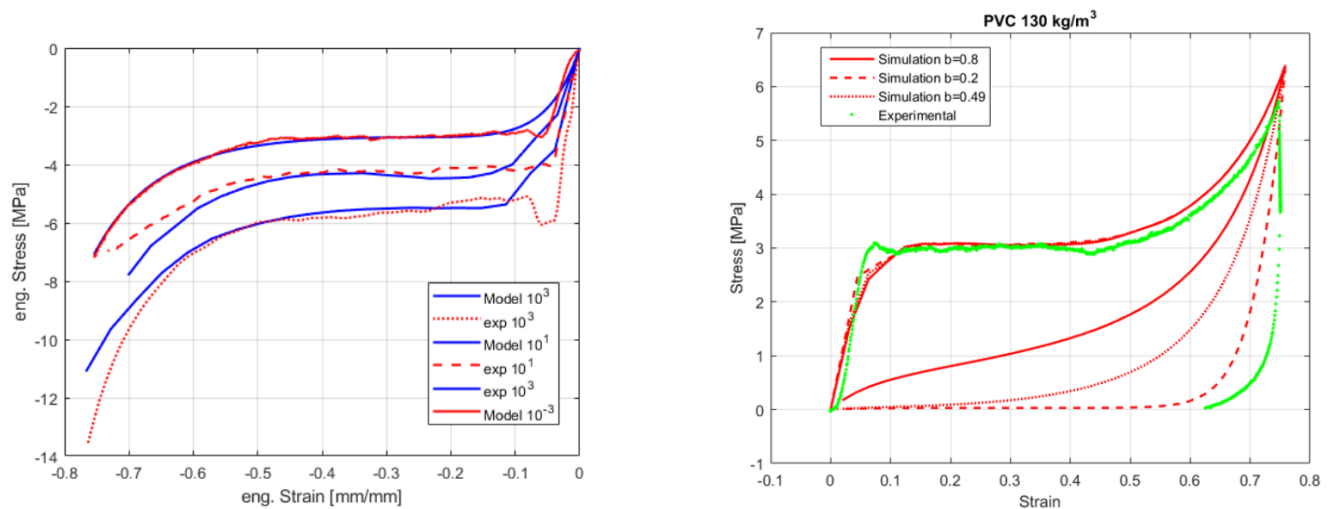
where  $\sigma(\lambda_i)$  is the stress corresponding to the primary behavior of the foam at the current strain level  $\lambda_i$ . Then, the stress is obtained by simply scaling the stress of the primary behavior of the foam by the damage variable  $\eta$ . Equation 9 is used when the material is at an energy potential  $W$  that is lower than the maximum energy potential  $W_m$  experienced by the material itself at the end of the loading phase. The damage variable is generally assumed to be represented by the so called “error function”:

$$\eta = 1 - \frac{1}{r} \operatorname{erf} \left( \frac{W_m - W}{m + \beta W_m} \right) \quad (10)$$

where  $r, m$  and  $\beta$  are material parameters that govern the shape of the unloading curve. Note that  $r > 1$ ,  $m \geq 0$ ,  $\beta > 0$  [1].

### Model validation

Before proceeding with the simulation of a puncture test, the proposed model was validated by reproducing the compression tests performed experimentally at different strain rates and densities. The specimen was modeled as a 2D body, constrained at the lower edge, applying the axial symmetry of the problem. Compression at various strain rates was applied by varying the velocity of the rigid body loading the PVC specimen. The simulation results for the density of 130 at strain rates of  $10^{-3}$ ,  $10^1$ , and  $10^3$  are shown in Fig. 3a.



**Fig. 3** (a)Hyperelastic and viscoelastic results,(b) Mullins effect result

The agreement between the experimental and numerical curves at the lowest strain rate validates the choice of the hyperelastic model parameters, while the good overlap of the curves at  $10^1$  and  $10^3$  demonstrates the ability of the Prony series model to capture the time-dependent behavior of the PVC foam. Fig. 3b shows the model behavior and the dependence of the Mullins effect on parameter  $\beta$ . Table 1 reports all the parameters used in the models.

**Table 1** 3<sup>th</sup> order Ogden hyperfoam parameters

| $\mu_1$  | $\alpha_1$ | $\mu_2$  | $\alpha_2$ | $\mu_3$  | $\alpha_3$ | $\beta_{1...4}$ |
|----------|------------|----------|------------|----------|------------|-----------------|
| 1.463029 | 0.211745   | -3.72722 | 0.859074   | 34.58817 | 21.63363   | 0               |

**Table 2** Prony series with 10 terms parameters

| $\tau_{min}$ | $\tau_{max}$ | $g_1$  | $g_2$ | $g_3$    | $g_4$    | $g_5$ | $g_6$    | $g_7$  | $g_8$  | $g_9$  | $g_{10}$ | $g_{\infty}$ |
|--------------|--------------|--------|-------|----------|----------|-------|----------|--------|--------|--------|----------|--------------|
| 1E-5.783     | 1E3.367      | 0.0002 | 0.68  | 1.79E-06 | 1.76E-06 | 0.079 | 2.22E-06 | 0.0443 | 0.0083 | 0.0008 | 9.20E-07 | 0.188        |

**Table 3** Mullins damage parameters

| r    | m      | $\beta$ |
|------|--------|---------|
| 1.01 | 0.0001 | 0.2     |

**Table 4** Density parameters

| $q_1$ | $q_2$ | $q_3$ |
|-------|-------|-------|
| 2.999 | 2.409 | 1.519 |

## Puncture Test

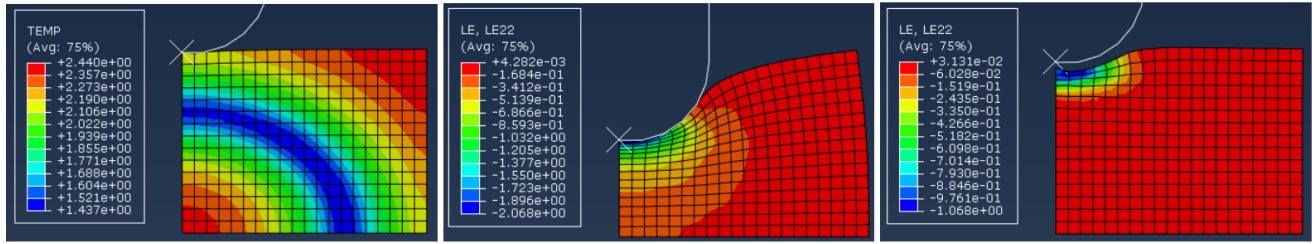
A puncture test simulation was chosen to investigate and quantify the benefits of using a structure with variable density, focusing on evaluating the peak acceleration measured on the impactor during the test. To generate a large set of possible density distributions, a law was defined to distribute the density in an elliptical shape.

$$\Omega = \left| \left( \frac{X - x_c}{\frac{a}{n}} \right)^2 + \left( \frac{Y - y_c}{\frac{b}{n}} \right)^2 - 1 \right| \quad (11)$$

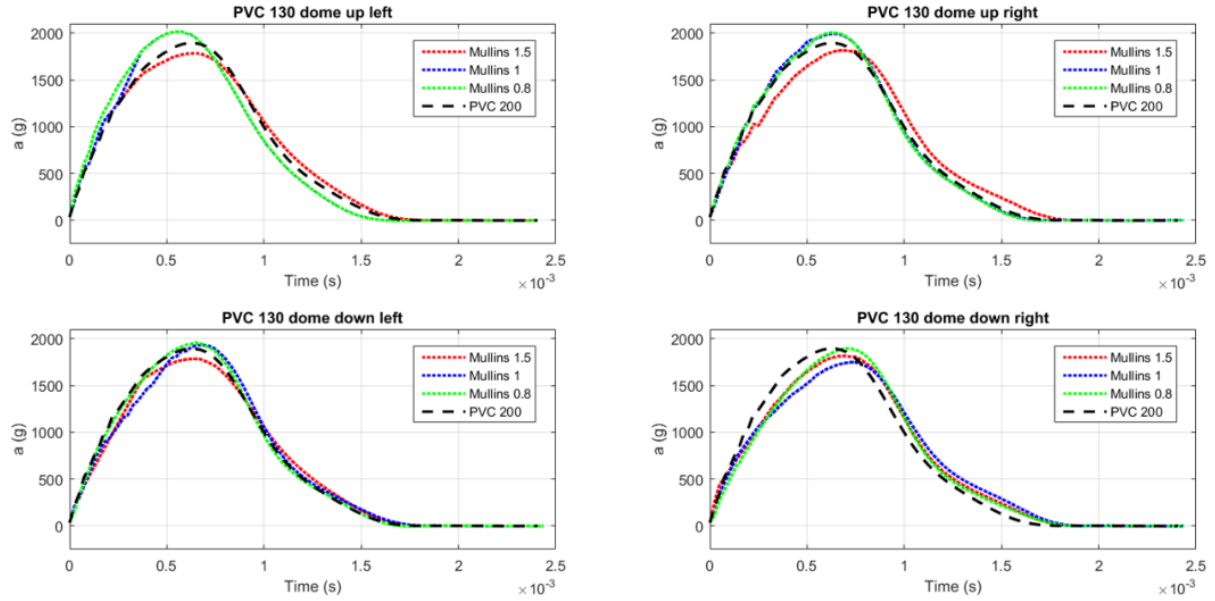
$$\rho_{min} + (\rho_{max} - \rho_{min}) \exp - \lambda \Omega \quad (12)$$

The first equation represent the location of the band with lower density, where  $x_c$  and  $y_c$  are the coordinates of the center of elliptical band,  $n$  is a factor that changes ellipse's axes dimensions and  $\lambda$  defines the band width. The Eq. 12 describe the density variation law. In order to compare the results obtained for various density distributions, it is necessary to keep the average density of the absorber unchanged and constant in every case. To achieve this, the parameter  $\lambda$  appearing in the distribution law was adjusted so that all density distributions had an average value of 200 kg/m<sup>3</sup>. The impact tests were simulated by means of commercial Finite element software. In particular, the Abaqus/Explicit code was adopted because it is well suited for implementing user defined constitutive models. A 2D axisymmetric model of the PVC block impacted area and of impactor has been built. The PVC block is fixed on the bottom edge and meshed and the impactor is discretized as a rigid surface, with a 0.8 kg mass and an initial velocity equal to 10 m/s. The Fig. 4 show an example of our elliptical density distribution and three different moment of simulation.

As the final result of the simulation, the plot of the acceleration curves, Fig. 5, measured on the impactor is shown for the various density distribution cases, always compared with the uniform density case equal to the imposed equivalent density.



**Fig. 4** Puncture test simulation (a)time = 0 with elliptical density distribution, (b) instant of maximum deformation, (c) residual deformation



**Fig. 5** Acceleration curves

## Conclusions

In this work, compression tests on PVC foams with different densities, namely 130,200 and 250 kg/m<sup>3</sup>, have been carried out at different strain rates, nominally from  $10^{-3}$  to  $10^3$  s<sup>-1</sup>. The tests were conducted by means of a servo pneumatic machine, exception made for the high strain rate tests performed by using a Split Hopkinson Pressure Bar, used to infer the stress-strain curves of the material.

Moreover, the stress-strain curves are used to calibrate a visco-elasto-plastic model, the minimization of an error function between the experimental and the analytical curves provides a reasonable estimation of parameters.

Once validated, the model is exploited to simulate the impact response of the foams under study. The simulation results in terms of acceleration curves show that the use of graded materials allows for improved energy absorption efficiency while keeping the average density of the absorber constant. Finally, the use of our defined model makes it possible to simultaneously implement both viscoelastic behavior and the Mullins effect, which is not always available in commercial Finite Element software.

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