Chapter 10
Data-Driven Nonlinear Reduced-Order Modeling for Flutter Analysis

Nami Mogharabin and Amin Ghadami

Abstract Flutter instability poses a serious threat to the reliability and safety of aerospace structures. While linear analysis tools have long been employed for their simplicity, they fail to capture key nonlinear behaviors that can emerge before or after the predicted flutter point. Nonlinear bifurcation-based stability analysis methods, however, often depend on explicit governing equations, limiting their applicability when closed-form equations are unavailable, such as in experimental studies or high-fidelity simulations of aeroelastic systems. Here, we propose a data-driven bifurcation analysis framework to analyze flutter instabilities in aeroelastic systems. The proposed method constructs a low-dimensional surrogate model directly from high-dimensional time-series measurements of system dynamics. Using measured time series, the framework maps the system's high-dimensional response to a reduced-order representation, enabling interpretable and efficient nonlinear stability analysis. By integrating data-driven dimensionality reduction with concepts from nonlinear dynamics, this approach offers a practical framework for analyzing the nonlinear stability and dynamics of aeroelastic systems in situations where traditional model-based methods are not feasible.

Keywords Flutter analysis · Reduced-order modeling · Deep learning · Bifurcation analysis

Introduction

Flutter instability is a critical aeroelastic phenomenon that can lead to catastrophic structural oscillations. The risk is amplified in modern aircraft, where the use of lighter and more flexible materials increases geometric and aerodynamic nonlinearities, making flutter behavior increasingly unpredictable and difficult to control [1]. Although classical linear flutter-prediction methods remain attractive for their computational efficiency, they cannot capture nonlinear behaviors such as supercritical limit-cycle oscillations that emerge gradually beyond the flutter speed or subcritical oscillations, where large amplitudes may arise even below the linear flutter speed. Nonlinear stability analysis is therefore essential to ensure safety. Bifurcation analysis provides a more complete picture of instability around non-hyperbolic equilibria. Existing approaches typically rely either on high-fidelity simulations and experiments [2], or on mathematical models [3, 4]. The first class requires repeated runs for different parameter sets, which is often expensive, while also offering limited interpretability in large-dimensional systems. The second class depends on explicit mathematical models, which are often unavailable or difficult to derive in many practical settings.

To overcome these challenges, we propose a data-driven reduced-order modeling (ROM) method for flutter analysis in aeroelastic systems [5]. The framework integrates autoencoders with principles from dynamical systems [6, 7]—specifically center-manifold and normal form theories [8]—to capture the essential reduced-order dynamics corresponding to the system nonlinear stability and dynamics. Using limited trajectory data, the method discovers low-dimensional coordinates and reduced-order dynamics that can predict and characterize flutter instabilities. Numerical demonstrations on a representative nonlinear airfoil validate the effectiveness of the approach for nonlinear stability analysis and flutter prediction.

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Methods

For the system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu)$, where $\mathbf{x} \in \mathbb{R}^n$ represent system states and $\mu \in \mathbb{R}$ is the bifurcation parameter, flutter instabilities typically arise through a Hopf bifurcation, where at equilibrium point $(\mathbf{x_0}, \mu_\mathbf{0})$ pair of complex-conjugate eigenvalues of the linear part $D_\mathbf{x}\mathbf{f}(\mathbf{x_0}, \mu_\mathbf{0})$ crosses the imaginary axis as the flow speed varies, leading to growing oscillations. According to the center-manifold theory, the behavior of such systems near critical point can be described on an invariant manifold that passes through the bifurcation point $(\mathbf{x_0}, \mu_\mathbf{0})$. Trajectories sufficiently close to the equilibrium rapidly converge to this manifold and subsequently evolve according to the reduced dynamics. As a result, the essential stability characteristics of the original high-dimensional system can be captured by a much simpler, second-order representation defined on the manifold. The normal form theory states that there is a transformation under which the dynamics restricted on the manifold can be represented in a minimal form including only necessary terms, called resonant terms. This representation captures the critical nonlinear behavior governing stability with the fewest possible terms, thereby facilitating analysis and interpretation of nonlinear stability. In the case of Hopf bifurcation relevant to flutter, the reduced dynamics can be written in the following extended normal form:

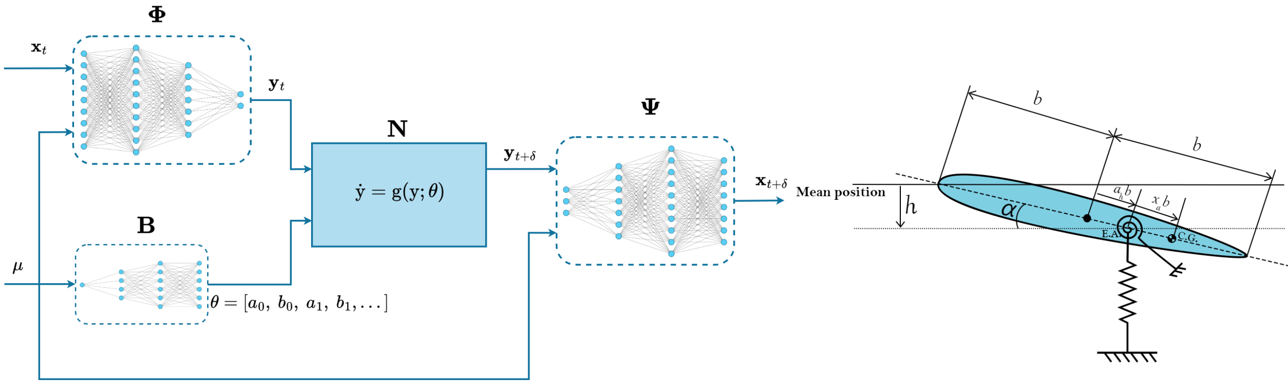
$$\begin{aligned}\dot{y}_1 &= a_0(\mu)y_1 - b_0(\mu)y_2 + (a_1(\mu)y_1 - b_1(\mu)y_2)(y_1^2 + y_2^2) + (a_2(\mu)y_1 - b_2(\mu)y_2)(y_1^2 + y_2^2)^2 + H.O.T \\ \dot{y}_2 &= b_0(\mu)y_1 + a_0(\mu)y_2 + (b_1(\mu)y_1 + a_1(\mu)y_2)(y_1^2 + y_2^2) + (b_2(\mu)y_1 + a_2(\mu)y_2)(y_1^2 + y_2^2)^2 + H.O.T\end{aligned} \tag{1}$$

The coefficients of this reduced-order representation are assumed to be dependent on the bifurcation parameter μ. In flutter analysis, μ is often selected as the flow speed. Analysis of the coefficients of this normal form characterizes the nonlinear dynamics and stability profile around the flutter speed, providing interpretable and tractable description of the dynamics near the critical flutter point. The proposed method tackles two main tasks: identifying the coefficients of the normal form dynamics in Eq. (1) and learning the nonlinear transformations between the observation space and reduced coordinates directly from trajectory data. Both tasks are addressed using a deep-learning framework based on autoencoders, which extract data-driven representations while preserving dynamical structure. The architecture couples autoencoder mapping with components that estimate normal-form coefficients and enforce constraints from center-manifold and normal form theory.

The neural network architecture consists of four main components (Fig. 1a). The encoder block $\mathbf{\Phi}$ maps the high-dimensional state $\mathbf{x}$, together with the bifurcation parameter μ, to a reduced two-dimensional coordinate system $\mathbf{y} = [y_1, y_2]^T$. The decoder block $\mathbf{\Psi}$ reconstructs the original state $\mathbf{x}$ from $\mathbf{y}$ and μ, ensuring an invertible transformation between the physical and reduced spaces. The parametric block $\mathbf{B}$ determines the μ-dependent coefficients of the normal form dynamics, including $a_i(\mu)$ and $b_i(\mu)$ ($i = 0, 1, 2, \ldots$), which characterize the nonlinear stability profile of the system. Finally, the normal-form block $\mathbf{N}$ advances the reduced coordinates $\mathbf{y}$ in time using an Euler integration scheme according to the normal form equations in Eq. (1), with the coefficients provided by $\mathbf{B}$. The training process is guided by a composite loss function that enforces both accurate reconstruction and theoretically consistent reduced-order dynamics. Each training sample consists of a state pair together with the corresponding bifurcation parameter $(x_t, x_{t+\delta t}, \mu)$. The total loss is composed of three terms. The first term, L_1, evaluates the static performance of the autoencoder by enforcing consistency between the original state and its reconstruction, i.e., $x_t \approx \Psi(\Phi(x_t, \mu), \mu)$. The second term, L_2, captures the dynamical performance of the encoder–decoder–normal form pipeline by requiring that the predicted future state matches the true state, i.e., $x_{t+\delta t} \approx \Psi(N(\Phi(x_t, \mu), B(\mu)), \mu)$. Finally, the third term, L_3, ensures that the normal-form block independently reproduces the manifold dynamics, enforcing $\Phi(x_{t+\delta t}, \mu) \approx N(\Phi(x_t, \mu), B(\mu))$. This separation prevents the decoder from masking errors in the latent dynamics and guarantees that the reduced-order model adheres to the theoretical normal form structure. Together, these three terms balance reconstruction, prediction, and latent-space consistency, enabling simultaneous identification of transformations and normal form coefficients.

Results and Discussion

The proposed framework is demonstrated on a two-degree-of-freedom (2-DOF) airfoil model (Fig. 1b), where the free-stream velocity U serves as the bifurcation parameter. The airfoil undergoes coupled pitch (α) and plunge (h) motions, with dynamics governed by nonlinear integro–differential equations that incorporate aerodynamic forces [9]. Depending on parameter selection, the system can exhibit either supercritical or subcritical flutter once U exceeds a critical threshold.



(a) Network architecture for data-driven reduced-order modeling

(b) Airfoil model [5]

Fig. 1 (a) Autoencoder architecture, and (b) Airfoil model

For demonstration, the structural stiffness parameters are chosen such that the system exhibits subcritical flutter. A total of 15 trajectories are simulated for values of U in the range $U \in [6.45, 6.65]$, which corresponds to the neighborhood of the bifurcation point. To ensure that the data reflect the dynamics on the manifold of interest corresponding to flutter, the network is trained on trajectories generated from initial conditions randomly sampled along a post-bifurcation limit cycle and evolved in the pre-bifurcation regime, promoting rapid convergence to the center-manifold. Initial transients are discarded, and only the converged portions are retained for training. Figure 2 illustrates the identified coefficients of the normal form. As expected, the coefficients a_1 and a_2 exhibit positive and negative signs, respectively, which is characteristic of subcritical flutter behavior. The ability to identify these coefficients in the reduced dynamics enables the detection of unstable limit-cycle oscillations (LCOs) directly in the latent space. By passing the coordinates of these unstable LCOs through the decoder, the corresponding oscillations are reconstructed in the original state space. This capability is particularly significant, as capturing unstable LCOs in the full system is typically a challenging task using conventional approaches. Results show that the identified LCOs closely match the results obtained with the MATCONT numerical continuation toolbox.

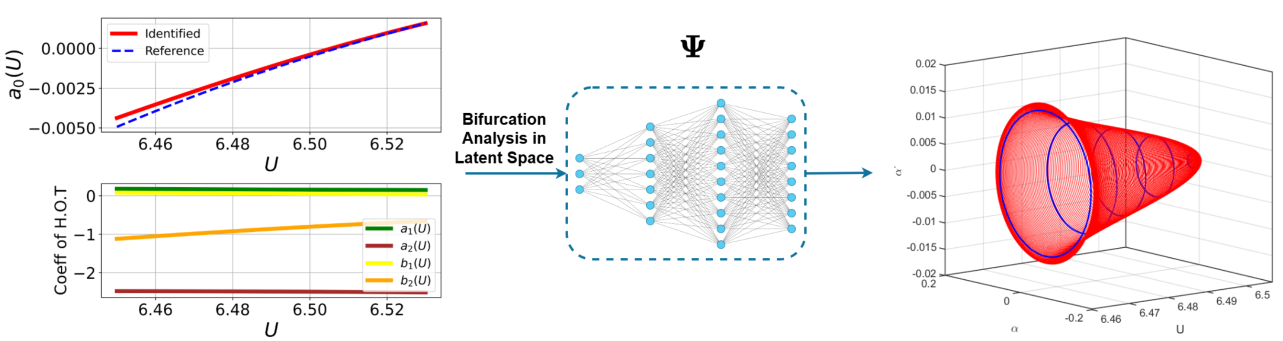


Fig. 2 Unstable limit cycles identified using reduced-order model (blue) closely match the reference limit cycles (red) calculated using MATCONT

Figure 3 presents the bifurcation diagram obtained from the analysis of the reduced-order model in the latent space. To evaluate the identification accuracy, three representative trajectories are simulated in the latent space using the initial conditions and flow speeds indicated in the figure. These trajectories are then mapped back to the original state space using the identified transformations (i.e., the trained decoder) and compared with trajectories generated directly using the full-order airfoil model. As shown in Fig. (3), the reduced-order predictions closely match the responses of the original system, highlighting the accuracy of the approach in reproducing the nonlinear dynamics and bifurcation characteristics of this aeroelastic system.

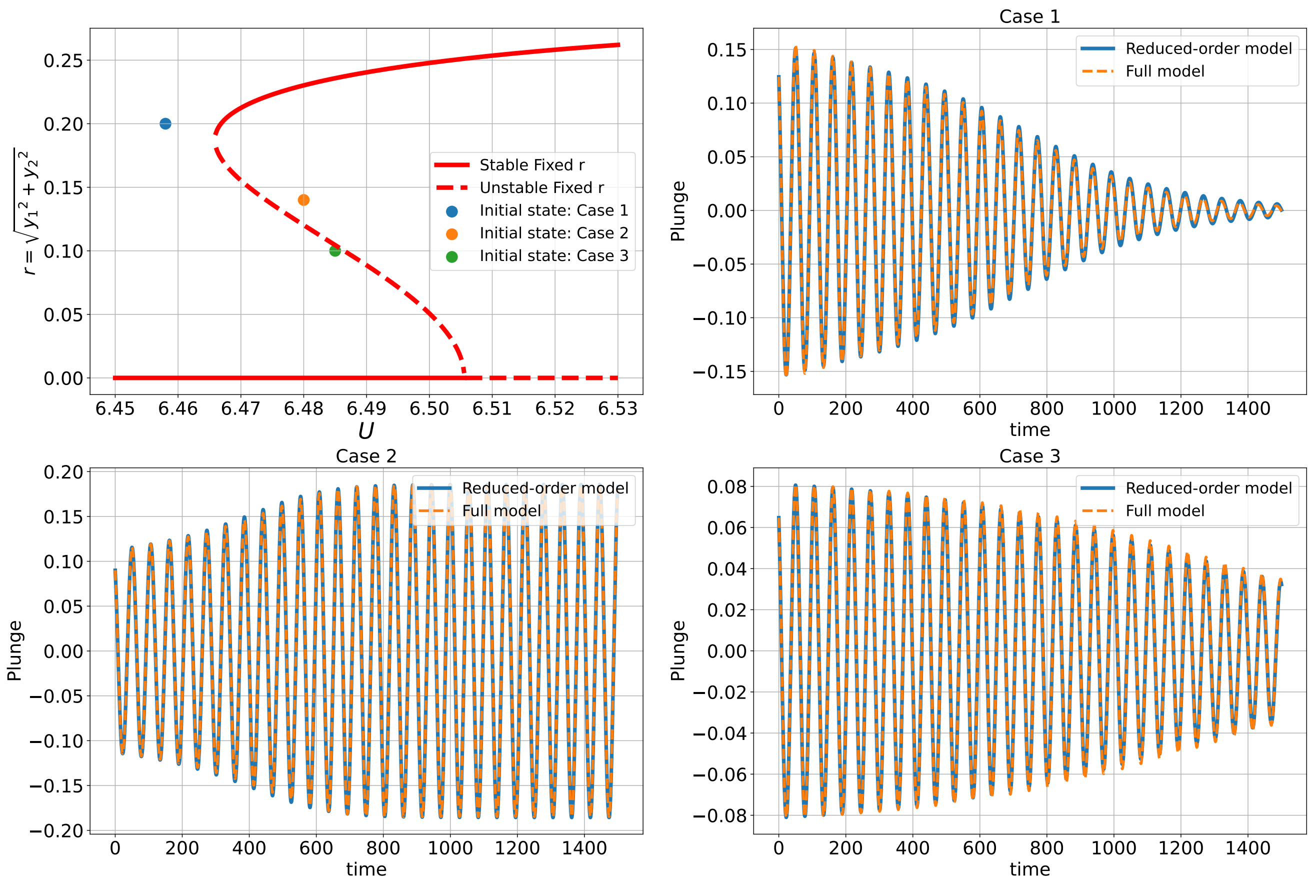


Fig. 3 Bifurcation analysis and trajectory reconstruction using the reduced-order model compared with reference trajectories

Conclusion

This work introduces a data-driven reduced-order modeling framework that integrates principles from nonlinear dynamics with deep learning for nonlinear flutter analysis. By combining autoencoder-based dimensionality reduction with center-manifold and normal form theories, the method extracts low-dimensional representations that capture bifurcation behavior and nonlinear stability using trajectory data. The proposed neural network architecture and training strategy enable identification of the key parameters governing flutter, while addressing the challenges posed by reliance on explicit mathematical models. Moreover, once trained, the reduced-order model provides an efficient and interpretable alternative to repeated high-dimensional simulations, enabling practical nonlinear stability analysis of aeroelastic systems.

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