Chapter 3
Amplitude Resonance Tracking Using Phase-Locked Loops

François Winand, Christophe Collette, and Gaëtan Kerschen

Abstract This work presents a hybrid control strategy for resonance tracking in nonlinear structural systems, combining classical phase-locked loop (PLL) control with an outer-loop extremum seeking control (ESC) scheme. Traditional PLL-based methods rely on enforcing phase quadrature between excitation and response—a condition that is not always satisfied for closely-spaced mode and high damping situations — leading to inaccurate backbone identification. To address this limitation, the proposed approach adaptively adjusts the PLL's reference phase in real time to directly maximize the response amplitude. The ESC-enhanced PLL is embedded in a continuation framework that incrementally increases the excitation level while tracking amplitude resonance at each step. The method is validated on a two-degree-of-freedom system and compared against reference solutions obtained via harmonic balance. Results show excellent agreement in both amplitude and phase, while highlighting the discrepancy between phase quadrature and true amplitude resonance. The approach enables accurate backbone extraction and shows strong potential for experimental implementation in nonlinear system identification and fatigue testing applications.

Keywords Nonlinear system identification · Experimental continuation · Phase-locked loop · Derivative-free optimization · Amplitude-based backbone curve

Introduction

In structural dynamics, identifying a system's behavior and assessing its nonlinearities are essential for validating models and ensuring reliable operation. While linear identification techniques are now well established, nonlinear systems require more advanced approaches due to effects such as amplitude-dependent resonance and mode coupling. Moreover, applications such as fatigue testing require the system to be maintained at resonance to accelerate damage accumulation, making adaptive resonance tracking critical for both identification and testing efficiency.

Over the past two decades, phase-locked loop (PLL) control has become a standard technique for tracking resonance under harmonic excitation. First applied in MEMS fatigue testing [1], PLLs were later extended to nonlinear structures, enabling real-time frequency adjustment by enforcing a fixed phase lag between excitation and response [2, 3]. This approach has been used to extract nonlinear modes and backbone curves. However, PLLs rely on the assumption that phase quadrature coincides with the amplitude maximum—an assumption that may fail in systems with damping, friction, or closely spaced modes. This limitation was evidenced in [4], where the identification of nonlinear modal parameters on an F-16 aircraft showed a mismatch between phase quadrature and actual amplitude peaks.

To overcome this limitation, we propose a hybrid strategy that combines PLL with an extremum seeking control (ESC) loop. Instead of locking to a fixed phase, the ESC adaptively adjusts the PLL reference to maximize amplitude directly, ensuring convergence to the true resonance peak. The method is model-free, robust to nonlinear effects, and particularly valuable when proper force appropriation cannot be guaranteed. Combined with a continuation strategy on the excitation level, it further enables automated construction of amplitude-based backbone curves. The effectiveness of the approach is demonstrated through numerical simulations.

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Background: Phase-locked loop testing with adaptive filters

Phase-locked loops are feedback control systems designed to enforce a prescribed phase lag Φ_{ref} between excitation and response. This principle exploits the unfolding of the nonlinear frequency response curve (NFRC) when parametrized in phase. A schematic implementation is shown in Figure 1.

The structure is excited by a sinusoidal force, where the excitation frequency $\omega(t)$ is dynamically adjusted through a proportional–integral (PI) control law:

$$\omega(t) = \omega_0 + K_p\big(\Phi_{\text{ref}} - \Phi(t)\big) + K_i \int_0^t \big(\Phi_{\text{ref}} - \Phi(\tau)\big)\, d\tau. \tag{1}$$

Here, ω_0 is typically initialized at the linear natural frequency, while the controller gains K_p and K_i determine stability and convergence properties. Tuning guidelines can be found in [5].

In this work, the instantaneous phase lag $\Phi(t)$ is estimated online using adaptive filtering [6]. The measured response $x(t)$ (and similarly the excitation $f(t)$) is reconstructed by a truncated Fourier series:

$$\hat{x}(t) = a_0(t) + \sum_{k=1}^{N_h} \Big[a_k(t)\sin(k\omega t) + b_k(t)\cos(k\omega t)\Big], \tag{2}$$

with the Fourier coefficients updated according to a least-mean-squares (LMS) adaptation law:

$$\begin{aligned} \dot{a}_k &= \mu\big(x(t) - \hat{x}(t)\big)\sin(k\omega t), \\ \dot{b}_k &= \mu\big(x(t) - \hat{x}(t)\big)\cos(k\omega t), \end{aligned} \tag{3}$$

where μ denotes the adaptation gain. The response phase Φ_x (and excitation phase Φ_f) is then extracted from the first harmonic as

$$\Phi_x = \text{atan2}(b_1, a_1), \qquad \Phi = \Phi_x - \Phi_f. \tag{4}$$

To trace the backbone curve of the system, the reference phase is commonly set to $\Phi_{\text{ref}} = \pi/2$, which corresponds to phase quadrature when measuring acceleration. Once the PLL has converged, the excitation amplitude U is progressively increased to continue the backbone.

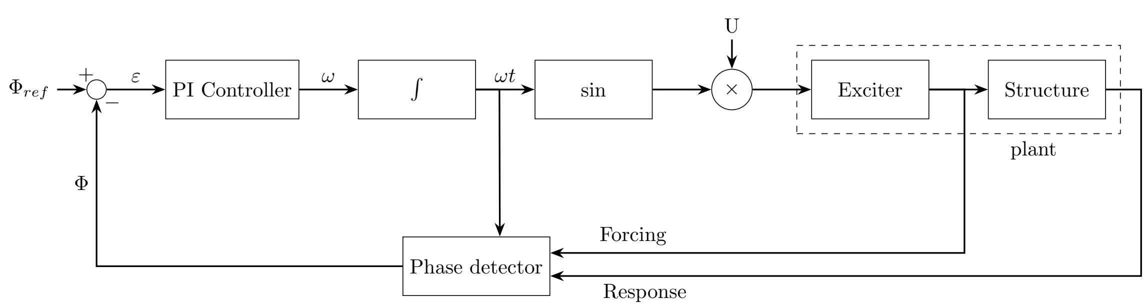


Fig. 1 Classical phase-locked loop implementation

Combining PLL with Extremum Seeking Control

Motivation for Amplitude-Based Tracking

Classical PLLs track resonance by enforcing a fixed phase lag between excitation and response, assuming that maximum amplitude occurs at phase quadrature. This assumption may fail when modes are closely spaced and the phase–amplitude relationship becomes distorted.

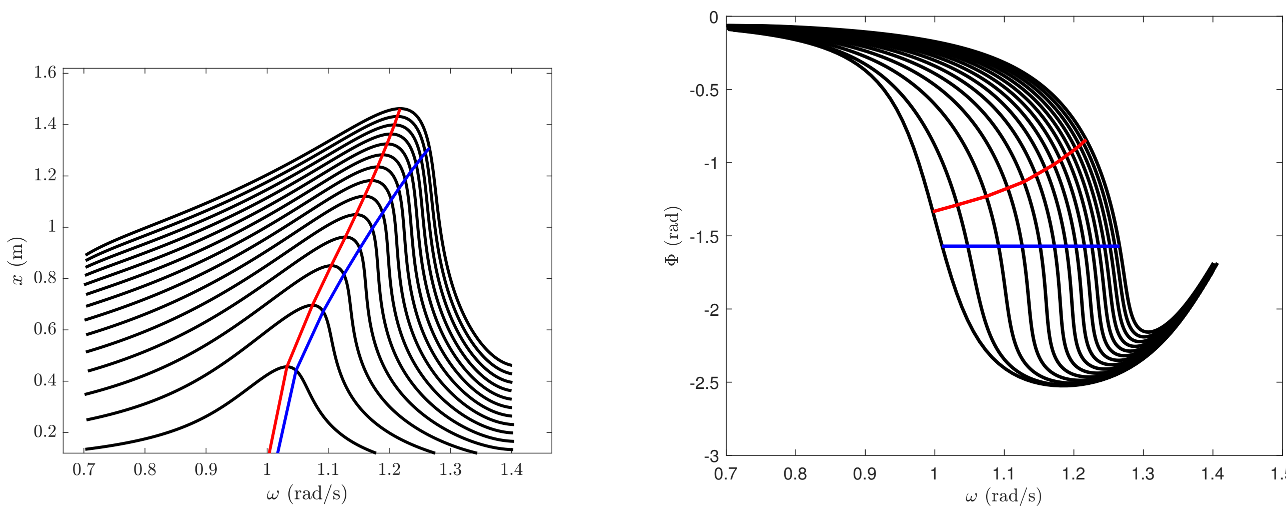
As an illustration, consider a two-degree-of-freedom mass–spring system with a cubic nonlinearity on the first mass:

$$\begin{aligned} &\ddot{x}_1 + 0.04\,\dot{x}_1 - 0.02\,\dot{x}_2 + 2x_1 - x_2 + x_1^3 = f\sin(\omega t), \\ &\ddot{x}_2 + 0.22\,\dot{x}_2 - 0.02\,\dot{x}_1 + 2x_2 - x_1 = 0. \end{aligned} \tag{5}$$

with linear natural frequencies at 1 and $\sqrt{3}$ rad/s. Due to modal proximity and $\sim$5% damping, the maximum amplitude no longer occurs at the nominal $-\pi/2$ phase lag.

This behavior is illustrated in Figure 2, which shows the frequency response curves of the first degree of freedom. As the excitation amplitude increases, the phase at which the maximum response occurs progressively departs from quadrature, exposing a key limitation of conventional PLL tracking. While multi-shaker force appropriation could effectively isolate the mode of interest and restore the quadrature condition allow the PLL to operate as intended, it is not always desirable in practice—for instance, industrial tests may require applying loads at fixed locations.

To address these limitations, the next section introduces an extremum seeking control (ESC) loop [7], which adaptively tunes the reference phase and drives the PLL toward the true amplitude resonance.



(a) Amplitude of the first harmonic

(b) Phase of the first harmonic

Fig. 2 Frequency response curves (amplitude (a) and phase (b) of first harmonic) computed via harmonic balance for excitation levels from 0.01 to 1.5 N. —— FRC, —— Amplitude resonance, —— Phase quadrature

ESC-Augmented PLL Architecture

Amplitude-based resonance tracking is achieved by augmenting the PLL with an outer-loop ESC that adaptively tunes the reference phase Φ_{ref}, as illustrated in Figure 3.

For this purpose, the reference phase is modulated by a small sinusoidal perturbation:

$$\tilde{\Phi}_{\mathrm{ref}}(t) = \bar{\Phi}_{\mathrm{ref}}(t) + A\sin\omega_{\mathrm{ESA}}t, \tag{6}$$

where A and ω_{ESA} denote the perturbation amplitude and frequency respectively. The variable $\bar{\Phi}_{\mathrm{ref}} := \Phi_{\mathrm{ref}} + \Delta\Phi$ becomes the control parameter to be optimized. The cost function is taken as the squared response amplitude, estimated in real time using adaptive filtering.

When $\Phi_{\mathrm{ref}} < \Phi_{\mathrm{ref}}^{\mathrm{opt}}$, the squared amplitude oscillates in-phase with the injected perturbation. Conversely, when $\Phi_{\mathrm{ref}} > \Phi_{\mathrm{ref}}^{\mathrm{opt}}$, it becomes out-of-phase. To exploit this relationship, the squared amplitude is high-pass filtered to remove its DC component, multiplied by the perturbation (demodulated), and then low-pass filtered. The resulting signal is integrated to produce the correction $\Delta\Phi$, which updates the reference toward the optimum. The gain K controls the adaptation rate, analogous to the learning rate in gradient descent. Convergence is smooth near the optimum, with residual oscillations determined by K.

This ESC-PLL scheme is structurally similar to a classical PLL yet enables online, derivative-free optimization. However, it introduces five additional tuning parameters: the learning rate K; the perturbation frequency ω_{ESA}; the perturbation

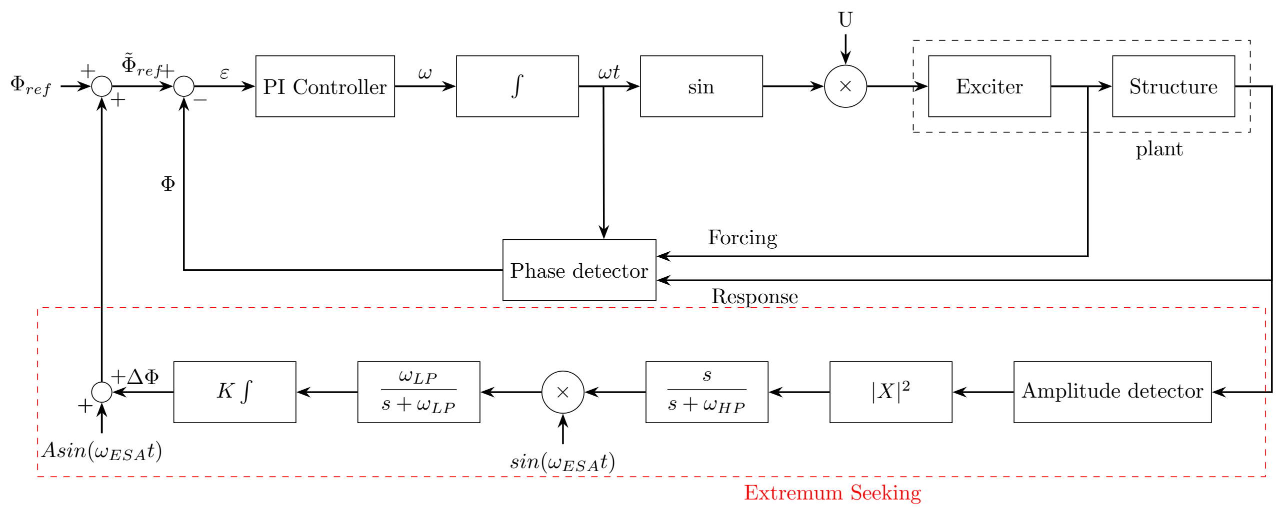


Fig. 3 ESC-augmented PLL scheme

amplitude A and the corner frequencies of the high-pass and low-pass filters, ω_{HP} and ω_{LP}, respectively. Although the tuning remains largely heuristic, some qualitative guidelines can be followed:

- ω_{ESA} should lie within the bandwidth of the PLL to preserve locking under modulation;
- ω_{HP} should be high enough to remove DC bias but low enough to retain the modulated signal;
- ω_{LP} should be low enough to suppress noise and higher harmonics;
- K is adjusted empirically, e.g. at low excitation levels where the optimal lag is well defined;
- A can be set around 0.05 rad.

In general, the ESC should evolve on a slower time scale than the PLL, and this scale separation should guide the choice of parameters.

Amplitude Backbone Identification Procedure

The proposed strategy begins by tuning a classical PLL to ensure stable frequency tracking in the linear regime. Appropriate PI gains are selected to lock the excitation frequency to the natural frequency using a fixed reference phase (typically enforcing quadrature). Once this base configuration is robust, the ESC loop is introduced and tuned at low excitation levels where the system behaves nearly linearly. This enables safe adjustment of ESC parameters (learning rate, dither frequency, and filter cutoffs) without risking instability.

After both loops are tuned, backbone identification proceeds incrementally. At each excitation level, the PLL is first allowed to lock to the current phase reference. Once convergence is achieved, the ESC loop adapts the reference phase to drive the system toward maximum amplitude. When convergence is detected—via amplitude stability or convergence of the reference phase—the ESC is paused and the optimal phase is stored.

The excitation amplitude is then incremented, and the reference phase is reinitialized using the previously optimized value. To maintain consistent optimization behavior across amplitude levels, the ESC learning rate is adjusted to account for the scaling of the cost function. Assuming linear behavior, the squared amplitude (the cost) grows quadratically with input amplitude; therefore, the learning rate is updated as

$$K \leftarrow K\left(\frac{U_{i-1}}{U_i}\right)^2. \tag{7}$$

While this rule is exact only in the linear regime, it remains a useful heuristic in nonlinear cases. Depending on the nature of the nonlinearity, the response amplitude may grow faster or slower than quadratically, which alters the effective learning rate. As a result, the scaling is not always ideal but still provides an adjustment that limits instability and prevents overly aggressive updates.

Repeating this process over successive amplitudes yields an automated construction of the amplitude-based backbone curve, summarized in Algorithm 1.

Algorithm 1 Amplitude Backbone Identification via PLL and ESC

Initialize: $\Phi_{\mathrm{ref}}^{(0)}$, $\omega_0^{(0)}$
Initialize (PLL): PI gains; adaptive filter gain μ and number of harmonics N_h at low amplitude U_0
Initialize (ESC): K, ω_{ESA}, ω_{HP}, ω_{LP} at U_0
for $i = 1$ to N **do**
  Set excitation amplitude U_i
  Update learning rate: $K \leftarrow K \left(\frac{U_{i-1}}{U_i}\right)^2$
  Initialize: $\Phi_{\mathrm{ref}}^{(i)} \leftarrow \Phi_{\mathrm{ref}}^{\mathrm{opt},(i-1)}$, $\omega_0^{(i)} \leftarrow \omega^{\mathrm{opt},(i-1)}$
  Run PLL until frequency convergence
  Activate ESC to adapt $\bar{\Phi}_{\mathrm{ref}}$ until convergence
  Pause ESC
  Store $\Phi_{\mathrm{ref}}^{\mathrm{opt},(i)}$, $\omega^{\mathrm{opt},(i)}$, and amplitude $|X|^{(i)}$
end for

Numerical Evaluation of the Amplitude Tracking Method

The performance of the proposed control scheme is assessed on system 5, following the continuation algorithm. Simulation parameters are listed in Table 1. Both PLL and ESC loops are first tuned at a low excitation level (0.01 N), ensuring stable frequency locking and sufficiently fast convergence of the optimization loop (Figure 4). At $t = 0$ cycles, the ESC is initialized at a reference phase of $-\pi/2$, the conventional starting point in PLL testing, and converges to the optimum value of -1.33 (corresponding to maximum amplitude) in less than 1000 oscillation cycles. To demonstrate stability, the loop is subsequently maintained at steady state for an extended period before being deactivated to collect the outputs.

The system is subsequently driven through 16 increasing excitation levels, from 0.01 N to 1.5 N. At each step, the resonance frequency is identified using the control strategy—typically within 2500 oscillation cycles, a practical limit chosen to keep simulation times reasonable. This procedure yields an amplitude-based backbone that follows the resonance peaks of the frequency response curves and deviates clearly from the quadrature backbone produced by a standard PLL (Figure 5).

All results are compared with harmonic balance reference solutions for validation. Both amplitude and phase of the first harmonic are in excellent agreement. In particular, the close match between frequency response curves measured with the PLL and those computed with HB strongly supports the applicability of the method in experimental contexts, where analytical solutions are unavailable.

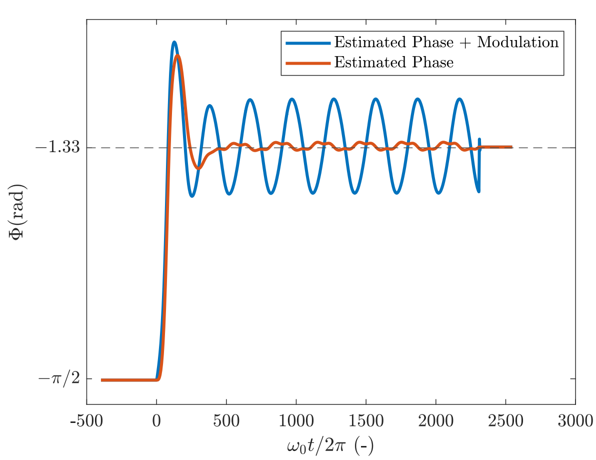
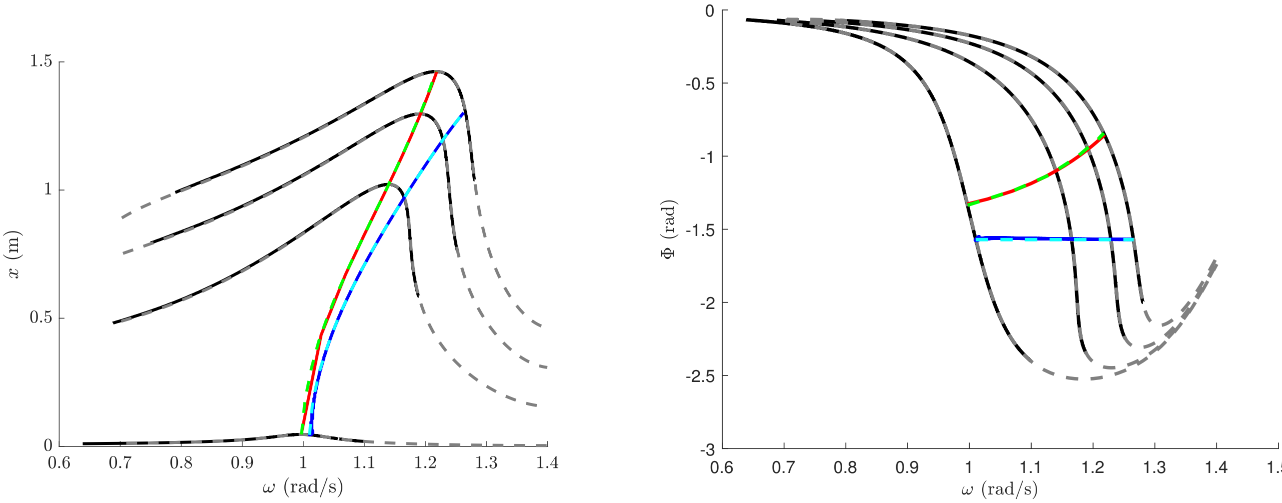


Fig. 4 Convergence of the reference phase Φ_{ref} toward the optimum at 0.01 N. Time is expressed in oscillation cycles

Parameter	Value
K_p	0.1813 s^{-1}
K_i	0.0065 s^{-2}
Adaptive filter gain μ	0.2 ($= \omega_0/5$) s^{-1}
Harmonics N_h	10
Perturbation amplitude A	0.05 rad
Dither frequency ω_{ESA}	0.00333 ($= \mu/60$) rad/s
Initial K	25
LPF cutoff ω_{LP}	10 rad/s
HPF cutoff ω_{HP}	0.00333 ($= \mu/120$) rad/s

Table 1 Simulation parameters for PLL and ESC



(a) Amplitude of the first harmonic

(b) Phase of the first harmonic

Fig. 5 Frequency response curves (amplitude (a) and phase (b) of first harmonic) and backbones for excitation levels from 0.01 to 1.5 N —— FRC (PLL), —— Amplitude resonance (ESC-PLL), —— Phase quadrature (PLL), - - - FRC (HB), - - - Amplitude backbone (HB), - - - Phase quadrature (HB).

Conclusion

This work presented a hybrid control strategy that enhances classical phase-locked loop resonance tracking by integrating an outer-loop extremum seeking control. The method adaptively adjusts the reference phase to locate amplitude resonance directly, avoiding reliance on the traditional quadrature assumption. A continuation scheme was further introduced to automate construction of amplitude-based backbone curves over increasing excitation levels.

The approach was validated numerically on a nonlinear two-degree-of-freedom system, showing excellent agreement with harmonic balance reference solutions. The observed divergence between amplitude and phase resonance highlights the importance of adaptive phase control. These findings support the applicability of the proposed strategy for experimental nonlinear system identification.

Future work will target improved efficiency and robustness of the ESC layer, with emphasis on faster convergence, refined continuation procedures, and systematic gain selection to facilitate experimental use.

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