Chapter 5
Data-Driven Method for Blisk Sector Dynamics with Resonant Vibration Absorber

Michael W. Reynolds, Mihai Cimpuieru, and Bogdan I. Epureanu

Abstract Blisks are critical to the safe operation of turbomachinery. Blisks are located in the compressor or turbine stages and are exposed to extreme forces and temperatures. They are manufactured as one piece and have low damping, making them susceptible to large vibrations, which can result in high cycle fatigue. Thus, it is essential to identify and mitigate blisk vibrations. In particular, Resonant Vibration Absorbers (RVAs) are used to reduce vibrations through energy absorption. These structures can further lower vibration amplitudes by dissipating energy through friction contacts. However, while beneficial to the safe operation of blisks, insertion of RVAs changes blisk dynamics. Hence it is paramount to accurately model vibration amplitudes and resonance responses of blisks with RVAs attached. State-of-the-art includes models for blisks with RVA attached and friction contacts use the Harmonic Balance Method (HBM). While HBM can predict blisk responses, it is not designed to use experimental data for enhanced prediction accuracy, and it can be very computationally expensive for complex contact properties. Hence, to address this gap, this paper proposes to use Control-Based Continuation (CBC), a model-agnostic technique that uses a proportional-derivative controller to cancel higher-order modes and a root-finding algorithm to reach the system's resonant peak, thus tracing out the 'backbone' curve of the response. Using CBC, the response of a single blisk sector with an RVA attached is tracked experimentally, leveraging physical data for enhanced prediction accuracy. The process is repeated by isolating each blisk sector, and the coupled predictions are used to compute the full blisk response. A Lumped Mass Model (LMM) with friction contacts is used to mimic the blade level dynamics. Results show that the proposed method accurately and efficiently predicts the backbone curve of this structure. Future work involves applying this method to a large scale as-manufactured blisk sector.

Keywords Turbomachinery · Nonlinear dynamics · Vibrations · Structural dynamics

Introduction

Turbomachinery blisks are key rotary components in gas turbines. They are composed of the disk section and blade components, with all the parts manufactured as one single piece. These structures are found in the compressor and turbine stages of gas turbines, rotating at high speeds, and being exposed to large pressure waves at extreme temperatures. Hence due to their low damping and how they are manufactured, blisks experience significant stress concentrations and vibrate at large amplitudes which in time can result in failure of the system due to high cycle fatigue. This effect is exacerbated by the inherent small mistuning, described as small variations from the nominal blade properties, which break the system cyclicity and can enhance vibration amplitudes. Furthermore, due to wear formation during operation or due to significant blade damage, large mistuning can further amplify this effect and increase the risk of failure. Therefore, it is critical to mitigate blade vibrations in turbomachinery blisks to ensure the safe operation of these structures. To do so, previous research has focused on utilizing auxiliary structures that are placed in the disk portion of the blisk. These structures are designed such that at specific target vibration excitation frequencies they absorb energy from the host structure (i.e., the blisk) significantly reducing the blade vibration. These so-called Resonant Vibration Absorbers (RVAs) can further reduce blisk vibrations through energy dissipation. Therefore, some RVAs are designed to include friction or impact contact surfaces to dissipate energy and enhance vibration mitigation. Generally, RVAs are designed for specific operating conditions (e.g., frequency range, RPM, tempera-

Michael W. Reynolds · Mihai Cimpuieru · Bogdan I. Epureanu
University of Michigan, Ann Arbor, MI 48109
e-mail: mwreynol@umich.edu; mcimpi@umich.edu; epureanu@umich.edu

Kristin B. Zimmerman, PhD (eds.), *Nonlinear Structures & Systems, Vol. 1*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/978-8-743-81440-5_5

ture etc.). Hence it is critical to have the computational capability to efficiently test the system (i.e., blisk with RVA attached) under different forcing excitations to predict variations in (i.e., blisk with RVA) resonance frequencies and amplitudes ahead of designing the final RVA to be used in production.

The state of the art includes multiple methods able to predict blisk vibrations with RVA attached. In particular, some methods utilize reduced order modeling techniques such as the Craig Bampton component mode synthesis (CB-CMS) [1] approach and treat the blisk and the RVA as different components of the same system retaining a smaller subset of primary degrees of freedom and secondary modes. However, when friction contacts are enabled between the RVA and the blisk to dissipate energy, multiple methods in the literature utilize the Harmonic Balance Method (HBM)[2, 3] to compute blade responses. While this method is highly efficient and accurate, it cannot utilize experimental data to further enhance the prediction and it can be very computationally expensive for specific contact dynamics.

Thus, this paper proposes a new approach using Control-Based Continuation (CBC), a model-agnostic technique that uses a proportional-derivative controller to cancel higher-order modes and a root-finding algorithm to reach the system's resonant peak, thus tracing out the 'backbone' curve of the response. To validate this method, a simple Lumped Mass Model (LMM) is utilized. Results show that this proposed method can accurately and efficiently predict the blisk-RVA system resonance amplitudes and frequencies at different forcing levels when friction contacts are engaged.

Methodology

A lumped-mass model (LMM) was employed as a simplified representation of the sector dynamics. The system consists of a primary (large) mass coupled to a secondary (smaller) mass, which is mounted on top of the primary mass. The secondary mass is governed by a linear spring–damper system referenced to the motion of the primary mass. In addition, a slip–stick friction contact is introduced between the two masses.

During sticking, the frictional restoring force is modeled as a linear elastic force proportional to the relative displacement between the two masses with respect to an evolving equilibrium position. When sliding occurs, this equilibrium position is updated accordingly. In this configuration, the secondary mass functions as a RVA for the primary mass.

The equations of motion for the coupled system can be expressed as:

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 - k_2\left(x_2 - x_1\right) = F sin\left(\omega t\right) - F_{nl} \\ m_2\ddot{x}_2 + c_2\left(x_2 - x_1\right) + k_2\left(x_2 - x_1\right) = F_{nl} \end{cases}, \tag{1}$$

where x_1 is the motion of the primary mass, x_2 is the motion of the secondary mass, F_{nl} is the nonlinear friction term, m_1 is primary mass, m_2 is the secondary mass, k_1 is the spring constant for the primary mass, k_2 is the spring constant for the secondary mass, c_1 is the primary damping coefficient, c_2 is the secondary damping coefficient, ω is the excitation frequency and F is the excitation force amplitude. This system is given by **Figure 1**.

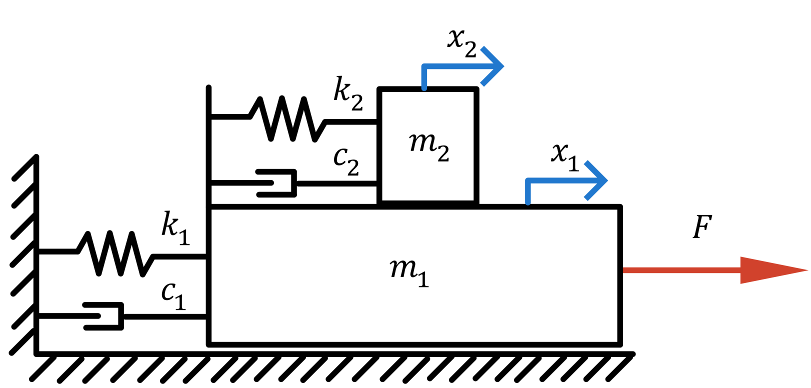

Fig. 1 LMM used for analysis. The friction force is not included but exists between at the interface between the primary mass (m_1) and the secondary mass (m_2)

Control-Based Continuation (CBC) has emerged as a powerful methodology for tracking vibrational characteristics in nonlinear systems [5, 6]. Assuming a general formulation for a linear system with nonlinear forcing, a mathematical formulation for a proportional–derivative (PD) controller can be achieved as seen in Eq. (1).

$$M\ddot{x} + C\ddot{x} + Kx + f_{nl}(\dot{x}, x) = f sin(\omega t) + k_d(\dot{x}^\star - \dot{x}) + k_p(x^\star - x) \; \#(2)$$

For a forced system, the fixed point corresponds to a multi-harmonic trajectory, which can be formulated as a reference (or target) trajectory for the nonlinear system.

$$x^\star(t) = \frac{A_0^\star}{2} + \sum_{i=1}^{m} A_i^\star \cos(i\omega t) \; + B_i^\star \sin(i\omega t) \; \#(3)$$

Assuming a similar form for the solution of the oscillator, the PD controller can be described as driving the system to the trajectory described by the Fourier coefficients of the reference. The PD controller can be used to stabilize the response, provided the variation in nonlinearities around the fixed point remains sufficiently small and the reference trajectory is chosen to coincide with the fixed point [5].

When no prior knowledge of the system is available, the fixed point can be identified through iterative computation or root-finding algorithms. The procedure follows 3 steps: (1) simulate the system response using an initial estimate of the Fourier coefficients, (2) compute the Fourier coefficients of the resulting steady-state trajectory, and (3) evaluate the error between the higher-order Fourier coefficients of the simulated response and the reference coefficients. If the error lies within a user-defined tolerance, the fixed point has been numerically identified and is represented by the corresponding Fourier coefficients. If the error exceeds the tolerance, the newly calculated Fourier coefficients are adopted as the updated reference trajectory, and the process is repeated until convergence is achieved.

The definition of a *backbone curve* varies across literature. A common approach defines the backbone as the locus of points in the Frequency Response Function (FRF) where the system response is in quadrature (90° phase lag) with the excitation [6]. In the present work, the backbone is defined as the curve constructed by connecting the resonant peaks of the FRF.

To estimate each resonant peak, an initial seed frequency was selected as a starting guess. Using this seed and two additional frequencies offset by a small increment above and below, a three-point bracket of steady-state response amplitudes was established. If the bracket encompassed the peak, the midpoint amplitude exceeded the endpoint amplitudes. If the peak was not contained within the bracket, the bracket was iteratively shifted toward the side corresponding to the larger amplitude until the peak was enclosed, assuming a smooth and well-behaved resonance.

Once the peak was bracketed, a quadratic fit was applied to the three points to obtain a parabolic estimate of the peak location [4]. This procedure, analogous to a bisection method but employing parabolic interpolation, was repeated iteratively, reducing the frequency interval until the bracket endpoints converged within a user-specified tolerance. After identifying two peaks, improved initial guesses for subsequent peaks were obtained by extrapolating from the Fourier coefficients and resonance frequencies of the previously identified peaks.

Analysis

The frequency response functions (FRFs) in this work were computed using a time-marching ODE45 solver in MATLAB 2024a (MathWorks, Natick, MA). To maintain accuracy while mitigating numerical issues associated with the discontinuity at the slip–stick transition in the friction law, the friction force was linearly smoothed within a 1% region surrounding the slip-to-stick boundary. Steady-state response amplitudes were extracted from the harmonic content of the solution, with Fourier coefficients obtained using the standard FFT routine in MATLAB 2024a. The resulting FRFs are presented in Figure 2.

From these FRFs, there is a clear peak in the middle at low forcing while at high forcing there are two very prominent peaks. A linear analysis of this system shows that the first and second peak at high forcing correspond to the in-phase and out-of-phase modes of the responses. Qualitative these show that as the forcing in the system approaches zero, a dominantly.

Using the CBC algorithm and conjunction with the peak estimation algorithm, a backbone trace was conducted on both mode starting from high forcing and going towards low forcing.

It is qualitative clear to see from the FRF, that the second peak slowly evolves into the dominant peak as the first peak slowly decays away. This conclusion can be seen from the trace of the backbone as well. As forcing diminishes, the out-of-phase mode smoothly transitions into the more prominent mode of the system. In contrast, the in-phase mode shifts down in frequency and amplitude until it merges in the single-peak response at the lowest forcing. Without knowledge of the system, the combination of CBC and the quadratic interpolation peak search algorithm can be leveraged to understand the motion of the vibrational modes a vibratory system.

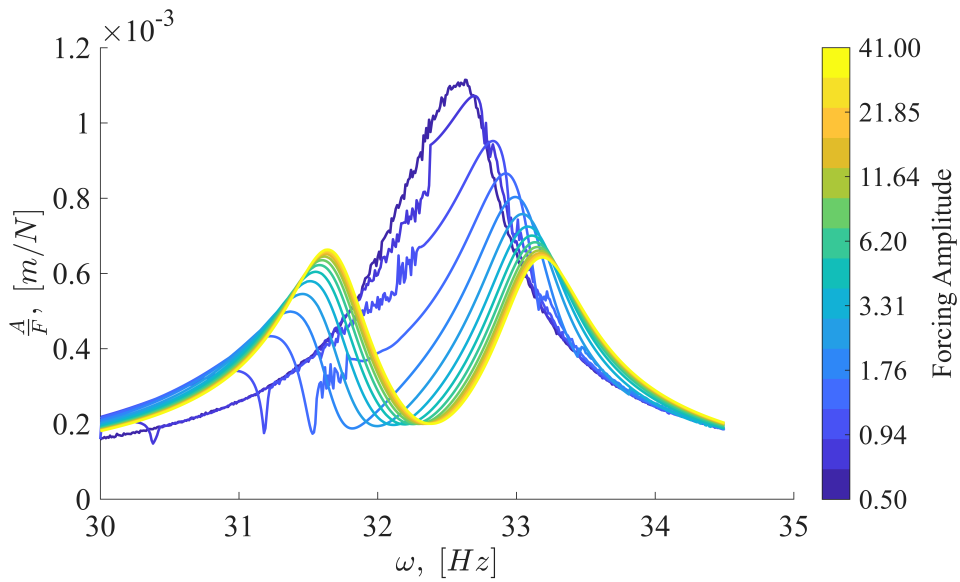

Fig. 2 Frequency response functions for the LMM. These are the amplitudes normalized by the excitation force amplitude. The color corresponds to the forcing value with yellow being 41 N and violet being 0.5 N. The forcing is logarithmically scaled from the low forcings to the high forcings

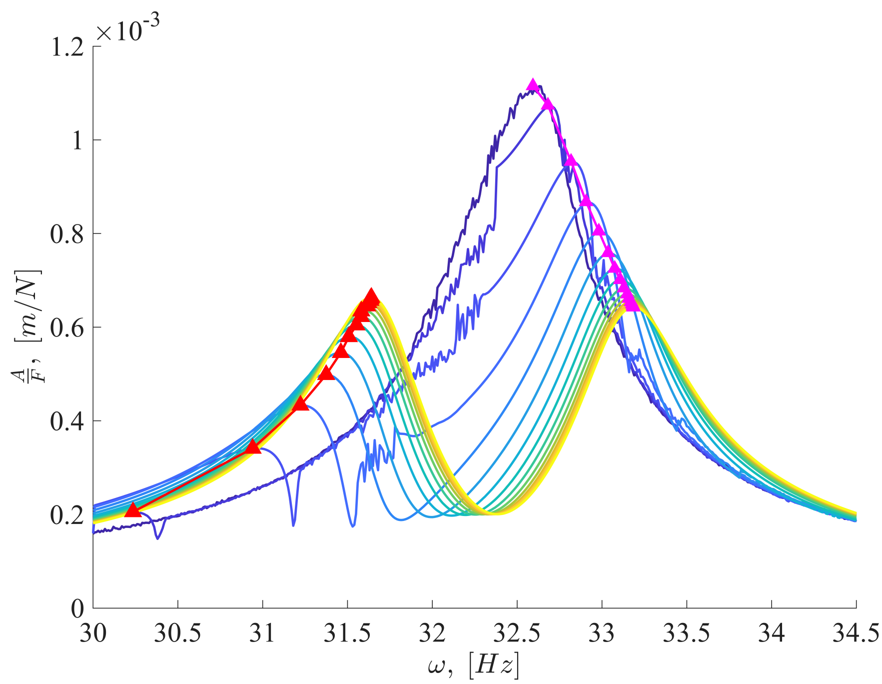

Fig. 3 Backbone trace for LMM model. Using the same FRF coloring scheme from before, yellow indicates strong forcing (41 N) and purple indicates weak forcing (0.5 N). The magenta line shows the backbone of the out-of-phase mode with triangles denoting the backbone points. Similarly, the red line and triangles show the backbone and backbone points respectively

Conclusion

To summarize, this paper proposes a novel technique to predict the backbone curve of a blisk with an RVA with friction contacts. A CBC technique is used on a LMM with 2 masses describing the blisk and the RVA motion when friction contacts are engaged. Results show highly accurate predictions, the CBC allowing efficient predictions of the system response under different coulomb friction conditions. Future work involves applying this technique to a Finite Element (FE) blisk model to further validate the approach.

References

1. Craig, R. R., and Bampton, M. C. C., 1968, "Coupling of Substructures for Dynamic Analyses.," AIAA Journal, **6**(7), pp. 1313–1319. https://doi.org/10.2514/3.4741.
2. Krack, Malte; Gross, Johann. *Harmonic Balance for Nonlinear Vibration Problems*. Springer (Cham), Mathematical Engineering series; ISBN 978-3-030-14022-9 (2019)
3. Lupini, Andrea; Epureanu, Bogdan I. *A Friction-Enhanced Tuned Ring Damper for Bladed Disks*. Journal of Engineering for Gas Turbines and Power, Vol. 143, Issue 1, 011002 (8 pages), 2021.
4. Gasior, Marek & González, José Luis. *Improving FFT Frequency Measurement Resolution by Parabolic and Gaussian Interpolation*. AIP Conference Proceedings 732 (Vol. 1), pp. 276–? (2004).
5. Barton, David A. W. *Control-based Continuation: Bifurcation and Stability Analysis for Physical Experiments*. Mechanical Systems and Signal Processing, Vol. 84 Part B, pp. 54–64 (2017).
6. Renson, L., Gonzalez-Buelga, A., Barton, D. A. W., & Neild, S. A. (2016). *Robust identification of backbone curves using control-based continuation*. Journal of Sound and Vibration, vol. 367, pp. 145-158.