Chapter 12
Modal Identification and MCMC-Based Bayesian Model Updating for Optimization of Material Parameters in Bridges

Sangwon Park and Minwoo Chang

Abstract As the number of aging bridges increases in Korea, the limitations of conventional visual inspection methods—particularly for third-class bridges—are becoming increasingly evident. Many of these structures lack original design drawings, highlighting the need for advanced structural health monitoring (SHM) techniques capable of quantitatively assessing structural integrity. Although image-based monitoring using drones or smartphones has recently gained attention, such methods have fundamental limitations in capturing time-dependent structural responses and internal damage influenced by environmental factors such as temperature. To address these challenges, this study proposes a framework based on Bayesian finite element model updating to identify and update key structural parameters. Ambient vibration and temperature data were collected over an eight-day period from a steel box girder bridge using accelerometers and thermometers. The subspace state-space system identification algorithm (N4SID) was applied to extract major dynamic parameters, including natural frequencies and mode shapes. Subsequently, Bayesian inference based on the Markov Chain Monte Carlo (MCMC) algorithm was employed to update the model while accounting for uncertainties in structural properties. Both natural frequencies and mode shapes were incorporated into the fitness function, with optimized weighting factors and sampling iterations. Furthermore, the choice of prior distribution functions was examined to improve the accuracy of model updating. Through this analysis, we aim to present a practical framework for effectively managing structural uncertainties in bridge condition assessment.

Keywords Structural health monitoring · Markov chain monte carlo · Bayesian model updating · System identification · Long-term monitoring

Introduction

Many bridges constructed during Korea's rapid urbanization have been exposed to decades of repeated traffic loads, harsh environmental conditions, and material deterioration, resulting in gradual degradation of structural performance. In particular, third-class bridges often lack original design drawings, which makes it difficult to quantitatively assess structural behavior over time or to detect internal damage caused by temperature variations using conventional visual inspection methods. As such, there is an increasing need for more systematic and quantitative structural health monitoring (SHM) technologies to ensure the long-term safety and reliability of aging infrastructure. Previous studies have attempted to assess structural condition using drone-based imaging, visual surveys, and time-series analysis. However, these approaches are mostly limited to static or short-term observations and are not well suited to capture long-term nonlinear variations in material properties or structural responses under changing environmental conditions. Furthermore, many of these methods do not account for the uncertainty of structural parameters in a quantitative manner, limiting their applicability in practice.

Sangwon Park
Engineer, Technology Development Center, Saman Corporation,
315, Macheon-ro, Songpa-gu, Seoul, Republic of Korea
e-mail: swpark3@samaneng.com

Minwoo Chang
Associate Professor, Department of Civil and Environmental Engineering,
Myongji University, 116 Myongji-ro, Cheoin-gu, Yongin-si, Gyeonggi-do, 17058 Republic of Korea
e-mail: cmw321@mju.ac.kr

Kristin B. Zimmerman, PhD (eds.), *Dynamics of Civil Structures, 2026, Vol. 2*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/97887-438-1441-2_12

To address these limitations, this study proposes an integrated framework for quantitatively estimating the time-varying dynamic characteristics of bridge bearings under real environmental conditions. For this purpose, eight days of high-frequency ambient vibration and temperature data were collected from a steel box girder bridge located in Gyeonggi Province, South Korea. Modal parameters, including natural frequencies and mode shapes, were extracted using an output-only subspace system identification method based on the Numerical Algorithms for Subspace State-Space System Identification(N4SID) algorithm. A Finite Element (FE) model was constructed to replicate the structural configuration, and its modal properties were compared with the identified data. Based on this comparison, Bayesian model updating was performed using the Markov Chain Monte Carlo (MCMC) algorithm. By simultaneously employing various prior distribution functions (such as normal, Weibull, and lognormal), adjusting the weighting coefficients between the errors in natural frequencies and mode shapes, and ensuring sufficient sampling, the uncertainty inherent in the model updating process could be quantitatively assessed. The proposed framework can be applied to structures with limited design information and serves as a practical alternative to conventional visual inspection methods, providing a solid foundation for systematic maintenance and long-term safety management of aging infrastructure.

Background

Accurate tracking of time-varying structural parameters is essential for assessing the long-term performance of aging bridges, particularly for components like bridge bearings that are highly sensitive to environmental influences. Bearings play a critical role in transferring loads between the superstructure and substructure and are prone to changes in stiffness and Young's modulus due to thermal fluctuations, aging, and material degradation. These variations can alter the global dynamic characteristics of the bridge, such as modal frequencies and mode shapes. Without proper consideration of such time-dependent behavior, modal interpretations may be inaccurate or misleading. Although output-only subspace identification techniques such as N4SID have proven effective in extracting modal parameters from ambient vibration data, the results can be significantly affected by uncontrolled variables especially temperature. In many small- and medium-span bridges, which often lack original design documentation, the effects of environmental conditions on structural dynamics are typically underestimated or neglected in practice. This limitation highlights the need for a continuous monitoring system capable of capturing changes in modal parameters and material characteristics arising from environmental conditions and parameter uncertainty.

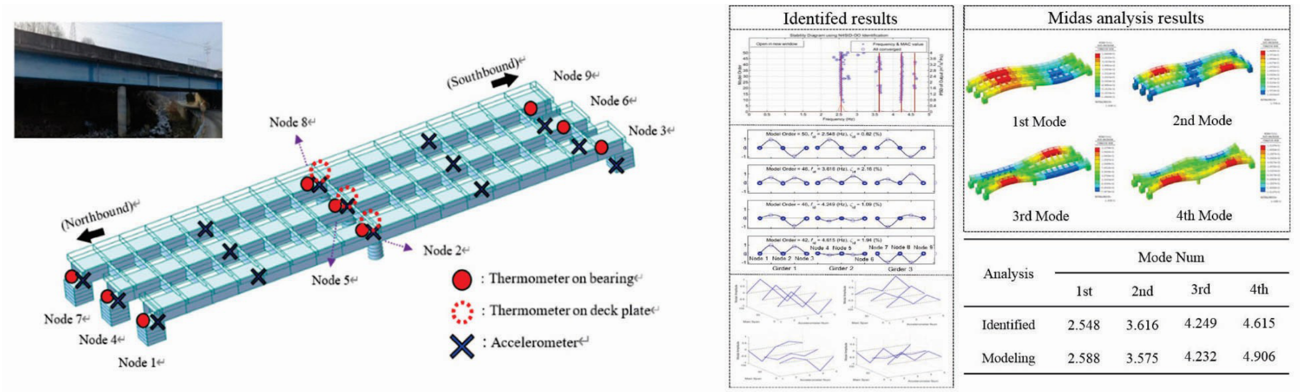
To address this challenge, Bayesian model updating has been introduced as an effective method for calibrating FE models by incorporating measured modal data and quantifying the uncertainty of physical parameters. This probabilistic framework allows for the continuous refinement of structural parameters such as bearing stiffness and elastic modulus based on field measurements, even under time-varying operational conditions. Recent efforts have emphasized the importance of selecting appropriate prior distributions and optimizing fitness functions to improve model accuracy and reliability. This study proposes a framework that combines output-only system identification with Bayesian finite element model updating, aiming to enable reliable tracking and updating of structural parameters in bridges. By effectively capturing changes in key dynamic properties such as bearing stiffness and elastic modulus, the proposed approach is expected to provide a more reliable and data-driven basis for structural performance evaluation and maintenance decision-making in bridges.

System Identification and Verification of Bridge Dynamics via FE Modeling

In this study, accelerometers and temperature sensors were installed at key locations of the bridge, as shown in Fig. 1(a), and dynamic responses were measured over a period of eight days at ten-minute intervals. The collected output data were processed using the N4SID algorithm, implemented in the Structural Modal Identification Toolsuite (SMIT). The measured output signals were organized into a block Hankel matrix according to time history, and an oblique projection Q_{Y_f/Y_p} was applied using past and future output matrices to extract modal information. This projection was subsequently decomposed using Singular Value Decomposition (SVD) as follows:

$$Q_{Y_f/Y_p} = U\Sigma V^T \tag{1}$$

In Eq. (1), Σ is a diagonal matrix containing the singular values, each of which represents the energy contribution of a corresponding mode and is used to estimate the system order. The matrices U and V contain the left and right singular vectors, respectively, and are used to transform the data into modal coordinates. A stabilization diagram was then used to select consistent modal responses over time, allowing the extraction of a total of 1,152 modal parameters, including natural frequencies, damping ratios, and mode shapes. As illustrated in Fig. 1(b), the extracted modal parameters were compared with the results obtained from a finite element model constructed using Midas civil based on the bridge's design specifications, confirming a high degree of similarity between the identified and simulated modal characteristics.



(a) Overview of Deokpyeong 1 bridge and sensor locations

(b) Identified modal parameters

Fig. 1 Sensor configuration of Deokpyeong 1 Bridge and comparison between identified and finite element modal results

MCMC based bayesian FE model updating

Bayesian model updating aims to reduce the discrepancy between measured data and the analytical model by estimating uncertain structural parameters. In this study, the posterior distribution of the parameters is derived by combining the prior distribution with the likelihood function based on modal properties such as natural frequencies and mode shapes. The posterior probability is given by:

$$p(\theta \mid K) \propto p(K \mid \theta) \cdot p(\theta) \tag{2}$$

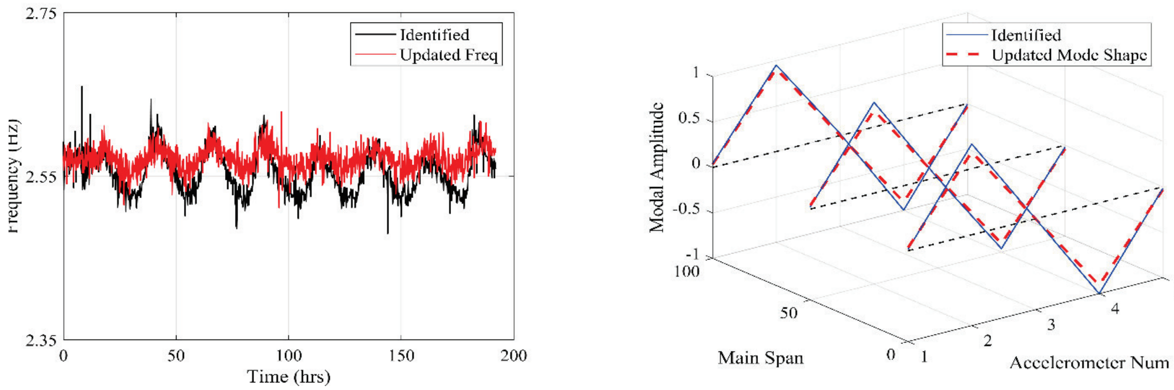
In Eq. (2), $p(\theta)$ is the prior distribution and $p(K \mid \theta)$ is the likelihood function representing the probability of observing the measured data given a parameter set θ. The likelihood is expressed as an exponential function of the fitness function $J(\theta)$, which quantifies the mismatch between measured and simulated modal responses.

$$p(K \mid \theta) = c_0 exp\left(-\frac{1}{2k^2} J(\theta)\right) \tag{3}$$

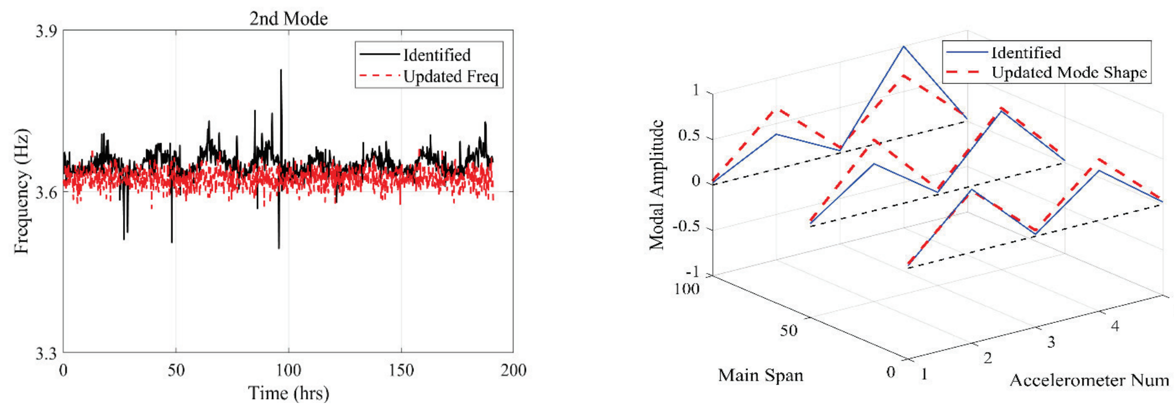
In Eq. (3), c_0 is a normalization constant that ensures the total value of the probability density function integrates to 1. The term k^2 represents the error variance, which controls the degree of sensitivity to modeling error. The function $J(\theta)$ is a fitness function used to quantitatively evaluate how well the predicted model matches the observed data. To efficiently sample from the posterior distribution defined in Eq. (2), the MCMC method is employed. Specifically, the Metropolis–Hastings algorithm is used to explore the parameter space by generating candidate values based on a proposal distribution and probabilistically accepting or rejecting them. The acceptance probability is calculated as follows:

$$p(\theta^* \mid \theta^t) = min\left(1, \frac{p(\theta^* \mid K)\, q(\theta^t \mid \theta^*)}{p(\theta^t \mid K)\, q(\theta^* \mid \theta^t)}\right) \tag{4}$$

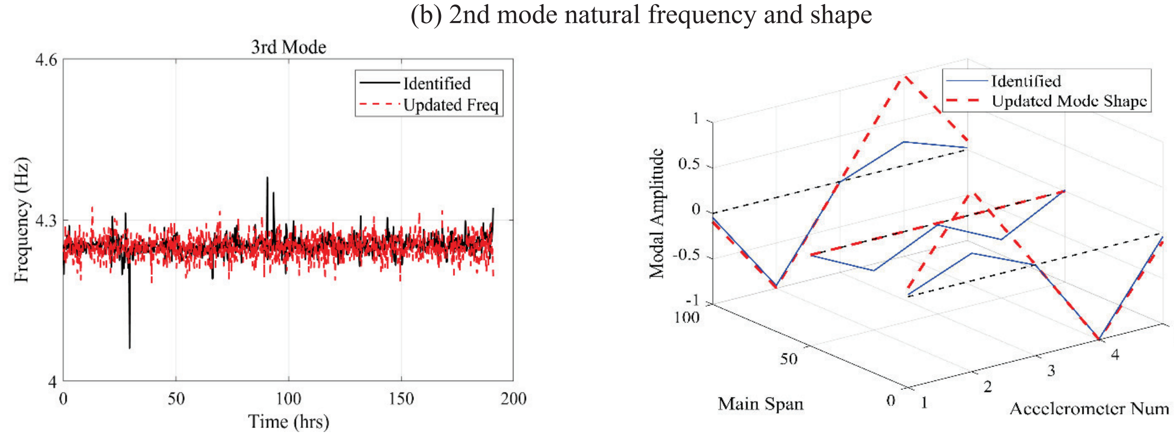
In Eq. (4), $p(\theta^* \mid K)$ denotes the posterior probability of the proposed parameter value θ^*given the observed data K while $q(\theta^t \mid \theta^*)$ is the proposal distribution that represents the probability of transitioning back to the current value θ^t from the candidate θ^*. The denominator $p(\theta^t \mid K)\, q(\theta^* \mid \theta^t)$ reflects the probability of the current state and the likelihood of proposing θ^* from it. The acceptance probability is thus determined by the relative plausibility of the proposed and current parameter values, enabling efficient exploration of the parameter space even when the posterior distribution is complex or analytically intractable.



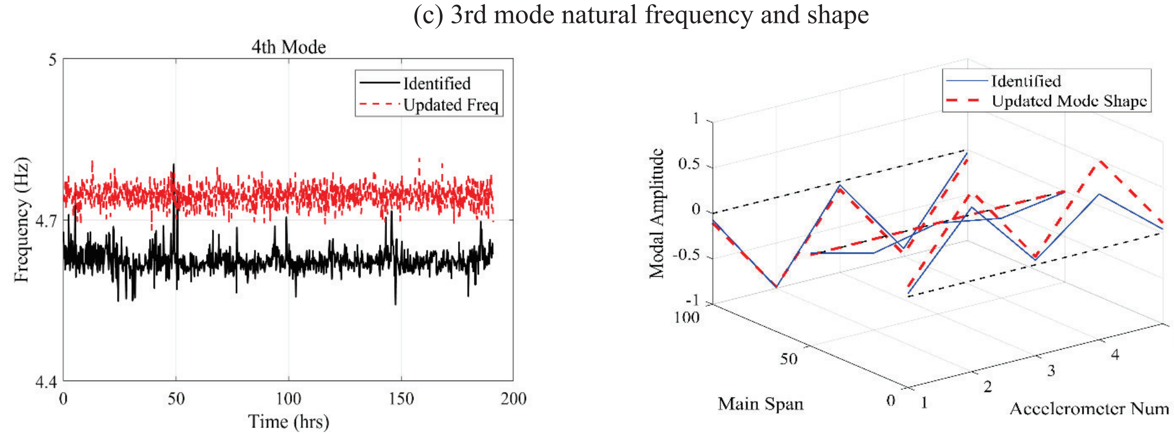
(a) 1st mode natural frequency and shape



(b) 2nd mode natural frequency and shape



(c) 3rd mode natural frequency and shape



(d) 4th mode natural frequency and shape

Fig. 2 Comparison of FE Model updating results with measured natural frequencies and Mode Shapes

The proposed framework for updating bearing stiffness and Young's modulus

To quantitatively compare the differences between measured and simulated modal responses, this study defines a fitness function based on natural frequencies and mode shapes. The fitness function is expressed as:

$$J(\theta) = \alpha \sum_{i=1}^{m} \left(\frac{v_i^{obs} - v_i^{sim}(\theta)}{v_i^{obs}} \right)^2 + \beta \sum_{i=1}^{m} \left(1 - MAC_i(\theta)\right) \tag{5}$$

In Eq. (5), v_i^{obs} and $v_i^{sim}(\theta)$ denote the observed and simulated natural frequencies for the i-th mode, respectively, and $MAC_i(\theta)$ represents the modal assurance criterion (MAC) value for the i-th mode. The coefficients α and β are weighting factors for natural frequency and mode shape, respectively. These weighting factors were adjusted to various values to optimize the fitness function, and the effects of different weightings on the model updating results were analyzed. The number of considered modes is represented by m. This fitness function $J(\theta)$ is incorporated into the likelihood term of the Bayesian posterior distribution and is used to determine sample acceptance in the Metropolis-Hastings MCMC algorithm. Lower values of $J(\theta)$ indicate higher consistency between the simulated and identified modal parameters.

In the Bayesian model updating framework, the prior distribution mathematically expresses pre-existing uncertainty in structural parameters before measured data are incorporated. While the Gaussian distribution is widely used, this study also employed alternative distributions such as Weibull and lognormal distributions to evaluate their impact on probabilistic parameter representation and estimation. The probability density function of the Weibull distribution is defined as:

$$f(\xi; \lambda, \eta) = \frac{\eta}{\lambda} \left(\frac{\xi}{\lambda} \right)^{\eta - 1} e^{-\left(\frac{\xi}{\lambda}\right)^{\eta}}, \ \ \xi \geq 0 \tag{6}$$

In Eq. (6), ξ a structural parameter such as bearing stiffness or Young's modulus. λ and η are the scale and shape parameters of the distribution, respectively. This distribution is suitable for modeling skewed uncertainty in positive-valued parameters. The probability density function of the lognormal distribution is given by:

$$f(\tau; \mu, \sigma) = \frac{1}{\tau\sigma\sqrt{2\pi}} e^{\left(-\frac{(ln\tau - \mu)^2}{2\sigma^2}\right)}, \ \ \tau \geq 0 \tag{7}$$

In Eq. (7), μ and σ are the mean and standard deviation of the underlying normal distribution for $ln\tau$. The lognormal distribution is useful for modeling physical quantities that are strictly positive and exhibit multiplicative variability.

Table 1 Bayesian model updating results under different hyperparameter settings

Hyperparameter set	Configuration	RMSE				MAC			
		1st	2nd	3rd	4th	1st	2nd	3rd	4th
Distribution function ($\alpha = 0.5$, $\beta = 0.5$)	normal (suggested)	0.044	0.039	0.027	0.124	0.996	0.760	0.698	0.949
	Weibull	0.051	0.268	0.207	0.212	0.980	0.865	0.775	0.895
	lognormal	0.028	0.144	0.048	0.041	0.992	0.782	0.729	0.927
Fitness function weight coefficient (normal distribution)	$\alpha = 1, \beta = 0$	0.043	0.031	0.024	0.135	0.995	0.761	0.698	0.951
	$\alpha = 0.8, \beta = 0.2$	0.038	0.033	0.024	0.133	0.996	0.761	0.698	0.951
	$\alpha = 0.2, \beta = 0.8$	0.037	0.032	0.025	0.132	0.996	0.759	0.699	0.950
	$\alpha = 0, \beta = 1$	0.043	0.035	0.029	0.135	0.996	0.761	0.699	0.951

Conclusion

This study proposed an integrated framework that combines consecutive system identification and Bayesian finite element model updating to quantitatively assess key structural parameters of a steel box girder bridge under in-situ conditions. Using

8 days of measured vibration and temperature data, a total of 1,152 modal parameters including natural frequencies and mode shapes were extracted via the N4SID algorithm and used for FE model calibration as illustrated in Fig. 2. To account for the uncertainty of physical parameters, a Bayesian model updating strategy based on the Metropolis Hastings MCMC algorithm was adopted. The prior distributions for bearing stiffness and Young's modulus were configured using Normal, Weibull, and lognormal forms. In addition, the fitness function was designed to balance modal frequencies and mode shapes with equal weighting, and the sensitivity to modeling errors was controlled by a fixed tolerance factor.

The analysis showed that the baseline configuration (normal prior, $\alpha = 0.5$, $\beta = 0.5$) produced low RMSE and high MAC values for the first and fourth modes, indicating reliable agreement between the updated and measured modal properties (Table 1). However, for the second and third modes, MAC values were relatively low, while the application of the Weibull prior notably improved MAC in these modes but resulted in significantly higher RMSE, suggesting that improvements in mode shape agreement can come at the expense of frequency prediction accuracy depending on the chosen prior. The lognormal prior yielded results similar to or slightly better than the baseline in certain modes, but overall differences were not substantial. Adjusting the weighting coefficients in the fitness function did not significantly affect the RMSE or MAC values, indicating that both natural frequency and mode shape information are important in the model updating process.

While the proposed framework demonstrated high accuracy, its practical real-time application may be limited by the high sensor density and computational demand. Future research should focus on enhancing efficiency, generalizability, and robustness to incomplete data by incorporating techniques such as data augmentation, uncertainty filtering, and adaptive model calibration. These advancements are expected to broaden the applicability of the framework for structural health monitoring across diverse bridge types.

Acknowledgments This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2025-00516404).

References

1. Andrieu, C., & Thoms, J. (2008). A tutorial on adaptive MCMC. *Statistics and Computing, 18*, 343-373. https://doi.org/10.1007/s11222-008-9110-y
2. Aria, M., & Akbari, R. (2013). Inspection, condition evaluation and replacement of elastomeric bearings in road bridges. *Structure and Infrastructure Engineering*, *9*(9), 918-934. https://doi.org/10.1080/15732479.2011.638171
3. Biondini, F., & Frangopol, D. M. (2016). Life-cycle performance of deteriorating structural systems under uncertainty. *Journal of Structural Engineering*, *142*(9), F4016001. https://doi.org/10.1061/(ASCE)ST.1943-541X.0001544
4. Chang, M., Leonard, R. L., & Pakzad, S. N. (2012). Structural modal identification toolsuite (SMIT) user's guide. *Lehigh University*, Bethlehem, PA
5. Chang, M., & Pakzad, S. N. (2014). Modal parameter uncertainty quantification using PCR implementation with SMIT. *In Model Validation and Uncertainty Quantification, Volume 3: Proceedings of the 32nd IMAC, A Conference and Exposition on Structural Dynamics*, 2014 (pp. 185-193). Springer International Publishing
6. Chang, M. (2018). Application studies on structural modal identification toolsuite for seismic response of shear frame structure. *Journal of the Earthquake Engineering Society of Korea*, 22(3), 201-210. https://doi.org/10.5000/EESK.2018.22.3.201
7. Chen, R., & Ni, Y. Q. (2023). A nonparametric bayesian approach for bridge reliability assessment using structural health monitoring data. *Structural Control and Health Monitoring*, *2023*(1), 9271433. https://doi.org/10.1155/2023/9271433
8. Cunha, Á., & Caetano, E. (2005). From input-output to output-only modal identification of civil engineering structures. In *1st International Operational Modal Analysis Conference (IOMAC)*. https://hdl.handle.net/10216/71418
9. Dutta, C., Kumar, J., Das, T. K., & Sagar, S. P. (2021). Recent advancements in the development of sensors for the structural health monitoring (SHM) at high-temperature environment: A review. *IEEE Sensors Journal, 21*(14), 15904-15916. https://doi.org/10.1109/JSEN.2021.3075535
10. Garcia-Palencia, A. J., & Santini-Bell, E. (2014). Structural model updating using dynamic data. *Journal of Civil Structural Health Monitoring, 4*, 177-194. https://doi.org/10.1007/s13349-014-0073-8
11. Keshmiry, A., Hassani, S., Mousavi, M., & Dackermann, U. (2023). Effects of environmental and operational conditions on structural health monitoring and non-destructive testing: A systematic review. *Buildings, 13*(4), 918. https://doi.org/10.3390/buildings13040918
12. Liu, X., Qiao, S. F., Chen, H., Tang, M. X., He-Song, H., & Zhu, Y. (2023). Prediction method of structural health monitoring data based on CNN-LSTM. In *Advances in Frontier Research on Engineering Structures Volume 2* (pp. 309-318). CRC Press. https://doi.org/10.1201/9781003363217-39
13. Ni, Y. Q., Ye, X. W., & Ko, J. M. (2012). Modeling of stress spectrum using long-term monitoring data and finite mixture distributions. *Journal of Engineering Mechanics*, *138*(2), 175-183. https://doi.org/10.1061/(ASCE)EM.1943-7889.00003
14. Midas (2018). Civil On-line Manual – Civil structure design system.
15. Park, S., Chang, M., & Yazdanpanah, O. (2025) "A condition monitoring of steel box girder bridges based on consecutive system identification and model updating using a long-term monitoring database", *Structures*, 80, 109779. https://doi.org/10.1016/j.istruc.2025.109779
16. Rocchetta, R., Broggi, M., Huchet, Q., & Patelli, E. (2018). On-line Bayesian model updating for structural health monitoring. *Mechanical Systems and Signal Processing, 103*, 174-195. https://doi.org/10.1016/j.ymssp.2017.10.015

17. Van Overschee, P., & De Moor, B. (1994). N4SID: Subspace algorithms for the identification of combined deterministic-stochastic systems. *Automatica*, 30(1), 75-93. https://doi.org/10.1016/0005-1098(94)90230-5
18. Xia, H. W., Ni, Y. Q., Wong, K. Y., & Ko, J. M. (2012). Reliability-based condition assessment of in-service bridges using mixture distribution models. *Computers & Structures*, *106*, 204-213. https://doi.org/10.1016/j.compstruc.2012.05.003
19. Zhang, H., Wang, L., & Liu, Y. (2025). A hybrid model-based and data-driven method for mechanical-thermal dynamic load identification considering multi-source uncertainties. Computer Methods in Applied Mechanics and Engineering, 435, 117662. https://doi.org/10.1016/j.cma.2024.117662