Chapter 3
Correlation & Orthogonality – What Do the Numbers Mean? – Use, Misuse and Confusion

Peter Avitabile

Abstract Experimental modal testing is performed for a variety of reasons – one of which is to validate or compare to a finite element model. Metrics such as the Modal Assurance Criteria and Mass Orthogonality are vector based techniques used to compare an analytically developed mode shape with an experimentally obtained mode shape. These vector based numerical values are then evaluated to some criteria that may be mandated by contract or imposed by some specification or maybe be a company internally agreed upon number. Beyond vector correlation metrics, the shapes can also be compared on a degree of freedom to degree of freedom (point to point) comparison to further interrogate the level of correlation – but often these further checks are not normally performed or mandated as part of the process. Unfortunately, these degree of freedom checks are very important to really understand the vector correlation results and without any assessment, the true correlation is really not clearly understood. This paper will discuss many of these aspects that are very important to the actual correlation achieved. Basic theoretical equations are presented and examples of many important issues are presented. In addition, general commentary is provided because there is a general misunderstanding and misconceptions as to what all the numbers really mean.

Keywords Orthogonality · Model validation · MAC · POC

Introduction

Experimental modal tests are performed for many purposes. Understanding the system frequencies and shapes is of paramount importance in designing better structures to allow for improved performance. But many times, a modal test may be conducted to help verify or update a finite element model. When this is the case, many correlation tools may be deployed to check both mode shapes as well as individual degree of freedom (DOF) correlation. At times, these correlation tools are deployed for the engineer to better understand their models and adjust or improve the model to better reflect the reality of the actual hardware. And at other times, these tools may be mandated by contract or agency responsible for the acceptance of particular hardware or components used as part of a larger assembly.

When correlation is mandated by contract, then an arbitrary value may be set. If the target is met then the structure is considered “validated” – if the target is not met, then the structure is considered to be “not validated”. But the specific number is somewhat arbitrary at times and then also, somewhat questionable”. The general theory is presented, and in some cases expanded, to clearly understand exactly how the numbers are computed and the actual contribution to the overall calculation to achieve a particular value; this helps in the ability to better understand the values and what they really mean in comparison to the stated “value to meet”.

This paper only addresses real normal modes and the understanding of the vector correlation metrics for real normal modes. The inclusion of complex modes only further complicates these issues. Once some of the misunderstandings and general misconceptions are clarified, the next step is to address this with complex modes as part of the mix. But this paper

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just takes the first step to clarify these points because there seems to be "old wives' tales" that have masked some of the reality of what really should be of concern when performing any type of correlation or orthogonality study.

Theory is summarized first followed by examples to illustrate many misconceptions. Some general observations are made followed by conclusions.

Theory Review

The very basic equations related to MAC, POC, CORTHOG and DCAF are summarized in this section; more detailed theory is contained in the references identified.

Modal Assurance Criteria (MAC)

The MAC [1] is really nothing more than a simple dot product that helps to identify how correlated two vectors are to each other (or not correlated to each other depending on how one looks at it). The MAC can be written as

$$\mathrm{MAC}_{ij} = \frac{\left[\{u_i\}^T \{e_j\}\right]^2}{\left[\{u_i\}^T \{u_i\}\right] \left[\{e_j\}^T \{e_j\}\right]} ; \mathrm{MAC}_{ij} = \frac{\left[\{e_i\}^H \{e_j\}\right]^2}{\left[\{e_i\}^H \{e_i\}\right] \left[\{e_j\}^H \{e_j\}\right]} \tag{1}$$

One formulation is used for real normal vectors (when comparing to analytical models) and the other is used for data that is comprised of complex vectors with both magnitude and phase. As the MAC approaches 1, there is a high indication that the two vectors are very correlated. As the MAC approaches 0, there is a high indication that the two vectors are very uncorrelated.

There are many reasons why the MAC may fall less than 1; some typical reasons usually cited are:

- The modal vectors extracted are different from each other
- There may be noise that contaminates the vectors being compared
- The vectors may be affected by nonlinearities in the measured data
- There is an insufficient number of measured points to characterize the shape

For all the cases presented in this paper, the modes shapes are considered real normal modes or proportional modes. A sample MAC matrix is seen in Figure 1 to show a correlation as typically observed comparing an experimental set of modal vectors to an analytical (FEA) set of modal vectors.

Notice that the diagonal has very strong values for the first seven modes indicating a high correlation between each of the mode shape pairs whereas the off-diagonal terms are much smaller approaching zero. Note that the MAC result is often presented in a matrix type display but the calculation is really just individual vector dot products with the results of each dot product arranged in a matrix format.

The biggest strength of the MAC is that the mass matrix is not required. However, that is also the biggest drawback to the MAC because the mass matrix is not used. Mass scaling is critically important to clearly and deeply understand the correlation between an analytical and experimental model. MAC is widely used and an extremely important tool for correlation assessment but does lack the important mass scaling.

Coordinate Modal Assurance Criteria (CoMAC)

The Coordinate Modal Assurance Criteria (CoMAC) [2] is nothing more than the vector comparison but without mass scaling and formulated in a similar fashion as the MAC for correlated mode pairs. Without mass scaling, the CoMAC (like the MAC) is not as robust a correlation tool. Interpretation is similar to MAC. The CoMAC is given as

$$CoMAC(k) = \frac{\left[\sum_{c=1}^{m} \left| u_k^{(c)} \cdot e_k^{(c)} \right| \right]^2}{\sum_{c=1}^{m} \left(u_k^{(c)}\right)^2 \cdot \sum_{c=1}^{m} \left(e_k^{(c)}\right)^2} \tag{2}$$

The interpretation follows in a similar fashion to the MAC.

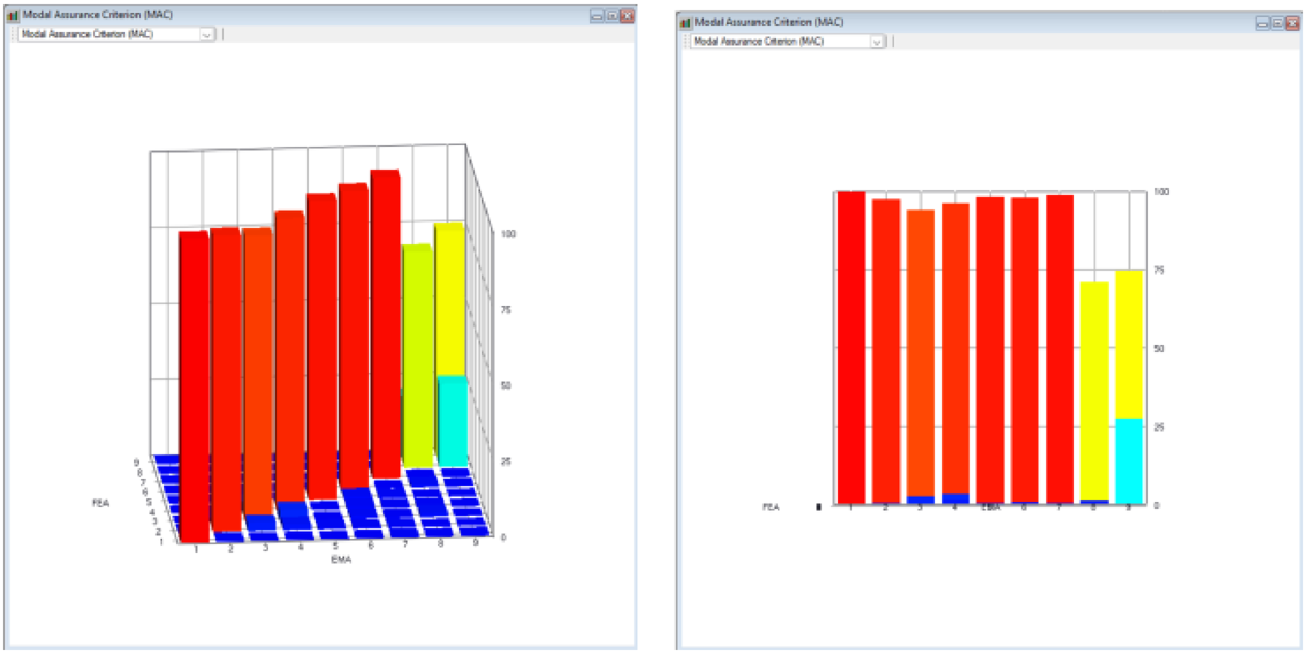


Fig. 1 An Example MAC Matrix – oblique view (left) and side view (right)

Test Response Assurance Criteria (TRAC)

At times, two different time responses may need to be compared. The MAC formulation can also be used for the comparison and is called the Test Response Assurance Criteria (TRAC) (3) and is given as

$$TRAC = \frac{\left[\{x_{n1}\}^T\{x_{n2}\}\right]^2}{\left[\{x_{n1}\}^T\{x_{n1}\}\right]\left[\{x_{n2}\}^T\{x_{n2}\}\right]} \tag{3}$$

The interpretation is the same but one additional item is that the plot can be formed like a regression analysis where a line fit and regression coefficient is shown. This provides significant additional graphical interpretation to the process as shown in Figure 2. Note that the regression coefficient is the calculation value from the TRAC (or MAC).

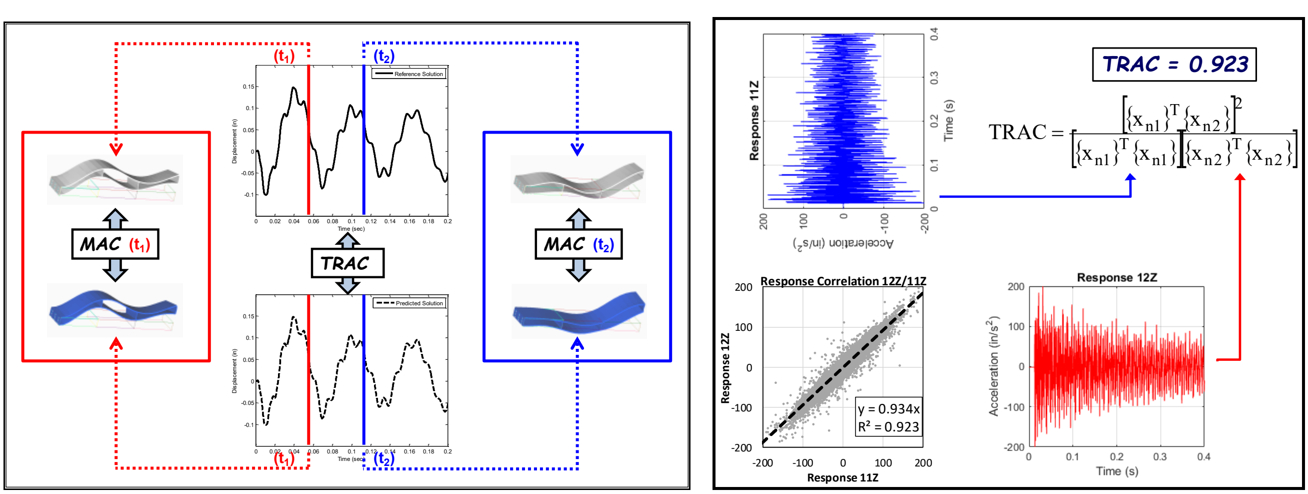


Fig. 2 Illustration of TRAC (with MAC) (left) and TRAC shown as a Regression Analysis (right)

Pseudo Orthogonality Criteria (POC)

The Mass Orthogonality can be stated as

$$\mathrm{ORT} = [\mathrm{U}]^{\mathrm{T}}[\mathrm{M}][\mathrm{U}] = [\mathrm{I}] \tag{4}$$

Now this leads to the Pseudo Orthogonality Criteria (POC) [4] stated as

$$\mathrm{POC} = [\mathrm{E}]^{\mathrm{T}}[\mathrm{M}][\mathrm{U}] \stackrel{?}{=} [\mathrm{I}] \tag{5}$$

This is stated to be "pseudo" or "false" because the definition of the orthogonality can only be with respect to the analytical mass with the analytical vectors.

A typical Pseudo Orthogonality is shown in Fig 3. Note that the Mass Orthogonality (Not shown) would clearly be the identity matrix as expected from the formulation in (4) whereas the POC is a similar result where the diagonals are close

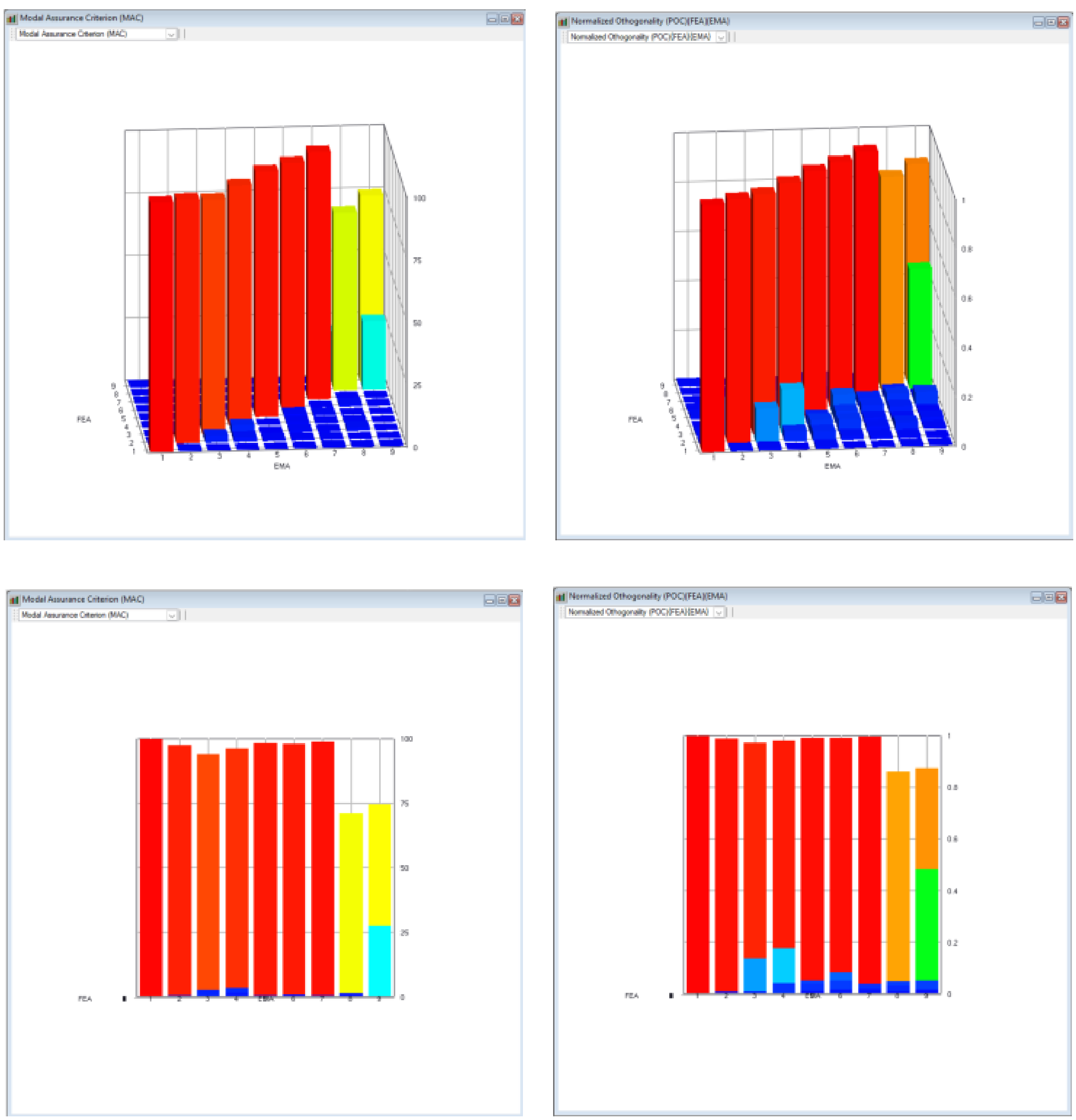


Fig. 3 Typical Comparison of MAC and POC for Two FEA Models (one with 4 times mesh density)
Top Left MAC oblique view – Lower Left MAC side view
Top Right POC oblique view – Lower Right POC side view

to 1.0 for a well correlated model with off-diagonal terms which are very low approaching zero. Figure 3 shows the POC with the MAC for the same models compared. Notice that the MAC values on the diagonal are slightly smaller than the POC values due to the inclusion of mass in the POC calculation; a side view is also shown to more clearly see these differences. And the off-diagonal terms for the POC value are larger than the corresponding MAC values, again due to the inclusion of mass in the POC calculation.

Now all individual terms of the resulting POC do come from a matrix calculation for each of the individual values. But there are many individual terms that are computed and added together to determine each term of the matrix.

Breakdown Orthogonality Calculation

So the Mass Orthogonality can be written out (for a lumped mass matrix) as

$$\mathrm{ORT_{ij}} = (\mathrm{u_{1i}\,m_{11}u_{1j} + u_{2i}\,m_{22}u_{2j} + u_{3i}\,m_{33}u_{3j}} + \ldots) = \begin{matrix} 1(\text{ for i} = \text{j}) \\ 0(\text{ for i} \neq \text{j}) \end{matrix} \tag{6}$$

And the POC can also be written in a similar fashion as

$$\mathrm{POC}_{ij} = (e_{1i}m_{11}u_{1j} + e_{2i}m_{22}u_{2j} + e_{3i}m_{33}u_{3j} + \ldots) = \begin{matrix} 1 & (\text{ for } i = j) \\ 0 & (\text{ for } i \neq j) \end{matrix} \tag{7}$$

When written this way, there is more clarity that there are many terms that form the entire resulting POC matrix. This leads the way to the Coordinate Orthogonality Check (CORTHOG) [5–7].

Reduction of Finite Element Mass/Stiffness Matrix and Expansion of Modal Vectors

There are many techniques available to do this. Guyan [8] and Improved Reduced System (IRS) [9] are possible choices. However, the System Equivalent Reduction and Expansion Process (SEREP) [10] has been shown to be the most accurate approach and is used here.

The reduced mass and stiffness transformation matrix (which is also used for expansion) can be formed from the full space eigenvectors obtained from an eigensolution using the finite element model. The transformation matrix [T] is used to project the full mass and stiffness matrices to a smaller size. The reduced matrices can be formulated as

$$[M_a] = [T]^T\,[M_n]\,[T] \tag{8}$$

$$[K_a] = [T]^T\,[K_n]\,[T] \tag{9}$$

The SEREP transformation is formed as

$$[T_U] = [U_n]\,[U_a]^g \tag{10}$$

Coordinate Orthogonality Check (CORTHOG)

The Coordinate Orthogonality Check (CORTHOG) [5–7] is simply the comparison of what should have been obtained analytically for each dof in an orthogonality check to what was actually obtained for each dof in a POC from test.

The most important thing to note is that each of the DOF are compared but the calculation includes the mass as a very important scaling component to the entire calculation. This mass scaling proves to be an important consideration that enables the proper scaling for each DOF.

Now if SEREP is used for the formulation of the reduced mass or the expanded vectors, then the orthogonality and pseudo-orthogonality become much simpler expressions. If the SEREP technique is utilized, then the standard mass orthogonality can be written as

$$\mathrm{ORT} = [\mathrm{U_n}]^{\mathrm{T}}\,[\mathrm{M_n}]\,[\mathrm{U_n}] = [\mathrm{U_a}]^{\mathrm{T}}\,[\mathrm{M_a}]\,[\mathrm{U_a}] = [\mathrm{U_a}]^{\mathrm{g}}\,[\mathrm{U_a}] \tag{11}$$

and the Pseudo Orthogonality Check can be written as

$$\mathrm{POC} = [\mathrm{U_n}]^{\mathrm{T}}\,[\mathrm{M_n}]\,[\mathrm{E_n}] = [\mathrm{U_a}]^{\mathrm{T}}\,[\mathrm{M_a}]\,[\mathrm{E_a}] = [\mathrm{U_a}]^{\mathrm{g}}\,[\mathrm{E_a}] \tag{12}$$

Now when considering one particular "ij" term, the orthogonality and pseudo-orthogonality can be easily written (using just the last term without the mass terms) as

$$\mathrm{ORT}_{ij} = u^g_{i1}u_{1j} + u^g_{i2}u_{2j} + u^g_{i3}u_{3j} + \cdots + u^g_{ia}u_{aj} = \begin{matrix} 1(\text{ for } i = j) \\ 0(\text{ for } i \neq j) \end{matrix} \tag{13}$$

$$\mathrm{POC}_{ij} = u^g_{i1}e_{1j} + u^g_{i2}e_{2j} + u^g_{i3}e_{3j} + \cdots + u^g_{ia}e_{aj} = \begin{matrix} 1(\text{ for } i = j) \\ 0(\text{ for } i \neq j) \end{matrix} \tag{14}$$

In this form, a simple comparison, term by term, shows the DOF correlation. The CORTHOG manipulation is the comparison of each of the terms from the relationships in Eq. 13 and Eq. 14 for corresponding degrees of freedom.

$$\boxed{\begin{array}{llllllll} \mathrm{POC}_{ij} & = u^g_{i1}e_{1j} & + u^g_{i2}e_{2j} & + u^g_{i3}e_{3j} & + \cdots & + u^g_{ia}e_{aj} & \\ & \Updownarrow & \Updownarrow & \Updownarrow & & \Updownarrow & \mathrm{CORTHOG}_{ij} \\ \mathrm{ORT}_{ij} & = u^g_{i1}u_{1j} & + u^g_{i2}u_{2j} & + u^g_{i3}u_{3j} & + \cdots & + u^g_{ia}u_{aj} & \\ & \vdots & \vdots & \vdots & & & \\ & \mathrm{dof1} & \mathrm{dof2} & \mathrm{dof3} & & & \end{array}} \tag{15}$$

Equation (13) and (14) above show the individual contribution for each DOF. The comparison of these (as a simple difference or other scaled forms) clearly shows the contribution that each DOF makes to the individual terms of the ORT or POC matrix and the comparison clearly show the differences between the FEA model and the Test results.

A simple 3 dof example will identify the concept very easily. One misconception is that the closer the off-diagonal terms are to zero, the less correlated the vectors are to each other. The problem is from the equation above, the sum of a large number of terms are required to obtain any one off-diagonal term. The numbers themselves can be large or small, but the sum of all of them will equal zero if the modes are orthogonal to each other. While this is true for analytical statement of orthogonality, this may not necessarily be true when combining analytical and experimental data sets. Two vectors are compared – one is the ORT and the other is the POC. Now in Figure 4, the individual terms for each calculation sum up to be zero but notice that the comparison on a term by term basis shows there are differences. So while the off-diagonal term may have a summed value of zero, this does not necessarily imply that the two vectors are indeed orthogonal to each other and there may be discrepancies looking on a term by term basis.

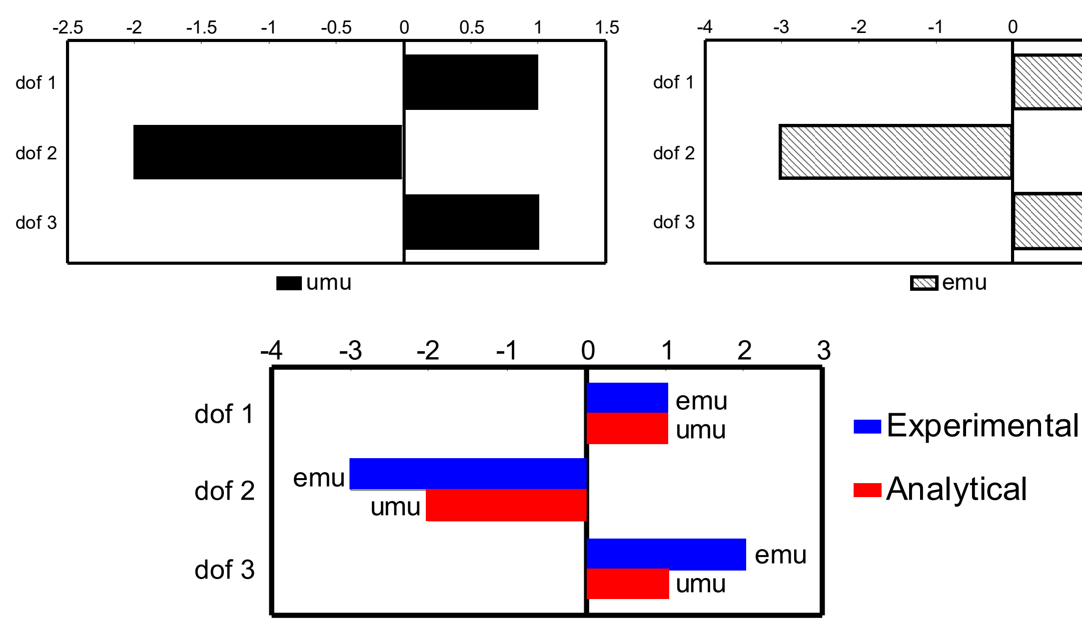


Fig. 4 CORTHOG for Simple 3 DOF System

Data Consistency Assessment Function (DCAF)

The Data Consistency Assessment Function (DCAF) [11, 12] is a tool that enables the identification of items in a data base that may not seem to be correlated with the rest of the data set. Basically, the DCAF systematically removes individual data points from a data set and uses synthesis to recreate the removed point which is then compared to the original data to see how well the removed data point compares. Obvious poor data points can be easily identified and bad data points can be removed from the data set. Figure 5 schematically shows the process for assessment of mode shapes for instance. (In essence, the DCAF is nothing more than removal of outliers as typically done in fitting of data.)

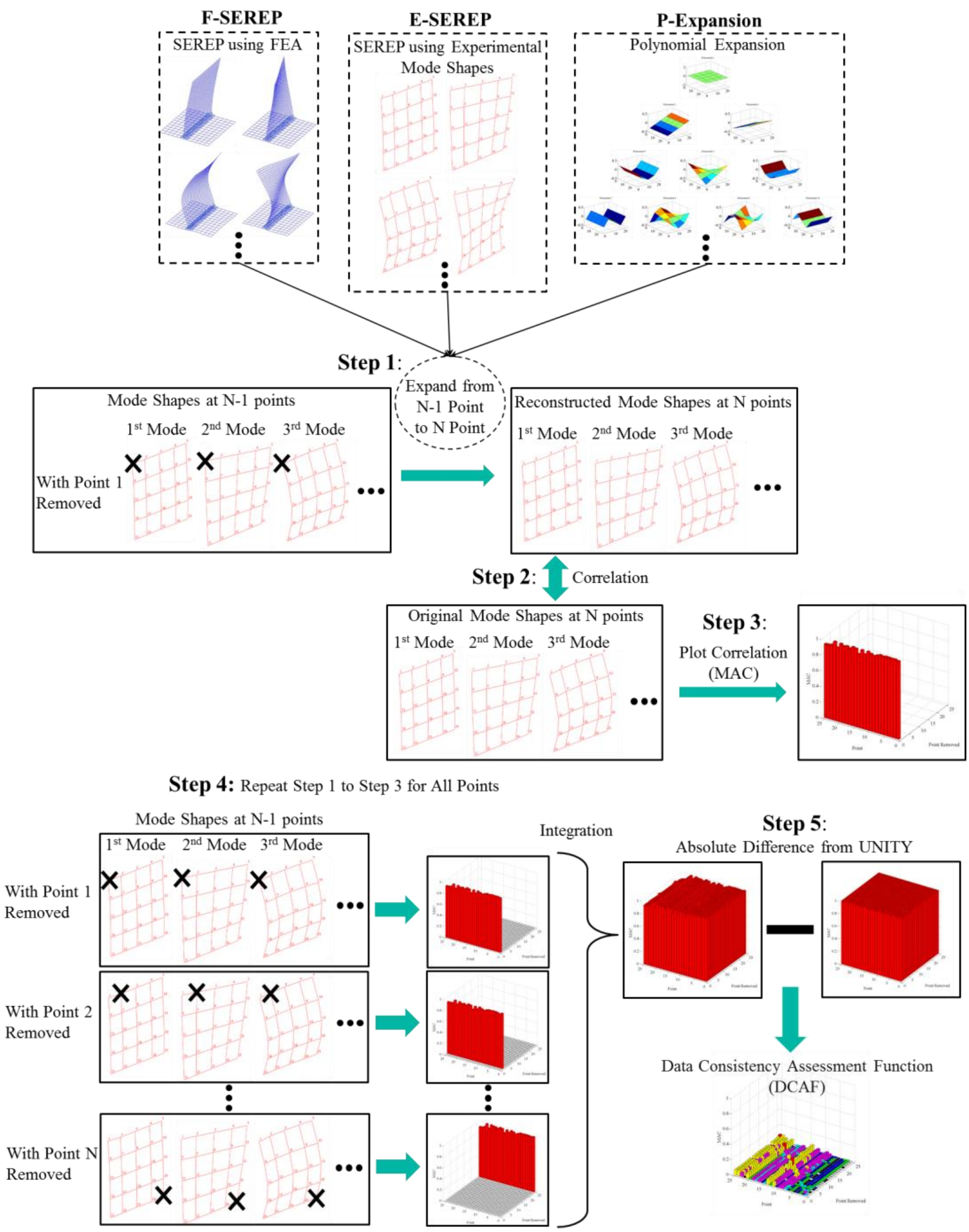


Fig. 5 Process of DCAF for mode shape assessment

Examples

In order to illustrate some of the differences and peculiarities of all the tools, several examples are included. In all cases, real normal modes are used to keep the numerics simple to show the strengths, benefits and quirks of the different tools.

Modal Assurance Criteria – Example of a Good MAC

The first example is one for a structure where the general MAC calculation produces results as would be expected. A MAC was performed for a finite element model for a simple Base Upright (BU) structure and the experimental modal test results obtained from a good modal test. Figure 6 shows the MAC matrix for an example where there are no major issues that arise and the MAC matrix provides good insight into the correlation between the test and FEA.

FEA Mode Number	Experimental Mode Number	FEA Frequencies (Hz)	Experimental Frequencies (Hz)	Frequency % Difference	MAC Value (%)
1	1	26.03	26.17	-0.54	99.8
2	2	70.70	67.34	4.75	98.5
3	3	77.69	75.89	2.32	99.3
4	4	108.80	104.48	3.97	97.1
5	5	158.01	161.55	-2.24	98.9
6	6	269.88	270.00	-0.04	98.9
7	7	304.40	299.93	1.47	96.5
8	8	350.45	354.60	-1.18	98.5
9	9	419.36	419.22	0.03	94.3
10	10	457.13	482.15	-5.47	92.7
11	12	541.29	547.84	-1.21	96.6
12	11	563.47	538.20	4.48	86.3
13	13	777.29	771.76	0.71	90.3
14	14	779.61	791.18	-1.48	97.7
15	15	862.76	871.90	-1.06	84.3

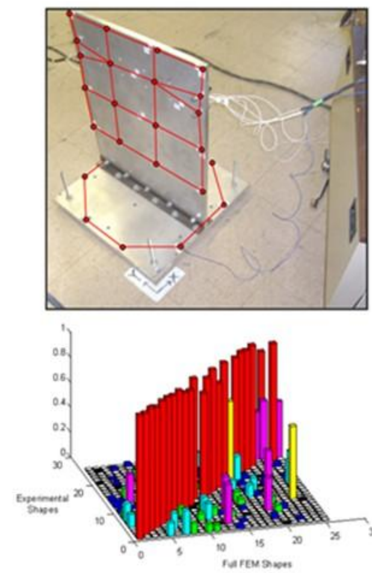


Fig. 6 Comparison of FEA and Modal Test for the Base-Upright Structure

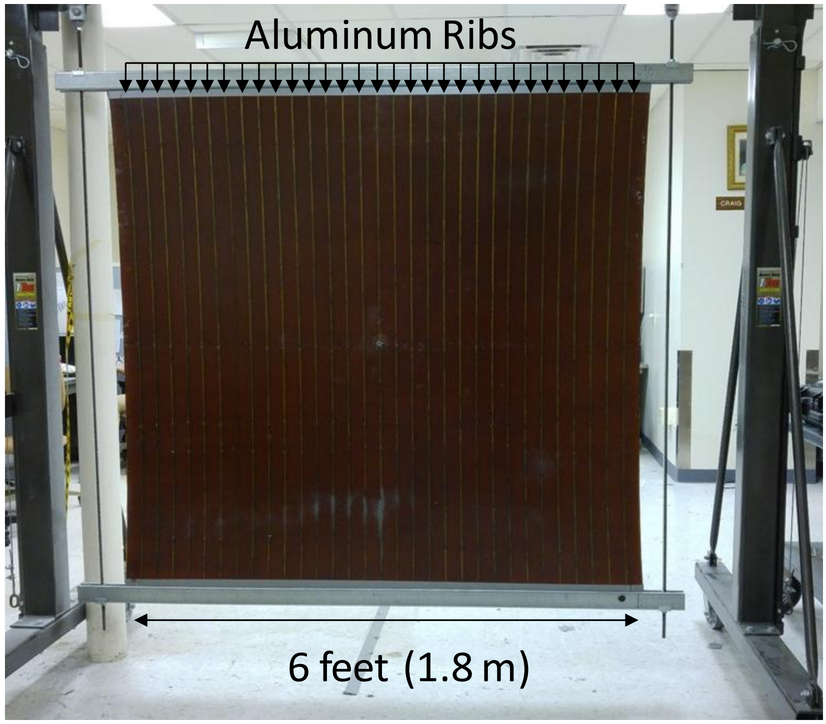
Modal Assurance Criteria – When Just a Portion of the Structure Lacks Correlation

So a question was raised as to how good a finite element model was that was used for predictions for Bragg scattering studies. A modal test was performed to compare to a very preliminary finite element model of a rib-stiffened urethane plate structure; the way the structure was designed and fabricated, the clamping arrangement (and modeling of that) was unclear and not straight-forward. The structure is shown in Figure 7 along with the mode shape of concern.

For one of the lower order modes, a MAC of 94% was identified from the correlation. Without really looking at the mode shapes in detail, a MAC value this high would generally indicate a good shape correlation. But looking at the mode shape more deeply and closely viewing the overlaid mode shapes, there appeared to be about 75% of the structure that was very correlated between the FEA and test results. But there was about 10% of the structure that really didn't show any correlation to the naked eye. And the portion that was most different was situated in the region where the clamping arrangement was of concern. So this is a good case where having a very high MAC value does not really guarantee that the entire mode shape is well correlated. (Of course, a mass orthogonality would be more telling but that was not done or required as part of the user initiated validation of the model.)

Modal Assurance Criteria – When Not All of the Structure is Measured

So a number of years ago, some work was done on jet engine turbine blades. As part of the certification of remanufactured jet engine turbine blades, 25 OEM blades and 25 re-engineered blades are subjected to correlation studies for the first 25 modes of the blades. This is typically up to 20 KHz to 30 KHz ranges. The criteria is that the MAC must be above 90% for all modes of all blades in order for the certification of the new blades to be accepted. Well, there was significant effort to accomplish



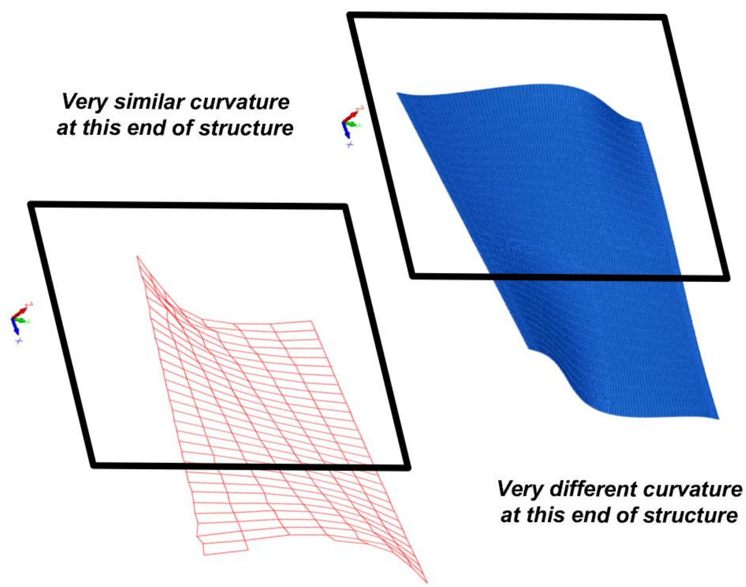
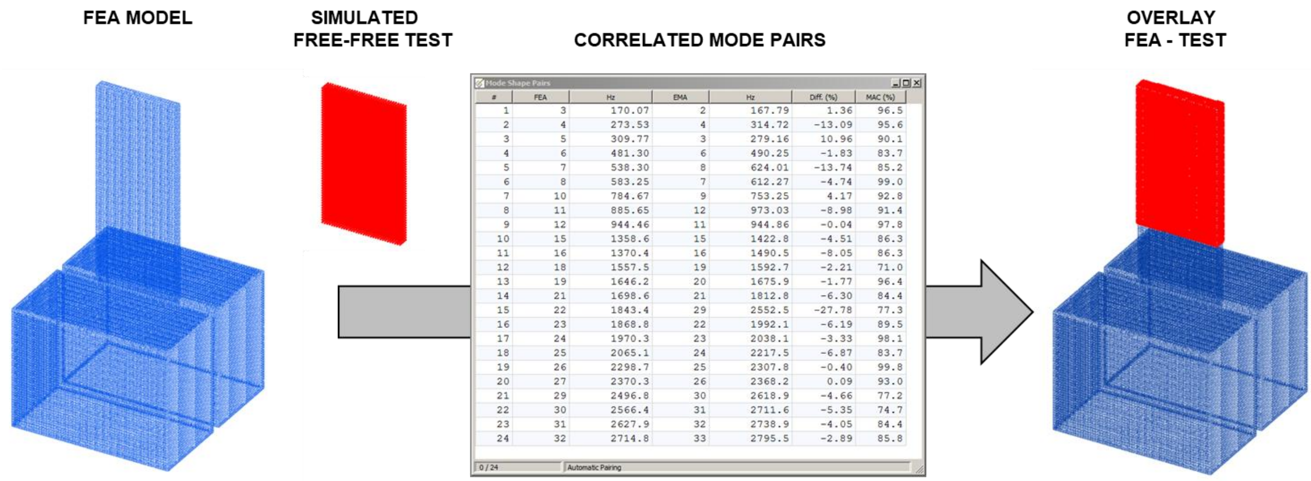


Fig. 7 Large Plate Structure with Mode Shape Comparison

this and along the way a number of questions arose as to how to handle variability in the blades and how address what the geometric correlation should be (considering that every blade is really slightly different) and how to actually perform the correlation. The FAA folks were adamant that a MAC of 90% was critical to assure that the blades were the "same" (whatever that means).

In an effort to help everyone understand MAC and level of correlation, a simple cantilevered plate FEA model was developed with a fixed BC. An equivalent FEA model was generated with a free-free BC to simulate test data. Because test typically cannot measure as many points, only the upper two-thirds of the model was used to simulate test data; this was mainly done because a laser holography system was being used to capture mode shapes and due to line of sight issues the entire blade could not be captured in the field of view of the optical system.

Now clearly comparing free-free modes to cantilever modes should result in an obvious conclusion that the modes are not very similar at all. However, the correlation showed something quite different. And this was intentionally done to highlight the fact that the MAC correlation might produce questionable results. Now the correlation results shown in Figure 8 shows that 20 of 24 modes correlated with a MAC of above 80% AND more than 10 modes correlated with MAC values above 90% with 7 modes above 95%. So using the MAC can provide questionable results when comparing modes from two very different configurations – one free-free and the other built in at one end.



#	FEA	Hz	EMA	Hz	Diff. (%)	MAC (%)
1	3	170.07	2	167.79	1.36	96.5
2	4	273.53	4	314.72	-13.09	95.6
3	5	309.77	3	279.16	10.96	90.1
4	6	481.30	6	490.25	-1.83	83.7
5	7	538.30	8	624.01	-13.74	85.2
6	8	583.25	7	612.27	-4.74	99.0
7	10	784.67	9	753.25	4.17	92.8
8	11	885.65	12	973.03	-8.98	91.4
9	12	944.46	11	944.86	-0.04	97.8
10	15	1358.6	15	1422.8	-4.51	86.3
11	16	1370.4	16	1490.5	-8.05	86.3
12	18	1557.5	19	1592.7	-2.21	71.0
13	19	1646.2	20	1675.9	-1.77	96.4
14	21	1698.6	21	1812.8	-6.30	84.4
15	22	1843.4	29	2552.5	-27.78	77.3
16	23	1868.8	22	1992.1	-6.19	89.5
17	24	1970.3	23	2038.1	-3.33	98.1
18	25	2065.1	24	2217.5	-6.87	83.7
19	26	2298.7	25	2307.8	-0.40	99.8
20	27	2370.3	26	2368.2	0.09	93.0
21	29	2496.8	30	2618.9	-4.66	77.2
22	30	2566.4	31	2711.6	-5.35	74.7
23	31	2627.9	32	2738.9	-4.05	84.4
24	32	2714.8	33	2795.5	-2.89	85.8

Fig. 8 Correlated Mode Pairs – Built-in Plate vs Free-Free Plate

Modal Assurance Criteria – Numbers can give a false impression (excerpt from Modal Space Articles)

But often times, people state that MAC values lower than 90% are acceptable with values as low as 80% being accepted; many state that real structures are much different than academic structures evaluated in an educational environment. Well, this researcher is very sensitive to the validity of this common statement heard mainly due to the fact that the mass matrix is not included in the evaluation.

Two examples are presented to show some MAC values that result from modes that are clearly not similar at all. The first case is to compare the rocking rigid body mode of a free free beam with the first cantilever mode of that beam. Figure 9 shows that there is about 60% similarity indicated by MAC.

These two vectors have nothing to do with each other but MAC shows 60% correlation. How could that possibly happen. Well looking at the lower right inset in Figure 9, notice that the values of the cantilever mode shape from the base to midspan of the beam are very small and their contribution in the MAC calculation are very small. And looking at the mode shape from the midspan to the tip of the beam, the values are much larger and with the naked eye you can see that the rocking rigid body mode looks quite similar. And that is what the MAC is indicating. But we know these two modes are not similar at all. Of course, there is no reason to compare these modes, but this was done to illustrate what a 60% MAC value indicates. So, the numbers can be very misleading.

Now let's proceed on with two cantilever beams – one with a uniform mass distribution and one with an additional lumped mass at the center of the beam (20% of the weight of beam). Now look at the MAC between Mode 1 and Mode 2 with the uniform mass distribution, the MAC will show little correlation as expected. But when the MAC is performed on the beam with the additional lumped mass, the MAC between Mode 1 and Mode 2 is almost 80%. *Now this is very disturbing!* But looking at the uniform beam modes in the upper right portion of Figure 2 and comparing them to the modes in the lower right portion in Figure 10, the same type of issue as discussed in the earlier case can be easily seen. The mode shape values close to the built in end have very little contribution to the MAC. And the values of the shape towards the end of the beam for Mode 1 and Mode 2 look very similar. And that is what MAC is indicating again. But these two modes are orthogonal to each other but the MAC completely breaks down here. However, the mass orthogonality clearly identifies the vector status because the proper mass distribution was included in the orthogonality check. The MAC has no way to account for the uneven mass distribution for this case.

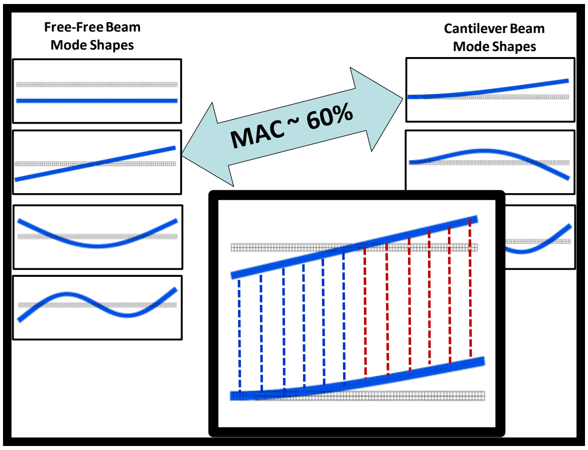


Fig. 9 Mode Shapes for Free-Free Beam (left) and Cantilever Beam (right)

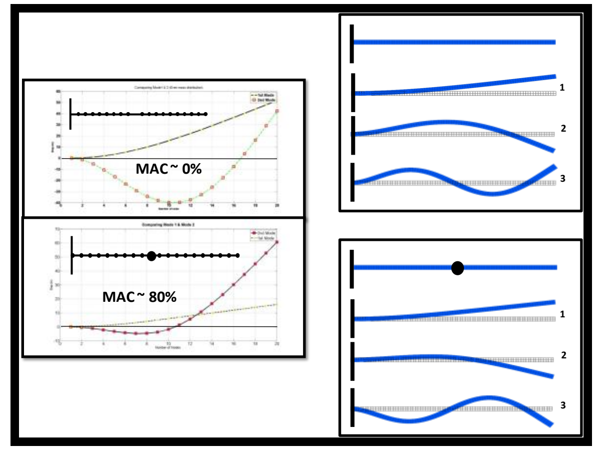


Fig. 10 Cantilever Beam Mode 1 and Mode 2 with Uniform Mass Distribution and Center Lumped Mass

Modal Assurance Criteria – When Geometry Doesn't Exactly Match

So back to the FAA turbine blade certification project. The earlier example showed a cantilever beam/plate structure compared to its free-free counterpart. Obviously, this was expected to show little correlation but the analysis showed something quite different. In this example [13], a simple cantilevered plate structure used for academic studies (related to fixture calibration work) is used to show correlation between an analytical model and a modal test data set. To illustrate the effects of geometry differences that were faced in regards to certifying 25 re-engineered blades, comparing the mode shapes amongst all the blades, the test model was dithered a small amount (one element width of the FEA) to illustrate how sensitive the MAC value can be. This is critically important when identifying node point pairs as part of the correlation process. Small geometry differences can affect the node correlation process when each of the test turbine blades are NOT the exact same dimensions. From an earlier study over 10 years ago, this simple graphic illustrated the problem faced when the geometry changes from blade to blade as shown in Figure 11. There is concern on exactly how to identify the node pair points for the correlation process. Four different cases were shown that, in general, had the same trends but the values for the correlated modes showed differences.

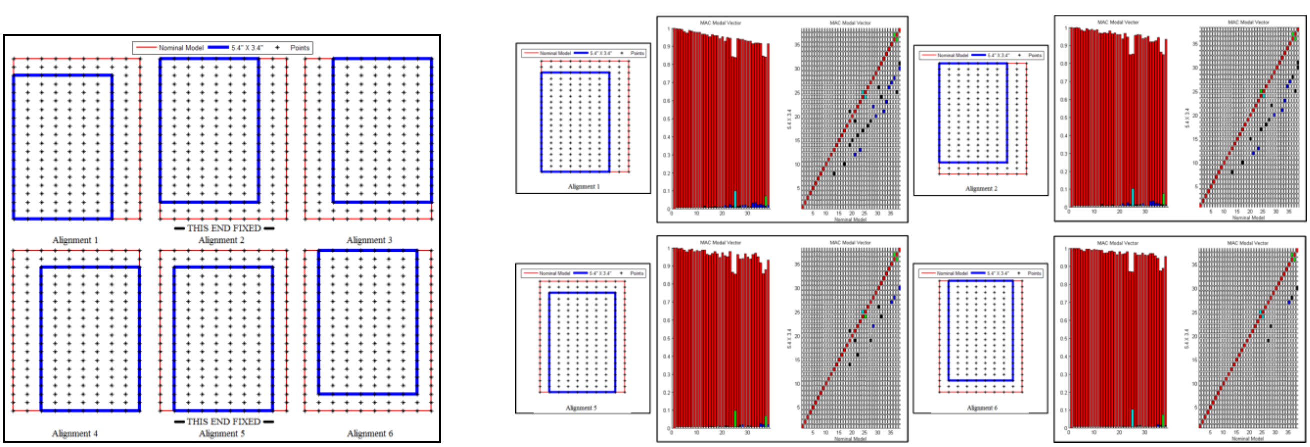
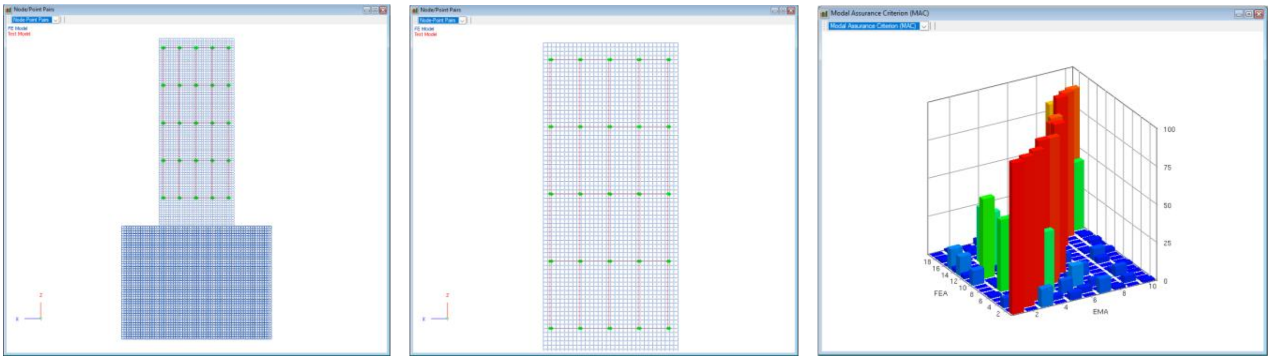


Fig. 11 Comparison of MAC Degradation Resulting from Node Pair Adjustments

In order to put numbers to the study a simple cantilevered plate, bolted to a larger more massive block, was used to simulate a cantilever boundary condition (which was similar to the manner in which the turbine blades are fixed and tested). The model was generated and an experimental modal test was performed. An initial correlation was performed with what seemed to be the proper node pair correlation based on the test performed. But a second correlation was performed where the test model was shifted one element up in the vertical direction. Node point pairs were identified and correlation was performed. The results are shown in Figure 12.



Orig Mode Shape Pairs

#	FEA	Hz	EMA	Hz	Diff.	MAC
1	1	364.65	1	370.75	-1.65	99.8
2	2	1586.43	2	1587.06	-0.04	98.9
3	3	1963.11	3	1966.74	-0.18	98.6
4	5	4994.55	4	4998.96	-0.09	98.8
5	6	5347.91	5	5351.97	-0.08	95.9
6	8	8714.53	6	9147.56	-4.73	98.5
7	12	10202.15	7	10322.81	-1.17	91.8
8	14	12668.24	8	13151.85	-3.68	98.5
9	16	14138.67	9	14274.81	-0.95	95
10	18	15048.8	10	15504.16	-2.94	90.7

Mode Shape Pairs - Element Shift

#	FEA	Hz	EMA	Hz	Diff.	MAC
1	1	364.65	1	370.75	-1.65	99.7
2	2	1586.43	2	1587.06	-0.04	98.9
3	3	1963.11	3	1966.74	-0.18	99.8
4	5	4994.55	4	4998.96	-0.09	99.5
5	6	5347.91	5	5351.97	-0.08	99.7
6	9	9103.49	6	9147.56	-0.48	99.5
7	12	10202.15	7	10322.81	-1.17	98.9
8	14	12668.24	8	13151.85	-3.68	98.8
9	16	14138.67	9	14274.81	-0.95	98.7
10	18	15048.8	10	15504.16	-2.94	92.7

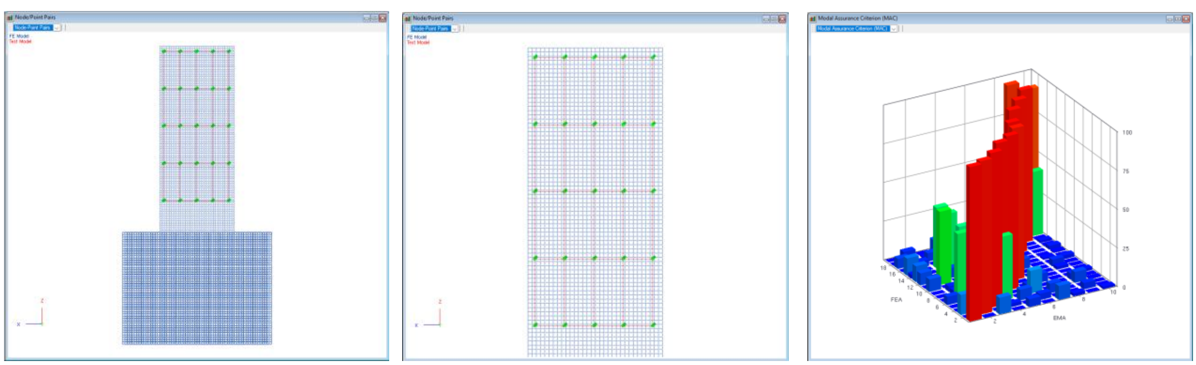


Fig. 12 Effect on MAC Values due to Shifting of Node Pairs By One Finite Element

So here out of the 10 correlated mode shape pairs, for 6 mode shape pairs, the MAC value changed by less than 2%. But 3 mode shape pairs changed by 2% to 4%. But most importantly 1 mode shape pair changed by over 7%. So, this seriously puts in question how good is a MAC threshold of 90% as a criteria for acceptance. Now this example simply illustrated with just a shifting of the model and test data by one element width in a very controlled environment how dramatic the results can be. But now consider the problem when many prototype re-manufactured blades are compared to each other. The manufacturing tolerances alone bring into view the fact that the node point paring can be arbitrarily "adjusted" to provide better correlation – or if care is not taken, blades that have so called good correlation may be questioned.

The MAC is a tool that is used often but these examples show just how much care and caution needs to be given in order to obtain useful results. For this blade correlation work, a different approach is needed which is not as sensitive to these geometric differences that can strongly affect the MAC results. Work on polynomial based correlation techniques provide an alternative for this type of correlation. Work done in this area has shown significant improvement in the correlation determination but "heritage" hinders the acceptance of newer approaches and concepts.

Orthogonality – Pseudo Orthogonality

Now orthogonality checks are a relatively simple calculation. However, the mismatch between the number of finite element DOF and number of measurement points is where the difficulty arises. There are essentially three options possible:

1. Reduce the finite element mass matrix down to the set of test DOF
2. Expand the mode shapes to the finite element set of DOF
3. A combination of reduction and expansion to a common set of DOF

A simple portal frame structure is used to present some test cases [14–16]. The model is shown in Figure 13 with some simulated measured data used for the evaluations presented here; one model uses a lumped mass formulation whereas the second model uses a consistent mass matrix. While the frequencies and mode shapes are very similar, this simple difference helps to see some very important aspects of correlation. Further discussion can be seen in the original reference papers covering these examples.

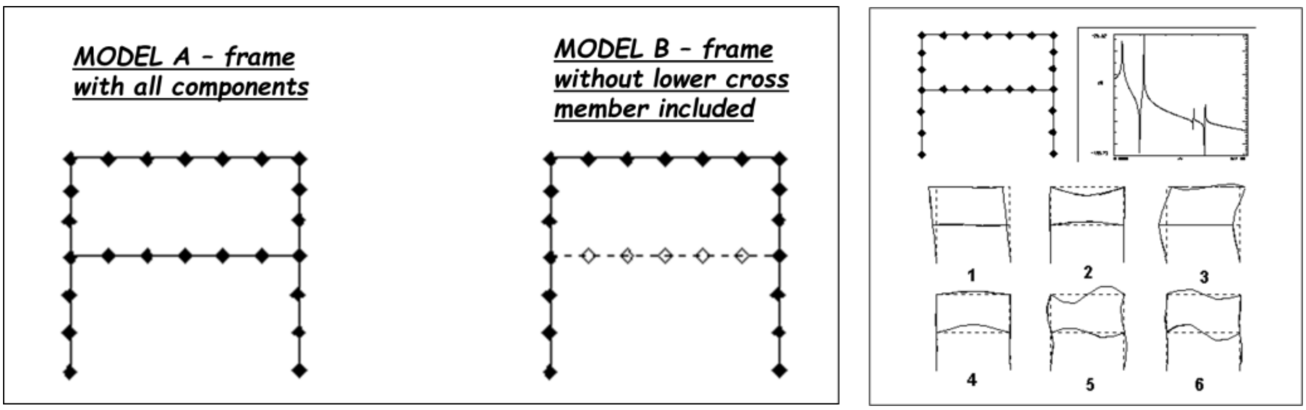
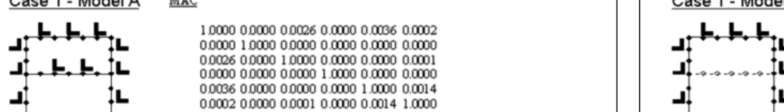


Fig. 13 Portal Frame with Measured FRFs

In the first case, all DOF available from test are used to correlate to the finite element model. The results are shown in Figure 14 on the left and without the interior cross piece on the right. Recall that these two models are essentially the same and only have differences due to the formulation of the mass matrix. The analytical vectors used here have purity in that there is no experimental variations seen.



Case 1 - Model A

MAC

1.0000	0.0000	0.0026	0.0000	0.0036	0.0002
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0026	0.0000	1.0000	0.0000	0.0000	0.0001
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0036	0.0000	0.0000	0.0000	1.0000	0.0014
0.0002	0.0000	0.0001	0.0000	0.0014	1.0000

GUYAN

0.9998	0.0000	0.0006	0.0000	0.0030	0.0095
0.0000	0.9894	0.0000	0.0001	0.0000	0.0000
0.0006	0.0000	0.9965	0.0000	0.0100	0.0107
0.0000	0.0001	0.0000	0.9850	0.0000	0.0000
0.0029	0.0000	0.0100	0.0000	0.8594	0.0136
0.0096	0.0000	0.0107	0.0000	0.0135	0.8513

SEREP

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0001	0.0000	0.0000	0.0000	1.0006	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0009

IRS

1.0000	0.0000	0.0000	0.0000	0.0002	0.0006
0.0000	0.9999	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0007	0.0008
0.0000	0.0000	0.0000	0.9999	0.0000	0.0000
0.0001	0.0000	0.0007	0.0000	0.9906	0.0015
0.0007	0.0000	0.0008	0.0000	0.0014	0.9896

HYBRID

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0001	0.0000	0.0000	0.0000	1.0006	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	1.0009

Analytical vs. Simulated Experimental Vectors
Full Frame Structure with Cross Member
(Every Other Point x and y)
(Modes 1 2 3 4 5 6)
MODEL A - Case 1

Table 1-a

Case 1 - Model B

MAC

1.0000	0.0000	0.0774	0.0000	0.0072	0.0234
0.0000	1.0000	0.0000	0.9309	0.0000	0.0000
0.0774	0.0000	1.0000	0.0000	0.0138	0.0050
0.0000	0.9309	0.0000	1.0000	0.0000	0.0000
0.0072	0.0000	0.0138	0.0000	1.0000	0.6381
0.0235	0.0000	0.0050	0.0000	0.6381	1.0000

GUYAN

0.9997	0.0000	0.0012	0.0000
0.0000	0.8659	0.0000	0.3342
0.0012	0.0000	0.9941	0.0000
0.0000	0.3342	0.0000	0.1498

SEREP

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0001	0.0000	0.0000	0.0000	1.0006	0.0000
0.0000	0.0000	0.0000	0.0000	0.0001	1.0013

IRS

1.0000	0.0000	0.0000	0.0000
0.0000	0.9674	0.0000	0.1164
0.0000	0.0000	1.0000	0.0000
0.0000	0.1164	0.0000	0.6357

HYBRID

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0001	0.0000	0.0000	0.0000	1.0006	0.0000
0.0000	0.0000	0.0000	0.0000	0.0001	1.0013

Analytical vs. Simulated Experimental Vectors
Frame Structure without Cross Member
(Every Other Point x and y)
(Modes 1 2 3 4 5 6)
MODEL B - Case 1

Table 1-b

Fig. 14 Portal Frame with Two Orthogonality Checks Using Simulated Experimental Data

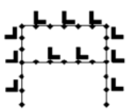
Several important items need to be noted here regarding the MAC and orthogonality checks shown [14–16].

- MAC provides useful results providing that sufficient dofs are used to spatially identify the modal vectors. Comparing the left plot the MAC shows very good correlation for the main modes of the system. However, on the right, the MAC shows significant correlation to the other modes. This is due to the fact that the cross piece, which has significant modal contribution, is not included in the measurement. The MAC is prone to this when the complete modal vector is not identified.

- As far as the different reduction techniques, there is a general improvement of the vector correlation identified using the IRS technique rather than Guyan. This is expected because the Guyan reduction is only based on the reduction of the stiffness matrix and the mass effects are not included. The IRS technique adjusts the reduced matrices to account for the embedded inertia related to the DOF that are not retained in the process.
- There is a general improvement of the vector correlation identified using the SEREP and Hybrid techniques rather than the IRS and Guyan techniques. There does not appear to be any benefit of the Hybrid technique rather than the SEREP technique; this discussion is beyond the scope of this paper (see references for additional info)
- One very important item to note is that the diagonal terms of this orthogonality check can actually be greater than 1.0. This is not true for an analytical mass orthogonality due to the inherent formulation. However, there is no guarantee as to how the experimental vectors are scaled; in addition, the analytical properties such as density and Young's modulus can introduce a scale that is clearly observed in this calculation. Some software "forces" the scale to be no greater than 1.0. This is not suggested because this can mask obvious material property differences that may exist in the model; rescaling the vectors tends to mask some of the critically important info.

In the second case, actual experimental data is used in Figure 15. Essentially the same remarks above are applicable here. The case above allows for very clear comments to be extracted from the case evaluated that is not affected by experimental variation that may occur in actual measurement scenarios. In this case experimental variations do exist but the same general trends are observed; but these are not a result of experimental variations and are due to expected differences.

Case 2 - Model A



MAC

0.9958	0.0000	0.0011	0.0000	0.0044	0.0056
0.0001	0.9698	0.0197	0.0008	0.0006	0.0005
0.0013	0.0000	0.9610	0.0000	0.0203	0.0067
0.0000	0.0027	0.0023	0.9951	0.0000	0.0000
0.0031	0.0000	0.0012	0.0000	0.9950	0.0007
0.0000	0.0000	0.0010	0.0000	0.0020	0.9954

GUYAN

1.1311	0.0001	0.0160	0.0000	0.0053	0.0667
0.0058	1.0099	0.1593	0.0191	0.0268	0.0276
0.0136	0.0003	1.1243	0.0001	0.1594	0.1054
0.0067	0.0529	0.0615	1.1305	0.0021	0.0026
0.0007	0.0057	0.0161	0.0013	0.9719	0.0049
0.0062	0.0000	0.0116	0.0044	0.0237	0.9783

SEREP

1.1305	0.0002	0.0192	0.0000	0.0098	0.0668
0.0058	1.0166	0.1574	0.0243	0.0298	0.0386
0.0226	0.0002	1.1193	0.0001	0.1666	0.1179
0.0069	0.0532	0.0603	1.1472	0.0021	0.0030
0.0008	0.0062	0.0323	0.0007	1.1308	0.0217
0.0185	0.0004	0.0249	0.0043	0.0068	1.1486

IRS

1.1284	0.0002	0.0142	0.0001	0.0123	0.0664
0.0007	1.0214	0.1631	0.0164	0.0277	0.0405
0.0184	0.0007	1.1264	0.0000	0.1859	0.1244
0.0038	0.0543	0.0659	1.1513	0.0007	0.0044
0.0018	0.0058	0.0255	0.0018	1.1405	0.0204
0.0112	0.0005	0.0217	0.0049	0.0223	1.0884

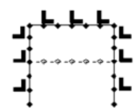
HYBRID

1.1307	0.0001	0.0177	0.0000	0.0116	0.0666
0.0043	1.0207	0.1584	0.0196	0.0310	0.0370
0.0162	0.0003	1.1244	0.0001	0.1759	0.1199
0.0064	0.0537	0.0616	1.1478	0.0019	0.0026
0.0032	0.0058	0.0283	0.0012	1.1331	0.0283
0.0177	0.0000	0.0243	0.0044	0.0089	1.1537

Analytical vs. Measured Experimental Vectors
Full Frame Structure with Cross Member
(Every Other Point x and y)
(Modes 1 2 3 4 5 6)
MODEL A - Case 2

Table 2-a

Case 2 - Model B



MAC

0.9983	0.0000	0.0706	0.0000	0.0144	0.0366
0.0015	0.9724	0.0158	0.9132	0.0003	0.0018
0.0703	0.0000	0.9408	0.0000	0.0096	0.0204
0.0016	0.8899	0.0170	0.9790	0.0002	0.0000
0.0051	0.0000	0.0178	0.0000	0.9947	0.6627
0.0110	0.0000	0.0116	0.0000	0.5899	0.9814

GUYAN

1.1313	0.0002	0.0096	0.0001
0.0016	0.8700	0.1667	0.3433
0.0126	0.0005	1.1226	0.0001
0.0027	0.3354	0.0677	0.1551

SEREP

1.1363	0.0001	0.0103	0.0011	0.0797	0.0554
0.0152	0.3765	0.1663	1.5569	0.1225	0.1250
0.0088	0.0093	1.1405	0.0225	0.4484	0.3826
0.0005	0.1663	0.0677	1.4266	0.0213	0.0431
0.0207	0.0731	0.0186	0.1661	1.0483	0.1780
0.0535	0.0425	0.0265	0.1104	0.2424	1.5750

IRS

1.1304	0.0008	0.0120	0.0021
0.0052	1.0401	0.1650	0.0036
0.0191	0.0058	1.1262	0.0114
0.0034	0.3555	0.0682	0.1425

HYBRID

1.1370	0.0003	0.0108	0.0011	0.0796	0.0547
0.0123	0.3810	0.1684	1.5575	0.1221	0.1254
0.0197	0.0094	1.1321	0.0225	0.4479	0.3764
0.0002	0.1676	0.0675	1.4287	0.0212	0.0432
0.0100	0.0729	0.0271	0.1667	1.0482	0.1858
0.0490	0.0436	0.0298	0.1119	0.2409	1.5780

Analytical vs. Measured Experimental Vectors
Frame Structure without Cross Member
(Every Other Point x and y)
(Modes 1 2 3 4 5 6)
MODEL B - Case 2

Table 2-b

Fig. 15 Portal Frame with Two Orthogonality Checks Using Actual Experimental Data

This particular paper [14–16] goes on to make several different evaluations in regards to the selection of DOF to include in the orthogonality checks and several items were noted (see additional content in the original papers).

- Selection of DOFs used for correlation studies be well thought out; the DOFs must be significant in all of the modal vectors
- This point is particularly important if the experimental modal vectors are used in an expansion process such as SEREP; inclusion of bad data points will have a direct effect on the accuracy of the expanded modal vector.
- Use of all measured dof may actually degrade the orthogonality check, resulting in poor vector correlation
- DOF which are points of high response for some modes may be poor for other modes, especially for structures with directional modes
- Using a carefully selected set of dof for each mode may result in a more accurate indication of correlation
- Due to the directional nature of the modes, confidence in the accuracy of ALL of the measured 'a' dof for ALL of the six modes is not high
- MAC is good correlation indicator for large shape components
- Expect the largest shape dofs are the most accurate
- Use of all dof for all modes for orthogonality checks in reduced space may not necessarily provide a good indication of vector correlation

- Difficulty in selecting an appropriate set of ‘a’ dof that are good for all modes evaluated
- Using a more select set of dofs may produce a better pseudo orthogonality check
- Correlation studies are most likely to produce the best results if the modes are expanded in sets using an optimum set of ‘a’ dofs for each set of modes

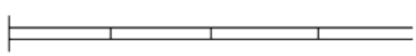
These results then pointed to the fact that a more elaborate assessment of DOF correlation needs to be performed. However, tools such as CoMAC do not include mass scaling. The CORTHOG was specifically developed to assist in a more robust evaluation of DOF contribution to the correlation process. A few examples in the next section highlight this.

DOF Correlation – CoMAC and CORTHOG

There are several misconceptions regarding the correlation of DOF for modal vectors. The next several examples [17–19] aim to clarify some of these.

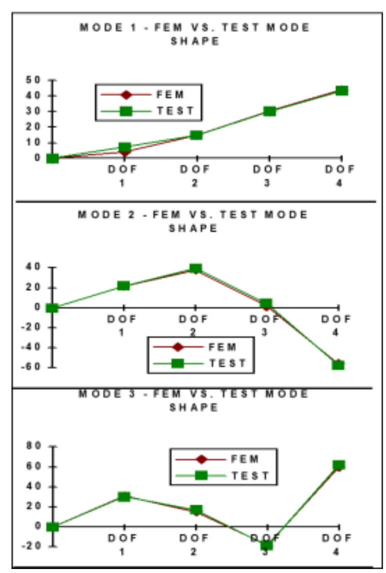
A simple 4 dof cantilever beam is used to quickly and easily gain an appreciation of contribution of DOF towards the Pseudo Orthogonality Check in Figure 16. The analytical vectors were perturbed with a slight amount of noise. The vectors themselves appear to look the same and the MAC values are almost identical. But the POC clearly shows one off-diagonal term (the 2-1 term with 0.056) that is very large. The CORTHOG reveals that DOF 3 has significant discrepancy. In fact, if the noise is removed from all DOF, except DOF 3, the off-diagonal term only changes from 0.056 to 0.054. This clearly shows how one DOF can have a dramatic effect on the off-diagonal terms. Note that this DOF is associated with the node of the second mode. This can be a more difficult DOF to measure and will not be identified with the MAC or CoMAC. Because the CORTHOG is mass weighted, the contribution of this very small mode shape value can be very important to the POC computed.

A second example involves a structure with very directional modes. The experimental modal test involved almost 100 measurement DOF (using 33 triaxial accelerometers. Looking at the mode shapes, there are clearly very directional modes



$$M = \begin{bmatrix} 0.28617 & & & \\ & 0.35664 & & \\ & & 0.7500 & \\ & & & 0.11082 \end{bmatrix} \mathrm{x}10^{-3}$$

$$K = \begin{bmatrix} 121 & -72 & 27 & -6 \\ -72 & 84 & -57 & 18 \\ 27 & -57 & 63 & 27 \\ -6 & 18 & 27 & 13 \end{bmatrix}$$



0.922	0.022	0.024	0.038
0.056	1.030	0.033	0.002
0.008	0.010	1.024	0.037
0.032	0.012	0.012	0.958

POC With Noise On All DOFS

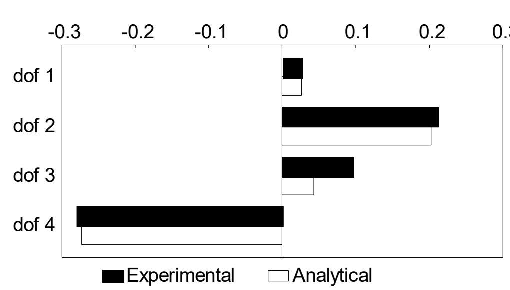
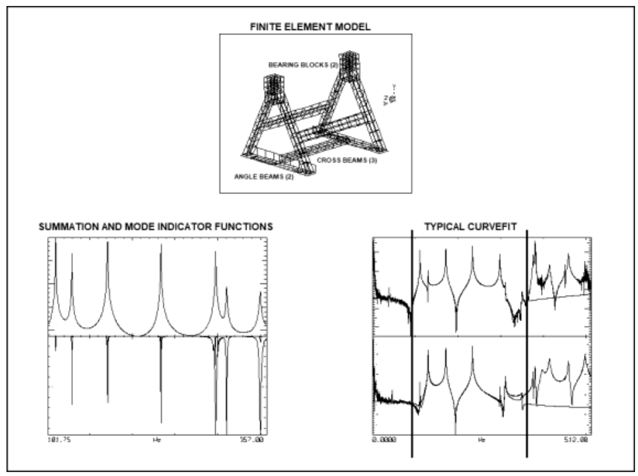


Fig. 16 Simple 4 DOF Cantilever Showing Mass Effect

related to the structure. Using triaxial accelerometers is common practice but obviously there are certain modes where one of the directions measured will not have significant response and will have small shape values more easily contaminated by noise. And each of the different directional modes will have different directions where this problem will occur. That is why an earlier the statement was made "due to the directional nature of the modes, confidence in the accuracy of ALL of the measured 'a' dof for ALL of the modes is not high". This is a common problem testing many engineering frame type structure – these directional modes will fall into this category all the time. The structure is shown in Figure 17 with sample FRFs and extracted mode shapes which are overlaid on the finite element model. Clearly the mode shapes appear to compare well.



MAC

0.85	0.00	0.06	0.00
0.00	0.94	0.00	0.00
0.00	0.00	0.87	0.00
0.00	0.00	0.00	0.94

Diagonal terms good with small off-diagonal terms

POC

1.138	0.048	0.009	0.026
0.008	1.169	0.007	0.004
0.054	0.025	1.257	0.042
0.205	0.045	0.039	1.169

Orthogonality diagonals greater than 1.0 with some off-diagonals values

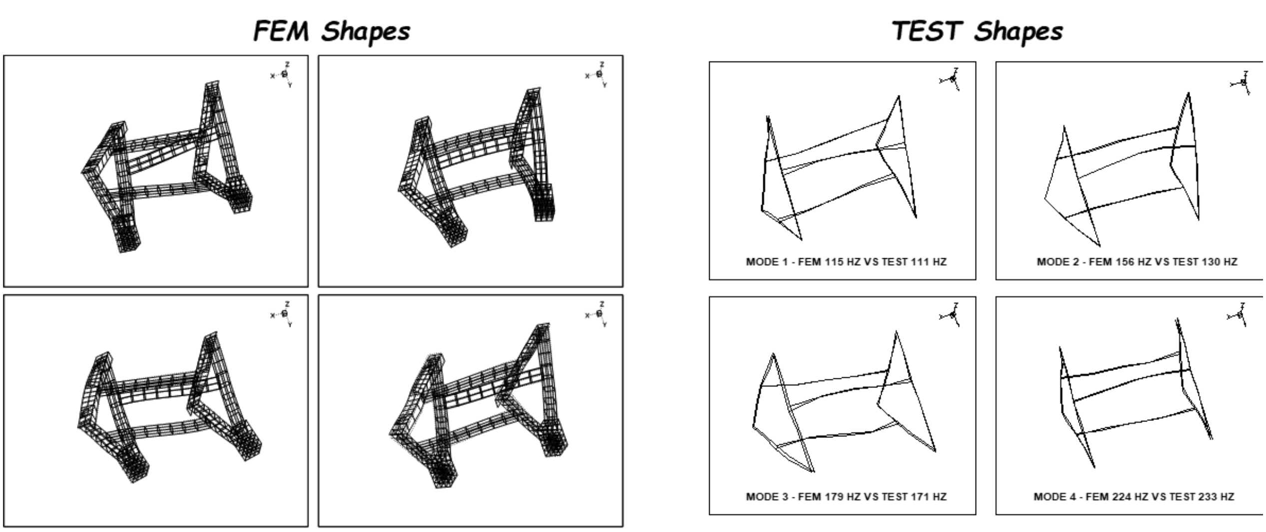


Fig. 17 GTE Frame with Directional Modes

Two modes are further studied using the CORTHOG. One is a comparison of two directional modes (4^{th} experimental mode with 1^{st} analytical mode). Here the off diagonal term showed a high value of 0.205. Certainly there are enough DOF to describe the mode shape. But the problem is that there are five locations on each leg that have essentially zero response but due to noise and mass scaling, their contribution to that off-diagonal term is significant. In fact, over 50% of the error in that off-diagonal term is related to these poorly measured DOF – that is less than 10% of the entire set of measurements which causes that poor off-diagonal term. And looking at the mode shapes, those particular DOF are expected to not be measured well at all. Including them in the POC calculation puts a heavy mark on that off-diagonal term. The CORTHOG clearly shows this as seen in Figure 18.

These results reveal that the DOF along both angle bracket legs have the most discrepancy and account for over 50% of the error in the total summation of absolute simple differences. These particular DOFs, which comprise over 50% of the total error, represent less than 10% of the total number of measured DOF. In reviewing mode 1 and mode 4, these two modes are somewhat directional in nature. Mode 1 is a twisting mode of the structure and mode 4 is a shearing mode. The main contribution to the off-diagonal term is due to these DOFs that are not measured well because of the directional nature of the modes. (The error is likely due to cross axis sensitivity of the transducers used.)

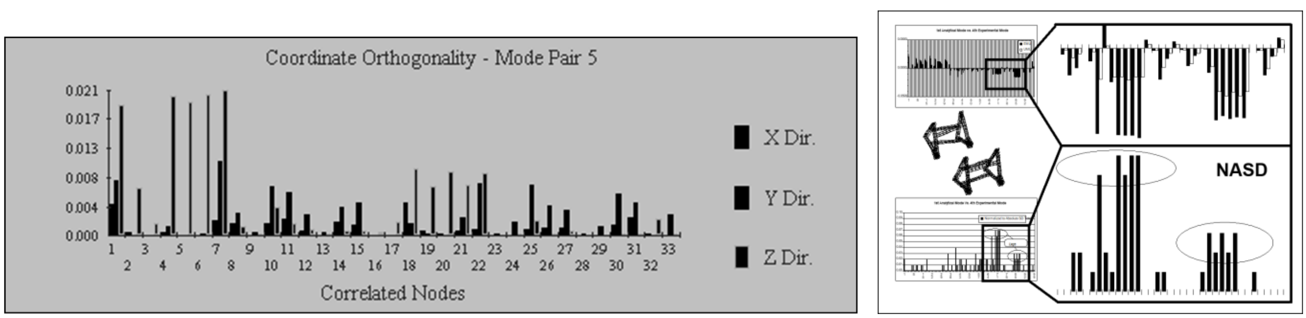


Fig. 18 GTE Frame with Directional Modes – CORTHOG Mode 1 to 4

The other comparison is in regards to the second bending mode of the system shown in Figure 19. The diagonal term shows difference. At first, consideration was given to the fact that the DOF geometric coordinates may not have been correct and the node correlation was to a different point in the model. But further investigation revealed that the mass in the pillar blocks were incorrectly input into the finite element model. The CORTHOG clearly showed this discrepancy; note that the CoMAC failed to identify this serious difference because mass scaling is not included in the CoMAC formulation.

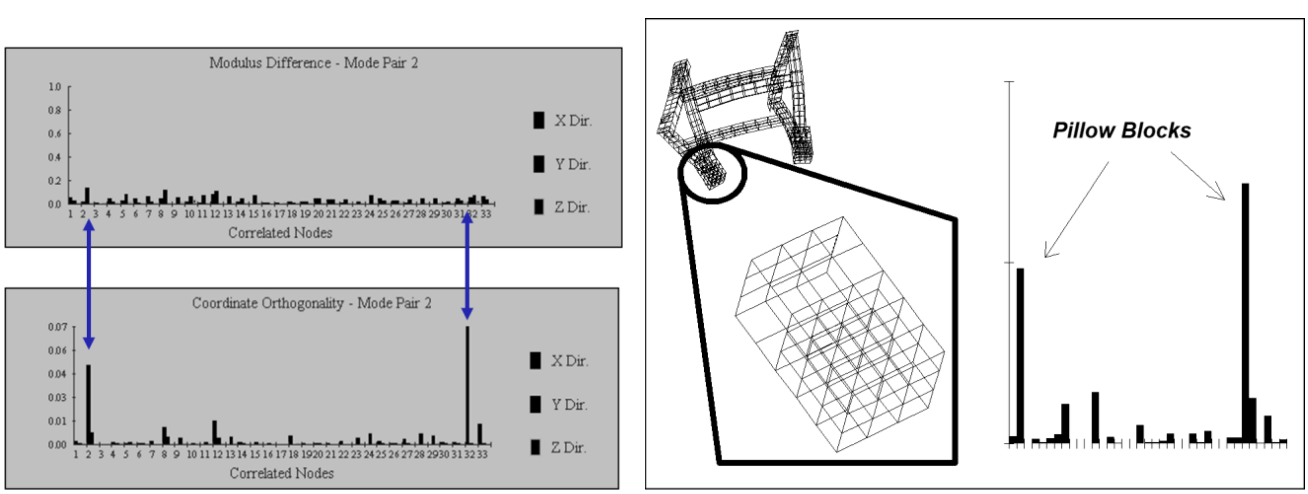


Fig. 19 GTE Frame with Directional Modes – CORTHOG Mode Pair 2

The CORTHOG plots clearly show the large discrepancy between the two degrees of freedom associated with the bearing blocks. This discrepancy may have occurred for several reasons such as the actual mass of the bearing blocks not being well represented, or the joint between the bearing block and the structure is not integral as modeled, or the geometric point for the analytical and experimental models are not truly at the same geometric point in space. While the difference between the mode shapes may not appear significantly different and cannot be identified using the CoMAC, ECoMAC or Modulus Difference, the CORTHOG shows that these two DOFs are significant contributors to the POC. This is due to the fact that these two DOFs have a significant amount of mass associated with them. While it is likely that the discrepancy is due to an analytical modeling error and not an experimental error, there is a definite indication that these two degrees of freedom are not correlated very well when compared to other degrees of freedom in the modal vector. Due to lack of mass scaling, both MAC and COMAC are unable to clearly identify this discrepancy.

Data Consistency Assessment Function (DCAF)

Tools are necessary to help interrogate the data collected. The measurement process is riddled with possible places where errors can crop into the measurement. Unfortunately, the measurement process is not perfect. So there needs to be a critical assessment of the experimental data acquired in order to assure its usefulness. Two specific tools used in the collection and reduction of data are discussed next but unfortunately they do not really interrogate the data in a significant meaningful fashion.

During the measurement process, the coherence is always evaluated. But that coherence may be deceptive because the coherence can be 1.0 and yet the measurement may be faulty. Issues such as phase distortion or phase lag between channels may cause a measurement to be faulty even though the coherence is 1.0; that is because all of the output was due to the measured input. A calibration error will never show up in the coherence function so the measurement may be wrong yet the coherence is 1.0. An accelerometer may be mounted in the wrong direction and the coherence could be 1.0 but the measurement is not associated with the proper point and dirrection. So each of these examples are a few of the common examples where the coherence may give a false sense of security as to the adequacy of the measurement.

Once data is collected and the data reduced with the modal parameter estimation tools, the next step is normally to synthesize the data from the model extracted and compared to the measured data. This provides feedback as to how well the data fits to the model. While this step is very important to the overall process, the problem is that each measurement is only compared individually; there is no association with all of the measurements included in the process. So while a measurement may have a very good synthesized comparison, there is no guarantee that the measurement is appropriate when compared to all the other measurement points describing the model. In a multi-reference test with many accelerometers, simple issues related to calibration, mounting direction, etc. fall into the same dilemma as the coherence.

Other tools are available to assist in further interrogation of the data such as Amplitude Weighted Assessment Function (AWAF) and Enhanced Amplitude Weighted Assessment Function (EAWAF) [20]. These tools help to more clearly show where errors may exist in the entire database describing the mode shapes for the structure. In a recent modal test on a fully dressed truck, there were two separate tests performed with different shaker configurations. Due to sensitivity of the data, a mock up structure [21] was developed to illustrate the situation at hand. One test using 4 vertical only shakers and the other test deployed skewed shakers which allowed for better excitation of all the modes of the structure. The EAWAF clearly showed where errors as part of the modal parameter estimation process were clearly evident. The modes extracted from the skewed shaker configuration were much better than the modes extracted with the vertical only shaker configuration. Two resulting EAWAF plots are shown to highlight the results obtained in Figure 20.

But one additional newer tool developed is the Data Consistency Assessment Function (DCAF) which interrogates all the modal data as a complete set to identify outliers in the data set. DCAF has been used on modal vectors, FRFs, operating data, optical data and general time data sets as well as strain data from optical systems. Basically, the DCAF systematically removes individual data points from a data set and uses synthesis to recreate the removed point which is then compared to the original data to see how well it compares. Obviously poor data points can be easily identified and bad data points can be removed from the data set. This is much different than other correlation tools which only assess individual measurements and never assess the relationship with the rest of the data set.

One recent application used an optical set of time response data which has thousands of data points. Subsets of data were evaluated and there were several "truth" measurement where the correlation was less than 90%. Using the DCAF, five points were clearly identified and removed from the data set thereby increasing the correlation significantly; additional passes on the data set removed additional outliers. Figure 21 shows the DCAF results from the first pass as an example.

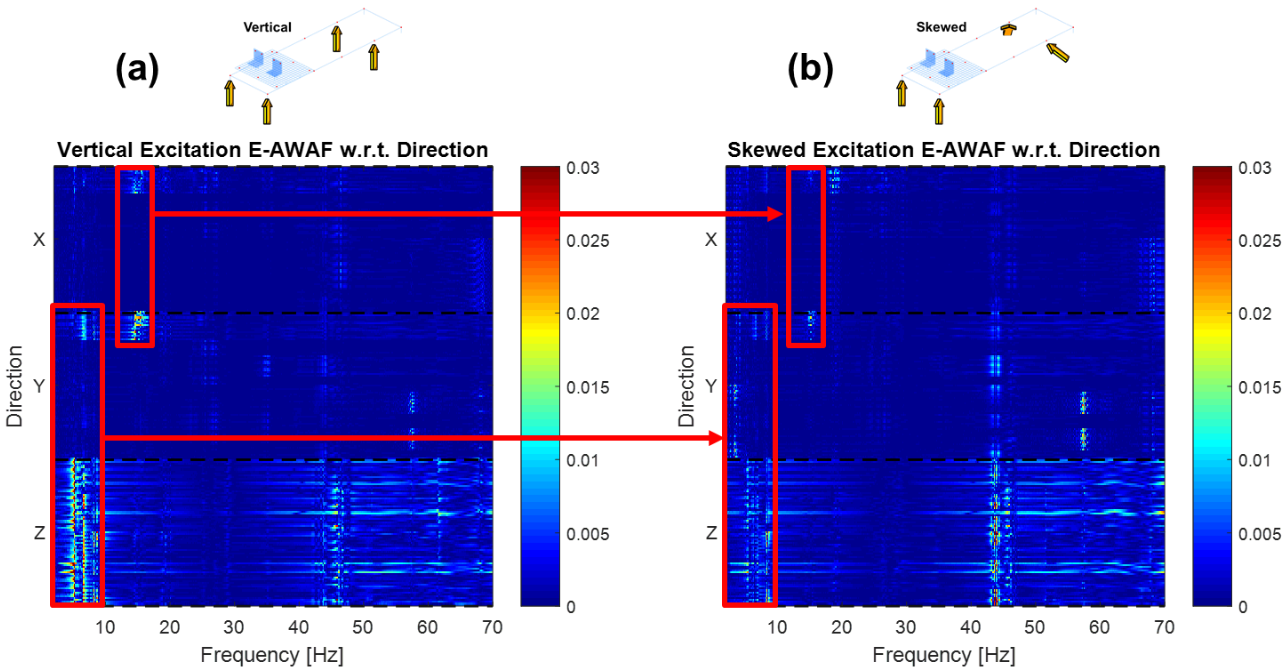


Fig. 20 Comparison of the E-AWAF computed from the (a) vertical configuration data and (b) skewed configuration data. Red boxes superimposed on the E-AWAF color maps to highlight areas of improvement using skewed excitation tests

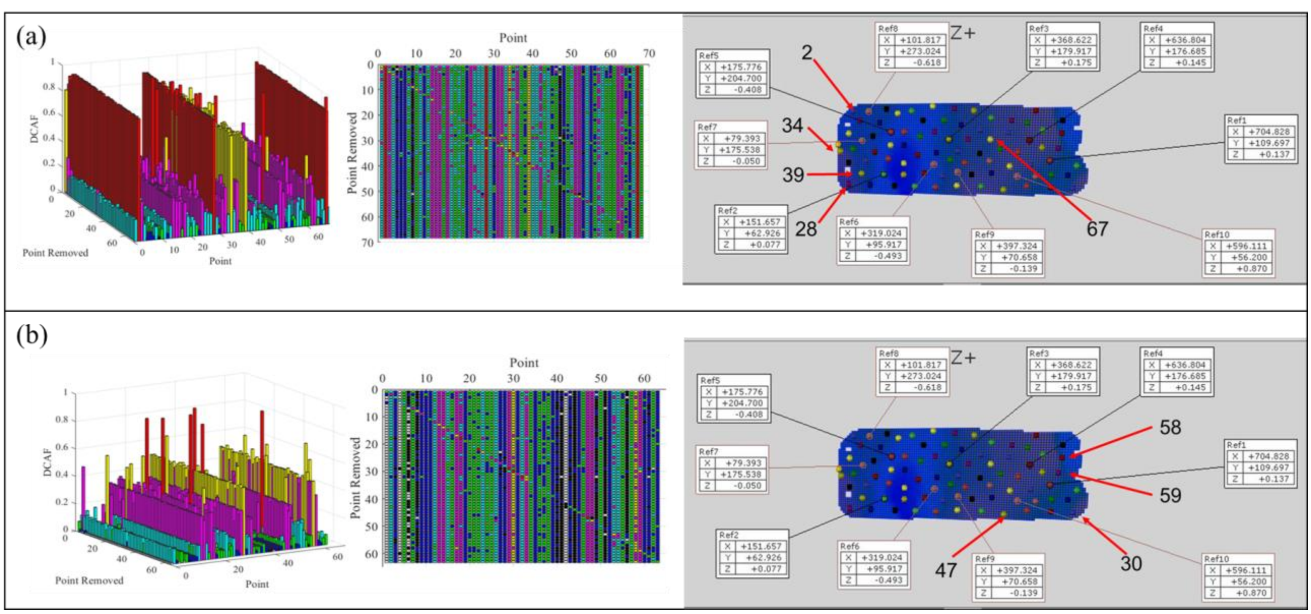


Fig. 21 (a) DCAF results for 68 active elements strain measurements with 5 inconsistent measurements identified (b) DCAF results for 63 active elements strain measurements with 4 inconsistent measurements identified

Observations and Misconceptions

So much has been presented in this paper to start to show some of the problems with the tools that are used for correlation. Using a simple number as a threshold may be necessary for contractual work but the actual correlation may be quite different. So the user must be very careful in the interpretation of the numbers. Suffice it to say that picking a threshold for accept or reject can be very difficult to say the least.

So this paper has presented several important concepts. But beyond that there are several areas where additional discussion is necessary to highlight additional problems extending from what has been presented

- Is all the measured data correct
- Test & Analytical Engineer awareness
- Complex modes – are they real? (no pun intended)

Is all the measured data correct?

Often times, data is collected with a contractual requirement or a mandate identified from above. Data, at times, may be collected without the real understanding of why the data is needed, how it is going to be used or what needs to be extracted from the study. When this is the case, the data may get acquired but the specific reason might cause the test engineer to acquire data in a different manner.

Most importantly, the consistency of the data to itself is rarely, if ever, checked. As mentioned in the paper, issues of coherence or synthesis of data as a validation tool is very important but this only checks the data with regards to itself. The coherence only shows that the particular measurement was collected in a way that the output is linearly related to the input signal. This just attests to the conditions of noise on the data or if a nonlinear measurement may be of concern. While this helps to validate the particular measurement, there is no consistency identified for that one data point in relation to all the other data points that are part of the entire data set. And while using synthesis following curvefitting is always a good step to take, this does not indicate if the data point is consistently related to all the other data points in the data set – this just validates this one individual point.

There are many issues related to measurements with a list too long to include here but a few glaring issues related to leakage, windows, time variance, DAQ dynamic effects, accelerometer cross axis sensitivity, inconsistent excitation (hammer hits for instance) are all very important issues that can have a significant effect on model correlation.

But at the end of the day, the analyst needs to use the data for the study necessary. Important details about the data may cause the test to be conducted differently or for the analysts to use the data in a more judicious manner.

And as stated in this paper before, "Are all the measured DOF for all modes measured with the same accuracy". Hopefully, from all the items discussed thus far, clearly the measured DOF are not measured with the same accuracy for all modes. Yet when an analyst uses the raw data (in its entirety) for a correlation study, this is absolutely assuming that all the measured data is equally accurate when in fact this is never the case. But almost all correlation software packages treat all the data in unison and the correlation tools are deployed with no understanding that some data points are very good and some data points are absolutely terrible.

Without a very judicious assessment of all the DOF for each modal vector, then all the data is considered equally valid. When correlation studies are performed in this manner then the results will be considered very questionable.

Test & Analytical Engineer awareness?

Often times, there is a disconnect between the test folks and the analytical folks. In most organizations they are in separate groups that may not even be in the same physical location. Both are very unaware of the problems faced by the other. Neither has done any detailed work in the other's area. There is often a complete lack of understanding of the problems faced by each other in the development of their model or test data. When this is the case, the tools are used in a very formal fashion but lacking the understanding of the limitations and without any respect for the detailed use of the tools. When this is the case, a correlation is preformed with numbers generated but without any real understanding of what the results mean.

Often times, the analytical engineer is passed data from the test engineer and then blindly starts to use that data without any real assessment of the validity of the data. On a recent project over a decade ago, test data was provided with an analytical model with the specific intent to develop a highly reduced order model to be updated with the measured experimental data and to predict accurate response with that measured data. The assumption was that the models and test data base were suitable to undertake the project. After many hours spent with various models, an acceptable model could not be developed. After extensive scrutiny of the test data, the measured data showed significant inconsistencies with itself. Of course, this was the root cause of all the difficulty with the project. Once the data was properly adjusted to maintain consistency, the analysis went forward without a hitch.

There was another case at the turn of the century when some test analysis correlation work was done on some helicopter missile firing systems. A consulting company was hired and spent several weeks evaluating the correlation between several sets of data and models developed. The level of correlation was poor and there was no insight as to what might be viable ways to achieve better correlation. All the typical software correlation tools were deployed but the software is just a tool and the insight must come from the engineer deploying the tools. A few months later different studies were performed but with a more hands-on approach to understanding the models developed and the test data acquired. Some very basic fundamental steps were taken to achieve far superior results within several days. In the earlier study, data was passed from test engineer to analyst but the data was never assessed before using it with the model. Correlation proved to be a difficult process until some of the basic aspects of the underlying test data were clearly understood and incorporated into the analysis.

The most critical item here is that the test and analytical engineers need to work in unison and very clearly identify all the particulars that can have a very distinct effect on the results of the correlation process. While this is obvious, at times there is a slight disconnect between the parties involved which can be detrimental to the project results.

Complex Modes – are they real?

Now in all the material presented here, real normal modes were assumed. This was necessary so that simple examples could be used to portray some of the difficulties with the tools. Complex modes just make all the numbers more difficult and more complicated but the same exact issues exist with complex modes.

But on the point of complex modes, there are several issues that must be addressed. First and foremost, are the measured modes really complex? Or are there other issues that may lead the engineer to believe the modes are complex, when in fact they may not be complex. Unfortunately, just because the software shows a complex mode does not really confirm the modes are complex – the software just indicates that with the measurements provided, the indication is that there is some complexity to modes based on the measured FRFs used to extract data.

In one recent large modal test on a fully dressed truck structure, complex modes were revealed. And with such a large structure, just about everyone was convinced that complex modes would exist. But that was mainly because everyone convinced themselves that complex modes would exist – but with no strong evidence that complex modes really did exist. The original test utilized a four vertical shaker configuration but the overall measurements were not of spectacular quality with MCOH that was not as good as needed. But everyone was strongly stating that complex modes for this type of structure were clearly expected. After much debate and much discussion, the shakers were arranged in a skewed orientation in the hopes to better excite all the modes of the structure. Overall, the FRFs were greatly improved and the MCOH was also greatly improved. Upon extracting modal parameters, the majority of modes that were classified as complex became far less complex and started to look more like real normal modes. So the way in which the structure is excited may have a direct effect on the measurements and ultimately the modes that are extracted.

This author has seen many, many, many instances where people insisted that complex modes existed. But with further scrutiny of the measurements, measurement technique, instrumentation used and techniques deployed, many of these so called "complex modes" are really just an artifact of the lack of consistent and proper measurement techniques. In the majority of the situation observed, there has almost always been some detail of measurement acquisition, manner of collection of data, and numerous other causes that have manifested themselves as complex behavior that is not related to the structure but more related to the lack of pristine measurements acquired.

Now certainly complex modes can exist from a mathematical standpoint and allowing for complex modes in the development of a modal model is always good practice. But before those so called "complex modes" are used in any further analysis (such as correlation) the experimental data needs to be scrutinized.

Attached here is a short list of some of the common issues observed (in close to half a century of modal testing) that have a strong influence on the extraction of "so-called" complex modes

- Leakage associated with measurement acquisition
- Inappropriate use of windows to minimize leakage
- Time invariance has been seen especially when tests are conducted over several days
- Suspension system (air bags) with bleeding air system
- Bungie cords stretching over time with an inconsistent stiffness
- Cords and cables not secured and allowed to freely vibrate
- Using different accelerometers where the phase roll-off varies from accel to accel
- Channel drift and phase distortion
- Accelerometer cross axis sensitivity
- Inconsistent hammer hits in impact testing

These are just a list of some of the more common problems that come to mind.

Conclusions

This paper attempts to address the use of correlation tools for model validation. The differences between vector correlation tools and DOF correlation tools was presented. The difference between mass scaled and non-mass scaled tools was also presented to illustrate the importance of mass scaling.

Many times there is a lack of understanding of what the tools can and cannot provide. Some of this stems from "old wives' tales" that do not have any credibility but have been blindly accepted for decades as truth.

A series of examples were presented to highlight some of the common misconceptions related to the tools.

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