Chapter 4
Anti-Resonance-Based Surrogate Modelling for Stochastic FRF Prediction of Finite Element Models

Enora Denimal Goy

Abstract Quantifying uncertainties in dynamic systems is crucial to ensure the safety and integrity of structures. When addressing real-world problems, numerous uncertain parameters exist, and the computational cost of determining the stochastic response increases significantly, potentially becoming unaffordable. This is particularly true when predicting the frequency response function (FRF). Surrogate models are a well-known strategy for reducing this numerical cost. However, direct application to the frequency response function results in convergence issues at the resonance peaks. This study proposes an advanced numerical framework for constructing of the surrogate models of FRF based on anti-resonance properties. Building on recent preliminary results obtained on a 2 dof system, this work presents an extension for finite element models, where the antiresonances may appear and disappear. The proposed approach is applied to a finite element model of a cantilever beam. It is demonstrated that the convergence issues at FRF peaks are eliminated.

Keywords Frequency response function · Uncertainty propagation · Antiresonance· Gaussian process · Finite element model

Introduction

The presence of uncertainties highly impacts dynamic response of mechanical structures, so they must be considered in the design stage. However, the computation of the stochastic response can quickly become expensive when the number of uncertain parameters increases or when one wants to predict the the structure full frequency response function (FRF). In this context, surrogate models are a powerful tool as they construct a mathematical approximation of the response, which can then be evaluated at almost no numerical cost. For FRF prediction, direct approaches based on constructing one surrogate for the full FRF directly proved to experience convergence issues at the resonance peaks [1]. In a recent work, a strategy where the FRF is cut in intervals based on numerical properties to construct one surrogate per interval [2]. However, those approaches fail to deal with complex mechanical structures with more complex dynamical behaviour. In [3], preliminary results prove that exploiting the resonance and antiresonance properties of the mechanical structures to cut the FRF in different intervals to construct surrogate models on each interval was a promising strategy. The results were limited to discrete systems, where the existence of antiresonance is always ensured and doesn't depend on coordinates. The current work proposes to extend this work to Finite Element Models (FEM) where antiresonance existence depends on the model parameters, but also on the location on the structure. The work proposes to answer this problematic with a two-steps surrogate modelling strategy. In a first time, based on classification surrogate modelling, the existence of the antiresonance is assessed. It makes possible to cut the FRF in different clusters. In a second time, a regressive surrogate model is constructed on each cluster to predict the FRF amplitude. The method is illustrated on the 3D FEM of a beam and gives promising results for more complex test cases.

Enora Denimal Goy
Inria, CMAP, Ecole Polytechnique, Palaiseau, France
e-mail: enora.denimal-goy@inria.fr

Kristin B. Zimmerman, PhD (eds.), *Model Validation and Uncertainty Quantification, 2026, Vol. 3*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/97887-438-1442-9_4

Proposed methodology

General objective

Let's consider a linear dynamical system described by the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices. $\mathbf{X}$ is the displacement vector, the dot denotes the derivative over time and $\mathbf{F}$ the excitation vector of the form $\mathbf{F}_e \cos \omega t$. The damping is supposed to be low. For such system, it is classical to study the Frequency Response Function (FRF), i.e. the response of the system when ω varies on an interval for each location, i.e. each dof. This work proposes a new and efficient strategy to predict the amplitude response at each dof, for any excitation frequency ω and when the system is uncertain (or depends on a set of parameters). The general workflow is explained first, and the different steps are then detailed.

General workflow

In the following, the uncertain parameters are denoted ξ and x denotes the coordinates of the point at which the FRF is computed. $\omega_R^{(i)}$ and $\omega_{AR}^{(j)}$ denote the i-th and j-th resonance and antiresonance, respectively. In the following, the surrogate of a quantity of interest z is referred with a: $\hat{z}$.

- Step 1: the surrogate models of the resonances $\hat{\omega}_R^{(i)}(\xi)$ are constructed using a GPR and a training set $\left\{ \left(\xi_k, \omega_R^{(i,k)} \right)_{k \in [1,N]} \right\}$.
- Step 2: the surrogate models of the antiresonances $\omega_{AR}^{(j)}(x, \xi)$ are constructed using a GPC (to predict the existence) and a GPR (to predict the pulsation frequency), using a training set $\left\{ \left((x_k, \xi_k), \omega_{AR}^{(i,k)} \right)_{k \in [1,N]} \right\}$.
- Step 3: a training set of the FRF is generated. It is composed of N_{FRF} input points $((\omega_k, x_k, \xi_k))_{k \in [1, N_{FRF}]}$ and their evaluation $\mathbf{y}_k$.
- Step 4: for each of this point, the resonance and antiresonance frequencies at (x_k, ξ_k) are computed. Comparing ω_k to those frequencies allow to attribute a cluster to the point.
- Step 5: for each cluster, a surrogate model of the FRF is constructed.

Antiresonance and resonance frequencies computation

The antiresonance at dof i can be obtained by resolving a modified eigenvalue problem where the i-th column of the sitffness matrix is replaced by the force vector, and where the i-th column of the mass matrix is replaced by a vector of $\mathbf{0}$ [4]. This strategy is employed in Step 2 to generate the training set of the antiresonance surrogates. For the resonance, a classical eigenvalue analysis is performed to obtain the natural frequencies of the system.

Gaussian process regression and classification

GPs are used for the creation of the different surrogate models. For GPR, the data are assumed to be generated by a process $Y_i = f(X_i) + \epsilon_i$ with $i \sim \mathcal{N}(0, \sigma^2)$. From a training set of N points (X_i, Y_i), a GP is constructed with the covariance function κ (which depends on hyperparameters θ) and the mean function m. It defines the matrix $K_{ij} = \kappa(\theta; X_i, X_j)$ and the vector $\mu = (f(X_1), \ldots, f(X_N))$. The hyperparameters θ are optimised to minimise the likelihood. Then, the prediction at a new points X_* is $\mu_* = K_*^T K^{-1} f$ with K_* the covariance matrix between X_* and the training set.

GPC are used for binary classification, to predict the existence or not of the different antiresonances. It is constructed with a training set of inputs $\mathbf{Z} = [\mathbf{z_1}, \ldots, \mathbf{z_N}]$ and outputs $\mathbf{y} = [\mathbf{y_1}, \ldots, \mathbf{y_N}]$ with $y_k \in \{0, 1\}$, which defines the class of membership of $\mathbf{z_k}$. The prediction of the membership of a new point is done by considering a latent function f approximated by a GP and mapped on the unitary interval with a sigmoid function.

The GPs are constructed with the GPflow python package [5].

Application on beam Finite Element Model

Model presentation

The model studied is a circular beam of radius R and length L simply supported at each end. It is shown in Figure 1. The Young modulus is 2e1 Pa, the shear modulus is 7.1e10 Pa, the density is 7800 kg/m^3, the Poisson ratio is 0.3, the radius of the cross section is 0.05 m, the length is 1 m. A Rayleigh damping is used with parameters $\alpha = 0.66$ and β =1.2e-6. The beam is discretised in 30 finite elements with four dof at each node. The excitation is located at dof 5 and is applied in both vertical and horizontal directions. The excitation amplitude is $1/\sqrt{2}$, and the frequency range of interest is $[100, 2500]$ rad/s. In the following, the Young modulus is considered as uncertain and can vary between $[1.75, 2.25\text{e}11$ Pa.

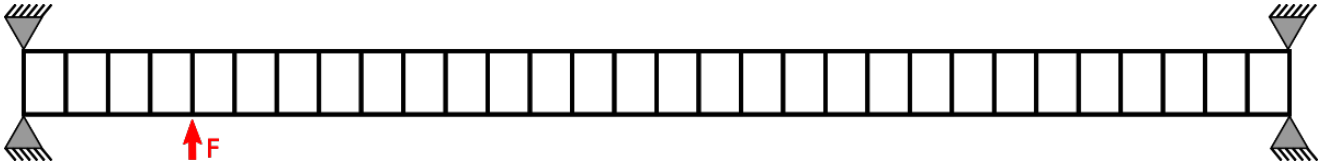
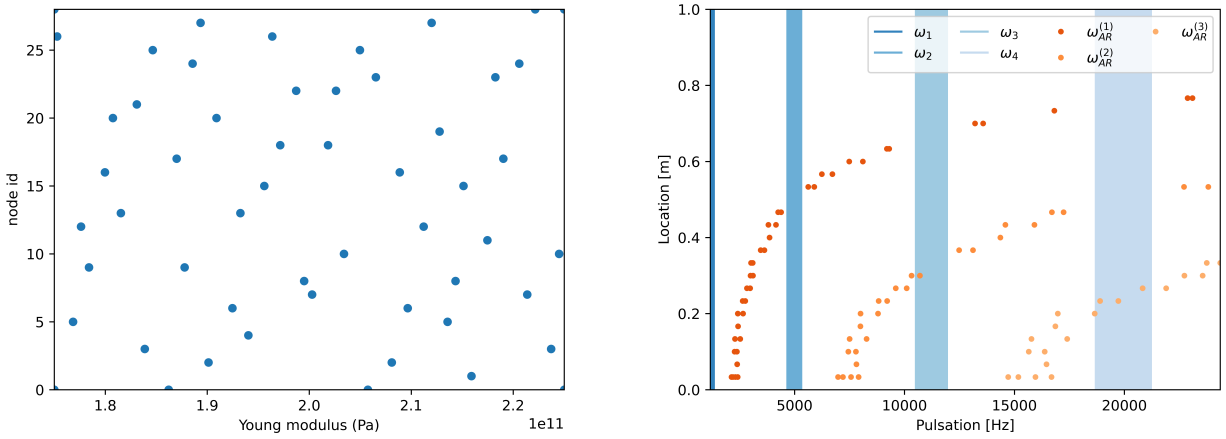


Fig. 1 Model under study

Results

In a first step, the training set for the resonance and antiresonance frequencies training are constructed. As an illustration, an input set of size 50 is shown in Figure 2(a). It is generated with a Halton sequence. For each of these points, the resonance and antiresonance frequencies are computed. They are displayedin Figure 2(b). As expected, the antiresonance frequencies depend on the location, and so are not always defined (e.g. near $L = 1$, there is no antiresonance).



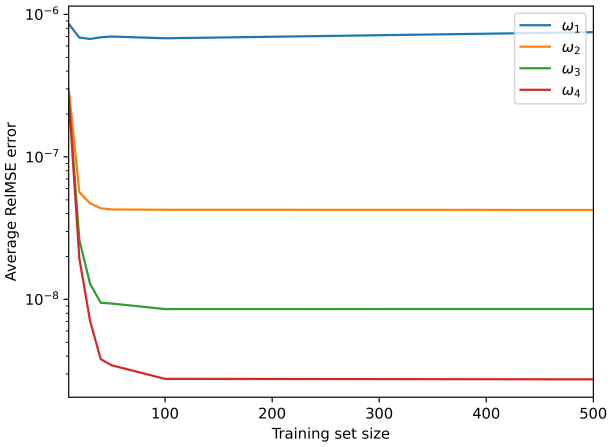
(a) Thrust dimensionless coefficient variation in inertial range

(b) Thrust dimensionless coefficient variation in stiffness range

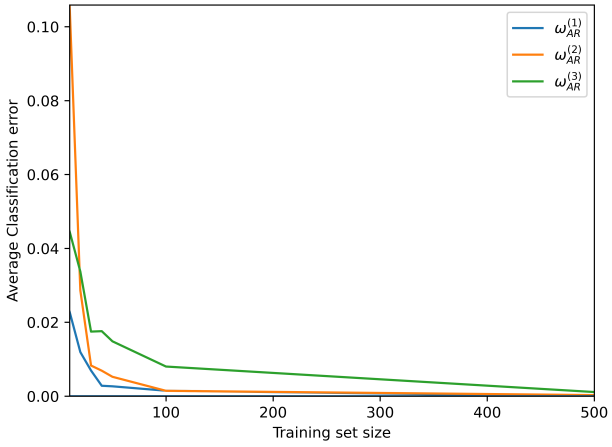
Fig. 2 Example of training set: input set of 50 points (left) and corresponding resonance and antiresonance frequencies (right)

GPR are constructed for each resonance frequencies, and for different size of training set. The latter can vary from 10 to 500 points, and for each size, 20 sets are generated. Each time, the relMSE error is computed using a validation set of 10000 points, and averaged over the 20 sets. The evolution of this error is given in Figure 3(a). One can see that the errors are always very low and that the convergence is quickly reached.

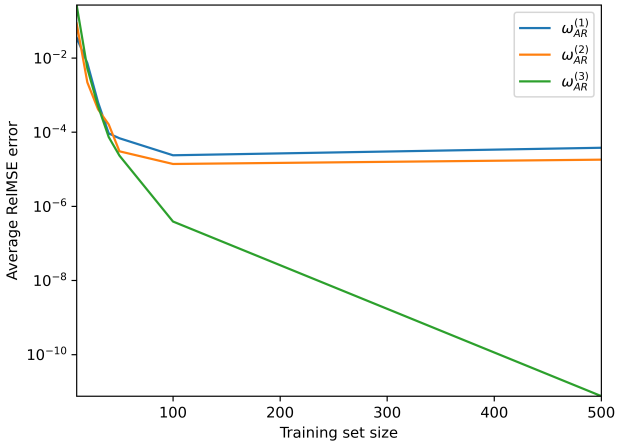
For the antiresonance frequencies, a first GPC is constructed to predict the existence or not the corresponding antiresonance. The error is computed as the average number of points misclassified on a validation set of 10000 points. Similarly to the resonance, training set size varies from 10 to 500 points, and 20 sets of each size are considered. The average error on



(a) Average error on resonance frequency prediction



(b) Average classification error on antiresonances

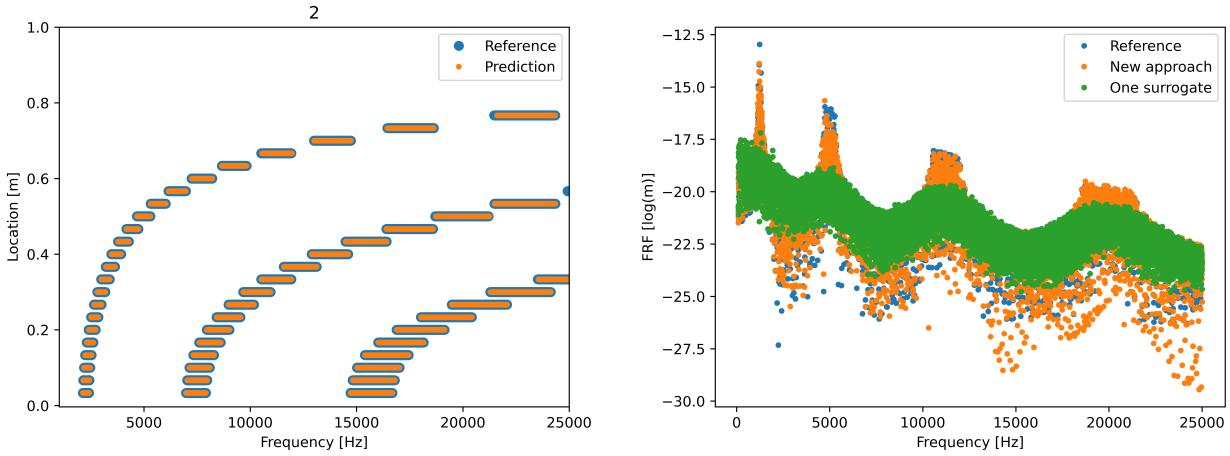


(c) Average regression error on antiresonances

Fig. 3 Evolution of the regression error on the resonance frequencies (a) and of the classification (b) and regression (c) error on the antiresonance frequencies vs the size of the training set

the 20 sets is computed and displayed in Figure 3(b). One can see that low classification error are reached with a low number of training points. Moreover, the convergence is quicker on the two first antiresonances. Finally, similarly to the resonances, the regression error is computed on the prediction of the antiresonance frequencies. The evolution is given in Figure 3(c). The error reached are low even with a low number of training points. As a validation, the predictions are compared to values from the validation set in Figure 4(a) for the three antiresonances. The surrogate used are constructed with 74 training points. Once can see that for each of them, prediction looks are very accurate.

Finally, the surrogate of the full FRF is constructed. The full FRF is divided in clusters for which boundaries are marked with the resonance and antiresonance frequencies (i.e. cut at the peaks). One GPR is constructed on each cluster. This strategy is compared to a strategy where all points from the training set are used to create a unique surrogate of the FRF. A validation set of 10000 points is validated and the average absolute error is computed, and with the proposed approach, the error of prediction is divided by 6 compared to the naïve approach. As a validation, the predicted FRF with both approaches are displayed in Figure 4(b), where they are compared to the reference values. Once can see that the proposed approach outperforms the one-surrogate approach as resonance peaks and anti-resonance peaks are well approximated. With the proposed approach and at high frequencies, the amplitude of the FRF near antiresonance is underestimated. This is due to some misclassification, which pushes the surrogate in extrapolation and bad prediction. This can be circumvented by adding more points to train the classification surrogate as small mistake there can have dramatic consequences.



(a) Antiresonance frequency predictions

(b) FRF prediction

Fig. 4 (a) Comparison of the reference and prediction antiresonance frequencies and (b) Comparison of the reference and predicted FRF with one surrogate and the new approach

Conclusion

The present study has for objective to propagate uncertainty for FRF prediction of linear mechanical system. A new strategy is proposed by cutting the training set in subsets based on physical properties, namely resonance and antiresonance, and creating a surrogate for each of this subset. The computation of the antiresonance is done with a modified eigenvalue analysis. As antiresonance frequencies are not always defined, their existence is learned with a classification surrogate and traditional GPR are used to predict their frequency. The strategy is compared to the case where a unique surrogate model is constructed to predict the full FRF. This is applied to a FEM of a beam, and the results proved the efficiency of the approach as lower error level and quicker convergence are observed.

Acknowledgments The author acknowledges the ANR – FRANCE (French National Research Agency) for its financial support of the ANR JCJC MeMoRa project n°ANR-23-CE51-0006.

References

1. Jacquelin, E., Adhikari, S., Sinou, J.J., and Friswell, M.I. "Polynomial Chaos Expansion and Steady-State Response of a Class of Random Dynamical Systems". *Journal of Engineering Mechanics*, 141(4):04014145 (2015)ifnextchar.gobble.
2. Yaghoubi, V., Marelli, S., Sudret, B., and Abrahamsson, T. "Sparse polynomial chaos expansions of frequency response functions using stochastic frequency transformation". *Probabilistic Engineering Mechanics*, 48:39–58 (2017)ifnextchar.gobble.
3. Goy, E.D. "A surrogate modelling approach based on anti-resonance properties for frf prediction of uncertain dynamical systems". In *International Modal Analysis Conference - IMAC*, Orlando, FL (2025)ifnextchar.gobble. Society for Experimental Mechanics.
4. Jung, J., Goo, S., and Kook, J. "Predicting anti-resonance frequencies using a novel eigenvalue formulation". *Finite Elements in Analysis and Design*, 191:103525 (2021)ifnextchar.gobble.
5. Matthews, A.G.d.G., Van Der Wilk, M., Nickson, T., Fujii, K., Boukouvalas, A., Le, P., Ghahramani, Z., Hensman, J., et al. "Gpflow: A gaussian process library using tensorflow". *Journal of Machine Learning Research*, 18(40):1–6 (2017)ifnextchar.gobble.