Chapter 5
Reliability Analysis of Nonlinear Dynamic Systems via Importance Sampling and Surrogate Modeling

Zhi-wei Wang, Michael D. Todd, and Zhen Hu

Abstract Reliability analysis of nonlinear dynamic systems under uncertainty remains a critical yet challenging task. Traditional methods, such as the Monte Carlo simulation (MCS) and analytical approximations, often suffer from limited accuracy and high computational cost, particularly when dealing with high-dimensional nonlinear dynamic systems. This study develops a surrogate-assisted importance sampling (IS) framework for reliability analysis of nonlinear dynamical systems subjected to stochastic excitations. The proposed method employs an attention long short-term memory (aLSTM) feature extractor combined with a spectral-normalized neural Gaussian process (SNGP) to form an uncertainty-aware surrogate model (aLSTM-SNGP). This model directly predicts the structural response and its uncertainty, thereby avoiding costly nonlinear time-domain simulations. A particle filtering approach is introduced to approximate the optimal proposal distribution, and an active learning strategy with fine-tuning is applied to iteratively improve surrogate accuracy in critical failure regions. Numerical studies on nonlinear shear-building models under stochastic seismic excitations demonstrate that the framework provides accurate failure probability estimates with greatly reduced computational cost compared to the MCS method. The results highlight the potential of integrating advanced surrogate modeling and adaptive sampling strategies for reliability analysis of complex structural systems.

Keywords Reliability analysis · Nonlinear dynamic systems · Importance sampling · Surrogate modeling · Uncertainty quantification

Introduction

The reliability analysis of nonlinear dynamic structural systems under stochastic excitations is a fundamental problem in structural engineering. Conventional Monte Carlo simulation (MCS) provides unbiased estimates of failure probability but suffers from prohibitive computational costs when dealing with rare-event probabilities and nonlinear dynamic responses [1, 10]. To improve efficiency, importance sampling (IS) has been widely adopted as it reallocates sampling effort toward critical failure regions while retaining unbiasedness [2, 9]. However, the efficiency of IS critically depends on constructing an effective proposal distribution, which remains challenging in high-dimensional and strongly nonlinear settings [6, 11].

To mitigate these challenges, surrogate models have been increasingly integrated into IS frameworks. Polynomial chaos expansions [5], Gaussian process regression [8], and Kriging-based meta-modeling [4] have been employed to approximate

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computationally expensive dynamic simulations. More recently, machine-learning-based surrogates such as NARX models [12], hybrid Kriging–NARX frameworks [3], and physics-informed neural networks [14] have been proposed to capture nonlinear system behavior with improved generalization. Deep learning models, including recurrent and convolutional architectures, have also demonstrated potential in approximating nonlinear dynamic responses under seismic or wind loading [13, 15]. Despite these advances, existing surrogates often face difficulties in maintaining accuracy near failure boundaries, where reliability estimates are most sensitive. In recent years, efforts have focused on adaptive and uncertainty-aware surrogate models. For example, active learning strategies have been used to enrich surrogate training datasets near critical regions, while Gaussian process (GP) models with uncertainty quantification have provided confidence-aware reliability estimates [7, 16]. Nevertheless, balancing surrogate accuracy, uncertainty quantification, and computational efficiency remains a pressing issue.

This study contributes to this field by proposing a surrogate-assisted IS framework tailored for nonlinear dynamic reliability analysis. The framework integrates an uncertainty-aware surrogate model with an adaptive sampling strategy, aiming to enhance accuracy near the failure boundary while retaining computational efficiency. A nonlinear shear-building model under stochastic seismic excitations is employed to demonstrate the effectiveness of the proposed method.

Methodology

IS-based Reliability Analysis for Nonlinear Dynamic Systems

In the reliability analysis of nonlinear dynamical systems, the structural response is obtained from a simulation model $\mathcal{M}$, which maps the stochastic dynamic excitation $\mathbf{s}$ (e.g., ground motions, wind loads) and random structural parameters $\boldsymbol{\theta}$ (e.g., stiffness, damping, and material properties) to the dynamic response time history $\mathbf{y}(t)$. Specifically, $\mathbf{y}(t) = \mathcal{M}(\mathbf{s};\boldsymbol{\theta})(t)$. Since the extreme responses are of interest, the performance (limit state) function is defined based on the maximum absolute value of the response $y_{\max}(\mathbf{s},\boldsymbol{\theta})$ within the time horizon $[0,T]$, namely,

$$g(\mathbf{s},\boldsymbol{\theta}) = d - y_{\max}(\mathbf{s},\boldsymbol{\theta}) = d - \sup_{t\in[0,T]} \left|\mathbf{y}(t)\right|, \tag{1}$$

where d is a prescribed threshold.

A straightforward approach to estimate the failure probability is the crude MCS, which draws samples $(\mathbf{s}^{(i)},\boldsymbol{\theta}^{(i)})$ from their true distributions and evaluates the failure probability, namely,

$$\widehat{P}_f^{\mathrm{MCS}} = \frac{1}{N}\sum_{i=1}^{N} I\left[g\left(\mathbf{s}^{(i)},\boldsymbol{\theta}^{(i)}\right) \le 0\right], \quad \mathbf{s}^{(i)} \sim f_{\mathbf{S}}(\mathbf{s}),\ \boldsymbol{\theta}^{(i)} \sim f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}). \tag{2}$$

where $I[\cdot]$ is the indicator function; $f_{\mathbf{S}}$ and $f_{\boldsymbol{\Theta}}$ are the probability density functions of the stochastic dynamic excitation and random structural parameters, respectively.

However, when the event of interest is rare, MCS requires an excessively large number of samples to obtain accurate estimates, leading to prohibitive computational costs in dynamic analyses. To overcome this limitation, the IS method is introduced here. The key idea is to sample from an auxiliary distribution that places more weight in regions contributing most to failure. In our formulation, the dynamic excitation samples are still drawn from their true distribution $f_{\mathbf{S}}(\mathbf{s})$, while the structural parameters are sampled from a proposal distribution $h(\boldsymbol{\theta})$. The failure probability can then be reformulated as

$$P_f = \int_{\mathbb{R}^{d_s}}\int_{\mathbb{R}^{d_\theta}} I[g(\mathbf{s},\boldsymbol{\theta}) \le 0]\ \frac{f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})}{h(\boldsymbol{\theta})}\ f_{\mathbf{S}}(\mathbf{s})\, h(\boldsymbol{\theta})\, d\mathbf{s}\, d\boldsymbol{\theta}. \tag{3}$$

Analogizing with the calculation formula of $\widehat{P}_f^{\mathrm{MCS}}$, the IS estimator of the failure probability is given by

$$\widehat{P}_f^{\mathrm{IS}} = \frac{1}{N}\sum_{i=1}^{N} I\left[g(\mathbf{s}^{(i)},\boldsymbol{\theta}^{(i)}) \le 0\right]\ \frac{f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}^{(i)})}{h(\boldsymbol{\theta}^{(i)})}, \quad \mathbf{s}^{(i)} \sim f_{\mathbf{S}}(\mathbf{s}),\ \boldsymbol{\theta}^{(i)} \sim h(\boldsymbol{\theta}). \tag{4}$$

By choosing a suitable proposal density $h(\boldsymbol{\theta})$, importance sampling greatly improves the efficiency of failure probability estimation compared to crude MCS, especially for rare-event problems in dynamic reliability analysis. Despite its advantages, the practical application of IS in the reliability analysis of nonlinear dynamic systems still faces several key challenges: (i) strong nonlinearity of structural response; (ii) high-dimensional random inputs; (iii) heavy computational burden; and (iv) design of the proposal distribution.

Surrogate-assisted IS Reliability Analysis Framework

aLSTM-SNGP Surrogate Model

To address the above challenges, we developed a surrogate-assisted IS reliability analysis framework based on the proposed aLSTM-SNGP model. A surrogate modeling strategy is adopted to approximate the complex input–output mapping of the nonlinear dynamic systems. Specifically, we develop a hybrid model termed the aLSTM-SNGP model, as illustrated in Fig. 1, which serves as an efficient and uncertainty-aware surrogate $\mathfrak{S}(\mathbf{s}, \boldsymbol{\theta})$ to the original simulation model $\mathcal{M}(\mathbf{s}; \boldsymbol{\theta})$. The proposed surrogate model $\mathfrak{S}$ is directly trained to approximate the maximum absolute response $y_{\max}$ and to provide an associated uncertainty measure for its prediction, that is,

$$\sup_{t\in[0,T]} \big|\mathcal{M}(\mathbf{s}; \boldsymbol{\theta})(t)\big| \;\approx\; \mathfrak{S}(\mathbf{s}, \boldsymbol{\theta}) \;=\; \big[\,\hat{y}_{\max}(\mathbf{s}, \boldsymbol{\theta}),\ \text{uncertainty}\,\big]. \tag{5}$$

The input of the aLSTM-SNGP model consists of the stochastic dynamic excitation $\mathbf{s}$ and random structural parameters $\boldsymbol{\theta}$. These inputs are first processed by an attention Long Short-Term Memory (aLSTM) network, which acts as a feature extractor. By compressing dynamic excitation histories and numerous structural parameters into compact latent representations, the aLSTM implicitly performs dimension reduction, which alleviates the curse of dimensionality that commonly arises in nonlinear dynamical reliability analysis. The Spectral-normalized Neural Gaussian Process (SNGP) module is employed as the probabilistic prediction component of the aLSTM-SNGP surrogate model. It receives compact feature representations from the aLSTM encoder and processes them through several residual layers with spectral normalization. These layers are followed by a random Fourier feature (RFF) mapping that approximates kernel functions. At the output stage, a neural Gaussian process (GP) layer is attached, which replaces the conventional dense output with a probabilistic predictor. Unlike a standard deterministic output, this layer produces not only the predictive mean $\hat{\mu}_{\max}$ but also the predictive uncertainty $\hat{\sigma}_{\max}$, effectively quantifying epistemic uncertainty in the model outputs. Formally, the surrogate model $\mathfrak{S}$ provides both the predictive mean estimate of the maximum absolute response $y_{\max}(\mathbf{s}, \boldsymbol{\theta})$ and the associated predictive uncertainty:

$$\hat{y}_{\max}(\mathbf{s}, \boldsymbol{\theta}) \;=\; \mathfrak{S}(\mathbf{s}, \boldsymbol{\theta}) \;\sim\; \mathcal{N}\Big(\hat{\mu}_{\max}(\mathbf{s}, \boldsymbol{\theta}),\ \hat{\sigma}^2_{\max}(\mathbf{s}, \boldsymbol{\theta})\Big). \tag{6}$$

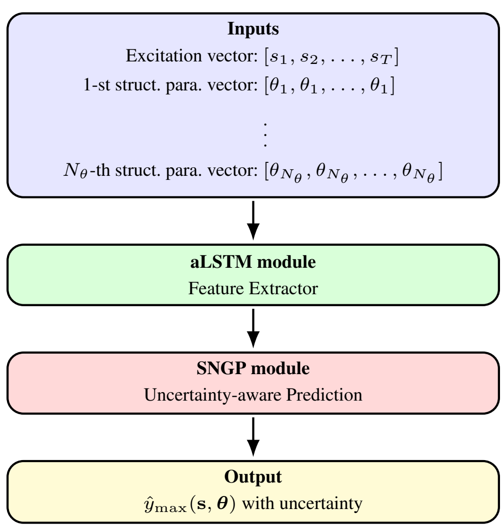

Fig. 1 Overall architecture of the proposed aLSTM-SNGP surrogate model

Particle Filtering for Approximating Proposal Distribution

A central difficulty in implementing IS for nonlinear dynamical reliability analysis lies in designing an effective proposal distribution $h(\boldsymbol{\theta})$. The distribution must concentrate sampling effort in regions that contribute significantly to the failure probability. We employ a surrogate-assisted particle filtering strategy, which adaptively updates the sampling distribution based on the surrogate model predictions. Specifically, we first draw a set of prior samples (particles) of structural parameters, $\mathcal{X}_{\text{prior}} = \{\boldsymbol{\theta}^{(i)}_{\text{prior}}\}_{i=1}^{N}$, $\boldsymbol{\theta}^{(i)}_{\text{prior}} \sim f_{\boldsymbol{\Theta}}(\boldsymbol{\theta})$, together with a pre-generated library of dynamic excitations $\mathcal{E} = \{\mathbf{s}^{(i)}\}_{i=1}^{N}$, $\mathbf{s}^{(i)} \sim$

$f_{\mathbf{S}}(\mathbf{s})$. For each particle pair $(\mathbf{s}^{(i)}, \boldsymbol{\theta}^{(i)}_{\text{prior}})$, the likelihood is evaluated through the surrogate model $\mathfrak{S}$. This likelihood is defined as:

$$\varphi(\mathbf{s}, \boldsymbol{\theta} \mid \mathfrak{S}) \;=\; \Phi\left(\frac{\hat{\mu}_{\max}(\mathbf{s}, \boldsymbol{\theta}) - d}{\hat{\sigma}_{\max}(\mathbf{s}, \boldsymbol{\theta})}\right), \tag{7}$$

where $\hat{\mu}_{\max}(\mathbf{s}, \boldsymbol{\theta})$ and $\hat{\sigma}_{\max}(\mathbf{s}, \boldsymbol{\theta})$ are the surrogate-predicted mean and standard deviation of the maximum absolute response, and d is the failure threshold. This quantity reflects the surrogate-informed conditional failure probability of a particle.

The normalized Bayesian weight of each particle is then defined as:

$$w(\mathbf{s}^{(i)}, \boldsymbol{\theta}^{(i)}_{\text{prior}} \mid \mathfrak{S}) \;=\; \frac{\varphi(\mathbf{s}^{(i)}, \boldsymbol{\theta}^{(i)}_{\text{prior}} \mid \mathfrak{S})}{\sum_{j=1}^{N} \varphi(\mathbf{s}^{(j)}, \boldsymbol{\theta}^{(j)}_{\text{prior}} \mid \mathfrak{S})}, \tag{8}$$

ensuring that particles closer to the failure region receive higher importance. By resampling according to these weights, the particle set gradually concentrates in the critical domain, resulting in the posterior set $\mathcal{X}_{\text{post}} = \{\boldsymbol{\theta}^{(i)}_{\text{post}}\}_{i=1}^{N}$ of structural parameters.

Next, an analytic approximation of the proposal distribution, denoted as $h^*(\boldsymbol{\theta} \mid \mathfrak{S})$, is constructed from $\mathcal{X}_{\text{post}}$, for example by fitting a Gaussian mixture model (GMM). This approximate distribution serves as a practical substitute for the unknown optimal $h(\boldsymbol{\theta})$. Drawing from this distribution yields a set of important samples of structural parameters, namely, $\mathcal{X}_{\text{IS}} = \{\boldsymbol{\theta}^{(i)}_{\text{IS}}\}_{i=1}^{N_{\text{IS}}}, \boldsymbol{\theta}^{(i)}_{\text{IS}} \sim h^*(\boldsymbol{\theta} \mid \mathfrak{S})$. Based on Eq. (4), the IS estimator is then evaluated as:

$$\begin{aligned} \hat{P}_f^{\text{IS}} \;=\; \frac{1}{N_{\text{IS}}} \sum_{i=1}^{N_{\text{IS}}} I\big[\hat{g}(\mathbf{s}^{(i)}, \boldsymbol{\theta}^{(i)}_{\text{IS}} \mid \mathfrak{S}) \le 0\big]\, \frac{f_{\boldsymbol{\Theta}}(\boldsymbol{\theta}^{(i)}_{\text{IS}})}{h^*(\boldsymbol{\theta}^{(i)}_{\text{IS}} \mid \mathfrak{S})}, \\ \boldsymbol{\theta}^{(i)}_{\text{IS}} \in \mathcal{X}_{\text{IS}},\;\; \mathbf{s}^{(i)} \in \mathcal{E}, \end{aligned} \tag{9}$$

where $\hat{g}(\mathbf{s}, \boldsymbol{\theta} \mid \mathfrak{S}) = d - \hat{y}_{\max}(\mathbf{s}, \boldsymbol{\theta}) = d - \hat{\mu}_{\max}(\mathbf{s}, \boldsymbol{\theta})$ is the surrogate-based limit state function.

Training Strategy for aLSTM-SNGP Model

The surrogate model $\mathfrak{S}$ trained using a limited initial dataset is often not sufficiently accurate in the critical failure regions (the initial dataset is generated by evaluating the simulation model $\mathcal{M}$ and denoted as $\mathcal{D}_0 = \{[(\mathbf{s}_i, \boldsymbol{\theta}_i),\; y_{\max,i}]\}_{i=1}^{N_0}, \mathbf{s}_i \in \mathcal{E}$, $\boldsymbol{\theta}_i \in \mathcal{X}_{\text{prior}}$, where the size N_0 is typically small, e.g., 120). Consequently, the approximated proposal distribution $h^*(\boldsymbol{\theta} \mid \mathfrak{S})$ and the corresponding IS estimates may lack precision. Hence, iterative training is necessary to progressively improve the accuracy of $\mathfrak{S}$ in the failure regions and enhance the precision of the IS estimates.

Active Learning-based Sample Augmentation. We adopt an active learning strategy to adaptively refine the surrogate by selectively adding new training samples that are most informative for failure probability estimation. Candidate samples are formed as pairs $(\mathbf{s}, \boldsymbol{\theta}) \in \mathcal{E} \times \mathcal{X}_{\text{IS}}$. The informativeness of each candidate is measured by the following learning function:

$$\mathcal{L}(\mathbf{s}, \boldsymbol{\theta} \mid \mathfrak{S}) \;=\; \frac{\big|\hat{\mu}_{\max}(\mathbf{s}, \boldsymbol{\theta}) - d\big|}{\hat{\sigma}_{\max}(\mathbf{s}, \boldsymbol{\theta})}. \tag{10}$$

Instead of selecting a single new sample, a batch of N_{new} candidates (e.g., $N_{\text{new}} = 5$) with the smallest values of $\mathcal{L}(\mathbf{s}, \boldsymbol{\theta} \mid \mathfrak{S})$ are selected at each iteration:

$$\{(\mathbf{s}^{(j)}_{\text{next}}, \boldsymbol{\theta}^{(j)}_{\text{next}})\}_{j=1}^{N_{\text{new}}} = \underset{(\mathbf{s},\boldsymbol{\theta}) \in \mathcal{E} \times \mathcal{X}_{\text{IS}}}{\overset{\text{select } N_{\text{new}}}{\arg\min}}\; \mathcal{L}(\mathbf{s}, \boldsymbol{\theta} \mid \mathfrak{S}) \tag{11}$$

so that the surrogate model is refined in regions where the prediction is close to the failure boundary and the uncertainty remains high. By repeatedly enriching the training dataset with samples near the failure boundary, the surrogate model $\mathfrak{S}$ gradually improves its predictive accuracy in critical failure regions, thereby improving the quality of the approximated proposal distribution $h^*(\boldsymbol{\theta} \mid \mathfrak{S})$ and enhancing the reliability of IS-based failure probability estimation.

Hybrid Training with Fine-Tuning. In addition to the active learning strategy, we adopt a hybrid training strategy to efficiently update the surrogate model $\mathfrak{S}$ during the iterative refinement process. Specifically, after the full training performed on the initial dataset, the training process alternates between the following two modes:

(i) periodic full retraining, where every fixed number of iterations (e.g., every 5 iterations) the surrogate $\mathfrak{S}$ is reinitialized and trained from scratch on the current entire dataset, thereby preventing error accumulation and model drift; and

(ii) lightweight fine-tuning, where in the remaining iterations the surrogate $\mathfrak{S}$ is refined using the newly augmented samples (or their cumulative set) after the last full training, starting from the most recent set of model weights obtained from the last full training, which allows efficient assimilation of new information while significantly reducing training budget.

This procedure is conducted until convergence of $\hat{P}_f^{\text{IS}}$ is achieved. The above hybrid training strategy achieves a balance between stability (through full retraining) and efficiency (through fine-tuning) in updating the surrogate model, allowing $\mathfrak{S}$ to remain accurate in the critical failure regions without incurring prohibitive computational cost.

Case Study: Nonlinear Shear-Building Model under Stochastic Seismic Excitations

Nonlinear Shear-Building Model Introduction

To validate the proposed surrogate-assisted IS reliability analysis framework, a six-story nonlinear shear-building model subjected to stochastic seismic excitations is investigated, as illustrated in Fig. 2. The governing equations of motion for an n-story shear building are given by:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{f}_s(\mathbf{u}(t), \mathbf{z}(t)) = -\mathbf{M}\mathbf{1}\ddot{u}_g(t), \tag{12}$$

where $\mathbf{u}(t) \in \mathbb{R}^n$ denotes the vector of floor displacements relative to the ground, $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the diagonal mass matrix, $\mathbf{C} \in \mathbb{R}^{n\times n}$ is the damping matrix, $\mathbf{f}_s(\mathbf{u}, \mathbf{z}) \in \mathbb{R}^n$ is the vector of nonlinear restoring forces, $\mathbf{z}(t) \in \mathbb{R}^n$ is the vector of hysteretic internal variables, and $\mathbf{1} \in \mathbb{R}^n$ is the vector of ones ensuring that all stories are subjected to the same ground acceleration input $\ddot{u}_g(t)$.

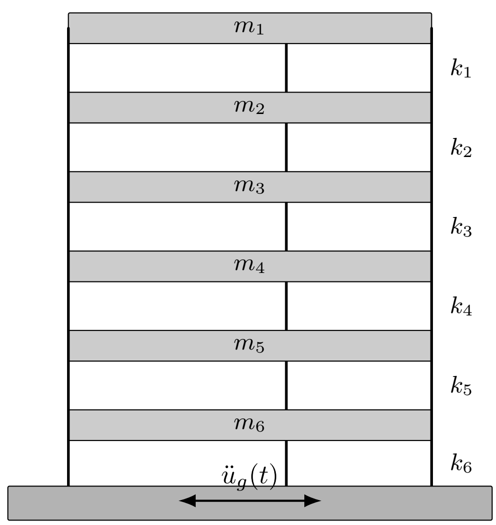

Fig. 2 Six-story nonlinear shear-building model under stochastic seismic excitation

The restoring force of the i-th story is modeled by the Bouc–Wen hysteretic model:

$$f_{s,i}(t) = k_i\big(\alpha u_i(t) + (1-\alpha)z_i(t)\big), \qquad i = 1, \dots, n, \tag{13}$$

with the hysteretic evolution equation:

$$\dot{z}_i(t) = A\dot{u}_i(t) - \beta|\dot{u}_i(t)||z_i(t)|^{\eta-1}z_i(t) - \gamma\dot{u}_i(t)|z_i(t)|^{\eta}, \tag{14}$$

where k_i is the inter-story stiffness, α is the post-yield ratio, and (A, β, γ, η) are Bouc–Wen parameters governing hysteresis shape. In this study, they are set as $\alpha = 0.3$, $A = 1.0$, $\eta = 1.0$, and $\beta = \gamma = A/(2x_y^\eta)$, and $x_y = 0.01$ m is the yield displacement.

In this case study, structural uncertainties are represented by the inter-story stiffness parameters $\{k_i\}_{i=1}^6$, which are modeled as independent Gaussian random variables with mean 4.5×10^7 N/m and standard deviation 9.0×10^6 N/m for each story. The lumped masses are deterministic and identical across all stories, each set to 3.4×10^4 kg. The performance measure for reliability analysis is defined as the maximum absolute top-floor displacement $y_{\max}$, and failure occurs if $y_{\max}$ exceeds a prescribed threshold d.

Results

In this example, the failure threshold $d = 0.060\,\mathrm{m}$ is considered. The aLSTM-SNGP surrogate model $\mathfrak{S}$ is trained initially on a dataset of $N_0 = 120$ samples. At each active learning iteration, a batch of $N_{\text{new}} = 5$ new samples is selected. The training procedure alternates between full retraining every 5 iterations and fine-tuning in the intermediate iterations. For the surrogate-assisted IS reliability estimation at each iteration, 100,000 Monte Carlo samples are employed for posterior sampling and 100,000 importance samples are used in the IS estimator. The convergence of surrogate model training and failure probability estimate over successive iterations is shown in Fig. 3. The results demonstrate that the surrogate-assisted IS estimator converges to the MCS reference $\widehat{P}_f^{\text{MCS}}$ within 50 iterations, using a total of 370 evaluations of the dynamic simulation model $\mathcal{M}$. The results confirm that the proposed method achieves accurate estimates with significantly reduced computational cost.

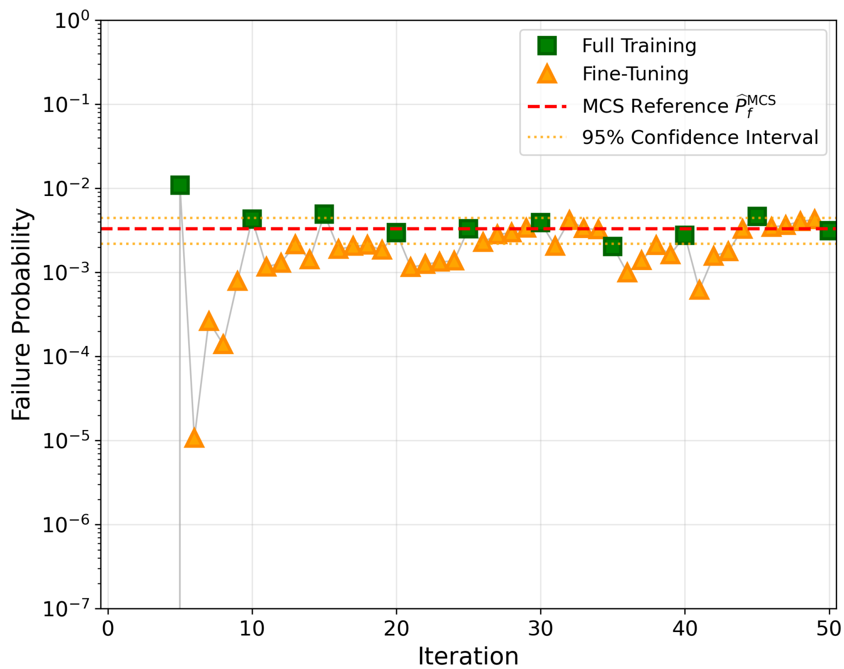

Fig. 3 Convergence of surrogate model training and failure probability estimate (Threshold $d = 0.060\,\mathrm{m}$)

To illustrate the approximation result of the proposal distribution $h(\boldsymbol{\theta})$, Fig. 4 shows the comparison of the prior distribution, the posterior distribution obtained from particle filtering, and the GMM-fitted proposal distribution for each stiffness parameter k_i. It can be clearly observed that the posterior distributions of the stiffness parameters k_i shift to the left compared to their priors, indicating smaller stiffness values. Physically, this corresponds to larger structural displacements under seismic excitations, which brings the system closer to the failure domain. This directional migration of the distributions confirms that the particle filtering process effectively concentrates the sampling effort in regions most relevant to failure events. Furthermore, the fitted proposal distribution provides a smooth and flexible representation that follows the posterior while reducing sampling variance, thereby ensuring both accuracy and efficiency in the IS estimator.

Figure 5 further illustrates the evolution of the surrogate model predictions on the important sample set drawn from the fitted proposal distribution across iterations. In the first iteration [Fig. 5(a)], the predictive variance $\hat{\sigma}_{\max}$ is relatively large, particularly near the failure threshold $d = 0.06\,\mathrm{m}$, indicating the limited accuracy of the initially trained surrogate in the critical region. After iterative active learning and fine-tuning, the variance is significantly reduced in the last iteration [Fig. 5(b)], especially in the vicinity of the failure boundary, which demonstrates the improved reliability of the surrogate for discriminating between safe and failure states. At the same time, the distribution of important samples is seen to migrate towards the failure boundary as iterations progress. This migration also reflects the fact that the surrogate-guided particle filtering adaptively allocates sampling effort to regions where the mean structural response $\hat{\mu}_{\max}$ approaches the failure threshold.

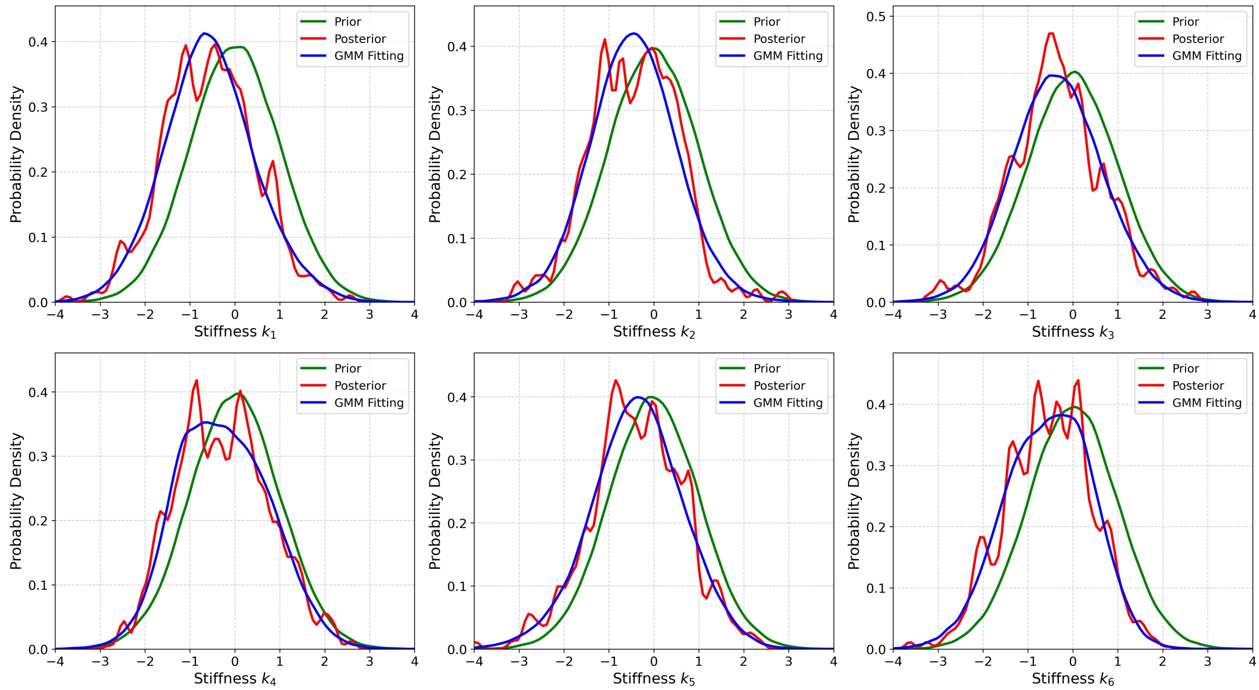

Fig. 4 Comparison of prior, posterior, and fitted proposal distributions of stiffness parameters at the last iteration

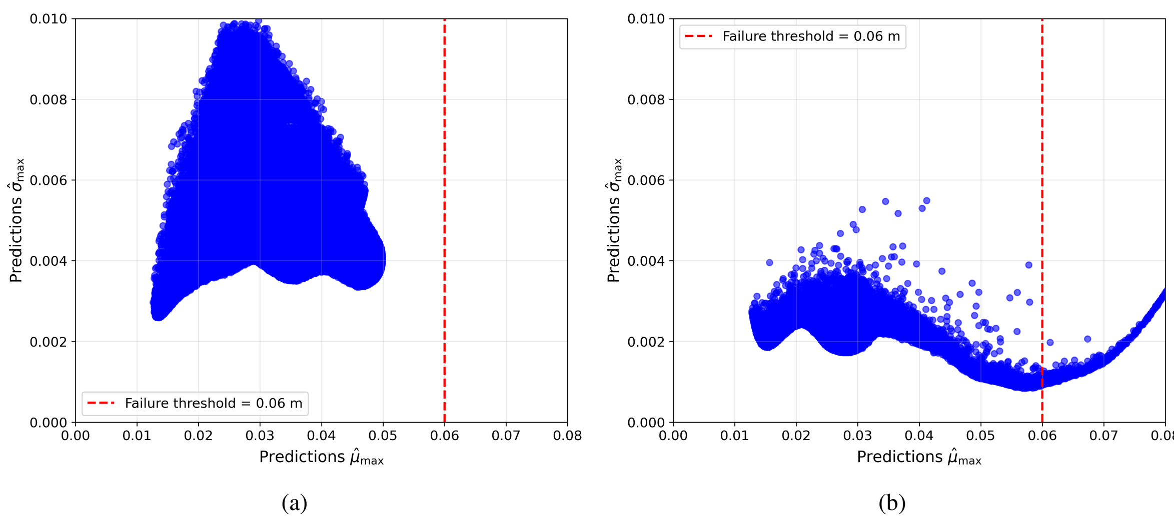

Fig. 5 The surrogate model predictions on the important sample set: (a) at the first iteration; and (b) at the last iteration

Conclusions

This work presents a surrogate-assisted importance sampling (IS) framework for efficient reliability analysis of nonlinear dynamic systems under stochastic excitations. The framework integrates three essential components: an aLSTM–SNGP surrogate model that predicts the maximum structural response together with predictive uncertainty, a particle filtering scheme that adaptively guides sampling toward regions relevant to failure, and an active learning strategy with hybrid training that incrementally improves surrogate accuracy. The numerical experiment on the six-story shear-building model confirms that the method can accurately estimate rare-event probabilities at a fraction of the cost of direct MCS. The study demonstrates that combining uncertainty-aware machine learning surrogates with adaptive sampling strategies offers a powerful tool for reliability analysis of dynamical systems, with potential applications to large-scale civil infrastructures under multi-hazard environments.

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