Chapter 7
Overcoming Data Scarcity for Load Identification and Full-Field Response Reconstruction

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Abstract Evidence of a substantial population of aging, deteriorated, or otherwise hazardous bridge infrastructure across the world has necessitated the development of new techniques for bridge health monitoring that can interpret behavior, localize damage, and prioritize repairs across bridge networks. Long-term continuous monitoring of bridges through a dense, stationary sensor network represents a common approach in the research community to rapidly interpret bridge condition, which has been shown to be successful in cases such as the Jindo Bridge (Korea) or the St. Anthony Falls Bridge (USA). However, achieving an estimate of full-field dynamic structural behavior using sparse and sub-optimal placed sensors remains a major challenge. This paper presents a novel approach that integrates a generalized structural model of a bridge based on the unit load method with non-negative Bayesian linear regression to enable real-time reconstruction of the full-field structural responses. We demonstrate this approach on a simulated 12-meter beam to estimate the loading scenario, bending moments and deflections using only sparsely deployed strain gauges in an underdetermined identification context. The proposed method is compared with traditional regularization methods and shows great accuracy in structural response reconstruction and load identification. This approach presents opportunities to reduce costs in long-term monitoring systems and allow for those systems to be more easily repurposed toward different monitoring goals.

Keywords Load identification · Sparse sensing · Non-negative regularization constraints · Response reconstruction

Introduction

Accurate load identification is important for structural health monitoring of infrastructure systems. Based on accurate load, the response of structure can be then reconstructed in real time to further monitor changes in structural parameters and prevent failure.

Indirect measurement methods based on strain response are commonly used in load identification. However, identifying or estimating structure input loads using system characteristics and responses is an inverse problem [1]. In practical deployments with sparse sensing, the number of parameters needed to characterize the load's spatial distribution and magnitudes often exceeds the number of available sensors, yielding an underdetermined, ill-posed inverse problem with non-unique and unstable solutions [2]. To address this problem, many regularization techniques have been utilized to achieve a unique and stable solution, like Tikhonov [3], Lasso [4] and Bayesian regularization methods [5].

Bayesian regularization methods are particularly useful because of their ability to estimate weights together with statistical information. A prominent instance is the Automatic Relevance Determination (ARD) method. Within a hierarchical Bayesian framework, ARD method employs evidence-driven hyperparameter learning to automatically select relevant variables, yielding sparse solutions that mitigate ill-posedness and enhance stability and identifiability in underdetermined settings [6]. However, to ensure conjugacy with the Gaussian noise in the likelihood, ARD method typically places a Gaussian prior on the weights, which allows the inferred weights to be negative. This characteristic hinders the applicability of ARD methods when non-negative parameters are required, such as in the case of identifying gravity induced loads, material densities, or strength parameters. Rather than imposing a non-negative prior distribution, we enforce non-negativity by modifying the feature-pruning scheme, resulting in a non-negative ARD (NN-ARD) approach.

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This study first formulates a least-squares problem for load identification with a non-negative support set via the unit-load method. These loads are then identified using NN-ARD approach, and model selection is performed by minimizing the negative log-likelihood (NLL). The structural displacement response is reconstructed using the response design matrix. Uncertainties in the load and measurement noise are estimated and propagated to the displacement response.

Unit load method

For load identification from structural responses, the possible loading scenario on the structure is first discretized into a series of finite-length uniformly distributed loads to approximate the true loading. As shown in Fig. 1, a beam under an arbitrary distributed load $W\left(x\right)$ with pinned support on the left and fixed support on the right is shown in the left figure and its discretized uniformly distributed loads is shown in the right figure.

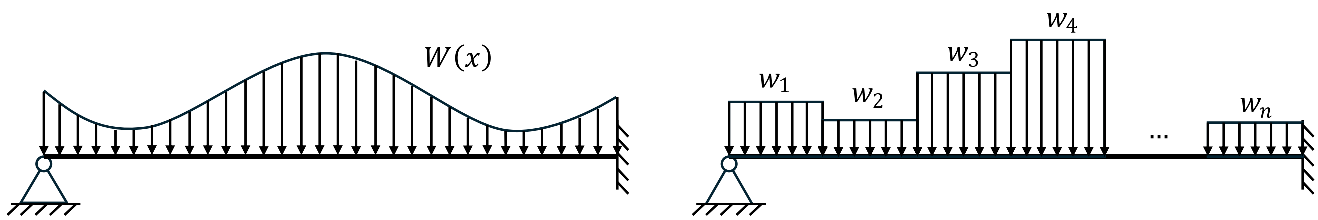


Fig. 1 Beam under an arbitrary distributed load $W\left(x\right)$ and beam under discretized uniformly distributed load segments $(w_1, w_2, ..., w_n)$

Under the assumption of linear elastic and small deformation (hence superposition holds), the beam's response under a series of loading segments $w_1,\ w_2,\ \cdots,w_n$ can be approximated as the sum of responses under each of discretized uniformly distributed load segments. Then, given linearity, the response of the beam to each discretized uniformly distributed load segment of intensity w_n is exactly the response of unit distributed load $\overline{w}_n = 1$ on that segment scaled by w_n. Therefore, the structural response can be represented as weighted summation of structural response under unit distributed loads, as shown in Eq. (1) below.

$$\boldsymbol{\mathcal{R}}\left(W\right) = \textstyle\sum_{i=1}^{n} \boldsymbol{\mathcal{R}}\left(w_i\right) = \sum_{i=1}^{n} w_i \boldsymbol{\mathcal{R}}\left(\overline{w}_i\right) \#\left(1\right) \tag{1}$$

Treating the structural response under unit distributed loads as a column vector, the response under unit distributed loads applied at different locations can be assembled into a design matrix $\boldsymbol{\Phi}$.

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\mathcal{R}}\left(\overline{w}_1\right) & \boldsymbol{\mathcal{R}}\left(\overline{w}_2\right) & \ldots & \boldsymbol{\mathcal{R}}\left(\overline{w}_n\right) \end{bmatrix} \#\left(2\right) \quad \boldsymbol{w} = \begin{bmatrix} w_1 & w_2 & \ldots & w_n \end{bmatrix}^{\top} \#\left(3\right) \tag{2}$$

where, $\boldsymbol{\mathcal{R}}\left(W\right) \epsilon \mathbb{R}^{m\times 1}$ is the structural response at m measurement stations under true distributed load $W\left(x\right)$, $\boldsymbol{\mathcal{R}}\left(\overline{w}_i\right) \epsilon \mathbb{R}^{m\times 1}$ is the structural response at m measurement stations under unit load $\overline{w}_i = 1$, $\boldsymbol{\Phi} \epsilon \mathbb{R}^{m\times n}$ is the design matrix with n segments along the beam, $\boldsymbol{w} \epsilon \mathbb{R}^{n\times 1}$ is the weight or scaling factor of n segments.

In our research problem, the objective is to identify the true load $W\left(x\right)$ acting on the structure. The structural responses $\boldsymbol{\mathcal{R}}\left(W\right)$ at the measurement stations are observable — for example, the bending moment of a beam at a given location can be inferred from strain. We approximate $W\left(x\right)$ by a linear combination of n uniformly distributed load segments with weights $\boldsymbol{w} \epsilon \mathbb{R}^{n\times 1}$. The design matrix $\boldsymbol{\Phi} \epsilon \mathbb{R}^{m\times n}$ is formed by taking the structural response $\boldsymbol{\mathcal{R}}\left(\overline{w}_i\right)$ to a normalized gravity-induced unit distributed load on segment i as its i-th column. In other words, each $\boldsymbol{\mathcal{R}}\left(\overline{w}_i\right)$ serves as a basis vector of the response space and directly constitutes one column of the design matrix. These responses can be obtained analytically, numerically or experimentally. This yields a general linear model, where ε represents the noise:

$$\boldsymbol{\mathcal{R}}\left(W\right) = \boldsymbol{\Phi}\boldsymbol{w} + \boldsymbol{\varepsilon},\ \boldsymbol{\varepsilon} \epsilon \mathbb{R}^{m\times 1} \#\left(4\right) \tag{3}$$

There are two important features for this specific general linear model. One is non-negativity on weights. The other one is underdetermined problem. For non-negativity, since the loads under consideration are gravity-induced and the columns of design matrix $\boldsymbol{\Phi}$ are constructed from responses to normalized gravity loads, the coefficients represent non-negative load intensities. Consequently, the admissible solution is supported on the non-negative domain, and we impose the constraint $\boldsymbol{w} \geq \boldsymbol{0}$. For underdetermined problems, with sparse sensing, the number of measurement stations m is limited. To better capture the spatial variation of $W\left(x\right)$, the load is discretized more finely so that $n > m$. The identification is therefore underdetermined and ill-posed. To stabilize the problem and obtain a unique, physically meaningful solution, different regularization methods are applied.

Regularization Method

The problem of load identification can be formulated as a least-squares (LS) optimization framework, wherein the objective function is defined by the discrepancy between the predicted structural responses, $\boldsymbol{\Phi w}$, and the experimentally measured responses, $\boldsymbol{\mathcal{R}}\,(W)$. However, the direct solution of such an LS formulation is often hindered by the intrinsic ill-posedness of inverse problems. To address this challenge, regularization methods are commonly employed, which enable the incorporation of prior knowledge to provide a stable approximation of ill-posed (pseudo-) inverses [7]. Different prior knowledge applied to solutions will lead to different regularization techniques:

Tikhonov regularization, also known as ℓ_2 regularization, is a widely used technique to stable inverses by penalizing the ℓ_2-norm on solution to the LS problem. The regularization parameter, denoted as λ, controls the penalty magnitude. In particular, ℓ_2 regularization penalizes large coefficient values, thereby preventing individual solution components from dominating the fit. This not only alleviates instability inherent to ill-posed problems but also promotes smoother and more robust solutions [8].

$$\min\nolimits_{\boldsymbol{w}} \|\boldsymbol{\mathcal{R}}\,(W) - \boldsymbol{\Phi w}\|_2^2 + \lambda\|\boldsymbol{w}\|_2 \;\#\,(5) \tag{4}$$

Lasso regularization, often referred to as ℓ_1 regularization, imposes a penalty proportional to the absolute value of the coefficients. Unlike ℓ_2 regularization, which shrinks all coefficients smoothly towards zero, ℓ_1 regularization encourages sparsity by driving some coefficients exactly to zero. This property not only stabilizes the solution of ill-posed problems but also enables automatic feature selection, leading to more interpretable models [9].

$$\min\nolimits_{\boldsymbol{w}} \|\boldsymbol{\mathcal{R}}\,(W) - \boldsymbol{\Phi w}\|_2^2 + \lambda\|\boldsymbol{w}\|_1 \;\#\,(6) \tag{5}$$

Bayesian regularization provides a probabilistic framework for stabilizing ill-posed problems by introducing prior distributions over model parameters. Instead of imposing penalties in an explicit optimization form (as in ℓ_1 or ℓ_2 regularization), Bayesian methods encode assumptions about parameter magnitudes and sparsity through priors and update these beliefs using observed data via Bayesian theorem. A notable example is the Automatic Relevance Determination (ARD) approach, which assigns each model weight an independent Gaussian prior with zero mean and its own precision (inverse variance) hyperparameter. By iteratively updating these hyperparameters, ARD suppresses irrelevant parameters while preserving influential ones, thereby achieving both regularization and feature selection within a Bayesian framework [6], [8]. An important advantage of this method is that it not only identifies the weights but also provides their associated statistical information, such as covariance of weights and noise. Furthermore, Bayesian regularization eliminates the need for complex tuning of regularization parameters, since the process naturally incorporates adaptive and automatic parameter selection.

Non-negative ARD Method

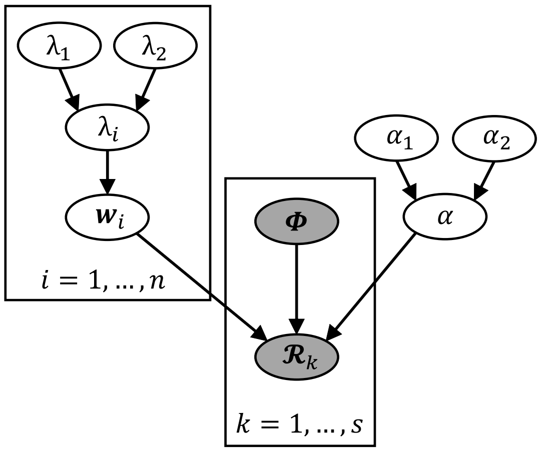
In the standard ARD framework, as shown in Fig. 2, each weight is assigned a zero-mean Gaussian prior with an individual precision hyperparameter. While this setting facilitates automatic feature selection, it also implies that the posterior estimates of the weights are distributed around zero. As a result, negative weight values may appear in the solution, which may be undesirable in applications where the underlying weights are inherently non-negative.

In the Bayesian framework, non-negativity can be introduced through the choice of priors, like the truncated Gaussian distribution [10] or the exponential distribution [11]. However, these priors break conjugacy, which means that the posterior distribution of the weights $\boldsymbol{w}$ no longer has a closed-form analytical expression. Consequently, approximate inference techniques such as variational inference (VI) are required to get posterior distribution of weights [10], [12]. This not only increases the methodological complexity but also entails substantial computational cost, making such non-negative Bayesian approaches difficult to apply in online identification tasks.

In this study, non-negativity is achieved by altering the feature-pruning rule rather than modifying the prior distribution of the weights. A zero-mean Gaussian prior is still adopted, which, combined with a Gaussian likelihood, guarantees the existence of a closed-form analytical posterior solution due to conjugacy. For the nth weight, let the posterior distribution in ARD method to be $w_n^+ \sim \mathcal{N}\,(\mu_n, \Sigma_{nn})$ with standard deviation $\sigma_n = \sqrt{\Sigma_{nn}}$. Then the standardized variable $Z = \frac{w_n^+ - \mu_n}{\sigma_n}$ follows the standard normal distribution, i.e. $Z \sim \mathcal{N}(0,1)$. Define the one-sided positivity probability as $P_n = P\,(w_n^+ > 0)$:

$$P_n = P\,(w_n^+ > 0) = P\left(\frac{w_n^+ - \mu_n}{\sigma_n} > \frac{0-\mu_n}{\sigma_n}\right) = P\left(Z > -\frac{\mu_n}{\sigma_n}\right) = 1 - \Phi\left(-\frac{\mu_n}{\sigma_n}\right) = \Phi\left(\frac{\mu_n}{\sigma_n}\right) \#\,(7) \tag{6}$$

Keeping the feature n if $P_n \geq \tau$; otherwise, the feature will be pruned. One choice, which will be used in the remainder of this study, is $\frac{\mu_n}{\sigma_n} = 0.5\;(\tau \approx 0.69)$.



Prior of weights:

$$\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Lambda}^{-1}), \qquad \boldsymbol{\Lambda} = diag(\boldsymbol{\lambda}), \qquad \boldsymbol{\lambda} = (\lambda_1, \dots, \lambda_n)$$

Likelihood:

$$\boldsymbol{\mathcal{R}}_k \mid \boldsymbol{w} \sim \mathcal{N}(\boldsymbol{\Phi w}, \alpha^{-1}\boldsymbol{I}), \qquad k = 1, \dots, s$$

Hyperparameters:

$$p(\alpha) = \text{Gamma}(\alpha \mid \alpha_1, \alpha_2) \propto \alpha^{\alpha_1} \exp\left(-\alpha_2 \alpha\right)$$

$$p(\lambda_i) = \text{Gamma}(\lambda_i \mid \lambda_1, \lambda_2) \propto \lambda_i^{\lambda_1} \exp\left(-\lambda_2 \lambda_i\right)$$

Fig. 2 Probabilistic graphical model and underlying principles of ARD method

Numerical Case Study

A constant cross-section beam with the left end pinned and the right end fixed is adopted as a numerical case study for this method. This beam has a length of 12m. Homogeneous material with elastic modulus of 2E11Pa is applied and the cross-sectional moment of inertia is 6.572E-5 m^4. Bending moments along the beam are inferred from strain measurements by considering the flexural rigidity of cross sections and the relative vertical positions between neutral axis and strain gauges. Positions of measurement stations along the beam are illustrated in Fig. 3.

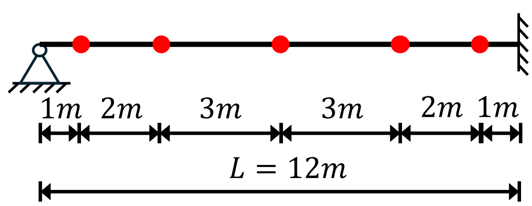


Fig. 3 Positions of measurement stations (strain gauges) along the beam

Different numbers of segments are examined, and the corresponding design matrices under unit uniform loads are obtained numerically. The structural responses considered include bending moment and deflection. Each line in Fig. 4 illustrates base vectors (BVs) of those responses produced by unit uniform loads applied at different positions for 1,2, and 3 segments.

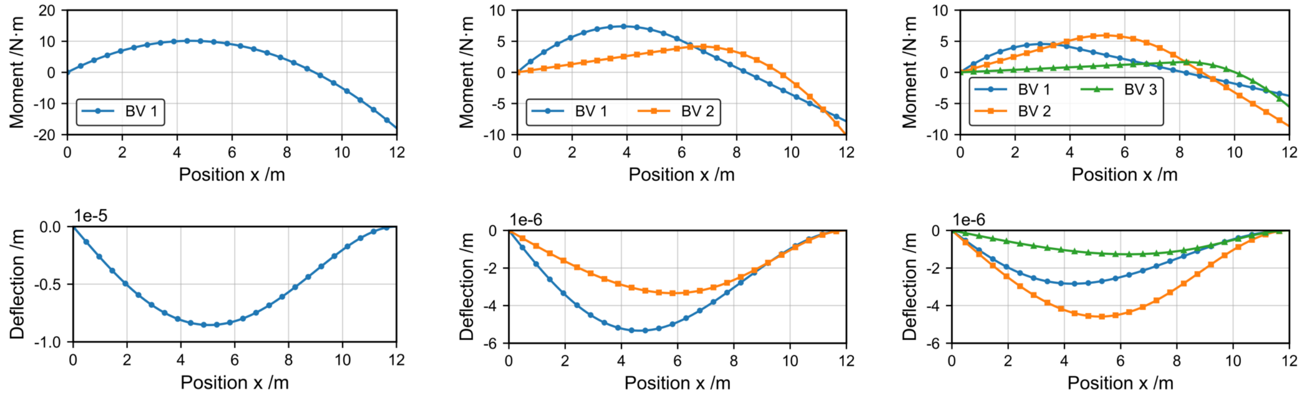
In the process of load identification, different numbers of segments can be applied to identify the loading scenario. To make the regularization method adaptive and select the optimal number of segments using only the identified result, an index called negative log-likelihood (NLL) is utilized [13]. NLL is the log form of likelihood of prediction model. The expression of NLL is presented in Eq. (7) under the assumption that the noise level is constant at different measurement stations and observations are independent and identically distributed:

$$NLL = \frac{1}{2}log(2\pi\hat{\sigma}^2) + \frac{1}{2ms\hat{\sigma}^2}\sum_{k=1}^{s}\ \sum_{j=1}^{m}\ (y_j^{(k)} - \hat{y}_j)^2 \# (8) \tag{7}$$

where $\hat{\sigma}^2$ is the estimation of noise variance, $\hat{y}_j$ is the model prediction at measurement station j, $y_j^{(k)}$ is the k^{th} observation at measurement station j, m is the number of measurement stations on the structure, and s is the number of observations in each measurement station. By taking the partial derivative of NLL to the estimation of noise variance $\hat{\sigma}^2$ and setting it to zero, we obtain:

$$\hat{\sigma}^2 = \frac{\sum_{k=1}^{s}\ \sum_{j=1}^{m}\ \left(y_j^{(k)} - \hat{y}_j\right)^2}{ms} \# (9) \tag{8}$$

Equation (8) demonstrates that the estimated variance of measurement noise in the NLL is interpreted as the mean discrepancy between the observations and the model predictions. A lower NLL then indicates that residuals are well explained by the noise model, implying better predictive fit. We fit models with varying numbers of segments and select the segment count that minimizes NLL. Computed via Eq. (7), NLL assesses consistency between the identified and measured responses under the estimated noise variance and does not require true response; by contrast, the root mean square error (RMSE),



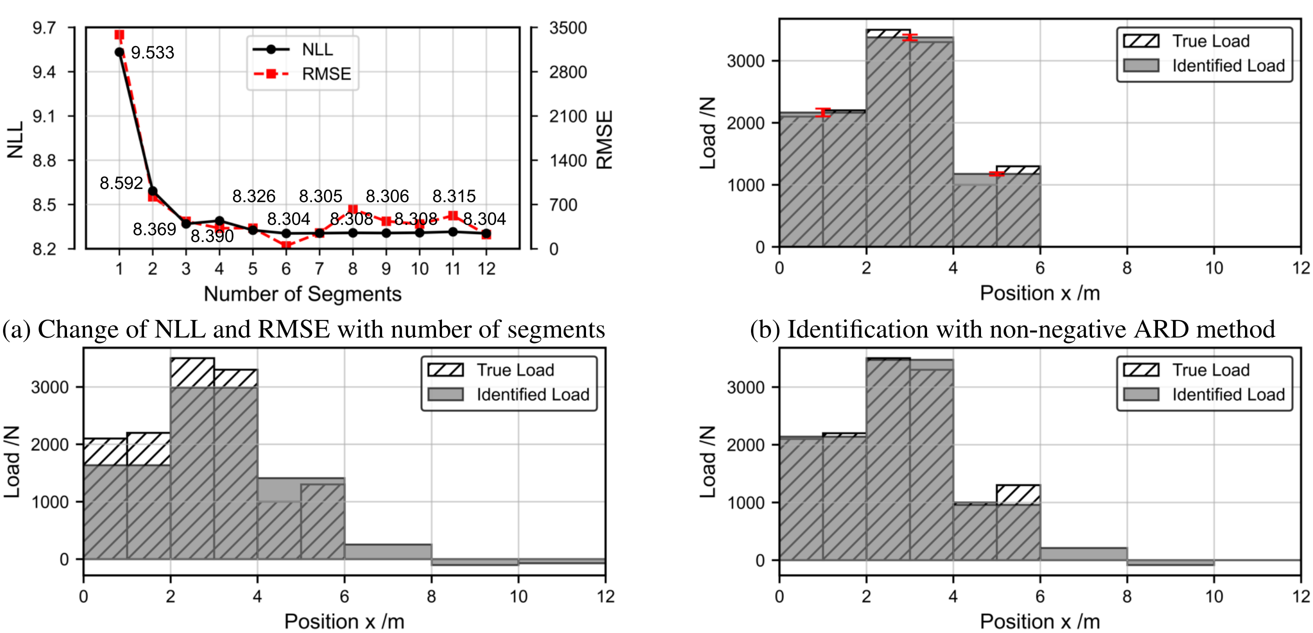
(a) BVs under 1 segment (b) BVs under 2 segments (c) BVs under 3 segments

Fig. 4 Base vectors of structural response under 1/2/3 segments

which is utilized for comparison, evaluates the difference between the true and identified responses and is often an infeasible metric in practice. However, NLL does not penalize model complexity and therefore cannot, by itself, prevent overfitting. In the case studies, we mitigate overfitting and improve generalization by promoting sparsity and preferring simpler models.

Case 1: Half span distributed load

Assume 500 sets of bending moments are inferred from strain measurements with an imposed additive noise sampled from $N(0, 1000\ (Nm)^2)$. The true load is discretized into 12 uniformly distributed loading sections across the beam. Models possessing between 1 and 12 segments are considered in the load identification process. The value of NLL and RMSE as functions of the number of segments are presented in Fig. 5(a), where values of NLL are recorded and presented at each number of segments. The true load and identification result by NN-ARD method are shown in Fig. 5(b). For comparison, the load identification results with ℓ_2 and ℓ_1 regularization are illustrated in Fig. 5(c) and (d) respectively. The regularization parameters λ are set to 1.0 in each case. Increasing λ strengthens the penalty, which driving ℓ_1 regularized solutions towards more sparsity and ℓ_2 regularized solutions towards more smoothness.



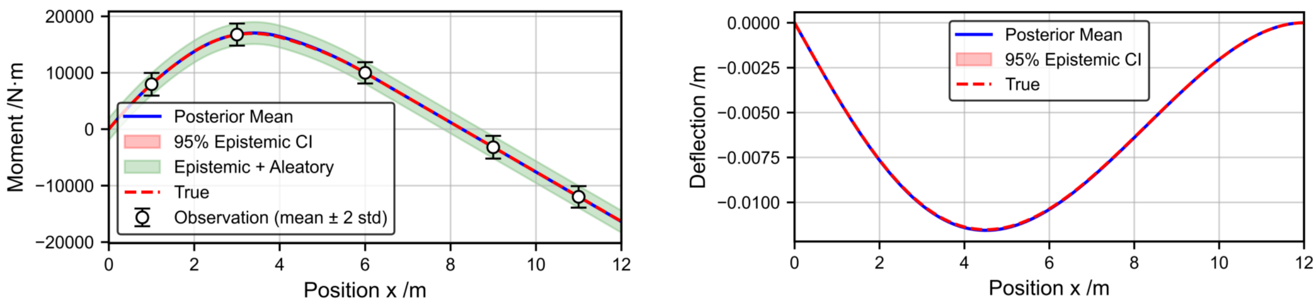
(a) Change of NLL and RMSE with number of segments (b) Identification with non-negative ARD method

(c) Identification with ℓ_2 regularization (d) Identification with ℓ_1 regularization

Fig. 5 Load identification results under half span distributed load

As shown in Fig. 5(a), both NLL and RMSE attain their minimum value when the number of segments is 6, indicating the NLL can correctly identify the number of segments that yields the best predictive performance. It's worth noting that, although the NLL is also comparatively low when the number of segments is 12, this comes with increased model complexity. In line with Occam's razor, we prefer a simpler model, i.e. a model with fewer segments, even if this requires accepting a marginal NLL increase to reduce the number of segments. The NN-ARD method yields sparse, non-negative load estimates with quantified uncertainty. By contrast, Tikhonov and Lasso produce physically inadmissible negative coefficients; with $\lambda = 1.0$, the $\ell 1$ penalty is too weak to induce meaningful sparsity, implying a larger λ is required.

The reconstructed response of moment and deflection and their propagated uncertainties from NN-ARD method are further shown in Fig. 6. Two kinds of uncertainties are considered here. Epistemic uncertainty corresponds to estimated variance from posterior distribution of weights which can be propagated into deflection by $\boldsymbol{\Phi}_D \boldsymbol{\Sigma} \boldsymbol{\Phi}_D^\top$, where $\boldsymbol{\Phi}_D$ is the design matrix of deflection and $\boldsymbol{\Sigma}$ is the posterior covariance of weights. Aleatory corresponds to identified variance of noise. The standard deviation of estimated noise is 978.12 Nm.

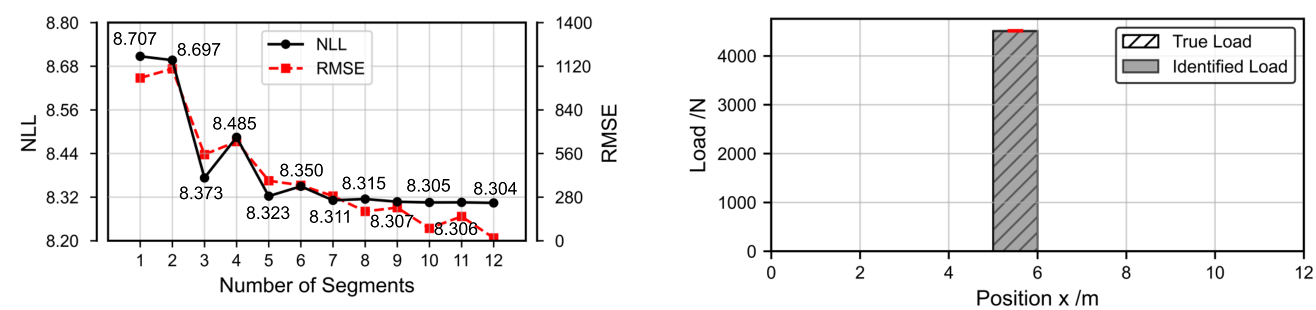


(a) Moment response reconstruction (b) Deflection response reconstruction

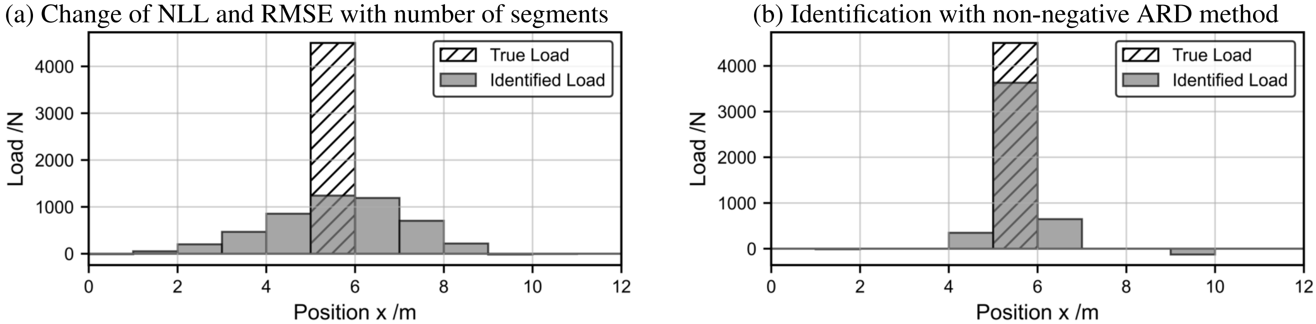
Fig. 6 Response reconstruction and uncertainty propagation under half span distributed load

Case 2: Concentrated load

Instead of a point load, we apply a uniformly distributed load over a 1-m segment to approximate a concentrated load. This setup enables a more direct and interpretable comparison between the true load and the identified load. As shown in Fig. 7(a),

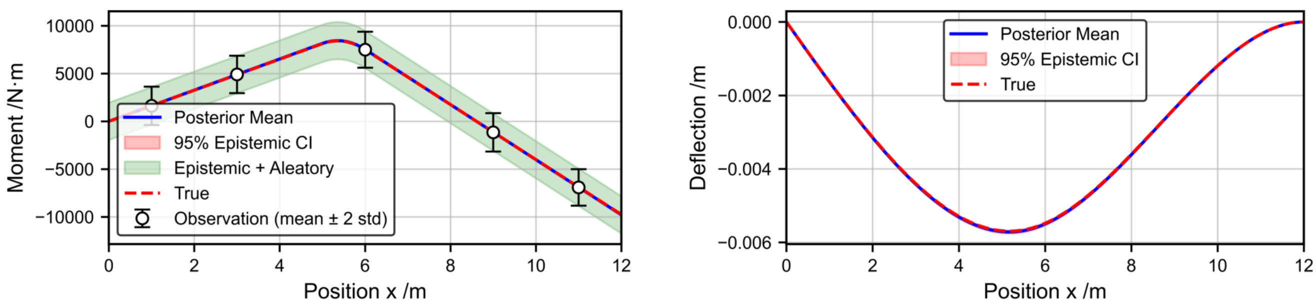


(a) Change of NLL and RMSE with number of segments (b) Identification with non-negative ARD method



(c) Identification with ℓ_2 regularization (d) Identification with ℓ_1 regularization

Fig. 7 Load identification results under concentrated load both NLL and RMSE attain their minimum value when the number of segments is 12. In Fig. 7(b), a sparse solution with estimated uncertainty in weights are obtained by NN-ARD method. Identification with ℓ_2 regularization tends to present a smooth solution, while a sparse solution is given under ℓ_1 regularization. The reconstructed response of bending moment and deflection and their propagated uncertainties are shown in Fig. 8.



(a) Moment response reconstruction (b) Deflection response reconstruction

Fig. 8 Response reconstruction and uncertainty propagation under concentrated load

Thanks to the accurate load estimation, the structural response can be reconstructed with high fidelity. As shown in Fig. 8(b), the posterior mean of the deflection almost coincides with the true response, resulting in a very narrow 95% credible interval for epistemic uncertainty. The standard deviation of estimated noise in this case is 977.88 Nm.

Conclusion

The proposed non-negative ARD (NN-ARD) method accurately identifies both the magnitude and location of loads while quantifying their uncertainties and eliminating the negative weights that often arise with conventional regularization methods. Given observed and estimated responses, the negative log-likelihood (NLL) criterion enables effective model selection by maximizing fitting likelihood. Moreover, full-field structural responses can be reconstructed with high fidelity from the response design matrix and the inferred weights, with proper propagation of uncertainty to the reconstructed responses. Future work will apply this method to real structures for in-situ load identification and full-field response reconstruction.

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