



Chapter 12

Evaluating Pseudo Forces Transferability in Transfer Path Analysis

Jacopo Brunetti, Walter D'Ambrogio, Annalisa Fregolent, and Andrea Soleri

Abstract Pseudo Force Transfer Path Analysis (TPA) is a key method within Component-Based TPA that uses a set of Pseudo Forces to reconstruct the vibratory response of a source. The mechanical system to be studied is divided into two parts: a receiving subsystem and a source subsystem, where the actual vibrational source is placed. All Component-Based TPA methods are characterized by the assumption that the forces used to reconstruct the vibrational response depend solely on the source subsystem. However, there is considerable debate in the literature regarding the transferability of Pseudo Forces and, thus, their true independence from the receiving subsystem. In practice, especially in real structures, the conditions under which this transferability holds are not well established. This work investigates the independence of Pseudo Forces from the receiving subsystem using the Dynamic Substructure Technical Division's Round Robin test bed. The goal is to provide guidelines for Pseudo Force transferability by analyzing Transfer Validation error and a new metric called Transferability Residue, using numerical and experimental results, and studying the effect of the noise in the data.

Keywords Transfer Path Analysis · Pseudo-Forces · Transferability · Inverse Source Estimation · Dynamic Substructuring

Introduction

The Pseudo Force Transfer Path Analysis (TPA) method was first proposed by Janssens and Verheij [1]. It consists of identifying unknown vibrational sources by reconstructing them using Pseudo Forces acting on the outer surface of a source. These Pseudo Forces are determined using an inverse method [2], based on the knowledge of the mechanical system's dynamics and its response at specific degrees of freedom (DoFs). Pseudo force TPA is historically considered part of the broader family of methods known as Component-Based TPA [3]. This family of methods is characterized by a series of forces, which reconstruct the true source action, that depend solely on the source itself. Therefore, such methods can be widely used in Virtual Prototyping [4–6] as the response of different mechanical systems to the same source can be studied with the same forces, without having to re-evaluate them.

Over the years, however, considerable debate has arisen in the literature regarding the actual transferability of Pseudo Forces. In the original papers by Janssens and Verheij [1, 7, 8], as well as in later work by other authors [3, 9, 10], it is stated that Pseudo Forces are solely source-dependent. However, some authors [11–13] have argued that Pseudo Forces are generally not transferable from one mechanical system to another except under certain unspecified conditions. Others [14] have stated that Pseudo Forces are not transferable at all. Janssens himself later expressed some doubt about the transferability of Pseudo Forces in [15], stating that the identification of a system's response may vary when the boundary conditions of the source change.

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The objective of this paper is to analyze the conditions for which the transferability of Pseudo Forces is theoretically verified, and to apply the method to a real structure. The study further evaluates the actual transferability through the Transfer Validation error and a new metric called the Transferability Residue.

Theoretical Background

Pseudo Force TPA

As depicted in Figure 1a, we consider an assembly of two linear, time-invariant mechanical subsystems coupled at the interface. Subsystem A acts as a source, with an excitation f_1 ¹, while subsystem B has no external excitations and represents the receiver substructure, where the responses u_3 , generated by the excitation f_1 , are measured.

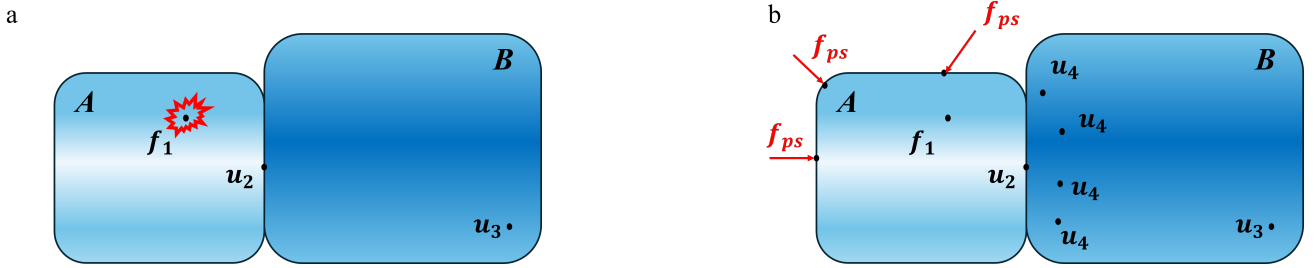


Fig. 1 (a) System with active vibrations source f_1 (b) System with the original excitation shut down reconstructed by Pseudo Forces f_{ps}

If f_1 was known, the response at the critical DoFs, u_3 , would be identified using the frequency response matrix $\mathbf{Y}_{31}^{AB}$, which relates the response at the critical DoFs to the known excitation.

$$u_3 = \mathbf{Y}_{31}^{AB} f_1 \quad (1)$$

However, excitations are often difficult to measure, especially when they act on the internal DoFs of the source subsystem. To overcome this limitation, the actual excitation is reconstructed using a set of Pseudo Forces f_{ps} acting on the outer surface of the source subsystem, as shown in Figure 1b. Pseudo Forces are identified by an overdetermined [16] inverse method, based on the knowledge of the system responses to the actual excitation in specific indicator DoFs, u_4 , located in the receiver subsystem near the interface [17, 18], and the frequency response matrix $\mathbf{Y}_{4ps}^{AB}$, which relates these indicator responses to the action of the Pseudo Forces:

$$f_{ps} = (\mathbf{Y}_{4ps}^{AB})^+ u_4 \quad (2)$$

Where the superscript + denotes a Moore-Penrose pseudo inverse [19]. These Pseudo Forces must completely describe the real source, meaning that if both the Pseudo Forces, in opposite verses, and the actual excitation act simultaneously on the source subsystem, the resulting response at any DoF in the receiver subsystem must be zero. If this condition is satisfied, then the Pseudo Forces can be used to predict the response of the system at the critical DoFs:

$$u_3 = \mathbf{Y}_{3ps}^{AB} f_{ps} \quad (3)$$

Pseudo Forces Transferability

As seen in the introduction, there is considerable discrepancy in the literature regarding the independence of Pseudo Forces from the receiving subsystem. Some papers have attempted to identify conditions under which this independence is verified.

In [10] is shown that if the response of the indicator DoFs is correctly reconstructed via Pseudo Forces such that:

$$u_4 f_1 = u_4 f_{ps} \quad (4)$$

¹ All frequency dependencies are omitted for better readability

then it can be inferred from the frequency based dynamic substructuring (FBS) that the Pseudo Forces are solely dependent on the source substructure and the original source:

$$\mathbf{f}_{ps} = (\mathbf{Y}_{2ps}^A)^+ \mathbf{Y}_{21}^A \mathbf{f}_1 \quad (5)$$

In [3] a similar condition is derived by rewriting the Eq. (4). From FBS it is true that:

$$\mathbf{Y}_{42}^{AB} (\mathbf{Y}_{22}^A + \mathbf{Y}_{22}^B) \mathbf{u}_{2,\mathbf{f}_1}^{\text{free}} = \mathbf{Y}_{42}^{AB} (\mathbf{Y}_{22}^A + \mathbf{Y}_{22}^B) \mathbf{u}_{2,\mathbf{f}_{ps}}^{\text{free}} \quad (6)$$

Thus, the only way in which Eq. (4) can be satisfied is if:

$$\mathbf{u}_{2,\mathbf{f}_1}^{\text{free}} = \mathbf{u}_{2,\mathbf{f}_{ps}}^{\text{free}} \quad (7)$$

The condition in Eq. (7) guarantees the independence of Pseudo Forces from the receiver subsystem, since it can be proven, from FBS, that it is equivalent to Eq. (5).

If Pseudo Forces are indeed independent of the receiver subsystem, they can be used to predict the response of the same source subsystem when coupled to different receiver subsystems, and thus Pseudo Forces can be transferred.

Therefore, by taking two different receiving subsystems, B and C, and assembling them to the same source subsystem, the set of Pseudo Forces identified on the two assemblies should be equivalent. When these sets are used to identify the response of the same mechanical system, for example the one with receiving subsystem B, they should yield identical results. Identifying the response in the critical DoFs we obtain:

$$\mathbf{u}_{3\mathbf{f}_{ps}}^{AB} = \mathbf{Y}_{3ps}^{AB} \mathbf{f}_{ps}^B \quad (8)$$

$$\mathbf{u}_{3\mathbf{f}_{ps}}^{AB} = \mathbf{Y}_{3ps}^{AB} \mathbf{f}_{ps}^C \quad (9)$$

Where $\mathbf{f}_{ps}^B$ and $\mathbf{f}_{ps}^C$ denote the Pseudo Forces identified on the assemblies with receiving subsystem B and C, respectively. Subtracting Eq. (8) and Eq. (9) we obtain:

$$\mathbf{u}_{3\mathbf{f}_{ps}}^{AB} - \mathbf{u}_{3\mathbf{f}_{ps}}^{AB} = \mathbf{Y}_{3ps}^{AB} (\mathbf{f}_{ps}^B - \mathbf{f}_{ps}^C) \quad (10)$$

If transferability between the two assemblies is achieved, the left-hand side of Eq. (10) must be zero, since $\mathbf{u}_{3\mathbf{f}_{ps}}^B = \mathbf{u}_{3\mathbf{f}_{ps}}^C$. If, for some reason, Pseudo Forces cannot be perfectly transferred between the two systems, a non-zero residue term is obtained that we can call Transferability Residue. The higher the residue value, the less transferable the Pseudo Forces are from the system with receiving subsystem C to the system with receiving subsystem B. This residue can also be evaluated on the indicator DoFs:

$$\mathbf{u}_{4\mathbf{f}_{ps}}^{AB} - \mathbf{u}_{4\mathbf{f}_{ps}}^{AB} = \mathbf{Y}_{4ps}^{AB} (\mathbf{f}_{ps}^B - \mathbf{f}_{ps}^C) \quad (11)$$

This latter term will be used throughout the remainder of this paper, linking it to Transfer Validation error, in order to effectively investigate transferability between multiple mechanical systems.

Test Structure

The structure studied is the Round Robin system of the Dynamic Substructures Technical Division of the Society for Experimental Mechanics [20]. Two versions of the system are used: the Thick Wing system, where the wing is an aluminum plate 0.25" thick, and the Thin Wing system, with the wing 0.125" thick. From these two configurations, another mechanical systems can be obtained, since these wings can be assembled to the frame using different number of bolts: a Thin Wing with One Bolt assembly.

The systems are excited by a force acting on the frame, in Figure 2 called Input, which acts as the source subsystem. Six Pseudo Forces are used to reconstruct this force, which is assumed to be unknown, distributed across the source subsystem's surface. The number of Pseudo Forces is chosen based on the hypothesis that the interface between the two subsystems behaves quasi-rigidly [3, 10]. The indicator responses are measured at four locations near the interface using triaxial accelerometers. This ensures that the inverse problem in Eq. (2) is overdetermined. The specific positions of the indicator sensors are determined using the effective independence (EFI) technique [21, 22]. The locations of each DoF is shown in Figure 2. Pseudo Forces are identified in all the systems, and used to evaluate the response at a critical DoF near one of the wing edges.

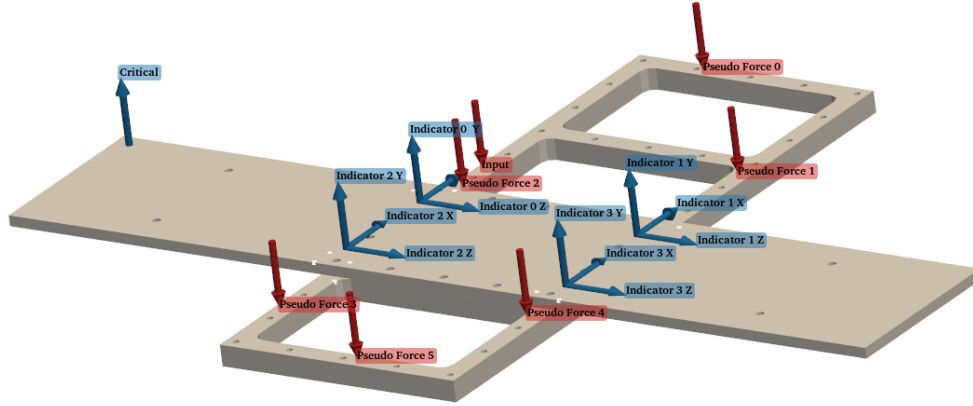


Fig. 2 Positions of input DoFs (red) and response DoFs (blue)

Results

The following sections are organized as follows: Pseudo Forces are identified on the three mechanical systems presented earlier. Then they are used to perform a response identification at the critical DoF, both on the system where they were identified (On-Board Validation) and on the other two (Transfer Validation). We then proceed with the calculation of Transferability Residues. Following Eq. (11), the AB system will be the one considered in each specific section, while the other two systems will have a different receiving subsystem indicated by C. The Normalized Root Square Error (NRSE) will also be computed, which returns a quadratic error value between the TPA identified response and the reference response. The error is computed at each frequency and normalized with respect to the maximum amplitude of the signal. Both of these metrics, which will be seen later to be related, can provide important information on the transferability of Pseudo Forces. Both numerical and experimental results are taken into account.

Numerical Results

Thick Wing Four Bolt Assembly

The numerical results of the identification in the system under consideration are shown in Figure 3.

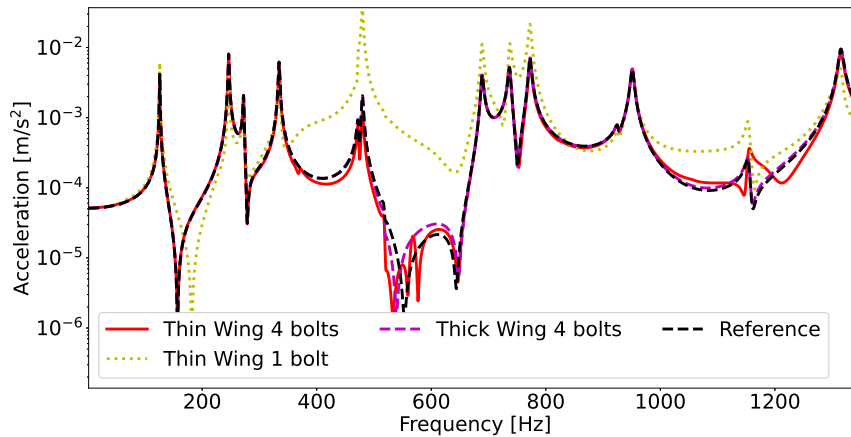


Fig. 3 Absolute value of the identified acceleration in the critical DoF (Thick Wing)

As can be observed, the identification of acceleration does not have the same accuracy for all sets of Pseudo Forces identified on the various systems considered. It can therefore be assumed that there is no perfect transferability between the Thick Wing system and the other mechanical systems. Following Eq. (11), we must expect, for every set of Pseudo Forces, the presence of a non-zero residue. Computing all the residues we obtain the results shown in Figure 4.

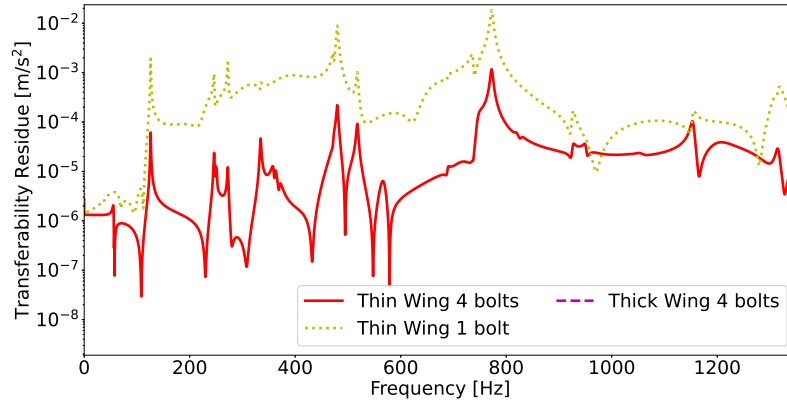


Fig. 4 Absolute value of the sum of Transferability Residues of each mechanical system (Thick Wing)

As we expected the residues are all non-zero, apart from the residue of the Thick Wing system, since in this case the two sets of Pseudo Forces on the right of Eq. (11) are the same. All the residues tend to be greater when the Pseudo Forces identified on the specific system are poorly transferred to the Thick Wing system. In fact, the values of the residues and those of the NRSE, shown in Figure 5, have similar trend.

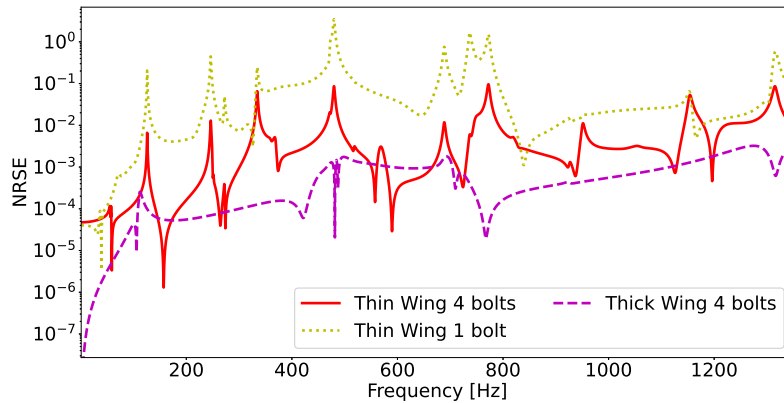


Fig. 5 Absolute value of the NRSE of acceleration identification in the critical DoF for each set of Pseudo Forces (Thick Wing)

Both metrics indicate that the best identification is obtained in the case of the On-Board Validation (purple), while the worse is obtained in the case of the Transfer Validation between the Thin Wing system with one bolt connection (yellow) and the system considered.

It can be concluded that, although the source subsystem is the same, the Pseudo Forces identified on some mechanical systems have poor transferability to the Thick Wing system.

Thin Wing Four Bolt Assembly

Numerical identification results and residues are shown respectively in Figures 6 and 7.

Analysis of the residues shows that, even in this case, the transferability of the various sets of Pseudo Forces seems to differ depending on the mechanical system in which they are identified. Also, the values of the residues of the Pseudo Forces

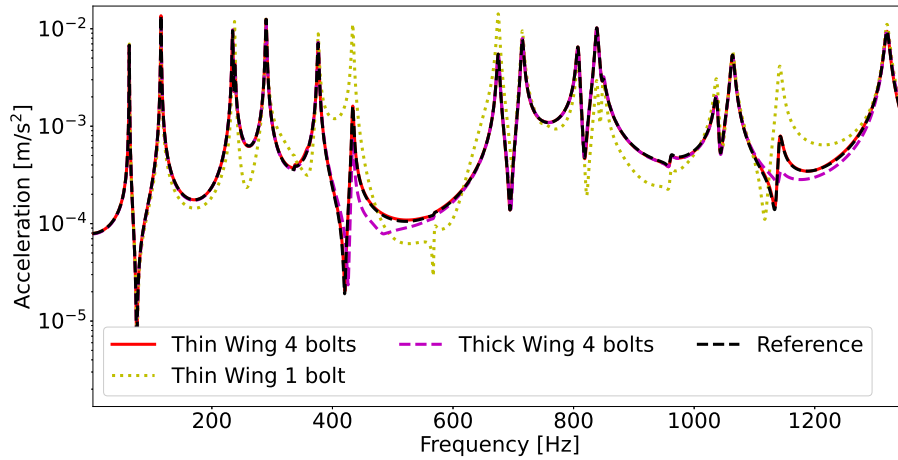


Fig. 6 Absolute value of the identified acceleration in the critical DoF (Thin Wing Four Bolts)

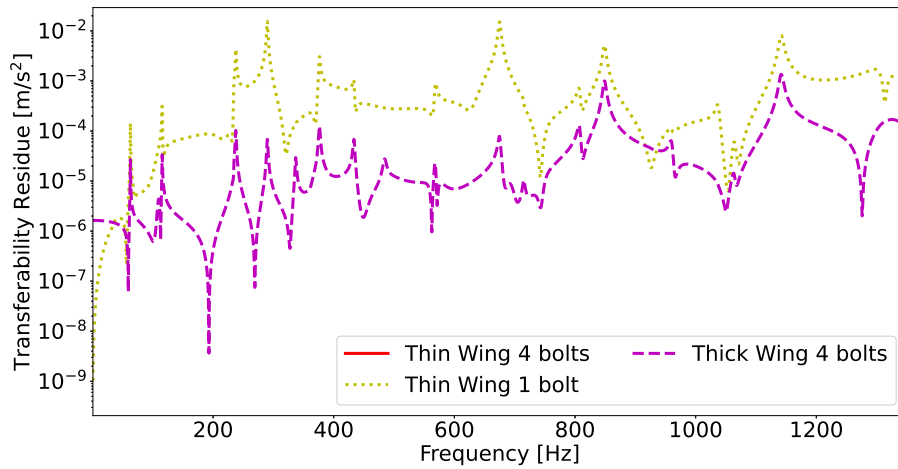


Fig. 7 Absolute value of the sum of Transferability Residues of each mechanical system (Thin Wing Four Bolts)

identified in the Thick Wing system differ from the values of the residues of the Pseudo Forces identified in the Thin Wing system with four bolt connection, that were seen in the previous section. Therefore, the transferability of the Pseudo Forces from the Thick Wing system to the Thin Wing system with four bolt connection is not equal to the transferability obtained in the opposite direction. In both this case and the previous one, the Pseudo Force sets with lower residues are those identified on the system that share interface conditions with the system under consideration.

Thin Wing One Bolt Assembly

Numerical identification results and residues are shown respectively in Figures 8 and 9.

Both the Thick Wing and Thin Wing with four bolt connection systems have absolute values of the residues that are very similar to each other. This similarity reflects the fact that the two systems share interface conditions, which are both different from those of the system under consideration. The relationship between Pseudo Force transferability and interface conditions is evident.

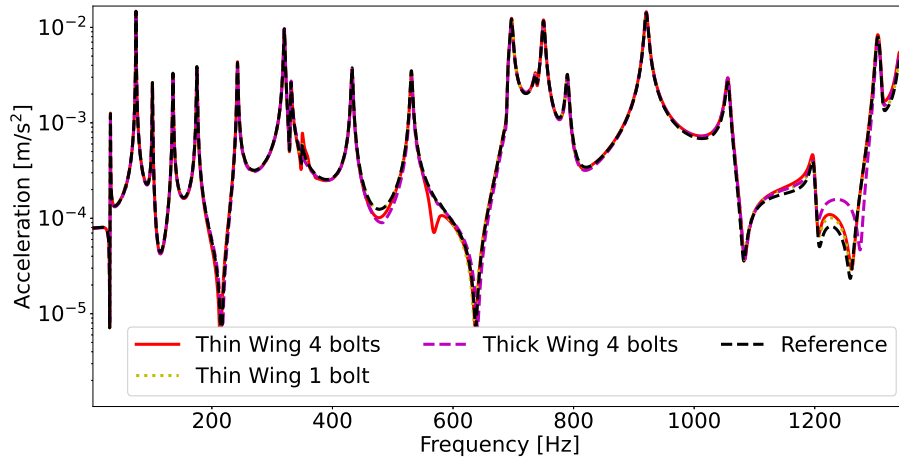


Fig. 8 Absolute value of the identified acceleration in the critical DoF (Thin Wing One Bolt)

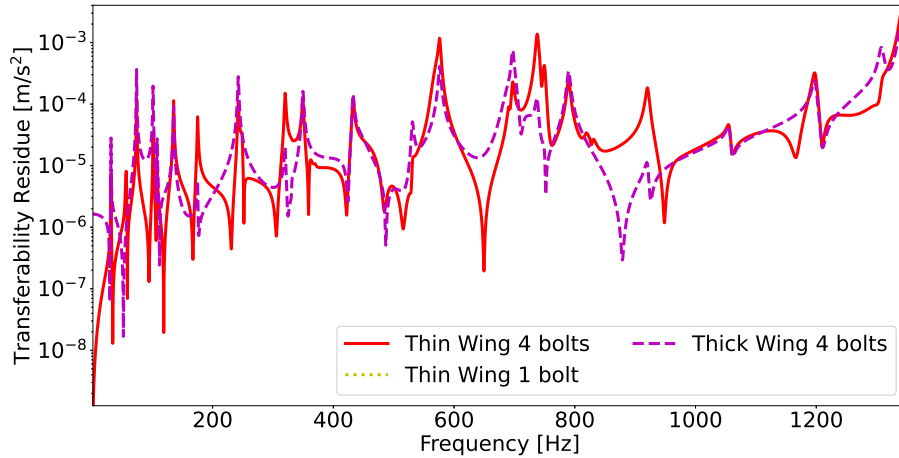


Fig. 9 Absolute value of the sum of Transferability Residues of each mechanical system (Thin Wing One Bolt)

Experimental Results

All results presented in the following sections are obtained by performing a Truncated Singular Value Decomposition (TSVD) [23] on the frequency response matrix in Eq. (2), in order to reduce noise effects. By observing the jump between singular values, only one singular value needed to be truncated.

Thick Wing Four Bolt Assembly

Experimental identification results in the system under consideration are shown in Figure 10.

Residue values are shown in Figure 11, while the identification error is shown in Figure 12. As already observed in the numerical results, residues and identification errors are closely related.

The residues obtained using experimental data are much closer than those obtained using numerical data (see Figure 4), making it difficult to determine which system transfers Pseudo Forces more effectively. The Thin Wing with one bolt connection system exhibits a much better transferability than hypothesized. Since the Thin Wing with four bolt connection and Thin Wing with one bolt connection systems share similar geometric characteristics, it can be inferred that also those play an important role in Pseudo Force transferability.

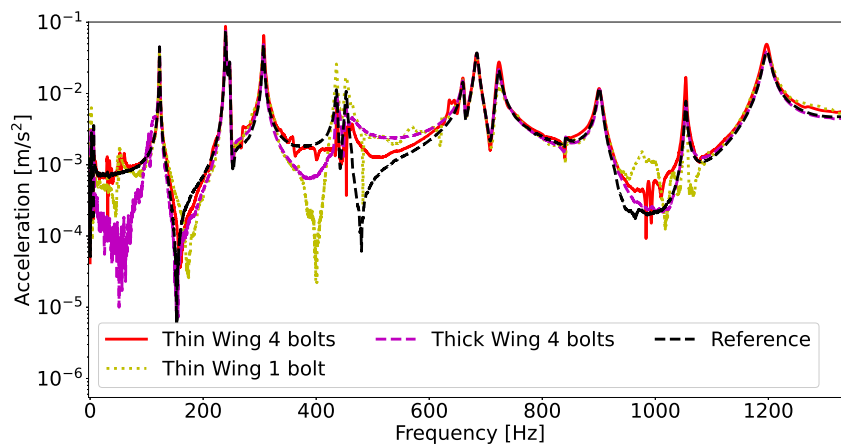


Fig. 10 Absolute value of the identified acceleration in the critical DoF (Thick Wing)

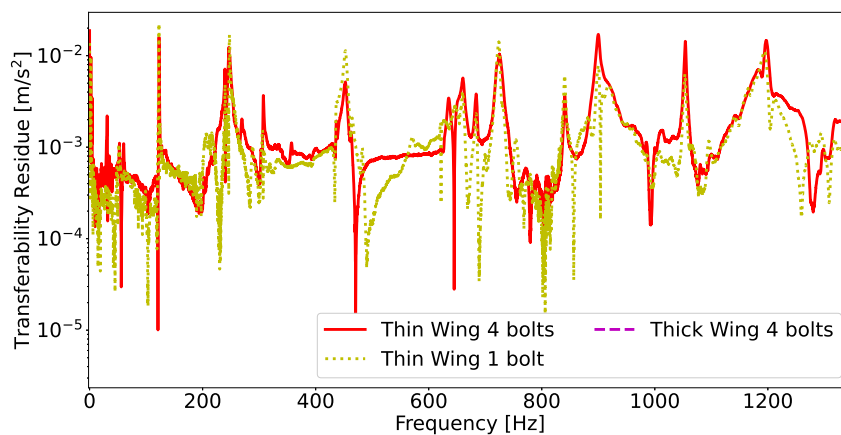


Fig. 11 Absolute value of the sum of Transferability Residues of each mechanical system (Thick Wing)

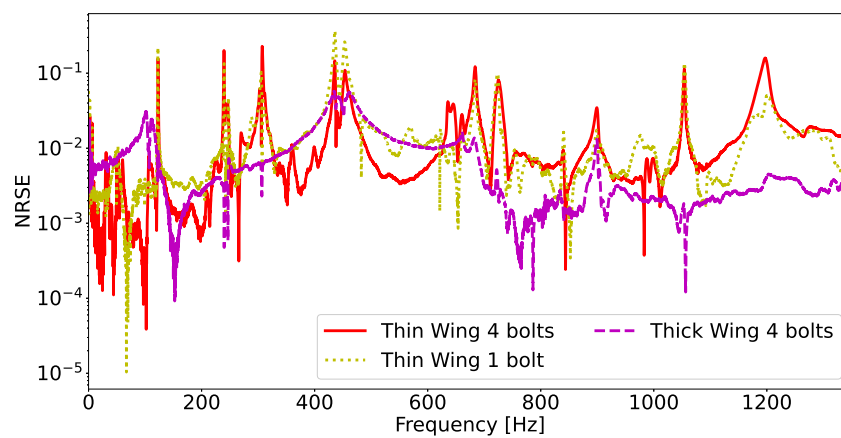


Fig. 12 Absolute value of the NRSE of acceleration identification in the critical DoF for each set of Pseudo Forces (Thick Wing)

Thin Wing Four Bolt Assembly

Experimental identification results and residues are shown respectively in Figures 13 and 14.

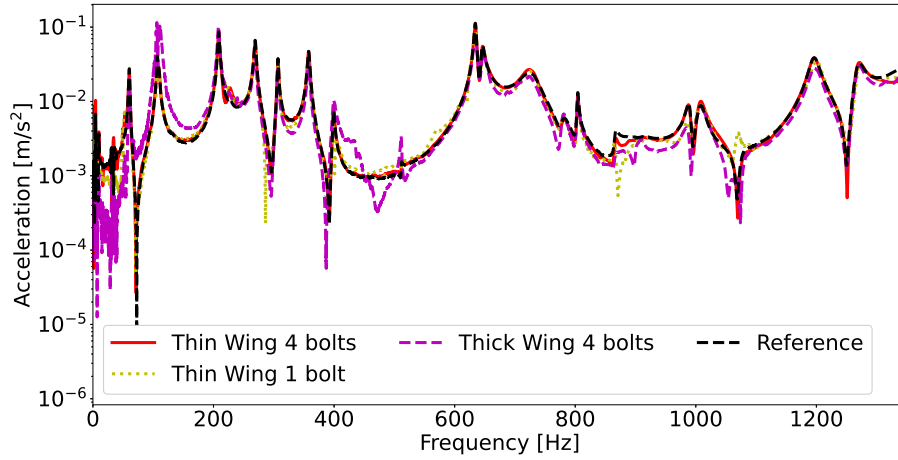


Fig. 13 Absolute value of the identified acceleration in the critical DoF (Thin Wing Four Bolts)

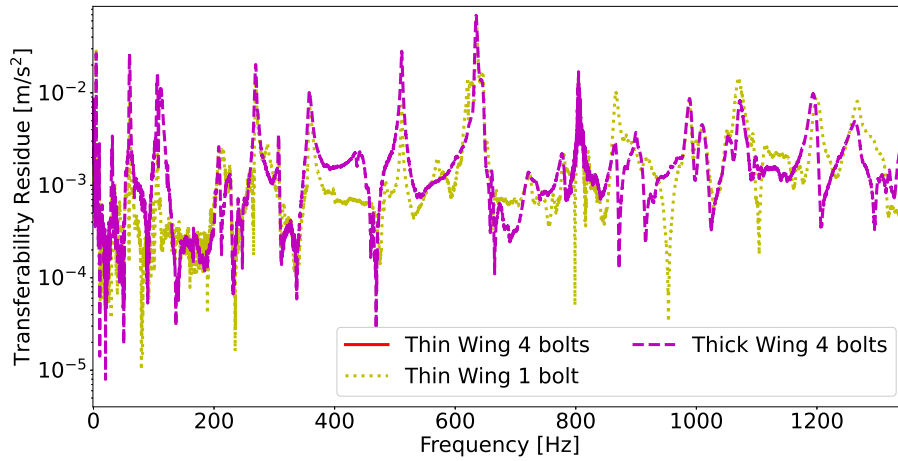


Fig. 14 Absolute value of the sum of Transferability Residues on each mechanical system (Thin Wing Four Bolts)

The residues are very close to each other. The Thin Wing with one bolt connection system (yellow) seems to have a better transferability especially at the lower frequencies, while the residue of the Thick Wing system (magenta) seems to get lower at the higher frequencies. It seems that both similarities in geometry, at lower frequencies, and interface conditions, at higher frequencies, play a role in the transferability of Pseudo Forces.

Thin Wing One Bolt Assembly

Experimental identification results and residues are shown respectively in Figures 15 and 16.

Here, the residues exhibit trends consistent with the numerical analysis (see Figure 9). Although the values are close, geometric similarities between the thin Wing one bolt connection and Thin Wing four bolt connection systems again appear to influence transferability at lower frequencies. Notably, the latter system exhibits lower residuals than the Thick Wing system, particularly below 400 Hz.

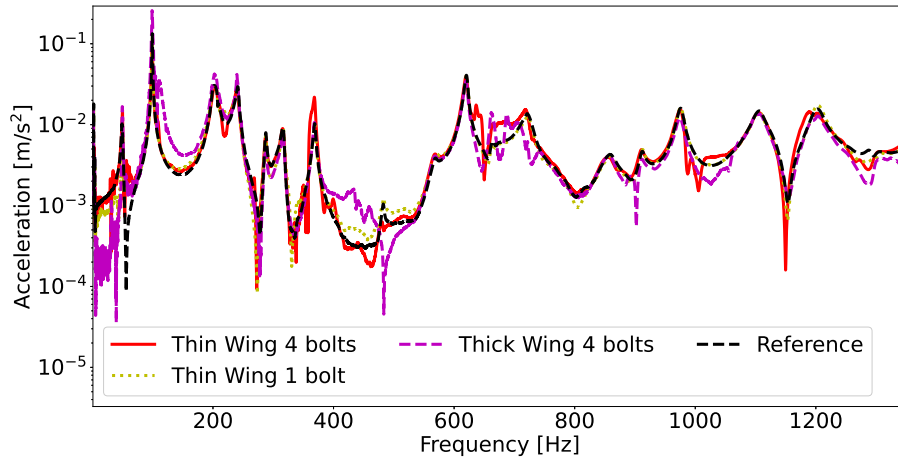


Fig. 15 Absolute value of the identified acceleration in the critical DoF (Thin Wing One Bolt)

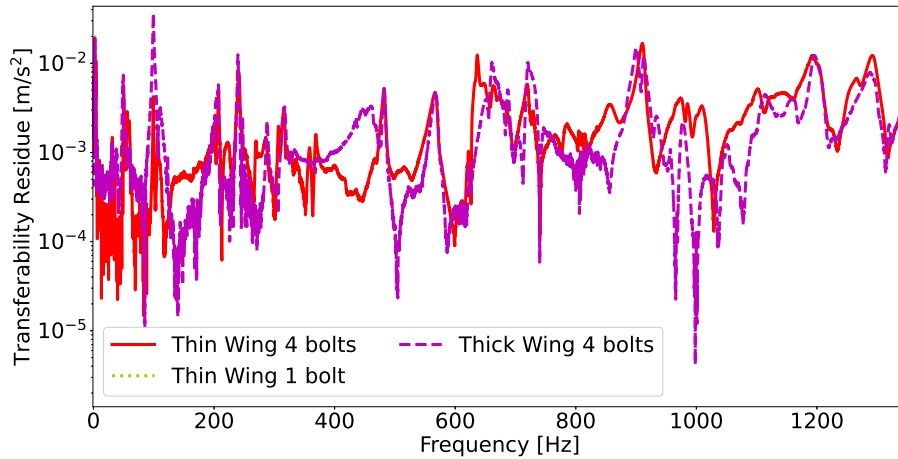


Fig. 16 Absolute value of the sum of Transferability Residues of each mechanical system (Thin Wing One Bolt)

Remarks

The following considerations can be drawn from the results presented:

- Pseudo Forces do not appear to be independent of the receiver subsystem: identifying them on one mechanical system does not generally yield the same results as identifying them on another system with the same source.
- The transferability of a set of Pseudo Forces from one system to a second system with a different receiver is not generally the same as the transferability in the opposite direction.
- In general, Pseudo Force TPA provides the most accurate response identification for a system when the Pseudo Forces are identified on that system itself. Thus, On-Board Validation typically yields a lower error than Transfer Validation.
- From the experimental results, the differences in transferability between the various systems are far less pronounced than in the numerical study. In particular, the Thin Wing system with a single connection exhibits much lower Transferability Residues than predicted numerically.
- Interface similarities appear to be crucial for transferability in numerical studies, while in the experimental study, geometric similarities also seem to play a significant role, especially at lower frequencies.

It is important to note that all sets of Pseudo Forces considered in the previous sections complied with the condition of Eq. (4). Consequently, Eq. (5) should theoretically have been valid and therefore these sets should have been independent of the receiving subsystem and thus completely transferable.

Conclusions

This paper investigates the transferability of Pseudo Forces.

The definition of Transferability Residue simplifies the analysis of Pseudo Force transferability compared to the error on Transfer Validation, since it is not necessary to know a reference response.

Thanks to this metric, transferability was studied in three mechanical systems. The results show that the transferability of Pseudo Forces varies between the various cases presented. In the numerical study interface similarities appear to play a crucial role in the transferability of Pseudo Forces. In the experimental study, however, it is also noted that geometric similarities have an effect on transferability, especially at low frequencies.

Transferability of Pseudo Forces between two mechanical systems is not guaranteed, and seems to depend on the degree of similarity between the systems in question. Further studies on other mechanical systems with different interface and geometric conditions are necessary to confirm these findings.

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