Chapter 2
Modal Coupling of Systems with Nonlinear Connections

M. Sezer Camcı, M. Bülent Özer, and H. Nevzat Özgüven

Abstract Substructuring has an important role in structural dynamics, offering benefits such as reduced computational cost in theoretical applications and the ability to incorporate experimental data. While established methods exist for linear substructuring, effective techniques for nonlinear substructuring are limited and relatively recent. This paper revisits the Nonlinear Modal Coupling (NLMC) Method, developed to compute the harmonic response of nonlinear systems consisting of linear substructures connected by nonlinear elements. As one of the early applications of quasi-linearization in nonlinear structural dynamics, the method employs Describing Functions to calculate the response amplitude-dependent Nonlinearity Matrix, which is then incorporated into the modal coupling equations. Linear substructures are modeled using modal parameters, enabling direct use of experimentally identified modal data. The resulting system of nonlinear algebraic equations in modal domain is solved iteratively, with the Nonlinearity Matrix numerically calculated at each step. The method assumes that sub- and super-harmonics are negligible, and that harmonic excitation results in harmonic response—an assumption valid for most practical applications. The method is validated on both a multi-degree-of-freedom discrete system and a continuous system modeled using finite elements. Harmonic responses obtained from the NLMC method are compared with results of the frequency- and time-domain simulations of the assembled system in physical coordinates, demonstrating the accuracy and efficiency of the method.

Keywords Nonlinear substructuring · Nonlinear coupling · Nonlinear modal coupling · Nonlinear dynamics · Nonlinear connections

Introduction

Substructuring techniques are widely used in structural dynamics, allowing complex systems to be divided into smaller, more manageable subsystems. These methods significantly reduce computational complexity, facilitate the reuse of previously validated subsystem models, and enable hybrid modeling by integrating analytical, numerical, and experimental domains. For linear dynamic systems, various substructuring strategies, such as component mode synthesis (CMS) [1] and frequency based substructuring [2], have reached maturity, supported by extensive theoretical foundations and numerous practical implementations [3, 4, 5]. However, many real-world engineering structures which can assumed to be linear are physically connected to each other with components containing inherent nonlinearities— including friction, clearance, or nonlinear stiffness—that violate linear assumptions of the assembled structure and introduce amplitude-dependent behavior. These nonlinear effects challenge the applicability of traditional substructuring techniques, creating a need for more robust methods that can accurately and efficiently capture the dynamic interactions between coupled subsystems.

In nonlinear substructuring with rigid or fixed interfaces, coupling between components is assumed to enforce strict kinematic continuity, with nonlinear effects confined to the substructures. A frequency response function (FRF)-based coupling methodology was developed for systems where a subsystem with a local nonlinear element is connected to a linear subsystem at a single degree of freedom [6]. This technique incorporates describing functions to handle the nonlinearities

M. Sezer Camcı · M. Bülent Özer · H. Nevzat Özgüven
Department of Mechanical Engineering Middle East Technical University, 06800 Ankara, Türkiye
e-mail: camci.sezer@metu.edu.tr; ozerb@metu.edu.tr; ozguven@metu.edu.tr

M. Sezer Camcı
REHİS Division ASELSAN Inc., 06830 Ankara, Türkiye
e-mail: camci.sezer@metu.edu.tr

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and is particularly suited for systems with localized nonlinear elements and rigid connections. In a later study, a nonlinear normal mode (NNM) approach was proposed to facilitate substructuring in nonlinear systems [7], and further advances introduced a nonlinear modal substructuring technique compatible with geometrically nonlinear finite-element models [8]. A hybrid framework, which extends a foundational linear structural modification technique based on FRFs [9] was presented, integrating FRFs from a linear substructure with system matrices from a nonlinear counterpart [10]. This method uses an iterative process based on the original FRF approach to model a nonlinear system connected to a linear system rigidly or via a nonlinear interface. A finite-element-driven nonlinear modal coupling approach was developed specifically for systems exhibiting geometric nonlinearities [11]. Complementary work applied a Rubin-based substructuring method enhanced with modal derivatives to handle geometrically nonlinear, flexible multibody systems [12].

A model reduction strategy was developed that extends traditional linear modal bases by incorporating discontinuous interface modes to effectively capture localized nonlinear behaviors [13]. Building on this, a hybrid reduction technique tailored for mistuned bladed disks featuring dry friction interfaces was proposed, enabling accurate prediction of their nonlinear forced response [14]. A coupling approach designed for substructures linked via nonlinear connections was introduced [15], while a global reduction method that embeds localized nonlinear interface characteristics within broader system-level models was formulated [16]. This coupling methodology was subsequently validated through experiments using modal coordinates [17]. A contact-focused reduction method based on massless boundary component mode synthesis was introduced, specifically aimed at elastodynamic problems [18]. Most recently, a frequency-domain substructuring approach was presented, combining nonlinear connection models with linear FRFs to offer computationally efficient formulations for systems characterized by localized nonlinear interfaces [19].

A promising direction within this field is modal substructuring, where each subcomponent is represented using a reduced set of vibration modes. One of the early and promising contributions to nonlinear modal substructuring is the NLMC method proposed by Cömert and Özguven [20]. This method, developed for linear substructures connected by nonlinear elements, uses describing functions to represent the nonlinear coupling forces, leading to a nonlinearity matrix that is dependent on response amplitude. The nonlinearity matrix is embedded in the modal equations, and the resulting nonlinear algebraic equations are solved iteratively under the assumption of harmonic excitation yields a harmonic response (that is, higher harmonics are negligible). In this study, the NLMC method is revisited and applied to both discrete systems and a continuous system modeled using the finite element method to compute the forced harmonic response under various nonlinear coupling conditions. Comparisons with full nonlinear time- and frequency-domain simulations are used to evaluate the accuracy, convergence behavior, and computational performance of the method. Results show that the NLMC approach remains a practical and effective tool for coupling systems with nonlinear connections.

Theory

The NLMC method proposed in [20] is revisited in this chapter.

Consider a harmonically excited nonlinear system consisting of two linear structures connected with a nonlinear connection, as shown in Figure 1.

Fig. 1 Two linear structures with a nonlinear connection

The differential equations of motion of the structurally damped substructures are given in the following equations

$$\boldsymbol{M}^A\ddot{\boldsymbol{u}}^A + i\boldsymbol{H}^A\boldsymbol{u}^A + \boldsymbol{K}^A\boldsymbol{u}^A + \boldsymbol{f}_N^A = \boldsymbol{f}^A = \boldsymbol{F}^A e^{i\omega t} \tag{1}$$

$$\boldsymbol{M}^B\ddot{\boldsymbol{u}}^B + i\boldsymbol{H}^B\boldsymbol{u}^B + \boldsymbol{K}^B\boldsymbol{u}^B + \boldsymbol{f}_N^B = \boldsymbol{f}^B = \boldsymbol{F}^B e^{i\omega t} \tag{2}$$

where $\boldsymbol{f}_N^A$ and $\boldsymbol{f}_N^B$ are the internal nonlinear forces of the assembly, but they are external forces to each sub-structure. Coordinate transformations using the mass-normalized modal matrices of the sub-structures, given in equations (3) and (4), can decouple the equations when each equation is pre-multiplied by the transpose of its modal matrix.

$$\boldsymbol{U}^A = \boldsymbol{\Phi}^A \boldsymbol{\eta}_0^A \tag{3}$$
$$\boldsymbol{U}^B = \boldsymbol{\Phi}^B \boldsymbol{\eta}_0^B \tag{4}$$

The uncoupled equations in the frequency domain will be as shown below (proportional damping is assumed)

$$-\omega^2 \boldsymbol{I}\boldsymbol{\eta}_0^A + i\left[\gamma_r^A\left(\omega_r^A\right)^2\right]\boldsymbol{\eta}_0^A + [(\omega_r^A)^2]\boldsymbol{\eta}_0^A + \left(\boldsymbol{\Phi}^A\right)^T \boldsymbol{F}_N^A = \left(\boldsymbol{\Phi}^A\right)^T \boldsymbol{F}^A \tag{5}$$
$$-\omega^2 \boldsymbol{I}\boldsymbol{\eta}_0^B + i\left[\gamma_r^B\left(\omega_r^B\right)^2\right]\boldsymbol{\eta}_0^B + [(\omega_r^B)^2]\boldsymbol{\eta}_0^B + \left(\boldsymbol{\Phi}^B\right)^T \boldsymbol{F}_N^B = \left(\boldsymbol{\Phi}^B\right)^T \boldsymbol{F}^B \tag{6}$$

where $\boldsymbol{\eta}_0^A$ and $\boldsymbol{\eta}_0^B$ are the amplitude vectors of modal coordinates, $\boldsymbol{F}_N^A$ and $\boldsymbol{F}_N^B$ are the amplitude vectors of internal nonlinear forces, $\boldsymbol{F}^A$ and $\boldsymbol{F}^B$ are the vectors of external force amplitudes, $\boldsymbol{I}$ is the identity matrix, and ω_r^A and ω_r^B denote the natural frequencies whereas γ_r^A and γ_r^B are the structural modal damping for subsystems A and B, respectively.

For ease of understanding the equations, the nodes of the linear structures are partitioned as shown in Figure 2.

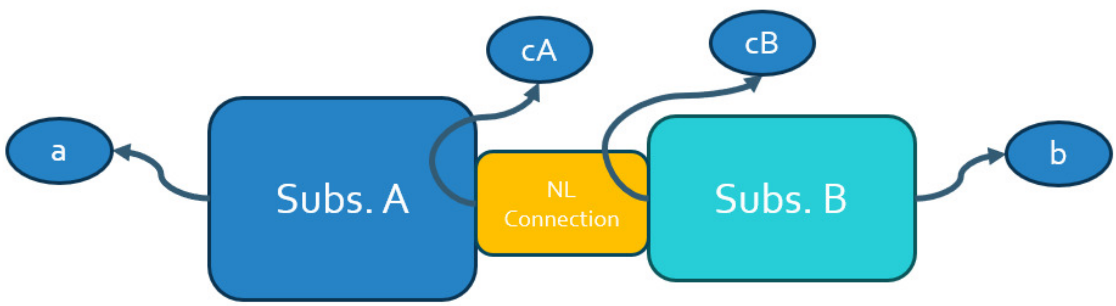

Fig. 2 Partitioned nodes of the nonlinear system

By defining the amplitude vector of the nonlinear joints forces as

$$\boldsymbol{F}_N = \begin{bmatrix} 0 \\ \boldsymbol{F}_{N_{cA}}^A \\ 0 \\ \boldsymbol{F}_{N_{cB}}^B \end{bmatrix} \tag{7}$$

equations (5) and (6) can be combined as follows

$$\begin{bmatrix} \left(1+i\gamma_r^A\right)\left(\omega_r^A\right)^2 - \omega^2 & 0 \\ 0 & \left(1+i\gamma_r^B\right)\left(\omega_r^B\right)^2 - \omega^2 \end{bmatrix} \boldsymbol{\eta}_0 + \dots$$
$$\dots + \begin{bmatrix} \left(\boldsymbol{\Phi}^A\right)^T & 0 \\ 0 & \left(\boldsymbol{\Phi}^B\right)^T \end{bmatrix} \boldsymbol{F}_N = \begin{bmatrix} \left(\boldsymbol{\Phi}^A\right)^T & 0 \\ 0 & \left(\boldsymbol{\Phi}^B\right)^T \end{bmatrix} \boldsymbol{F} \tag{8}$$

where $\boldsymbol{\eta}_0$ and $\boldsymbol{F}$ are the amplitude vectors of the modal coordinates and the vectors of external force amplitudes of the coupled system, respectively, as shown below

$$\boldsymbol{\eta_0} = \begin{bmatrix} \boldsymbol{\eta_0^A} \\ \boldsymbol{\eta_0^B} \end{bmatrix} \tag{9}$$
$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{F^A} \\ \boldsymbol{F^B} \end{bmatrix} \tag{10}$$

The equations in (8) can be coupled through the nonlinear joint forces given in equation (11).

$$\boldsymbol{F}_N = \begin{bmatrix} 0 \\ \boldsymbol{F}_{N_{cA}}^A \\ 0 \\ \boldsymbol{F}_{N_{cB}}^B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{\Delta}_{cA,cA} & 0 & \boldsymbol{\Delta}_{cA,cB} \\ 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{\Delta}_{cB,cA} & 0 & \boldsymbol{\Delta}_{cB,cB} \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_a \\ \boldsymbol{U}_{cA} \\ \boldsymbol{U}_b \\ \boldsymbol{U}_{cB} \end{bmatrix} = \boldsymbol{\Delta}'(\boldsymbol{U})\boldsymbol{U} \tag{11}$$

where coordinates a and b do not include the coupling coordinates, cA and cB, as can be seen from Figure 2, and the nonlinear joint forces are represented using Nonlinearity Matrix [21] as shown below

$$\boldsymbol{\Delta}\left(\boldsymbol{U}\right) = \begin{bmatrix} \boldsymbol{\Delta}_{cA,cA} & \boldsymbol{\Delta}_{cA,cB} \\ \boldsymbol{\Delta}_{cB,cA} & \boldsymbol{\Delta}_{cB,cB} \end{bmatrix} \tag{12}$$

The combined equations can be expressed in compact form by using the Nonlinearity Matrix as follows:

$$\begin{bmatrix} \left(1+i\gamma_r^A\right)\left(\omega_r^A\right)^2-\omega^2 & 0 \\ 0 & \left(1+i\gamma_r^B\right)\left(\omega_r^B\right)^2-\omega^2 \end{bmatrix} \boldsymbol{\eta}_0 + \dots$$
$$\dots + \boldsymbol{\Phi}^{\boldsymbol{T}} \boldsymbol{\Delta}'\left(\boldsymbol{U}\right) \boldsymbol{\Phi}\boldsymbol{\eta}_{\boldsymbol{0}} = \boldsymbol{\Phi}^T \boldsymbol{F} \tag{13}$$

where $\boldsymbol{\Phi}$ is

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}^A & 0 \\ 0 & \boldsymbol{\Phi}^B \end{bmatrix} \tag{14}$$

Then, the solution of equation (13) yields $\boldsymbol{\eta}_{\boldsymbol{0}}$, from which the response in physical coordinates can be found using equations (3) and (4) as follows

$$\boldsymbol{U} = \boldsymbol{\Phi}\boldsymbol{\eta}_0 = \begin{bmatrix} \boldsymbol{\Phi}^A & 0 \\ 0 & \boldsymbol{\Phi}^B \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_0^A \\ \boldsymbol{\eta}_0^B \end{bmatrix} \tag{15}$$

Note that the solution requires iteration, since the nonlinearity matrix is response amplitude dependent. However, the number of equations to be solved is very small (as many as the total number of modes considered in the analysis) regardless of the sizes of the substructures, as they are in the modal domain.

Numerical Validation

Discrete System

This section presents the validation of the NLMC method using both direct frequency-domain and time-domain simulations on discrete nonlinear systems. Two structurally similar systems, shown in Figure 3, each composed of 3-DOF viscously damped substructures coupled through a hardening cubic nonlinear spring, are analyzed. The frequency response of the coupled system is evaluated through both NLMC method and direct solution approaches.

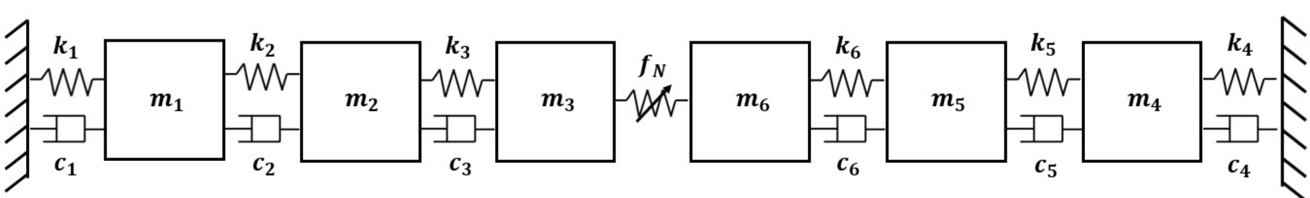

Fig. 3 Schematic diagram of the discrete system

The system parameters of each substructure are given as:

$$\boldsymbol{M}^A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} kg \quad \boldsymbol{K}^A = \begin{bmatrix} 11000 & -10000 & 0 \\ -10000 & 20000 & -10000 \\ 0 & -10000 & 10000 \end{bmatrix} N/m \quad \boldsymbol{C}^A = \begin{bmatrix} 11 & -10 & 0 \\ -10 & 20 & -10 \\ 0 & -10 & 10 \end{bmatrix} N.s/m$$

$$\boldsymbol{M}^B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} kg \quad \boldsymbol{K}^B = \begin{bmatrix} 11000 & -10000 & 0 \\ -10000 & 20000 & -10000 \\ 0 & -10000 & 10000 \end{bmatrix} N/m \quad \boldsymbol{C}^B = \begin{bmatrix} 11 & -10 & 0 \\ -10 & 20 & -10 \\ 0 & -10 & 10 \end{bmatrix} N.s/m$$

The nonlinearity is defined as an internal coupling force between the third and sixth coordinates using the Nonlinearity Matrix:

$$\boldsymbol{\Delta}\left(\boldsymbol{U}\right) = \begin{bmatrix} \nu_D & -\nu_D \\ -\nu_D & \nu_D \end{bmatrix} \tag{16}$$

where ν_D is the describing function for cubic stiffness, given as:

$$\nu_D = k_l + \frac{3}{4}k_c|u_3 - u_6|^2 \tag{17}$$

Here, $k_l = 500\ N/m$ and $k_c = 2.5x10^6\ N/m^3$ are the linear and cubic stiffness coefficients, respectively. u_3 and u_6 are the displacements of the coupled coordinates. The excitation is applied as a harmonic forcing of 1 N on coordinate 3. Firstly, the equations of motion of the substructures are transformed into the modal domain. Natural frequencies and normalized mode shapes are derived for each substructure. The system is then coupled via the describing function representation of the nonlinearity with the Nonlinearity Matrix using the NLMC method. The resulting coupled equations are solved iteratively using Newton's method with arc-length continuation to obtain the frequency response. Secondly, the coupled system is solved directly in the frequency domain without modal reduction. The nonlinearity is defined using the describing function, given in equation (17).

To further validate the method, the same discrete system is modeled in the ANSYS Mechanical finite element environment. Masses are treated as rigid bodies, and damping is incorporated using viscous elements proportional to stiffness. The nonlinear coupling is introduced using the 'COMBIN39' nonlinear spring element, with its force–displacement curve discretized in accordance with the hardening cubic stiffness relationship, shown in Figure 4. The system is excited harmonically in the frequency range of interest, using small frequency increments. At each frequency, time-domain simulations are carried out with carefully selected time steps and durations to ensure a steady-state response.

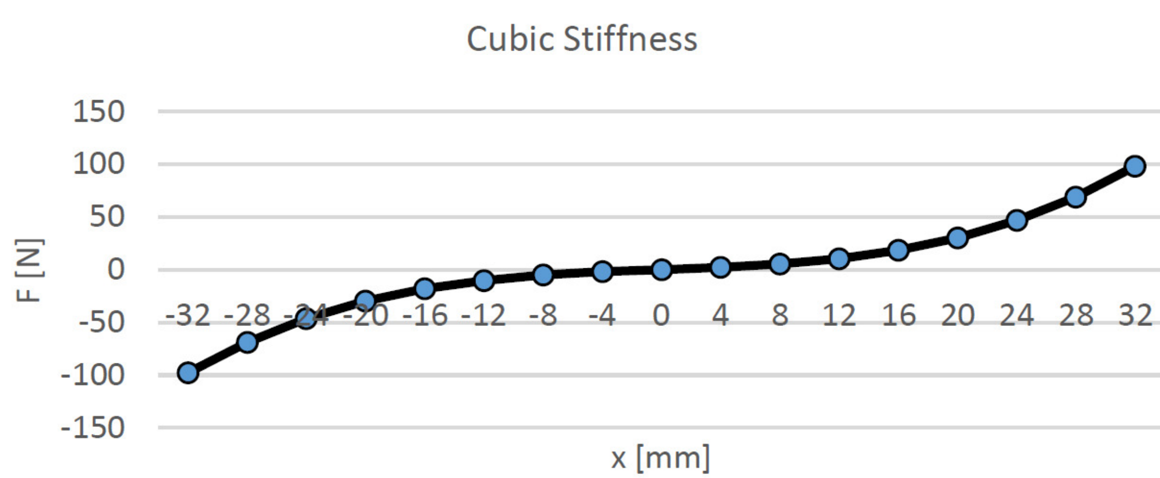

Fig. 4 Nonlinear coupling force-displacement curve

The results from the NLMC method are compared with the driving point frequency responses obtained from both the direct frequency-domain approach and time-domain simulations. As shown in Figure 5, the agreement among all three approaches is excellent, which demonstrates the accuracy of the NLMC method in predicting the nonlinear dynamic behavior of the systems coupled with nonlinear connection elements.

Continuous System

This section presents the validation of the NLMC method on a continuous nonlinear structure using both direct frequency-domain and time-domain simulations. The system consists of two axially vibrating bars, substructures A and B, with fixed supports, coupled by a hardening cubic stiffness element as shown in Figure 6. The system parameters are given in Table 1. A harmonic force of 80 N is applied in the middle of substructure A (at $l = 80\ mm$), and the response is taken from the

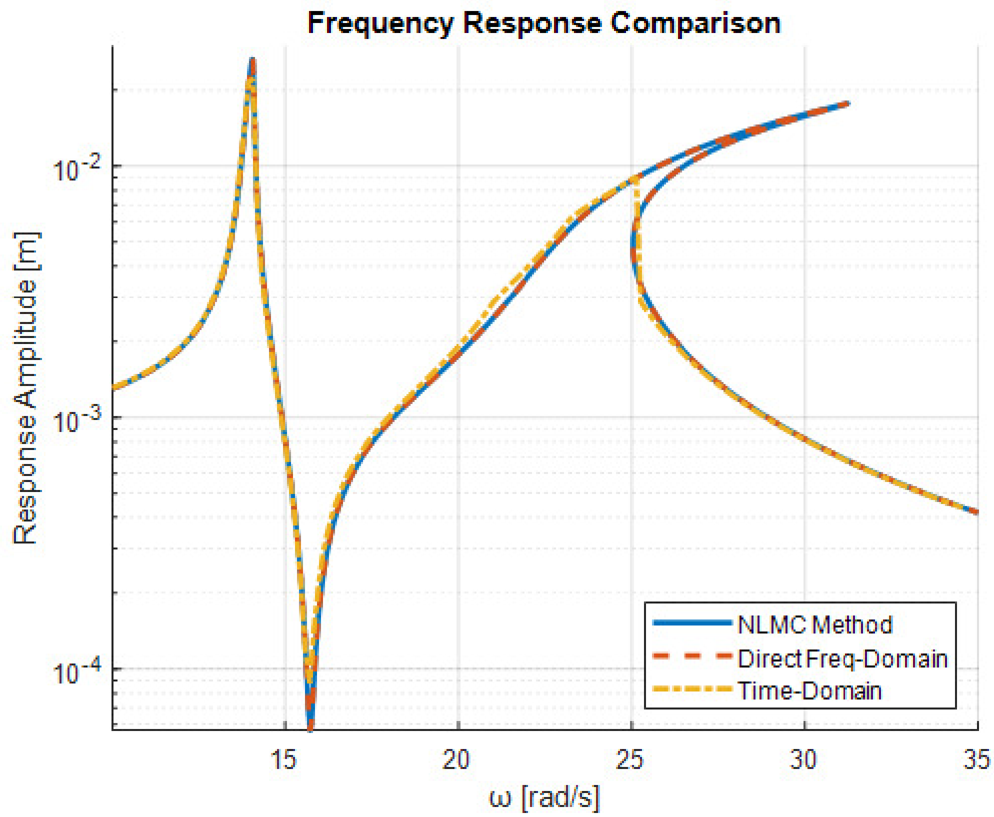

Fig. 5 Comparison of driving point frequency responses obtained using different approaches ($F = 1\ N,$ at coordinate 3)

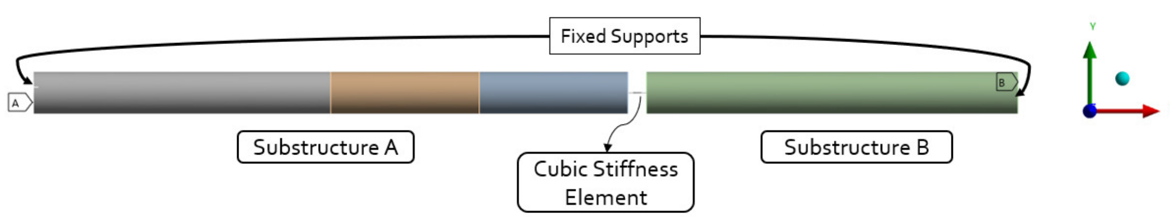

Fig. 6 Schematic diagram of the continuous system

cross-section at $l = 120\ mm$ as indicated in the schematic in Figure 7. The frequency responses of the coupled system are evaluated via the NLMC method and direct solution approaches.

The finite-element model employs linear solid elements with a nominal mesh size of 10 mm and an axisymmetric sweep method; mesh details are shown in Table 2. Viscous damping is included as a proportional damping with constant $\beta = 1.3x10^{-6}\ s$. Mass and stiffness matrices containing the DOFs of each substructure are exported from ANSYS Mechanical and used in the analytical studies.

Table 1 System parameters of the continuous system

	Substructure A	Substructure B
Elastic Modulus $[GPa]$	2	7
Density $[kg/m^3]$	1100	1800
Length $[m]$	0.16	0.10
Cross-Sectional Area $[m^2]$	1x10^{-4}	1x10^{-4}

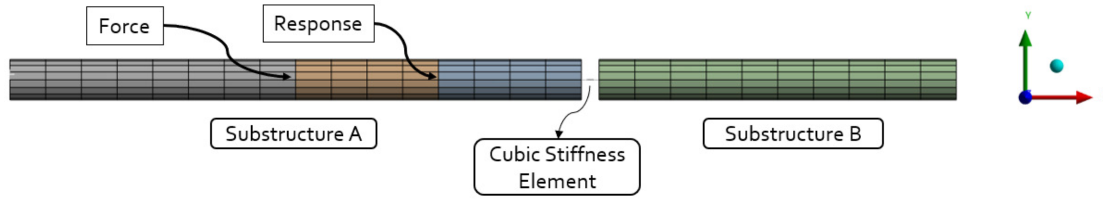

Fig. 7 Details of the continuous system

Table 2 Mesh details of the continuous system

Nodes	Elements	DOFs
1596	1248	4446

Firstly, the continuous equations are converted into modal domain. The NLMC method is applied by retaining the first 10 modes of each substructure, and the nonlinearity is introduced between the DOFs of substructures A and B as a hardening cubic stiffness element represented by the Nonlinearity Matrix, given in equation (18).

$$\boldsymbol{\Delta}(\boldsymbol{U}) = \begin{bmatrix} \nu_C & -\nu_C \\ -\nu_C & \nu_C \end{bmatrix} \tag{18}$$

where ν_C is the describing function for cubic stiffness, given as:

$$\nu_C = k_l + \frac{3}{4} k_c |u_A - u_B|^2 \tag{19}$$

where $k_l = 500\ N/m$ and $k_c = 5x10^9\ N/m^3$ are the linear and cubic stiffness coefficients, respectively. u_A and u_B are the displacements of the coupled DOFs. The resulting coupled equations are solved iteratively using Newton's method with arc-length continuation to obtain the frequency response.

Secondly, a direct frequency-domain solution is performed without modal reduction. Since the coupled system has a large number of DOFs, the global mass and stiffness matrices are partitioned to move the coupling DOFs to the first rows/columns, and the calculation procedure proposed in [9] is adopted in order to avoid matrix inversion for all DOFs.

To further validate the method, the system is simulated in the time domain using ANSYS Mechanical finite element environment (full transient analysis). The nonlinear coupling is implemented as a 'COMBIN39' spring between the nodes of substructures A and B with its force–deflection curve, as shown in Figure 8, discretized to match the hardening cubic stiffness relation. Three parameters; excitation frequency, time step, and analysis duration, are defined parametrically to automate the runs at each frequency. Transient analyses are executed at small frequency increments; time steps are adjusted to capture peak harmonic responses accurately, and durations are selected carefully to ensure steady state response.

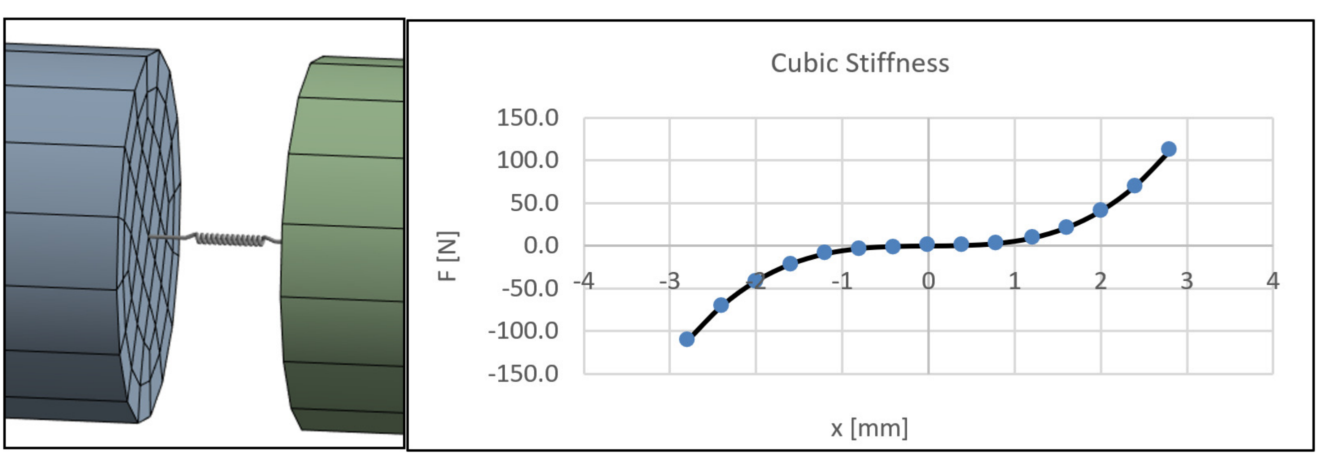

Fig. 8 Nonlinear coupling element and force–displacement curve

The frequency response from the NLMC method is compared with those from the direct frequency-domain solution and time-domain simulations. As shown in Figure 9, the close agreement across all three approaches demonstrates the accuracy of the NLMC method for continuous systems.

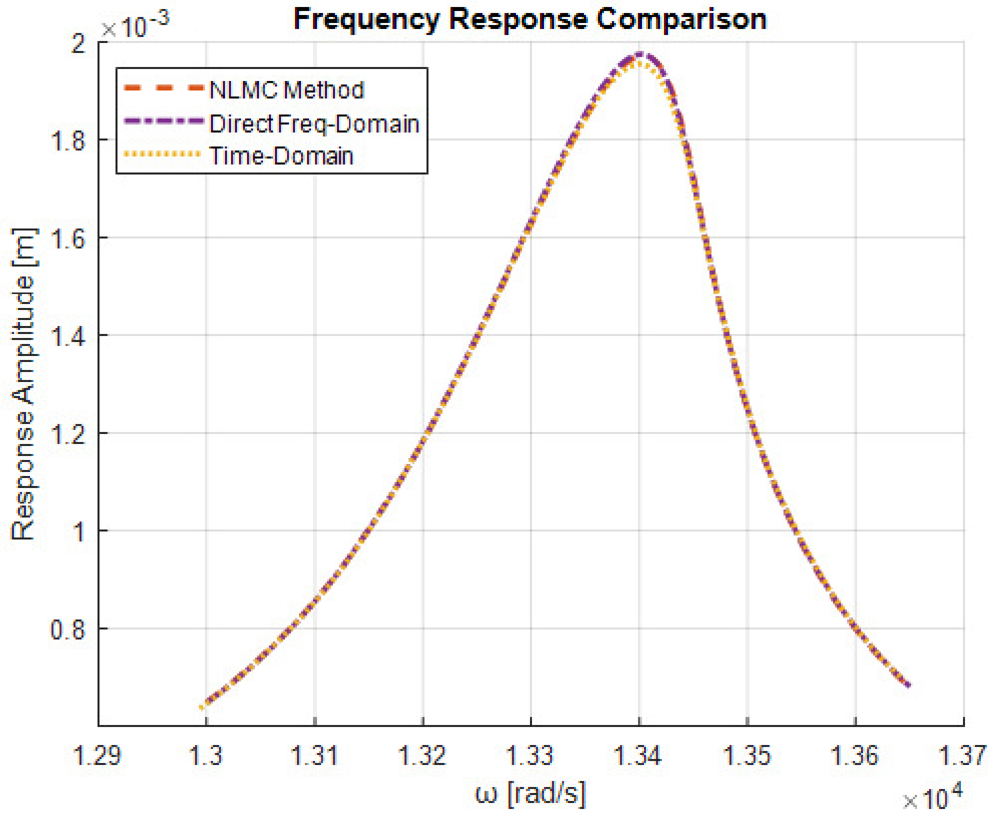

Fig. 9 Comparison of frequency responses obtained from different approaches ($F = 80\ N$)

Table 3 Amplitude and frequency differences of the NLMC method with other results for continuous system

	Amplitude Difference (%)	Frequency Difference (%)
Comparison with Direct Frequency-Domain Solution	0.160	0.015
Comparison with Time-Domain Solution	0.829	0.034

In order to assess the performance of the NLMC method, two different criteria are used to quantify the differences in the results. First one is the amplitude difference, which is the difference of the maximum displacement amplitudes obtained by using limited number of modes (20 modes in this case) in NLMC method and other solutions. Second one is the frequency difference, which is the deviation between the frequencies at which the maximum displacement amplitudes occur (i.e., the resonance frequencies). The differences are presented in Table 3, and the results reveal the accuracy of the NLMC method. Furthermore, when the full system responses are considered, the computation time for the NLMC method is approximately 1.5% of that of the direct frequency-domain solution, showing the efficiency of the method.

Conclusion

This study revisited the NLMC method to assess its effectiveness for predicting the forced harmonic response of nonlinear systems consisting of linear substructures coupled by nonlinear elements. The technique employs describing functions to represent nonlinear coupling forces, leading to a response-dependent Nonlinearity Matrix that is embedded into the modal equations. The resulting nonlinear algebraic system is solved iteratively in the modal domain, offering a computationally efficient framework that remains independent of substructure size.

Validation was carried out on both a discrete multi-degree-of-freedom system and a continuous finite element model. For both systems, frequency responses obtained using the NLMC method showed very close agreement with those from direct frequency-domain analysis and nonlinear time-domain simulations. For the continuous system, a quantitative comparison revealed very small differences, with amplitude and frequency deviations below 0.829%. These results confirm that the NLMC method can reliably capture the nonlinear dynamic behavior of coupled systems.

The main limitation of the method lies in its reliance on the assumption that sub- and super-harmonics are negligible, which may restrict applicability if this assumption is not satisfied. Nevertheless, the demonstrated accuracy, efficiency, and flexibility of the NLMC method highlight its potential as a practical tool for nonlinear substructuring and hybrid modeling in engineering applications. As future work, it is planned to extend the method to nonlinear coupling of subsystems exhibiting inherent nonlinearities.

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