Chapter 7
Preparing Data for Impulse Response Based Substructuring using Observer/Kalman Filter Identification

Christopher Page and John Seymour

Abstract Impulse response based substructuring (IBS) is a time-domain approach to dynamic substructuring that uses impulse response functions or Markov parameters to couple substructures. Although the technique has been demonstrated numerically, relatively few examples using measured data have been presented. Preliminary experimental efforts have highlighted practical challenges when applying IBS with measured data. For example, measuring Markov parameters of a component is difficult because mechanical application of true unit impulse is not possible in practice, e.g., impact from a modal hammer. Markov parameters could be estimated from measured frequency response functions (FRFs) with the inverse Fourier transform (IFFT), but these Markov parameters are generally corrupted by exponentially growing spurious oscillations associated with unstable poles introduced by the FRF averaging process and spectral truncation, i.e., "time-domain leakage". This paper shows how the Observer/Kalman Filter Identification (OKID) technique can be used to estimate Markov parameters from general input-output data such as those acquired during a typical modal impact test and applied to IBS applications. The paper also demonstrates that the OKID algorithm is theoretically equivalent to methods for smoothing measured frequency response functions (FRFs), highlighting the OKID as a domain agnostic tool for preparing measured data for dynamic substructuring use cases.

Keywords Dynamic Substructuring · Time Domain · IBS · OKID

Introduction

Impulse-based substructuring (IBS), introduced by Rixen [1], can be viewed as the time-domain counterpart of Lagrange Multiplier Frequency Based Substructuring (LM-FBS). Its main advantages are efficiency for impulsive loads, since only the first time steps are needed, and the ability to handle nonlinear components via numerical models.

Early applications, such as predicting the axial response of a clamped-free rod [2], revealed practical challenges. IRFs measured from impact tests are not true impulses, as a hammer strike cannot replicate a Dirac delta function. When used directly in IBS, these data can lead to artificially smoothed predictions. Moreover, the IBS procedure is ill-conditioned, making it highly sensitive to measurement noise and small initial values at the start of the impulse response time block.

Other work has noted that impulse response functions (IRFs) obtained from inverse Fourier transforms of frequency response functions (FRFs) often exhibit instabilities due to non-causality introduced by averaging and spectral truncation [3, 4, 5]. These effects, sometimes described as 'time-domain leakage,' can corrupt the Markov parameters needed for IBS. Van der Seijs et al. proposed an inverse force filter to mitigate these issues, though it has not yet been applied in IBS coupling [5].

Christopher Page
Noise Control Engineering, 85 Rangeway Road
Billerica, Massachusetts 01862
e-mail:page.a.chris@gmail.com

John Seymour
Applied Mechanics Engineering, 202 Burlington Road
South Dartmouth, Massachusetts 02748
e-mail: seymour.a.john@gmail.com

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To address these broader challenges, Juang et al. introduced the Observer/Kalman Filter Identification (OKID) algorithm [6]. OKID estimates the Markov parameters of an observer system—the original system augmented with proportional feedback that shifts eigenvalues to enforce rapid decay of the impulse responses. This modification ensures a well-posed and better conditioned deconvolution problem. The observer's Markov parameters are obtained from a least-squares solution using input–output data, and those of the original system are recovered from their relationship to the observer system [7]. In this way, OKID can generate stable Markov parameters directly from general experimental data, including typical impact tests, making it a promising tool for preparing measured data for IBS applications.

In what follows, the theoretical development of IBS is reviewed using dually assembled state-space formulation. The review highlights the importance of having a well conditioned and invertible feed through matrix when coupling time domain models. Next, OKID is presented using the approach of Juang et. al. [6, 7] and applied to numerically synthesized data containing artifacts common to impact and shaker tests. Finally, OKID is shown to be equivalent to the Left Matrix Factorization Description (LMFD) of the FRF, demonstrating that recently proposed FRF smoothing techniques are analogous to OKID [8].

Substructure Coupling in the Time Domain

State-space models are a natural choice for describing the time-domain dynamics of the components. The dually assembled system of equations in discrete-time state-space form can be expressed as

$$x[k+1] = \mathbf{A}x[k] + \mathbf{G}p[k] \tag{1a}$$

$$y[k] = \mathbf{D}x[k] + \mathbf{J}p[k] \tag{1b}$$

where the state-vector $x \in \mathbb{R}^{2n}$ is expressed in terms of the N_s components' displacement and velocity vectors as

$$x[k] = \begin{bmatrix} u^{(1)}[k] & u^{(2)}[k] & \ldots & u^{(N_s)}[k] & \dot{u}^{(1)}[k] & \dot{u}^{(2)}[k] & \ldots & \dot{u}^{(N_s)}[k] \end{bmatrix}^T \tag{2}$$

and $\mathbf{A}$, $\mathbf{G}$, $\mathbf{D}$ and $\mathbf{J}$ are block-diagonal matrices containing each component's state-transition matrix, input matrix, measurement matrix and feed-through matrix respectively. Data are assumed to be uniformly sampled at an interval of Δt with all time dependent quantities described in terms of a length l sample index $k = \{0, 1, 2, \ldots, l-1\}$. The input vector $p \in \mathbb{R}^{r+c}$ is expressed in terms of the r external forces, f, and c Lagrange multipliers, λ by

$$p[k] = \begin{bmatrix} \mathbf{S}_f & -\mathbf{B}^T \end{bmatrix} \begin{bmatrix} f[k] \\ \lambda[k] \end{bmatrix} \tag{3}$$

where $\mathbf{S}_f \in \mathbb{R}^{n\times r}$ is a transformation matrix that describes how the external forces are applied to the n structural degrees of freedom (DoF) and $\mathbf{B} \in \mathbb{R}^{c\times n}$ is a signed Boolean matrix representing the interface compatibility constraint. The output vector $y \in \mathbb{R}^m$ contains the measured data, which could be displacements velocities, accelerations, strains or any other linear combination of the state vector.

With $x[0] = 0$, successive application of (1a) and (1b) produces a system of equations for $y[k]$ of the form

$$\begin{bmatrix} \mathbf{J} & 0 & 0 & \cdots & 0 \\ \mathbf{DG} & \mathbf{J} & 0 & \cdots & 0 \\ \mathbf{DAG} & \mathbf{DG} & \mathbf{J} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \mathbf{DA}^{l-2}\mathbf{G} & \mathbf{DA}^{l-3}\mathbf{G} & \cdots & \mathbf{DG} & \mathbf{J} \end{bmatrix} \begin{bmatrix} p[0] \\ p[1] \\ p[2] \\ \vdots \\ p[l-1] \end{bmatrix} = \begin{bmatrix} y[0] \\ y[1] \\ y[2] \\ \vdots \\ y[l-1] \end{bmatrix} \tag{4}$$

which is a time-discretized version of the convolution integral. The matrix on the left-hand side of (4) and the entries of the column define a sequence of Markov parameters or the impulse response functions. By incorporating the compatibility constraint and expressing the input in terms of the externally applied loads and Lagrange multipliers, the dually assembled state-space model can be expressed more compactly as

$$y[k] = y_\Delta[k] - \mathbf{JB}^T\lambda[k] \tag{5a}$$

$$\mathbf{B}y[k] = 0 \tag{5b}$$

where the term $y_\Delta[k]$ represents the response of the structure due all external loads up to time k and all interface forces up to time $k-1$, i.e.,

$$y_\Delta[k] = \sum_{i=0}^{k-1} \mathbf{D}\mathbf{A}^{(k-i-1)}\mathbf{G}\left(f[i] - \mathbf{B}^T\lambda[i]\right) + \mathbf{J}f[k] \tag{6}$$

The Lagrange multipliers at the k-th can be expressed in terms of y_Δ after applying the computability constraint to (5a) as

$$\begin{aligned}\mathbf{B}y[k] &= \mathbf{B}y_\Delta[k] - \mathbf{B}\mathbf{J}\mathbf{B}^T\lambda[k] = 0\\ \lambda[k] &= \left(\mathbf{B}\mathbf{J}\mathbf{B}^T\right)^{-1}\mathbf{B}y_\Delta[k]\end{aligned} \tag{7}$$

The coupled response at time-step k is obtained by substituting (7) into (5a) resulting in

$$y[k] = \left(\mathbf{I}_m - \mathbf{J}\mathbf{B}^T\left(\mathbf{B}\mathbf{J}\mathbf{B}^T\right)^{-1}\mathbf{B}\right)y_\Delta[k] \tag{8}$$

From (7), the interface forces can be estimated only if $\left(\mathbf{B}\mathbf{J}\mathbf{B}^T\right)^{-1}$ is invertible, which is true if $y[k]$ contains acceleration measurements. For displacement, velocity or strain measurements the feed-through matrix $\mathbf{J} = 0$, thus the measurement at time-step k contains no information about the interface force at time-step k. In this scenario, the interface force, $\lambda[k-1]$ is estimated from the measurement, $y[k]$ according to

$$\lambda[k-1] = \left(\mathbf{B}\mathbf{D}\mathbf{G}\mathbf{B}^T\right)^{-1}\mathbf{B}y_\Delta[k] \tag{9}$$

and the coupled response at time-step k becomes

$$y[k] = \left(\mathbf{I}_m - \mathbf{D}\mathbf{G}\mathbf{B}^T\left(\mathbf{B}\mathbf{D}\mathbf{G}\mathbf{B}^T\right)^{-1}\mathbf{B}\right)y_\Delta[k] \tag{10}$$

Observer Kalman Filter Identification (OKID)

The Observer/Kalman Filter Identification (OKID) algorithm estimates a system's Markov parameters directly from input–output data. By introducing proportional feedback, OKID stabilizes the deconvolution problem and avoids the "time-domain leakage" often encountered when recovering Markov parameters from the inverse Fourier transform of FRFs. This section summarizes the formulation of Juang et al. [7, 9].

For a discrete-time system with zero initial conditions, the output can be written as

$$\mathbf{y} = \mathbf{h}_l\mathbf{P} \tag{11}$$

where $\mathbf{h}_l \in \mathbb{R}^{m\times rl}$ is the matrix of Markov parameters, $\mathbf{P} \in \mathbb{R}^{rl\times l}$ the input matrix, and $\mathbf{y} \in \mathbb{R}^{m\times l}$ the measured output vector:

$$\mathbf{P} = \begin{bmatrix} p[0] & p[1] & \cdots & p[l-1] \\ 0 & p[0] & \cdots & p[l-2] \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & p[0] \end{bmatrix}, \tag{12a}$$

$$\mathbf{y} = \begin{bmatrix} y[0] & y[1] & \ldots & y[l-1] \end{bmatrix}. \tag{12b}$$

With a single input, i.e., $r = 1$, $\mathbf{P}$ is square and the Markov parameters can be obtained uniquely. With multiple inputs, or when $p[0] = 0$, the problem is ill-posed or ill-conditioned. In practice, low damping also forces long sequences, making direct inversion of $\mathbf{P}$ computationally expensive. OKID addresses this by introducing an observer with feedback gain, $\mathbf{F} \in \mathbb{R}^{2n\times m}$, shifting the eigenvalues of the system matrix $\mathbf{A}$ and enforcing rapid temporal decay of the impulse response. The observer system is

$$x[k+1] = \bar{\mathbf{A}}x[k] + \bar{\mathbf{G}}\nu[k], \tag{13a}$$

$$y[k] = \mathbf{D}x[k] + \mathbf{J}p[k], \tag{13b}$$

with

$$\bar{\mathbf{A}} = \mathbf{A} + \mathbf{F}\mathbf{D}, \tag{14a}$$

$$\bar{\mathbf{G}} = \begin{bmatrix}\mathbf{G} + \mathbf{F}\mathbf{J} & -\mathbf{F}\end{bmatrix}, \tag{14b}$$

$$\nu[k] = \begin{bmatrix} p[k] \\ y[k] \end{bmatrix}. \tag{14c}$$

The observer's Markov parameters are collected in

$$\mathbf{y} = \bar{\mathbf{h}}_s \bar{\mathbf{P}}, \tag{15}$$

where

$$\bar{\mathbf{h}}_s = \begin{bmatrix}\mathbf{J},\ \mathbf{D}\bar{\mathbf{G}},\ \mathbf{D}\bar{\mathbf{A}}\bar{\mathbf{G}}, \ldots, \mathbf{D}\bar{\mathbf{A}}^{s-1}\bar{\mathbf{G}}\end{bmatrix} \in \mathbb{R}^{m\times(m+r)s+r}$$

and

$$\bar{\mathbf{P}} = \begin{bmatrix} p[0] & p[1] & p[2] & \ldots & p[s] & \ldots & p[l-1] \\ 0 & \nu[0] & \nu[1] & \ldots & \nu[s-1] & \ldots & \nu[l-2] \\ 0 & 0 & \nu[0] & \ldots & \nu[s-2] & \ldots & \nu[l-3] \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ldots & \vdots \\ 0 & 0 & 0 & \ldots & \nu[0] & \ldots & \nu[l-s-1] \end{bmatrix} \in \mathbb{R}^{(m+r)s+r\times l}$$

with $s << l$ is the number of time samples until the Markov parameter sequence decays to zero resulting in system of equations with more equations than unknowns. The observer's Markov parameter sequence can be estimated from a least squares solution to Equation (15). Once observer's Markov parameters are identified, the system Markov parameters are recovered using

$$\mathbf{h}[0] = \mathbf{J}, \tag{16a}$$

$$\mathbf{h}[k] = \bar{\mathbf{h}}^{(1)}[k] + \bar{\mathbf{h}}^{(2)}[k]\mathbf{J} + \sum_{i=0}^{k-1} \bar{\mathbf{h}}^{(2)}[i]\,\mathbf{h}[k-i-1]. \tag{16b}$$

where

$$\bar{\mathbf{h}}^{(1)}[k] = \mathbf{D}\bar{\mathbf{A}}^{k-1}(\mathbf{G} + \mathbf{F}\mathbf{J}) \tag{17a}$$

$$\bar{\mathbf{h}}^{(2)}[k] = -\mathbf{D}\bar{\mathbf{A}}^{k-1}\mathbf{F} \tag{17b}$$

Impulse Response Function Recovery using OKID

In this section, OKID algorithm is used to recover the impulse response function from simulated input-output data. The data are simulated from lumped parameter models and include artifacts typical of real data, e.g., small initial values and noise, that render the data unsuitable for IBS applications.

Figure 1 shows the simulated drive point acceleration response due to an impact on mass 1. The simulated input force time history is not a perfect impulse. The force and acceleration time histories are initially zero as would occur with a pretrigger. The force has a finite duration and the system response is lightly damped, having barely decayed over the two second data acquisition period. Figure 2 shows the input and output response for a simulated hammer impact on mass 2.

Pretrigger effects render these data unusable for IBS, since the feed-through matrix $\mathbf{J}$ is zero at the start of the block. Manually identifying the correct starting time to ensure invertibility would be ad hoc and error-prone. Moreover, the input force has a finite duration and is not an instantaneous pulse introducing bias errors in the coupled system prediction.

Figure (3) is a graphical depiction of the OKID process described in Equation (15). On the left hand side, the imperfectly measured impulse response are assembled into the matrix $\mathbf{y}$. On the right hand-side the imperfect impulse responses and measured input forces are assembled into the block Toeplitz matrix $\bar{\mathbf{P}}$

The Markov parameters or impulse response functions for the system are obtained by pre-multiplying the matrix $\mathbf{y}$ by the left-generalized inverse of the block Toeplitz matrix $\bar{\mathbf{P}}$. Figure (4) compares the IRFs recovered from the OKID process with the ground truth IRFs. In the case of noiseless data, the exact IRFs are recovered from the general input-output data

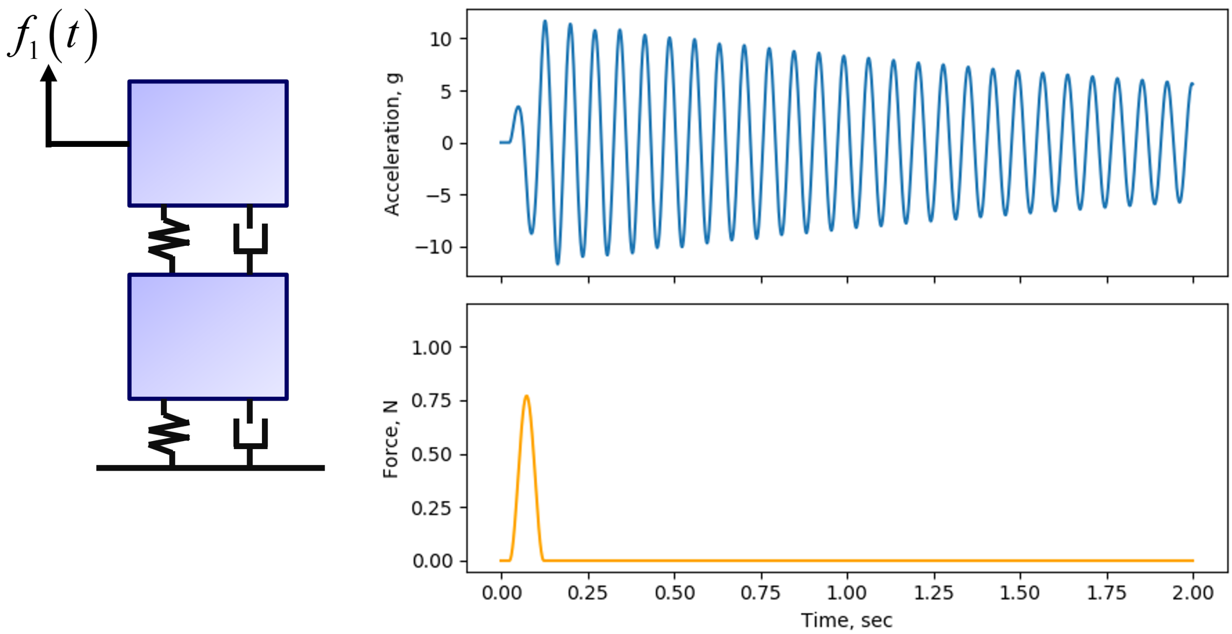

Fig. 1 Simulated Input-Output data from a Hammer Impact on Mass 1

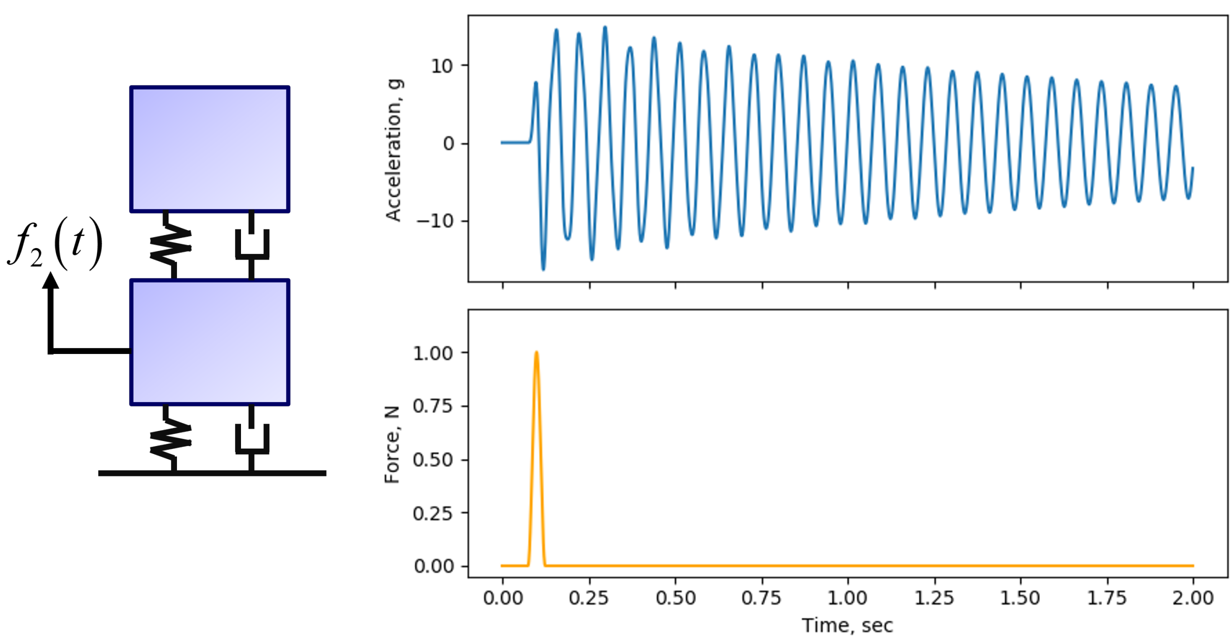

Fig. 2 Simulated Input-Output data from a Hammer Impact on Mass 2

Figure (5) presents the results of applying the OKID process to noisy input-output data. The data were simulated using a single degree-of-freedom lumped parameter model subject to periodic bursts of band-limited noise. These simulated data are typical of data collected during a shaker test. The dark lines in the plots on the left panel represent the noisy measured data. The dashed lines show the noise-less data for reference. The panel on the right compares the IRF obtained from the OKID process with the ground truth IRF, showing excellent agreement with true IRF.

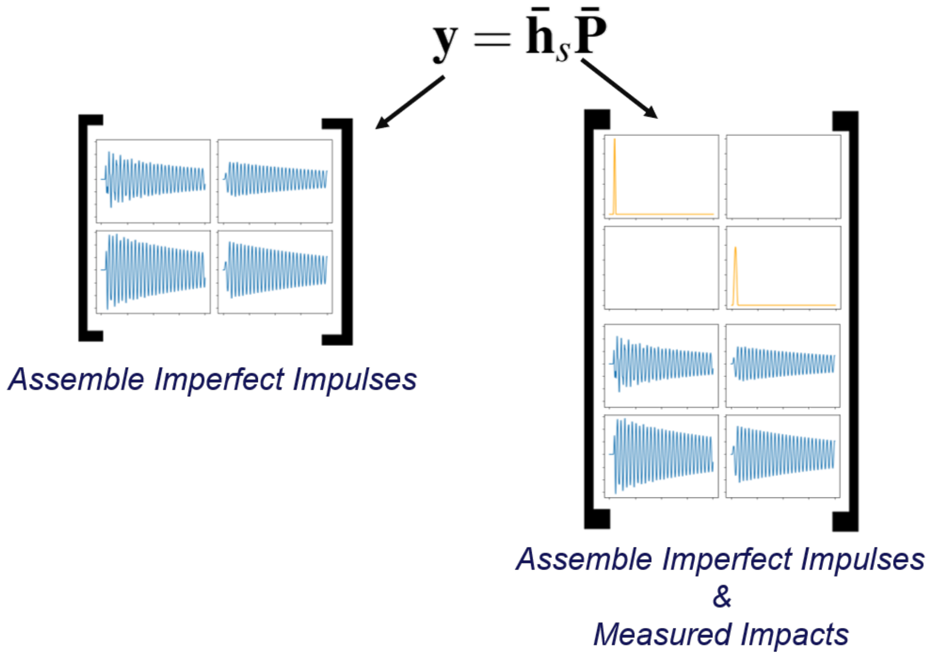

Fig. 3 Graphical Depiction of the OKID Process

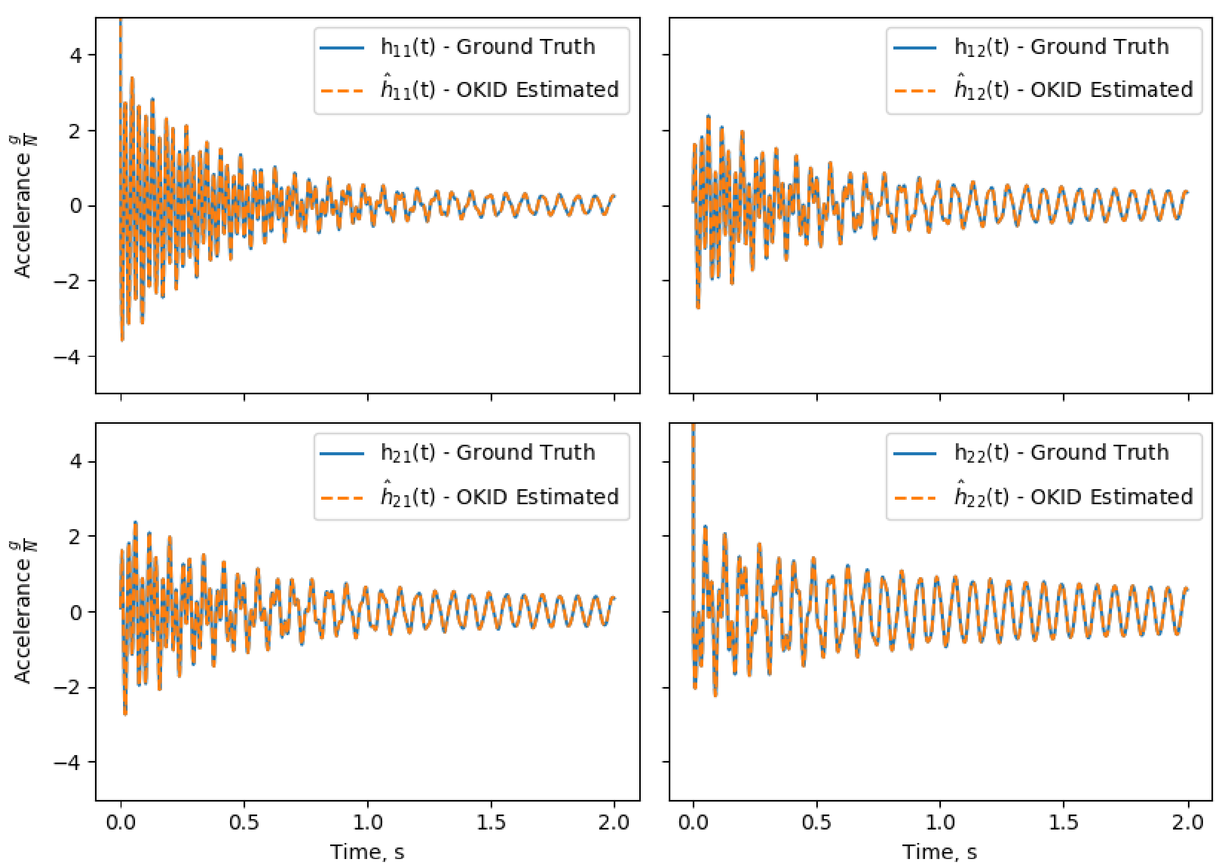

Fig. 4 IRFs Recovered from Noiseless Generalized Input-Output Data

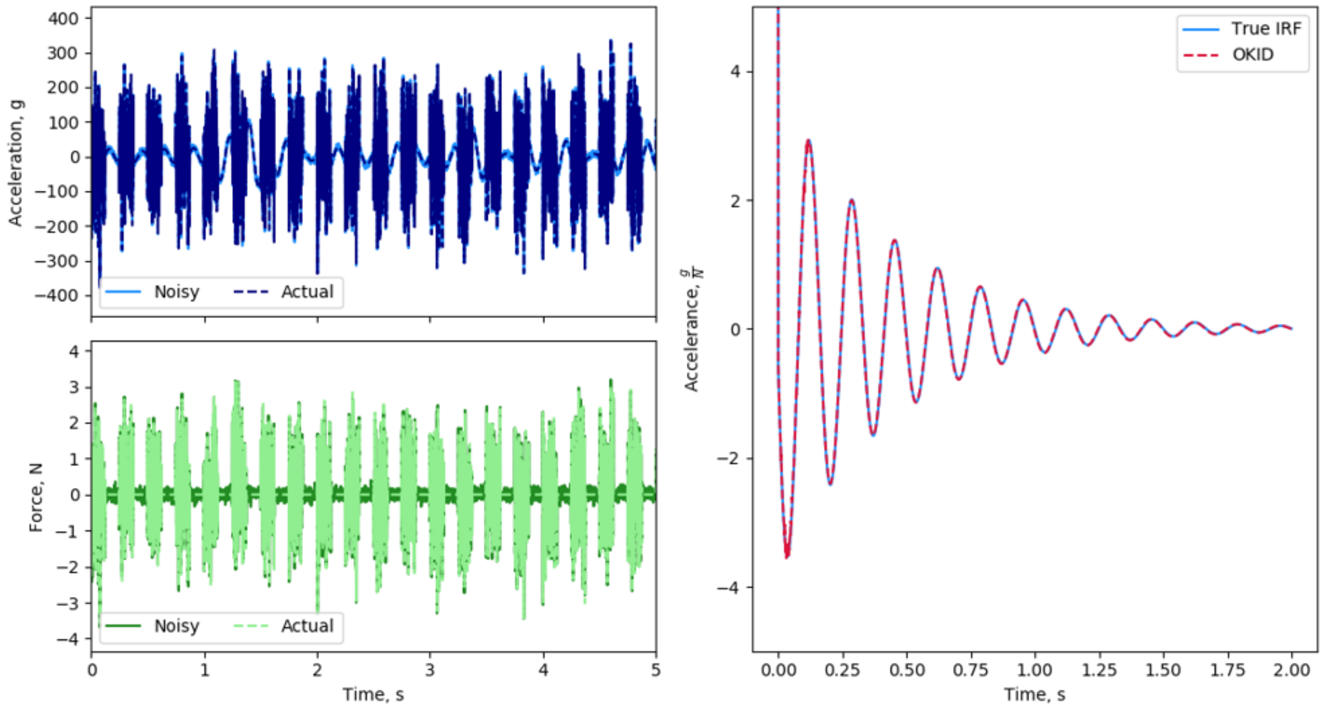

Fig. 5 IRF Recovered from Noisy Burst Random Data

OKID, Left Matrix Factorization Description and FRF Smoothing

In this section, the z-domain characteristics of the observer system are examined and connections to the Left Matrix Factorization Description (LMFD) of the frequency response function are highlighted. The LMFD of the FRF is shown to be exactly identical to a z-domain linear difference model for the observer. In this way, OKID is shown to be equivalent to FRF smoothing procedures that employ matrix fraction parameterizations, and identical to those using the left matrix fraction parameterization

The z-transform of the observer system output given in Equation (13b) is given by

$$Y[z] = \mathbf{D}\left(z\mathbf{I} - \bar{\mathbf{A}}\right)^{-1}\bar{\mathbf{G}}V[z] + \mathbf{J}P[z] \tag{18}$$

which after substituting terms from (14a), (14b) and (14c), can be expressed as a linear difference model of the form

$$\left(\mathbf{I}_m + \sum_{k=0}^{\infty} z^{-(k+1)}\mathbf{D}\bar{\mathbf{A}}^k\mathbf{F}\right)Y[z] = \left(\mathbf{J} + \sum_{k=0}^{\infty} z^{-(k+1)}\mathbf{D}\bar{\mathbf{A}}^k\left(\mathbf{G} + \mathbf{FJ}\right)\right)P[z] \tag{19}$$

The infinite series in Equation (19) can be truncated to a finite number of samples, s, due to the rapid Markov parameter decay provided by the observer feedback gain. Moreover, if the input force is an impulse function, i.e., $p[k] = \delta[k]$, then $P[z] = 1$ then $Y[z]$ represents the FRF, $\mathbf{H}[z]$, such that

$$\left(\mathbf{I}_m + \sum_{k=1}^{s} z^{-k}\mathbf{D}\bar{\mathbf{A}}^{k-1}\mathbf{F}\right)\mathbf{H}[z] = \left(\mathbf{J} + \sum_{k=1}^{s} z^{-k}\mathbf{D}\bar{\mathbf{A}}^{k-1}\left(\mathbf{G} + \mathbf{FJ}\right)\right) \tag{20}$$

The matrix on the right-hand side of Equation (20) and the matrix pre-multiplying the FRF matrix on the left-hand side are both s-th order matrix polynomials. The coefficients of the matrix polynomials are the Markov parameters of the observer system.

Equation (20) is identical to the Left Matrix Factorization Description (LMFD) of the FRF, where the FRF is parameterized as the product of a denominator matrix $\mathbf{Q}[z] \in \mathbb{C}^{m\times m}$ and a numerator matrix $\mathbf{R}[z] \in \mathbb{C}^{m\times r}$ according to

$$\mathbf{H}[z] = \mathbf{Q}^{-1}(z)\,\mathbf{R}(z) \tag{21a}$$

$$\mathbf{Q}[z] = \mathbf{I}_m + \sum_{k=1}^{s} z^{-k}\mathbf{Q}_k \tag{21b}$$

$$\mathbf{R}[z] = \mathbf{R}_0 + \sum_{k=1}^{s} z^{-k}\mathbf{R}_k \tag{21c}$$

where both $\mathbf{Q}[z]$ and $\mathbf{R}[z]$ are s-th order matrix polynomials. Pre-multiplying Equation (21a) by $\mathbf{Q}[z]$ and expanding terms gives

$$\left(\mathbf{I}_m + \sum_{k=1}^{s} z^{-k}\mathbf{Q}_k\right)\mathbf{H}[z] = \mathbf{R}_0 + \sum_{k=1}^{s} z^{-k}\mathbf{R}_k \tag{22}$$

Comparing of Equations (20) and (22) shows that the the z-domain linear difference model for the observer system and LMFD description of the FRF are identical and that the Markov parameters of the observer system can be obtained from the denominator and numerator matrix polynomial coefficients as

$$\mathbf{Q}_k = \mathbf{D}\bar{\mathbf{A}}^{k-1}\mathbf{F} \tag{23a}$$

$$\mathbf{R}_0 = \mathbf{J} \tag{23b}$$

$$\mathbf{R}_k = \mathbf{D}\bar{\mathbf{A}}^{k-1}\left(\mathbf{G} + \mathbf{F}\mathbf{J}\right) \tag{23c}$$

which have the following equivalency to Equations (17a) and (17b)

$$\bar{\mathbf{h}}^{(1)}[k] = \mathbf{R}_k \tag{24a}$$

$$\bar{\mathbf{h}}^{(2)}[k] = -\mathbf{Q}_k \tag{24b}$$

so that the impulse response function $\mathbf{h}[k]$ can be recovered from denominator and numerator matrix polynomial coefficients using Equation (16a) and (16b).

For FRF smoothing, the numerator and denominator matrix polynomial coefficients are estimated in a least squares sense, by expressing Equation (22) as

$$\Psi = \Theta\Phi \tag{25}$$

solving for Θ, reconstructing the smoothed FRF using left matrix parameterization given by Equation (21a). The terms in (25 are

$$\Psi = \begin{bmatrix}\mathbf{H}[z_0] & \mathbf{H}[z_1] & \dots & \mathbf{H}[z_l]\end{bmatrix} \tag{26a}$$

$$\Theta = \begin{bmatrix}-\mathbf{Q}_1 & -\mathbf{Q}_2 & \dots & -\mathbf{Q}_s & \mathbf{R}_0 & \mathbf{R}_1 & \dots & \mathbf{R}_s\end{bmatrix} \tag{26b}$$

$$\Phi = \begin{bmatrix}\mathbf{H}[z_0]z_0^{-1} & \mathbf{H}[z_1]z_1^{-1} & \dots & \mathbf{H}[z_l]z_l^{-1}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{H}[z_0]z_0^{-s} & \mathbf{H}[z_1]z_1^{-s} & \dots & \mathbf{H}[z_l]z_l^{-s}\\ \mathbf{I}_r & \mathbf{I}_r & \dots & \mathbf{I}_r\\ z_0^{-1}\mathbf{I}_r & z_1^{-1}\mathbf{I}_r & \dots & z_l^{-1}\mathbf{I}_r\\ \vdots & \vdots & \ddots & \vdots\\ z_0^{-s}\mathbf{I}_r & z_1^{-s}\mathbf{I}_r & \dots & z_l^{-s}\mathbf{I}_r\end{bmatrix} \tag{26c}$$

Given the equivalency of the matrix fraction parameters and the Markov parameters of the observer system, the smoothing process is revealed to be the OKID algorithm applied to frequency domain data. For impulse based substructuring applications, suitable IRFs can be extracted from frequency domain FRFs, by first estimating matrix polynomial coefficients of the LMFD and applying the OKID Markov parameter recovery process outlined by Equations (16a) and (16b)

Conclusions

This paper illustrated that the OKID, introduced by Juang in the early 1990's, can be an effective tool for conditioning IRFs for IBS applications. Moreover, the paper highlighted the equivalency of the observer system to the LMFD in the frequency domain, showing that FRF smoothing procedures that employ matrix factorization parameterizations are frequency domain implementations of OKID. In this sense, OKID provides a general framework for conditioning measurements in both time- and frequency-domain substructuring applications.

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