Chapter 13
Assessing Floquet Theory as an Alternative to Multi-Blade Coordinate Transformation in Wind Turbine Stability Analysis

Jose J. Mora Amaro, Emmanuel Branlard, and Riccardo Riva

Abstract We investigate the use of Floquet theory as an alternative to the Multi-Blade Coordinate transformation for wind turbine stability analysis. A simplified 5-DOF rotor model demonstrates the method and supports the development of an open-source Floquet analysis tool for future integration with multi-physics platforms such as OpenFAST.

Keywords Stability analysis · Coleman transformation · Floquet theory

Introduction

In the analysis of rotormachinery, the Multi-Blade Coordinate (MBC) transformation [1–3] is widely used to decouple periodic systems into fixed-frame representations, facilitating modal and stability analyses. However, MBC has limitations—particularly for anisotropic rotors and for systems with strong coupling or time-periodic behavior not well represented in the fixed frame [3, 4]. Current state-of-the-art wind turbine analysis tools typically perform stability assessments under isotropic assumptions, neglecting gravity effects and azimuthal wind variations [5]. However, as field observations reveal emerging instability issues, the importance of accounting for anisotropy and exploring alternatives to MBC is increasingly recognized.

In this work, we focus on exploring the application of Floquet theory [6] to perform stability analyses, and setting the stage for future application to multi-physics wind turbine softwares such as OpenFAST [5]. Our objective is to investigate the strengths and weaknesses of Floquet-based approaches relative to MBC and develop an open-source Floquet analysis tool tailored for wind turbine applications. Beyond establishing a stability criterion, our goal is also to extract modal characteristics (frequencies, damping ratios, and mode shapes) to guide design iterations when instability regions are identified. While related studies have addressed periodic stability analysis [7, 8], key questions remain regarding the generalization of the method, and, to the authors' knowledge, no open-source implementation is currently available.

In this preliminary work, we use a simplified 5-degree-of-freedom system, consisting of two degrees of freedom for the support structure and one edgewise degree of freedom per blade, following the structure of the original Coleman model. Using this model, we investigate the differences between Coleman-based and Multi-Blade Coordinate (MBC) transformations. Our goal is to identify the conditions under which Floquet theory provides clear advantages and to lower the barrier for its adoption in the wind energy modeling community.

Jose J. Mora Amaro · Emmanuel Branlard
Wind Energy Center, University of Massachusetts - Amherst, Amherst, MA 01002, USA
e-mail: jmoraamaro@umass.edu; ebranlard@umass.edu

Riccardo Riva
Technical University of Denmark, Roskilde, Denmark
e-mail: ricriv@dtu.dk

Kristin B. Zimmerman, PhD (eds.), *Special Topics & Experimental Techniques in Structural Dynamics, 2026, Vol. 5*, Conference Proceedings of the Society for Experimental Mechanics Series, https://doi.org/10.1007/978-8-743-81444-3_13

Summary of Floquet Theory

We consider a periodic coefficient system of period T defined as:

$$\underline{\dot{x}} = \underline{\underline{A}}(t)\underline{x} \tag{1}$$

where the periodicity implies $\underline{\underline{A}}(t+T) = \underline{\underline{A}}(t)$. The Floquet transformation [6], performs a change of coordinates from $\underline{x}$ to a new set of coordinates $\underline{z}$, assuming that in this system, the state-space equation is a linear time invariant (LTI) system. The transformation writes:

$$\underline{x}(t) = \underline{\underline{Q}}(t)\,\underline{z}(t) \tag{2}$$

where $\underline{\underline{Q}}(t) = \underline{\underline{X}}(t)e^{-\underline{\underline{B}}t}$ is the Floquet transformation matrix, $\underline{\underline{B}} = \frac{1}{T}\log(\underline{\underline{X}}(T))$ is the matrix of Floquet factors [9], $\underline{\underline{X}}$ is the fundamental matrix, and $\underline{\underline{X}}(T)$ is the monodromy matrix (the fundamental matrix evaluated at T). After applying the transformation given in Equation 2 to Equation 1, we obtain the LTI system [9]:

$$\underline{\dot{z}} = \underline{\underline{B}}\,\underline{z} \tag{3}$$

Simple stability analysis can be performed by looking at the eigenvalues of the monodromy matrix, called the characteristic multipliers. The eigenvectors of the monodromy matrix are written $\underline{\underline{S}}$, and its eigenvalues $\underline{\theta}$. The eigenvalues of $\underline{\underline{B}}$, denoted by $\underline{\eta}$, are called the Floquet exponents, and its eigenvectors are denoted by $\underline{\underline{V}}$. It can be shown that $\underline{\eta}$ and $\underline{\theta}$ are related as [9]:

$$\theta_j = e^{\eta_j T}, \qquad \Rightarrow \qquad \eta_j = \frac{1}{T}\ln\theta_j \tag{4}$$

The complex logarithmic is multivalued and not invertible. Therefore, there is an indeterminacy when finding the Floquet exponents. Resolving this indeterminacy is challenging: stability can be determined directly, but obtaining mode shapes, frequencies, and Campbell diagrams requires additional effort.

Methods

Implementation details

The indeterminacy in Equation 4 can be lifted by considering the physical meaning of harmonic participation of each mode. A complete discussion of the significance of harmonic indeterminacy is given in [4]. The most accepted resolution is done by quantifying the modal participation across each possible frequency [7, 8, 10]. Riva et al. [7, 11] consider the relationship between the Floquet multipliers and exponents, directly constructing the eigenvalues of the LTI space by inference using Equation 4. To evaluate the modes in the frequency domain, the author infers a state space constructed on an equivalent eigenvector basis as the initial system, considering that $\underline{\underline{X}}(0) = \underline{\underline{I}}$. In this space, $\underline{\underline{V}} = \underline{\underline{S}}$. Based on that, the eigenvector basis for the LTP is:

$$\underline{\underline{\Xi}}(t) = \underline{\underline{Q}}(t)\underline{\underline{V}} \tag{5}$$

Using a Fourier decomposition of $\underline{\underline{\Xi}}$ and normalizing the coefficients, they quantify the modal participation of each frequency, thereby identifying the dominant oscillation frequency of each mode.

In this work, we solve the stability problem directly on the LTI system (Equation 3) by finding the eigenvalues of $\underline{\underline{B}}$ (i.e., $\underline{\eta}$) in that space. Similar to the work of Riva, we project the eigenvectors on the rotating frame to quantify the modal participation. However, in our framework, we lift the inference made on the prior implementations by projecting the eigenvectors of the floquet exponent matrix rather than the eigenvectors of the monodromy matrix, generalizing the stability analysis to the LTI system.We will publish our framework on an open-source GitHub repository. Floquet analysis tool tailored for wind turbine applications.

5 DOFs system

We use the 5 degrees of freedom (DOF) system from Coleman [1] as a test case for our library. The model consists of two ground frame DOFs for the base motion for the rotor, and one edgewise degree of freedom for each blade.

Results

In Figure 1, we illustrate the application of our framework to the problem given in subsection 3.2. On this figure, we compare the MBC transformation and the Floquet theory results, focusing on the dominant frequencies.

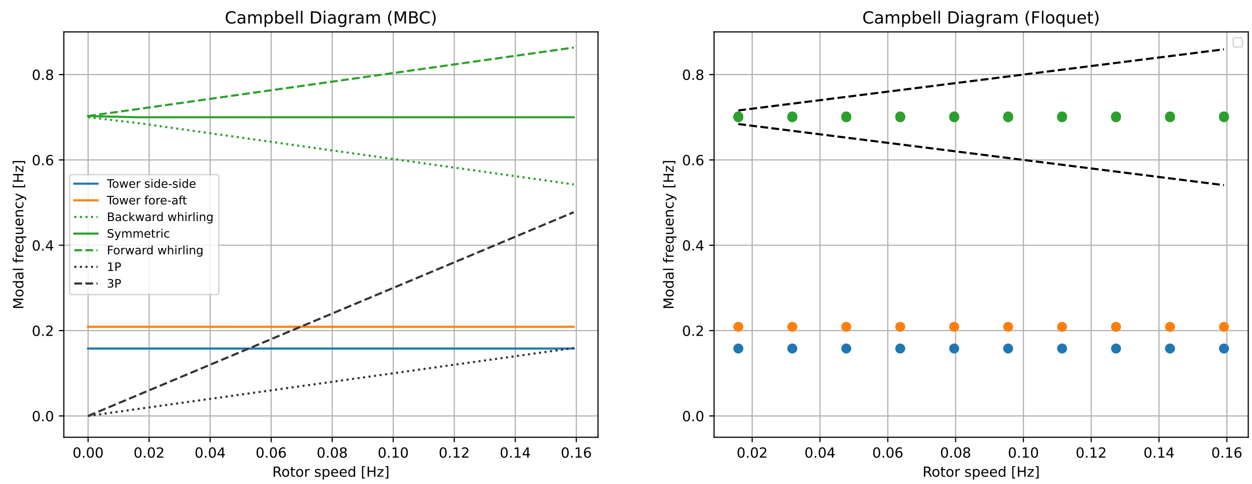

Fig. 1 Comparison between symmetric modes obtained by MBC vs Floquet stability analysis

In the figure, the regressive and progressive modes of the blades are obtained by inference, adding ± 1 frequency period to the dominant frequency. And overall agreement with the MBC method is confirmed.

Conclusion and future work

We verified our methodology and will publish our framework as a open-source repository before the conference. We intend to apply our library to study wind turbine anisotropy further. In particular, we plan to apply this framework to OpenFAST, an open-source multi-physics tool to simulate wind turbines. This will enable use to conduct stability analyses of full wind turbine models capturing aero-hydro-servo-elastic interactions in a high-fidelity setting, and investigating where current MBC methodology fails.

Acknowledgments We would like to thank Justin Porter from National Renewable Energy Laboratory for his continuous support and fruitful discussions.

References

1. R. P. Coleman and A. M. Feingold. *Theory of Self-Excited Mechanical Oscillations of Helicopter Rotors with Hinged Blades*. Tech. rep. TR 1351. National Advisory Committee for Aeronautics (NACA), 1958.
2. W. Johnson. *Helicopter Theory*. Princeton, NJ: Princeton University Press, 1980.
3. G. S. Bir. "Multi-Blade Coordinate Transformation and Its Application to Wind Turbine Analysis". In: *Proceedings of the AIAA/ASME Wind Energy Symposium*. Reno, NV, Jan. 2008.
4. D. A. Peters, S. M. Lieb, and L. A. Ahaus. "Interpretation of Floquet eigenvalues and eigenvectors for periodic systems". In: *Journal of the American Helicopter Society* 56.3 (2011), pp. 1–11. DOI: 10.4050/JAHS.56.032002.
5. J. Jonkman and el. al. *OpenFAST v3.5.1. Open-source wind turbine simulation tool, available at http://github.com/OpenFAST/OpenFAST/*. 2023.
6. G. Floquet. "Sur les équations différentielles linéaires à coefficients périodiques". In: *Annales scientifiques de l'École Normale Supérieure* 12.2 (1883), pp. 47–88. DOI: 10.24033/asens.220. URL: https://doi.org/10.24033/asens.220.
7. R. Riva, S. Cacciola, and C. L. Bottasso. "Periodic stability analysis of wind turbines operating in turbulent wind conditions". In: *Wind Energy Science* 1.2 (2016), pp. 177–203. DOI: 10.5194/wes-1-177-2016.

8. B. Pamfil, H. Bredmose, and T. Kim. “Coleman-free aero-elastic stability methods for three- and two-bladed floating wind turbines”. In: *Wind Energy Science* 10.4 (2025), pp. 827–856. DOI: 10.5194/wes-10-827-2025.
9. E. Folkers. *Floquet’s Theorem*. Bachelor’s Thesis. 2018.
10. C. L. Bottasso and S. Cacciola. “Model-independent periodic stability analysis of wind turbines”. In: *Wind Energy* 18.5 (2015), pp. 865–887. DOI: 10.1002/we.1735.
11. R. Riva. *System ID library, last accessed September 2025*. URL: https://gitlab.windenergy.dtu.dk/wtstab/sysid.