Chapter 5
Correlating Damage in Vibration Environments Using Fixed-Base Modal Responses

Marcus Behling, Matthew S. Allen, Randy Mayes, and Washington DeLima

Abstract A significant challenge in dynamic environment testing is understanding how accurately the in-flight stress is recreated in the lab test. The RMS dB error metric is often used to gauge test success, and while it accurately quantifies the spectral differences between the flight and lab responses at various DOF, its correlation with stress is poorly understood. This study proposes an alternative metric and compares it to RMS dB error by simulating SDOF, 3DOF, 6DOF, and IMMAT tests using a finite element model case study. The proposed metric is shown to be better correlated with RMS Von Mises stress than RMS dB error in these simulated tests. It is observed to be best correlated to stress when modal responses are accurately reconstructed, though it marks an improvement over RMS dB error even when this is not the case.

Keywords Random vibration · Error metric · Fixed-base modes · Multi-axis testing · finite element modeling

Introduction

Vibration qualification tests are performed to verify a component's durability in its intended operating environment. The original military standard for these tests [1] specified enveloped acceleration power spectral densities (PSDs) to be controlled near the base of the device under test (DUT) in a single axis. Multi-axis testing methods, such as 6DOF [2] and Impedance Matched Multi-Axis Testing (IMMAT) [3], have grown in popularity in the past decade and have demonstrated the potential to improve environment reconstruction [4]. The preferred control target for these tests is an acceleration PSD matrix that is defined across various degrees of freedom (DOF) on the DUT. In single-axis and multi-axis tests, the controller replicates the environment as closely as possible. The RMS dB error metric, described in [5], is commonly used to quantify how accurately the flight environment is reconstructed. It is a measure of the area, in dB, between the flight and lab environments and thus is closely correlated with a visual comparison of the two environments plotted in dB. If the RMS dB error is low and the DUT functions properly during the test, it is deemed ready for flight.

Even though this metric is commonly used, the correlation between RMS dB error and stress is poorly understood. A test with a low RMS dB error might recreate stress inaccurately, in other words; a DUT that passes the test could fail in flight, and one that fails the test might have been flight-ready but would be redesigned and retested unnecessarily. One study suggested that RMS dB error is better correlated to stress than a variety of other metrics [6], though the two were found to have an

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average Pearson correlation magnitude of only 0.11 across 100 simulated tests. This value increased to 0.26 when modal rather than physical acceleration responses were used, suggesting that modal responses are better correlated to stress. Others have similarly investigated modal metrics: Harvie [7] and Schoenherr [8] used fixed-base and free-free modes, respectively, to compare flight and test environments. Harvie's study showed that modal responses provide valuable insight that physical responses lack, and Schoenherr proposed comparing RMS free modal responses.

RMS dB error can also pose logistical problems to a testing program as the test environment often has to meet error requirements. A common requirement is that all spectral lines lie within 3 or 6 dB of the specification, and a similar requirement could be placed on the RMS dB error. If the error exceeds the maximum acceptable value, the test setup or environment must be modified, though doing so is not necessarily trivial; Schultz proposed a method for modifying a given environment to be more controllable in the lab [9], though it does not seem to have been extensively tested at this point. One could also use Daborn's buzz test method [10] to redefine the cross-PSDs, though the environment may remain challenging to recreate in the lab, and this procedure is likely to modify the reconstructed stress in ways that are poorly understood.

This study proposes using a DUT's RMS fixed-base modal responses to compare flight and lab environments. The concepts behind fixed-base modal responses are elaborated in the section that follows. As suggested by Beale [6], this metric may be better correlated to stress than methods based on physical responses. Additionally, it could help improve test program efficiency, as a pre-test could be performed, the RMS fixed-base modal responses calculated, and the control PSD matrix scaled up or down to reach equivalent RMS fixed-base responses in flight and lab. The test could be easily modified to meet the specification, in other words. The test workflow changes from a potentially iterative, costly procedure to a straightforward, linear one that yields more confidence that stress has been accurately reconstructed as shown in Figure 1.

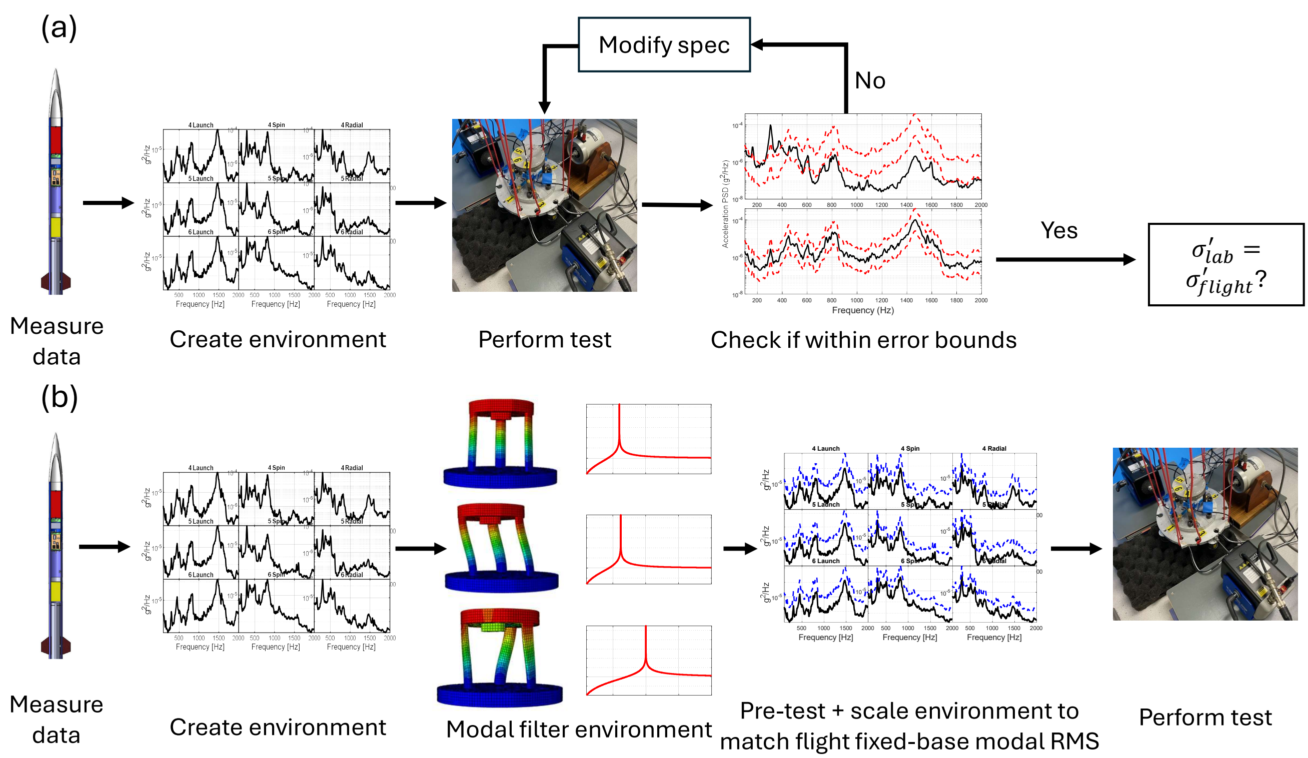


Fig. 1 Traditional test workflow (a) vs. proposed RMS fixed-base modal response workflow (b)

Damage Metric Calculation

The procedures for calculating the RMS dB error and RMS fixed-base modal response metrics are briefly described in the sections that follow.

RMS dB Error Metric

The calculation of the RMS dB error is described in [5] and reproduced here for convenience. A single error value is obtained at each frequency line by taking the RMS of the dB differences at each channel according to

$$e_{\text{PSD}}(\omega_i) = \sqrt{\frac{1}{n_{\text{accels}}} \sum_{k=1}^{n_{\text{accels}}} \left[\text{dB}\left(\mathbf{S}^r_{x_k x_k,\text{fl}}(\omega_i)\right) - \text{dB}\left(\mathbf{S}^r_{x_k x_k,\text{lab}}(\omega_i)\right) \right]^2}, \tag{1}$$

where $\mathbf{S}_{xx}$ is the acceleration PSD matrix, r denotes a set of reference DOF over which error is calculated, and fl denotes the flight environment. Ideally, the reference DOF contain flight measurements at uncontrolled DOF on the DUT, though in practice, measurements may be sparse, and error may be calculated at the control DOF on the DUT and / or test fixture. The RMS is then taken across frequency lines f to yield a single error value for the test,

$$e_{\text{PSD}} = \sqrt{\frac{1}{n_f} \sum_{i=1}^{n_f} \left(e_{\text{PSD}}(\omega_i)\right)^2}. \tag{2}$$

RMS Fixed-Base Modal Response Metric

The Hurty / Craig-Bampton method, developed in [11, 12], divides a system into boundary and interior DOF. For qualification tests, the interior DOF can be defined as those on the DUT, and the boundary DOF contain all other DOF of the system, e.g., those of the test fixture. The responses of the interior DOF are calculated according to

$$\mathbf{x}_i = \mathbf{\Phi}_i^{\text{FB}} \mathbf{p} + \mathbf{\Psi}_i \mathbf{x}_b, \tag{3}$$

where i and b are the interior and boundary DOF, respectively, $\mathbf{\Phi}_i^{\text{FB}}$ is the mode shape matrix of the DUT with a fixed-base boundary condition, $\mathbf{p}$ is the vector of fixed-base modal responses, and $\mathbf{\Psi}_i$ is the constraint shape matrix which relates the displacements of the boundary DOF to those of the interior DOF. This equation separates the rigid body and elastic portions of the DUT's response into $\mathbf{\Psi}_i \mathbf{x}_b$ and $\mathbf{\Phi}_i^{\text{FB}} \mathbf{p}$, respectively. Since the DUT's stress is caused by its elastic deformation, comparing $\mathbf{p}$ in the flight and lab should better reflect how accurately stress is recreated than comparing physical responses.

In this study, the modal responses are calculated using

$$\begin{bmatrix} \mathbf{s} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_d^{\text{RB}} & \mathbf{\Phi}_d^{\text{FB}} \end{bmatrix}^+ \mathbf{x}_d, \tag{4}$$

where $\mathbf{s}$ is the vector of rigid body modal responses, $\mathbf{\Phi}_d^{\text{RB}}$ are the rigid body mode shapes at a set of DUT measurement DOF d, $\mathbf{\Phi}_d^{\text{FB}}$ are the fixed-base mode shapes at the same DOF, $+$ denotes the pseudoinverse, and $\mathbf{x}_d$ is the vector of measured accelerations on the DUT. One could also use a combination of DUT and next assembly measurements, though this requires including the next assembly's mode shapes and following a slightly different procedure such as that outlined in [13].

There must be at least as many measurements as the total number of modes to accurately estimate modal responses, and even more accelerometers might be needed in practice; they must also be placed so that $\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_d^{\text{RB}} & \mathbf{\Phi}_d^{\text{FB}} \end{bmatrix}$ is well conditioned. $\mathbf{\Phi}$ should include the modes that are needed to describe the system's motion in the frequency band of interest, and it is common to include all modes below 1.5 times the highest frequency of interest to account for residual effects.

Each fixed-base modal response can be described by a single RMS value calculated according to

$$\mathbf{p}_{k_{\text{RMS}}} = \sqrt{\delta f \sum_{i=1}^{n_f} \mathbf{S}_{p_k p_k}(\omega_i)}, \tag{5}$$

where δf is the frequency spacing and $\mathbf{S}_{p_k p_k}$ is $(k, k)^{th}$ element of the fixed-base modal acceleration PSD matrix. The RMS of the resulting values can then be taken according to

$$\mathbf{p}_{\text{RMS}} = \sqrt{\frac{1}{n_{FB}} \sum_{k=1}^{n_{FB}} \mathbf{p}^2_{k_{\text{RMS}}}}, \tag{6}$$

yielding a single fixed-base RMS modal acceleration value. Dividing this value in the lab and flight configurations provides a ratio which is the desired metric. If this ratio equals one, then the RMS accelerations are equal, and one might assume that stress is accurately recreated; conversely, stress reconstruction is expected to be overly severe when greater than one and not severe enough when less than one. The primary goal of this study is to understand if the fixed-base modal response metric accurately reflects the stress imparted in a test, i.e., to understand if $\mathbf{p}^{\mathrm{lab}}_{\mathrm{RMS}}/\mathbf{p}^{\mathrm{fl}}_{\mathrm{RMS}} = \sigma'^{\mathrm{lab}}_{\mathrm{RMS}_{\max}}/\sigma'^{\mathrm{fl}}_{\mathrm{RMS}_{\max}}$, where $\sigma'_{\mathrm{RMS}_{\max}}$ is the maximum RMS Von Mises stress on the DUT. To this end, a set of tests is simulated, and the correlation between this metric and maximum RMS Von Mises stress is studied in section . The maximum RMS Von Mises stress is calculated using Segalman's method [14] which is reviewed in appendix .

Advantages of Fixed-Base Modal Response Metric

The RMS fixed-base modal response value provides a few significant advantages as an error metric:

- **Fixed-base modes define how the DUT elastically deforms.** When responses are compared in physical coordinates, it is difficult to tell how significant differences between environments are. Harvie simulated a test that seemed to be conservative from the observed physical response [7]. In reality, one of the elastic modal responses was reconstructed conservatively while the other was under-excited. Fixed-base modes provided this crucial insight that physical responses could not.
- **Fixed-base modes do not change between the flight and lab setups** when the DUT has the same connection DOF in both cases. This means that fixed-base modal responses can be compared between the flight and lab setups. The free modes of the DUT and transmission simulator (attached to each other) are usually different in flight and in the lab and cannot generally be compared. An accurate comparison would require that the free modes happen to be similar as Schoenherr noted [8]. Even if the next assembly elastically deforms while the fixture remains rigid, the fixed-base modes stay the same.
- **The DUT's elastic motion can often be described with only a few fixed-base modal responses.** In addition, there are typically fewer fixed-base modes than free modes of the DUT and transmission simulator. A flight vehicle may have hundreds of modes, and it would be challenging to estimate their responses. In contrast, a DUT might have a few fixed-base modes which are easier to estimate and analyze. Fixed-base modes provide an effective way to separate the DUT's motion from the rest of the flight vehicle and reduce the complexity of the analysis.
- **Accurate RMS response reconstruction is easier than accurate spectral reconstruction.** The RMS of a signal weights peaks more heavily than the rest of the signal. This means that errors in reconstructing the environment at low response levels do not significantly affect the RMS; if only the peaks of the environment are matched, the RMS will be matched reasonably well. This could reveal that requirements for an accurate test are less stringent than previously thought; perhaps fewer shakers are required to accurately capture the RMS of the fixed-base modal responses, for example. The ability to meet test specifications more consistently would also improve test program efficiency as previously mentioned.

Estimating fixed-base modal responses generally requires accurate response spectra at various locations on the DUT. When this data is not available, this metric cannot be accurately used. In addition, rigid body motion causes damage to some components. The solder joints on a PCB might not have resonances in the frequency band of interest, though they may still experience high stresses due to the inertial forces caused by the rigid body motion. Since the rigid body modes must be included in the modal filtering process, the RMS rigid body modal responses can be readily calculated and analyzed if needed without much additional effort.

Method for Evaluating the Metrics

To understand the relationship between the damage metrics and stress, qualification tests are simulated using a finite element model case study. The RMS dB error, RMS fixed-base modal response, and maximum RMS Von Mises stress are calculated for each test case and compared. In the sections that follow, the finite element model case study is introduced, and the steps of the simulated experiment are described in further detail.

Flight and Lab Configurations

The "flight vehicle" of interest is shown in Figure 2(a), where the DUT is the stool at the top of the vehicle. This vehicle, which will be referred to as the wedding cake, has 13 free modes below 2000 Hz, corresponding to six rigid body modes, three fixed-base modes of the DUT, and four elastic modes of the next assembly. The four columns near the bottom of the wedding cake induce elasticity into the plates above them, though these plates appear to be rigid when the columns are absent. This boundary condition mismatch between configurations makes the case study relevant to real hardware, while the relative simplicity of the FEM also makes it a useful testbed for verifying the damage metrics.

A flight environment was created by applying a 1 N^2/Hz force PSD at 14 different DOF on the wedding cake simultaneously, and a random value between $\pm$ 10 percent of the PSD magnitude was added at each frequency line to simulate sensor noise. Four simulated tests were performed to capture a variety of qualification testing methods that are currently practiced, namely SDOF, 3DOF, 6DOF, and IMMAT. These test setups are shown in Fig. 2(b) with shakers shown in blue and accelerometers in red. One percent modal damping was assumed for each mode in the flight and lab configurations. In the SDOF test, rigid body motion is fixed in all but the translational direction of interest, and all rigid body rotation modes are constrained in the 3DOF test. A condition number threshold of 0.01 was applied to the control FRF in each of the tests to reduce the shaker forces to levels that could feasibly be reached in an actual test. The condition number threshold sets all FRF singular values that are less than one percent of the largest singular value to zero at each frequency line, eliminating the ill-conditioned portions of the FRF matrix. This procedure is further described in [15].

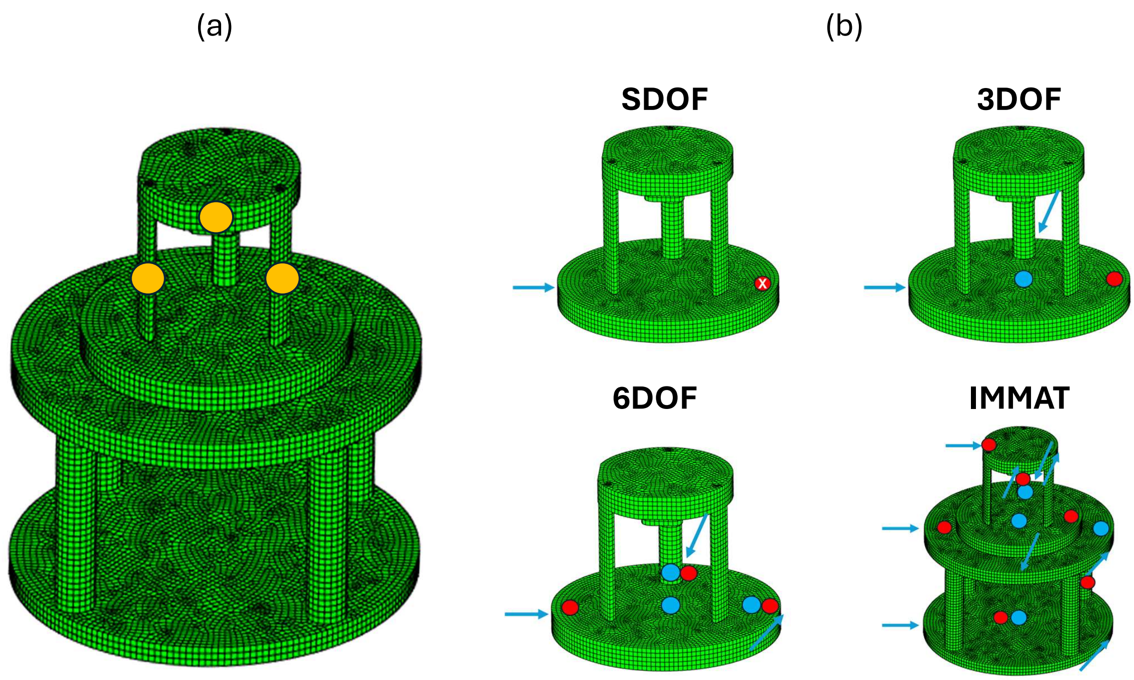


Fig. 2 Flight vehicle (a) with reference triaxial accelerometers in orange and lab test setups (b), where blue arrows represent shakers, blue circles represent shakers that apply force in the launch direction, and red circles represent triaxial control accelerometers

Comparing the Metrics to Stress

The RMS dB error for each test is calculated over the 3 orange triaxial accelerometers in Fig. 2. These locations are spread across the DUT to observe its dynamics and global response. They are not controlled in the test, but are rather reference DOF at which the environment is known. To calculate the RMS modal responses, a modal filter must first be applied to the test results. There are 9 modes to be filtered - six rigid body modes, and three fixed-base modes which are shown in Fig. 3 - so there must be at least 9 accelerometer channels on the DUT to observe these modes. 24 channels at various locations on the DUT are used in the filtering process to ensure that its dynamics are observed.

Fixed-base modal responses can be accurately estimated when the transmission simulator elastically deforms, though doing so requires including the transmission simulator modes and measurements on the transmission simulator in the filtering process. In this study, only DUT measurements are used to estimate the fixed-base modal responses, so only the rigid body modes and fixed-base modes are included. Accurate modal filtering requires that the rigid body and fixed-base modes fully span the response space of the DUT below 2000 Hz in each test configuration. This is expected to be the case since the plate directly below the stool appears to remain rigid in each of the test configurations, but Schoenherr's modal projection error

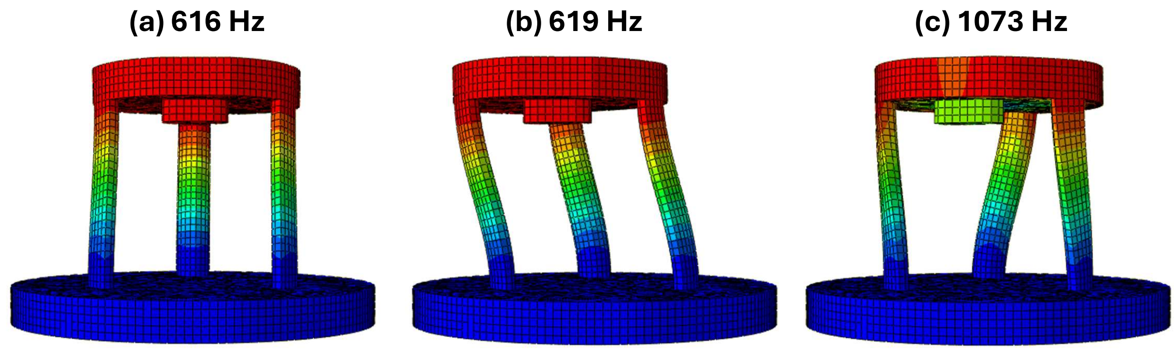


Fig. 3 616 Hz bending, 619 Hz bending, and 1073 Hz torsion fixed-base modes

metric [16] can be used to verify the quality of modal filtering more rigorously. This metric is not directly computed here, but similar insight is gained by expanding the estimated modal responses back to physical DOF and comparing them to the original environment.

The most significant modal projection error is expected in the wedding cake configuration as the plate experiences a slight bubbling motion in its 13th mode at 1882 Hz; this could cause the plate to dynamically interact with the DUT, potentially introducing new motion into the DUT that is not spanned by the rigid body and fixed-base modes. The flight modal responses are estimated using 21 of the aforementioned filtering channels and are then expanded to physical DOF and compared to the flight environment on the three DUT channels not used in the filtering process. As seen in Fig. 4, there is generally close agreement between the flight and filtered environment, suggesting that the fixed-base modes span the DUT's motion in the wedding cake assembly fairly well. This process was repeated for the other test setups and even less error was observed. Visually, the most significant errors occur below 600 Hz and above 1800 Hz, where there is a plate bubble mode that causes the DUT to deform in a shape not spanned by the fixed-base modes. These errors are unlikely to have a significant effect on the RMS of the fixed-base modal responses since the response magnitude is much smaller at these frequencies than between 600 and 1200 Hz, where most of the fixed-base modal energy is concentrated. In this frequency band, the radial response is conservative around 900 Hz, and that likely affects the RMS somewhat. Most of the energy in this band is captured well, though, so the fixed-base modes appear to be a good basis for the elastic motion of the DUT in the three configurations.

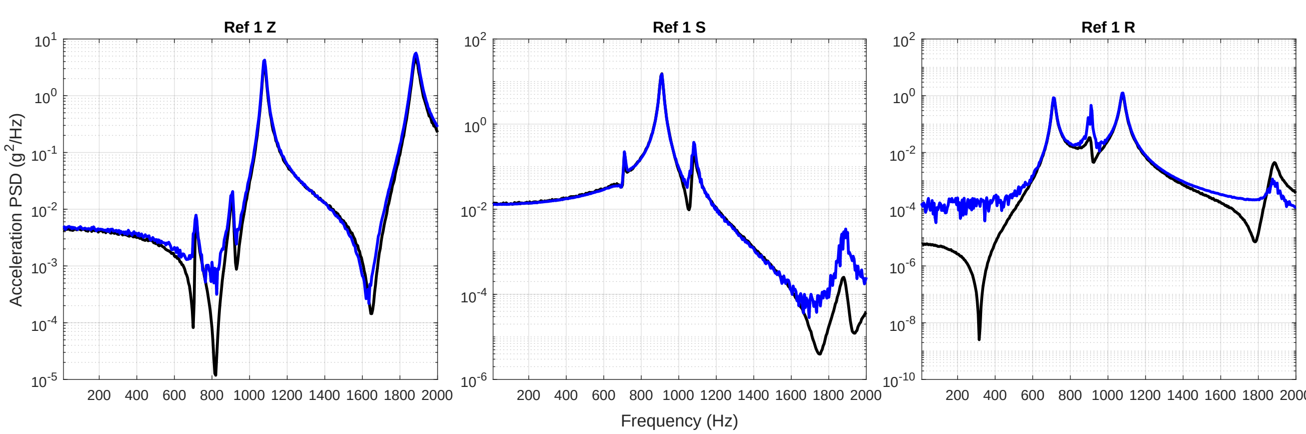


Fig. 4 Flight (black) and modal filtered (blue) environments for first 3 DUT reference channels

In this study, the RMS Von Mises stress is calculated at various points of interest on the DUT. Observation of the stress modes reveals that the top of the stool and the plate both appear to remain rigid below 2000 Hz, suggesting that the most significant stresses occur in the stool legs. The RMS Von Mises stress is calculated at the 36 candidate locations in Figure 5 to capture these areas of most probable failure. These same stress locations are used for each model with negligible variation because each model has the same mesh seed size. The final values reported are the maximum of the RMS Von Mises stresses across these locations, as that is where failure would first occur in flight.

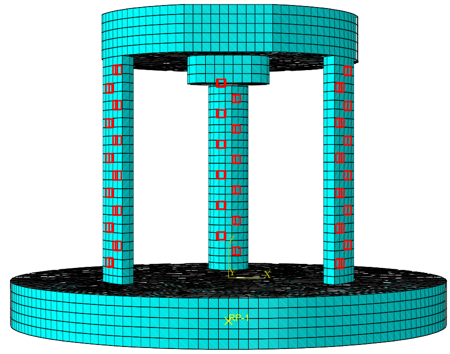


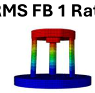
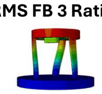
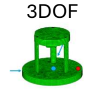
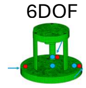
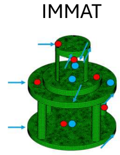
Fig. 5 Locations where RMS Von Mises stress was calculated (red)

To summarize, tests are simulated using the finite element model case study. The RMS dB error and RMS fixed-base modal response values are then calculated and compared to the maximum RMS Von Mises stress values to understand how closely each of these metrics correlates to stress.

Results

The maximum RMS Von Mises stress ratios, RMS dB errors, and RMS fixed-base modal response ratios for the simulated tests are shown in Table 1. In each case, the ratio is the lab value divided by the flight value, so a stress ratio lower than one denotes an under-test. Similarly, a fixed-base response ratio of less than one predicts that a test is not severe enough. All fixed-base modal response ratios are reported, though the analysis will primarily focus on the combined ratios which were calculated using Eq. (6).

Table 1 Comparison of stress and error metrics in SDOF, 3DOF, 6DOF, and IMMAT Tests, where all ratios are the lab value divided by the flight value

Type of test	Max RMS VM Stress Ratio	RMS dB Error	RMS FB 1 Ratio 	RMS FB 2 Ratio 	RMS FB 3 Ratio 	RMS FB Combined Ratio
SDOF 	0.81	38.7	0.02	0.85	0.01	0.38
3DOF 	3.0	15.2	1.46	0.83	0.02	1.17
6DOF 	1.69	6.6	0.91	0.93	2.53	1.44
IMMAT 	1.07	4.5	0.99	0.93	0.98	0.97

Discussion of RMS dB Error

It is often desired that each spectral line lie within 3 or 6 dB of the specified environment, and the same criterion could be applied to the RMS dB error metric. Looking at the stress ratios and RMS dB errors, it becomes apparent that the RMS dB error metric does not predict stress reconstruction very accurately. It is most accurate in the 6DOF test, where the RMS dB error is 6.6 and the stress ratio is 1.69, and the IMMAT, where the RMS dB error is 4.5 and the stress ratio is 1.07. This suggests that the error in the IMMAT may be acceptable while that of the 6DOF test would not be. In reality, both tests recreate stress conservatively and could be acceptable depending on the desired level of margin in the test. Hence, the RMS dB error metric could result in a reasonable test being rejected, redesigned, and repeated unnecessarily.

In the SDOF and 3DOF cases, the RMS dB error metric is far less accurate. The SDOF test results in a stress ratio of 0.81 and an RMS dB error of 38.7, while the 3DOF test results in a stress ratio of 3.0 and an RMS dB error of 15.2. The SDOF test RMS dB error suggests that it is inaccurate by almost four orders of magnitude when, in reality, it is only a slight under-test. The 3DOF test error value similarly predicts that the test is off by more than an order of magnitude though it is actually just three times as severe as it ought to be.

In summary, the RMS dB error metric seems to over-predict test error, and this might cause accurate tests to be needlessly redone. Significant resources could be dedicated to modify the environment to be more replicable in the lab, though this is challenging and still might not meet the error requirements. This metric likely overestimates error because it calculates the difference between response magnitudes at each frequency line. It is difficult to accurately recreate a set of response spectra in general, so it makes sense that this value would often be large. Error at lower response levels is likely not important, though, as most of the stress is caused by the peaks in the environment. In short, the RMS dB error metric weights errors at low response magnitudes equally to high ones, and this is part of the reason that it is poorly correlated to stress.

It is worth repeating that the RMS dB error was calculated over a set of 9 uncontrolled reference accelerometer channels. In practice, these channels may not be available and the metric could be calculated over the control accelerometer responses instead. This would reduce the RMS dB error values which would make them closer to the stress ratios in this case, though the correlation with stress would still be poor as accurate reconstruction of the control responses does not necessarily mean that the rest of the structure's response is accurately recreated. In the 3DOF test, for example, the RMS dB error on the control channels is zero even though stress reconstruction is very inaccurate.

Discussion of Fixed-Base Modal Response Metric

The RMS fixed-base modal response ratios match the maximum stress ratios much more closely than RMS dB error does. Like RMS dB error, this metric is most accurate in the 6DOF and IMMAT cases. In the 6DOF case, the maximum stress ratio is 1.69 and the fixed-base ratio is 1.44, while the stress ratio is 1.07 and the fixed-base ratio is 0.97 in the IMMAT case. This agreement is excellent, particularly compared to the RMS dB error predictions. The RMS fixed-base response ratio is less accurate in the SDOF and 3DOF cases, however. In the SDOF case, the max stress ratio is 0.81 and the fixed-base response ratio is 0.38, while the 3DOF max stress ratio is 3.0 and the fixed-base response ratio is 1.17. While still more accurate than the RMS dB error predictions, the level of disagreement between the proposed metric and stress reconstruction is significant in these cases.

It should also be noted that the fixed-base metric would result in a conservative test in both the SDOF and 3DOF cases since the max stress ratios are larger than the combined RMS fixed-base response ratios. Scaling environment magnitude to make the fixed-base response ratio equal one would result in a stress ratio greater than one, in other words. This is not expected to be the case in general, though.

To understand why the RMS fixed-base modal response metric is accurate in some cases but not others, it is helpful to consider the criterion $\mathbf{p}^{lab}/\mathbf{p}^{fl} = \sigma'^{lab}_{max}/\sigma'^{fl}_{max}$ for a system with two fixed-base stress modes ψ_1 and ψ_2:

$$\frac{{p_1^l}^2 + {p_2^l}^2}{{p_1^{fl}}^2 + {p_2^{fl}}^2} = \frac{(\psi_1 p_1{}^l + \psi_2 p_2{}^l)^{\mathrm{T}} \mathbf{A} (\psi_1 p_1{}^l + \psi_2 p_2{}^l)}{(\psi_1 p_1{}^{fl} + \psi_2 p_2{}^{fl})^{\mathrm{T}} \mathbf{A} (\psi_1 p_1{}^{fl} + \psi_2 p_2{}^{fl})} \tag{7}$$

Note that the free modes of the lab and flight configurations were actually used to compute stress to eliminate modal projection error, but the right side of this equation is accurate when fixed-base modal responses are perfectly known and when fixed-base modes span the free modes of the flight and lab configurations. Making those assumptions, it is apparent that both sides of this equation will equal one if the flight fixed-base modal responses $\mathbf{p}$ are reconstructed perfectly. When there is significant error in response reconstruction, the two sides of the equation are no longer guaranteed to be equal and can be quite different due to the stress modes and interaction terms - e.g., $\psi_1\psi_2 p_1 p_2$ - on the right side. The two sides of the equation might happen to be similar when the fixed-base modal responses are recreated with error, though they are only guaranteed to be equal with accurate modal response reconstruction.

Conclusion

In this study, qualification tests were simulated to attempt to accurately recreate in-flight stress due to vibration. The RMS dB error metric, which is often used to compare environments across multiple accelerometers, was calculated for each case. The RMS fixed-base modal response ratio metric was proposed as an alternative and was also calculated for each test. These were compared to the maximum RMS Von Mises stress ratios to understand how accurately the two metrics captured stress.

The RMS dB error metric was shown to over-estimate the error in tests because it accounts for every spectral difference between environments, some of which can be large in magnitude while not significantly contributing to the stress state of the DUT. The fixed-base modal response ratio metric was shown to represent stress far more accurately in the simulated tests. It was shown to be particularly accurate when modal response spectra were closely matched in the lab. When fewer shakers and / or accelerometers were used, the modal responses were less accurate, and the complex relationship between the modal responses and Von Mises stress resulted in an increased discrepancy between the two. Even with this discrepancy, the proposed metric marked a significant improvement over the RMS dB error metric in every case observed in this study.

Appendix: RMS Von Mises Stress Calculation

RMS Von Mises stress is calculated using the method presented in [14]. While the Von Mises stress mode shapes can be obtained directly from a finite element analysis, modal superposition cannot be applied using these shapes because Von Mises stress is a nonlinear function of the elements of the stress tensor. Because the elements of the stress tensor, $\boldsymbol{\sigma} = \left[\sigma_{xx}\ \sigma_{yy}\ \sigma_{zz}\ \sigma_{xy}\ \sigma_{xz}\ \sigma_{yz}\right]^{\mathrm{T}}$, are linearly related to strain - which is linearly related to derivatives of displacement - modal superposition can be applied to the stress tensor mode shapes, so these are the desired output of the FEM. The RMS Von Mises stress at a single point z on the structure is computed according to

$$\sigma'_{\text{RMS},z} = \sqrt{\delta f \sum_{k=1}^{n_f} \sum_{i=1}^{m} \sum_{j=1}^{n} \mathbf{S}_{\sigma_i \sigma_j, z}(\omega_k) \mathbf{A}_{i,j}}, \tag{8}$$

where $\mathbf{S}_{\sigma_i \sigma_j, z}(\omega)$ is the i, j^{th} element of the stress tensor PSD matrix at some location z. The elements of the stress tensor are related to Von Mises stress according to $\sigma'(\omega) = \sigma(\omega)^{\mathrm{T}} \mathbf{A} \sigma(\omega)$ where

$$\mathbf{A} = \begin{bmatrix} 1 & -0.5 & -0.5 & 0 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & 0 \\ -0.5 & -0.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}. \tag{9}$$

The stress tensor PSD matrix at point z is calculated according to

$$\mathbf{S}_{\sigma\sigma,z}(\omega) = \mathbf{H}_{\sigma f}(\omega) \mathbf{S}_{ff}(\omega) \mathbf{H}_{\sigma f}(\omega)^*, \tag{10}$$

where $\mathbf{H}_{\sigma f}(\omega)$ is a $(6, n_{\text{forces}})$ matrix relating applied force to the elements of the stress tensor and $\mathbf{S}_{ff}(\omega)$ is the force PSD matrix. In flight, the force PSD matrix contains the forces applied to the vehicle, while it contains shaker forces in the lab. $\mathbf{H}_{\sigma f}(\omega)$ can be computed using modal analysis theory based on the free displacement mode shapes at the shaker DOF and the free stress tensor mode shapes at the response point of interest. The maximum RMS Von Mises stress of the structure is obtained by calculating the RMS Von Mises stress at various points on the DUT and retaining the largest of them. Accurately calculating the maximum value requires that the point of highest stress is included in the set of points at which stress is evaluated. This may be challenging to identify, though intuition for potential failure points can be gained by looking at the stress mode shapes, and it also helps to use a large set of candidate locations.

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