Chapter 15
Tips, Tricks, and Obscure Features for Operational Modal Analysis with Cross Spectra

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Abstract While it has recently been proffered that cross spectra can be used for operational modal analysis, this alternative approach has revealed some unanticipated nuances in the modal parameter estimation process. This paper discusses some auxiliary procedures and obscure features that can be used to address these pertinent issues.

Keywords Operational modal analysis · Cross spectra

Introduction

In [1] it was demonstrated that using cross spectra for operational modal analysis (OMA) is a viable third option (in addition to correlations functions and positive power spectra), with just some slight alterations to algorithms that have traditionally estimated poles and shapes from frequency response functions (FRFs). However, as the authors gained some experience with this alternative approach for OMA, we discovered some new challenges that haven't usually been a consideration when using FRFs for experimental modal analysis (EMA). A few tips, tricks, and obscure features intended to redress these issues are discussed in this paper using data from a direct field acoustic noise (DFAN) test [2].

For a DFAN test, the tested structure is surrounded by an array of loudspeakers as an alternative to testing in a reverberant chamber for acoustic qualification tests. The test article for this DFAN test, shown in Fig. 1(a), was intended to represent a satellite with acoustically active panels mounted on an environmental shaker. There were 34 accelerometers mounted on the test article, as shown in Fig. 1(b), and the test article was surrounded by twelve Acoustic Research Systems (ARS) Neutron™ acoustic sources. The purpose of this test was to demonstrate that DFAN could be used for in situ acoustic qualification testing as well as for modal survey tests.

Virtual References in the Frequency Domain

When using cross spectra for OMA, there is a quartet of poles for each mode, which means that the model order of the modal parameter estimation (MPE) algorithm is double of what it would be for FRFs. The number of poles from a solution of an MPE algorithm is equal to the model order times the size of the polynomial matrix coefficients, which for a high-order model such as the Laplace-domain orthogonal polynomial (LDOP) algorithm [3] or the z-domain orthogonal polynomial (ZDOP) algorithm [4] is the number of references. For an FRF dataset from an EMA test, the number of references is typically between one and ten, which produces a manageable number of poles for moderate model orders. However, for a cross-spectra dataset from an OMA test, the number of references is the same as the number of responses, which could be several score to a few hundred. Clearly the references would need to be down-selected to a more reasonable number. This could be done by intuition, experience, and judgement based on the locations of the response degrees of freedom (DOFs) and the expected mode shapes or by inspection of the response data as exemplified in Fig. 2. The power spectral mode indicator functions (PSMIFs) for each reference, which is the summation of the cross spectra of all responses for each reference,

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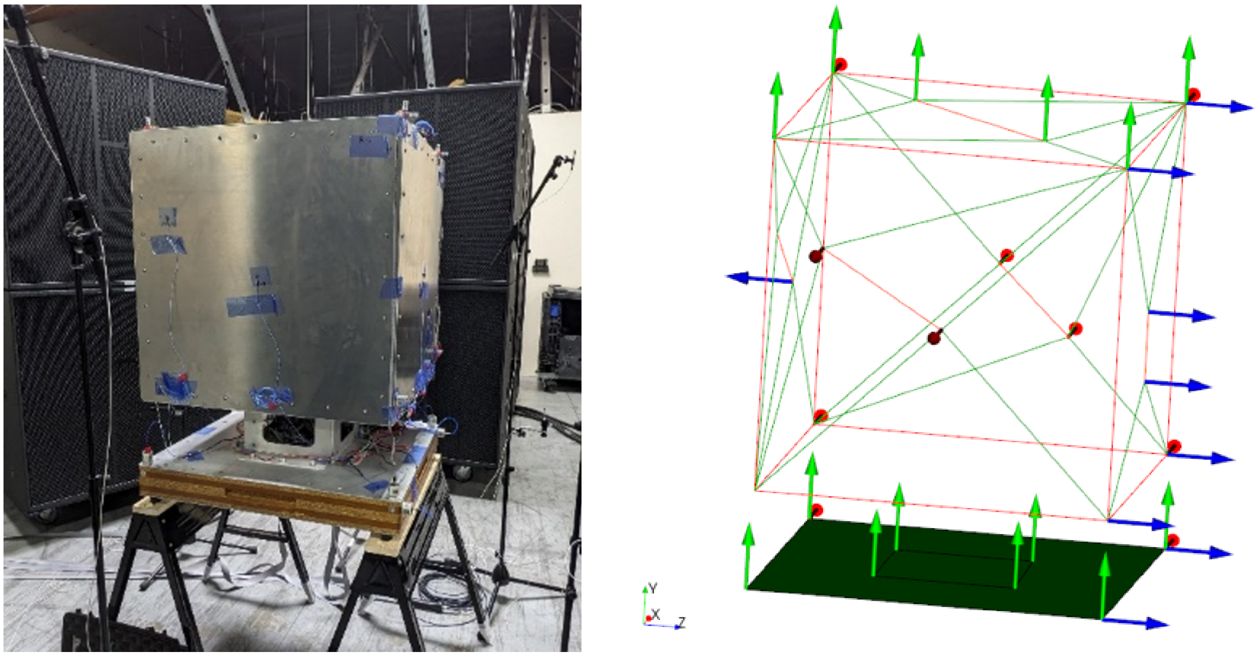
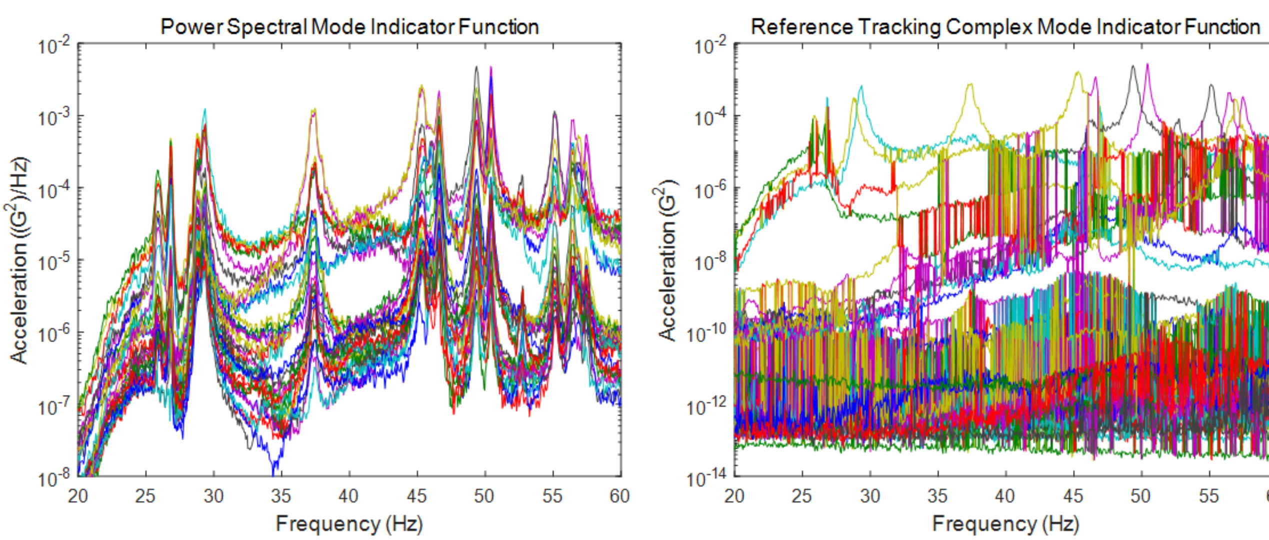


(a) (b)

Fig. 1 (a) Example DFAN test article; (b) test display model with accelerometer measurement degrees of freedom

are plotted in Fig. 2(a) and show which modes are observed by each DOF. The reference tracking (or color-coded [5]) complex mode indicator functions (CMIFs), which evaluate the right singular vectors to sort the functions according to each reference's relative ability to observe the modes, are plotted in Fig. 2(b).



(a) (b)

Fig. 2 (a) PSMIF of all references; (b) reference tracking CMIF

After some tedious contemplation, you could probably decipher a reduced set of references that capture all of the modes of interest from either of these two plots. Or you could forgo the physical realm of the measured DOFs and condense the references into fewer *virtual references* instead. Transforming the $N_o \times N_o$ cross-spectra matrix G_{XX}, where N_o is the number of responses, into an $N_o \times N_v$ matrix with N_v virtual references starts with the singular value decomposition (SVD) of a concatenated cross spectra array containing N_s spectral lines in a frequency band:

$$\underset{N_o N_s \times N_o}{\tilde{G}_{XX}} = \begin{bmatrix} G_{XX}(\omega_1) \\ G_{XX}(\omega_2) \\ \vdots \\ G_{XX}(\omega_{N_s}) \end{bmatrix} \tag{1}$$

Then the economy-size SVD of $\tilde{G}_{XX}$ results in

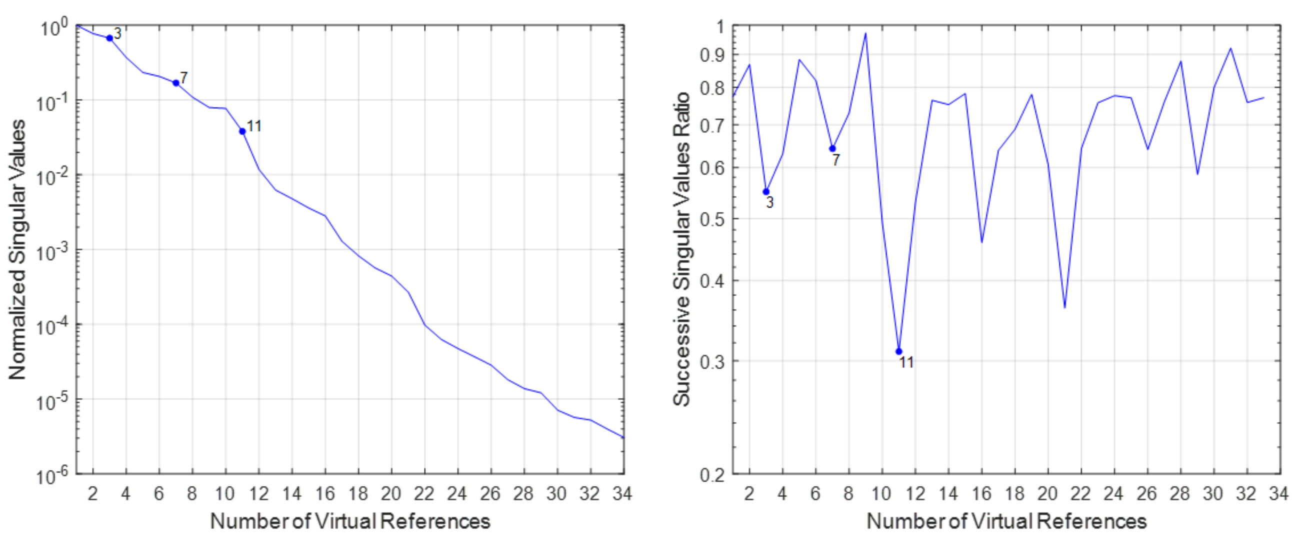
$$\underset{N_o N_s \times N_o}{\tilde{G}_{XX}} = \underset{N_o N_s \times N_o}{U} \underset{N_o \times N_o}{\sum} \underset{N_o \times N_o}{V^H} = \sum_{k=1}^{N_o} u_k \sigma_k v_k^H \approx \sum_{k=1}^{N_v} u_k \sigma_k v_k^H = \underset{N_o N_s \times N_v}{\tilde{U}} \underset{N_v \times N_v}{\tilde{\Sigma}} \underset{N_v \times N_o}{\tilde{V}} \tag{2}$$

One of the popular usages of SVD is to represent the primary content of a matrix with fewer components by discarding those that contribute less to the first summation in Eq. (2), where the singular values are customarily sorted from the largest to the smallest. Thus, a subset of the right singular vectors in $\tilde{V}$ can condense the N_o columns of G_{XX} to N_v virtual references in $\tilde{G}_{XV}$:

$$\underset{N_o \times N_v}{\tilde{G}_{XX}}(\omega) = \underset{N_o \times N_o}{G_{XX}}(\omega) \underset{N_o \times N_v}{\tilde{V}} \tag{3}$$

In addition, virtual drive point responses, which are useful for evaluating which virtual reference best identifies particular modes, can also be created by using a subset of left singular vectors from a horizontally-concatenated array of $\tilde{G}_{XX}$ as spatial filters.

The number of virtual references (N_v) is determined from the number of significant singular values, which can be selected from a plot of the singular values. Two renditions of the singular values for the cross spectra in the frequency band of 23–59 Hz are shown in Fig. 3. The singular values normalized by the first (and largest) value are plotted in Fig. 3(a). Since there isn't always a distinctive knee in the curve demarcating the significant singular values, the successive ratio of the singular values, where each value is divided by the preceding value, is plotted in Fig. 3(b) for an alternative perspective. From this plot, eleven virtual references are the apparent choice, although seven or three might also be worth considering.



(a) (b)

Fig. 3 (a) Normalized singular values of cross spectra in 23–59 Hz frequency band; (b) successive ratio of singular values of cross spectra

To evaluate if any of these choices recreates the important aspects of the original data, cross spectra with virtual references, i.e., G_{XV} from Eq. (3), were generated for $N_v = 3$, 7, and 11. Fig. 4 compares the CMIF of the reduced number of virtual references to that of all of the original measured references (where only the first five CMIFs are plotted). The eleven virtual references match the original at all of the primary peaks, as do the seven virtual references, except with a slightly reduced amplitude for the peaks at 26 and 27 Hz. The three virtual references capture most of the primary peaks, although with reduced amplitude for the peaks at 26, 27, 29, 37, and 55–57 Hz, but might be missing a mode at 28 Hz. The conclusion here would be that while eleven virtual references would certainly be a good choice, seven could also be sufficient.

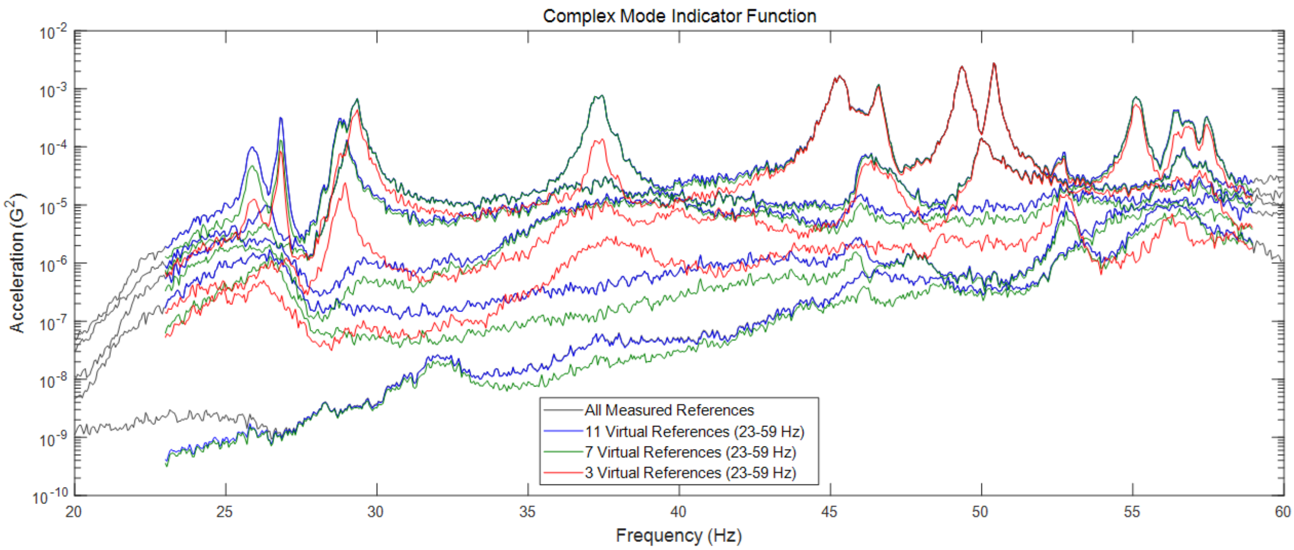


Fig. 4 CMIFs of cross spectra for 3, 7, and 11 virtual references in 23–59 Hz frequency band compared to CMIF of cross spectra for all measured references

In the preceding example, a set of virtual references was found for the entire 23–59 Hz frequency band. However, the example in Fig. 5 shows that it could be divided into several smaller frequency bands, with fewer virtual references for each band. In this case, three or four virtual references are sufficient to match the peaks in each of the three frequency bands.

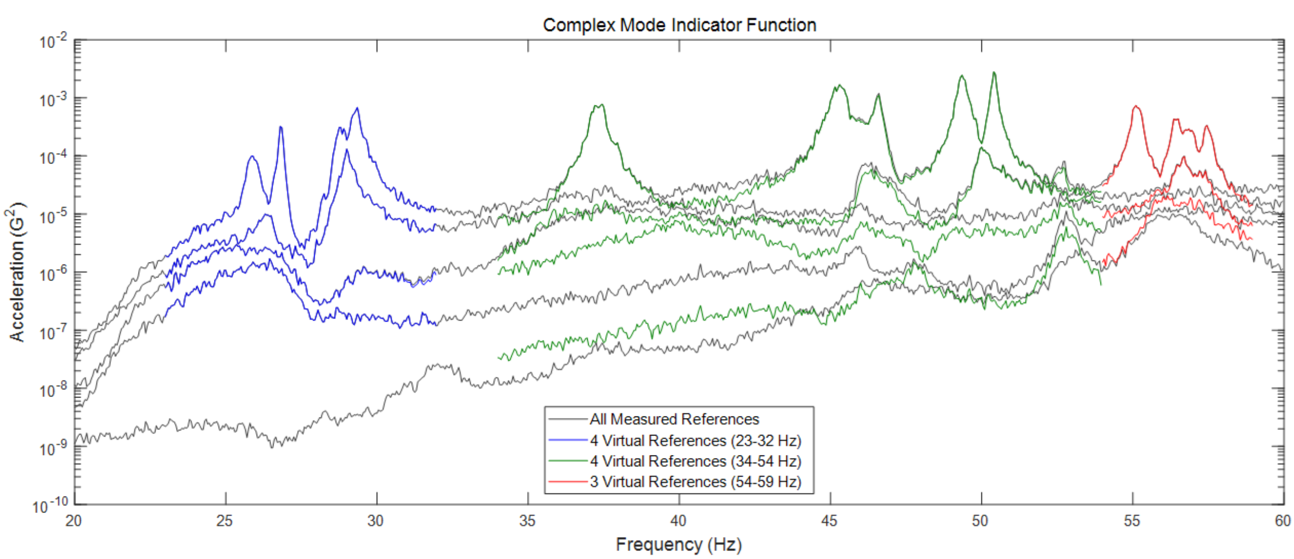


Fig. 5 CMIFs of cross spectra for virtual references in three frequency bands compared to CMIF of cross spectra for all measured references

Virtual References in the Time Domain

For the examples in the preceding section, we started with the 34×34 cross-spectra matrix from the DFAN test, which is not an excessive number of functions. However, for a larger number of responses, generating the entire $N_o \times N_o$ cross-spectra matrix could be avoided by creating the virtual references a few steps upstream in the time domain. To do this, you would first compute the response power spectral densities (PSDs) and plot a PSMIF of these functions to select the frequency bands, as shown in Fig. 6. Then bandpass-filter the time-response data to a frequency band. Virtual-reference time-response functions are generated by spatially filtering the measured time response with a subset of the singular vectors from the SVD of an $N_o \times N_p$ array of the measured time response, where N_p is the number of time points. Then compute the cross spectra between the measured responses and the virtual references.

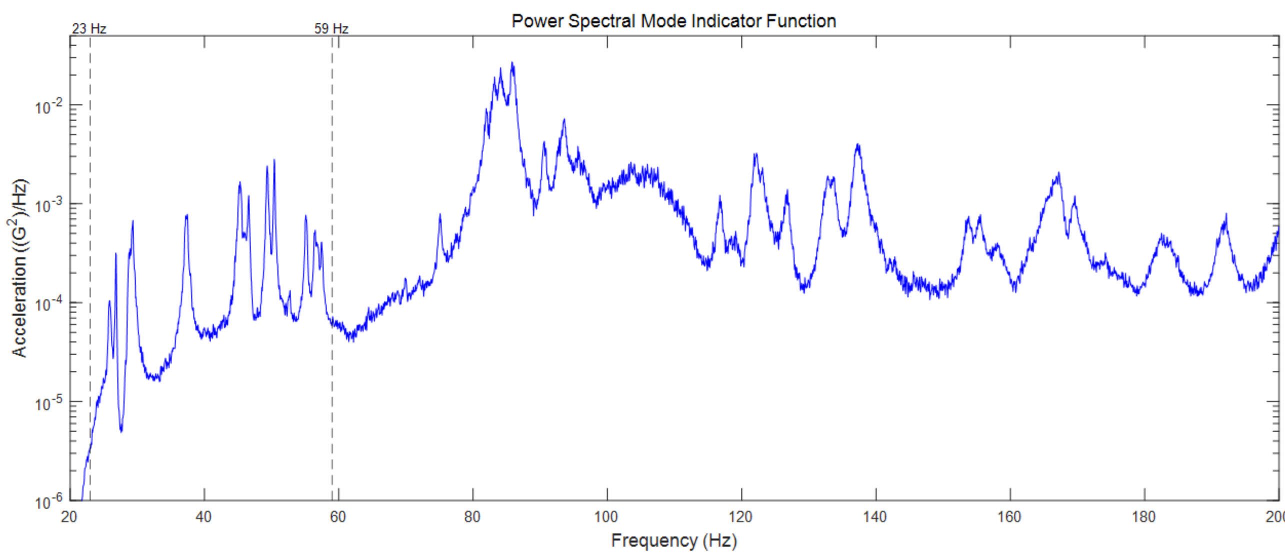


Fig. 6 PSMIF of response PSDs with the 23–59 Hz frequency band denoted by vertical, dashed lines

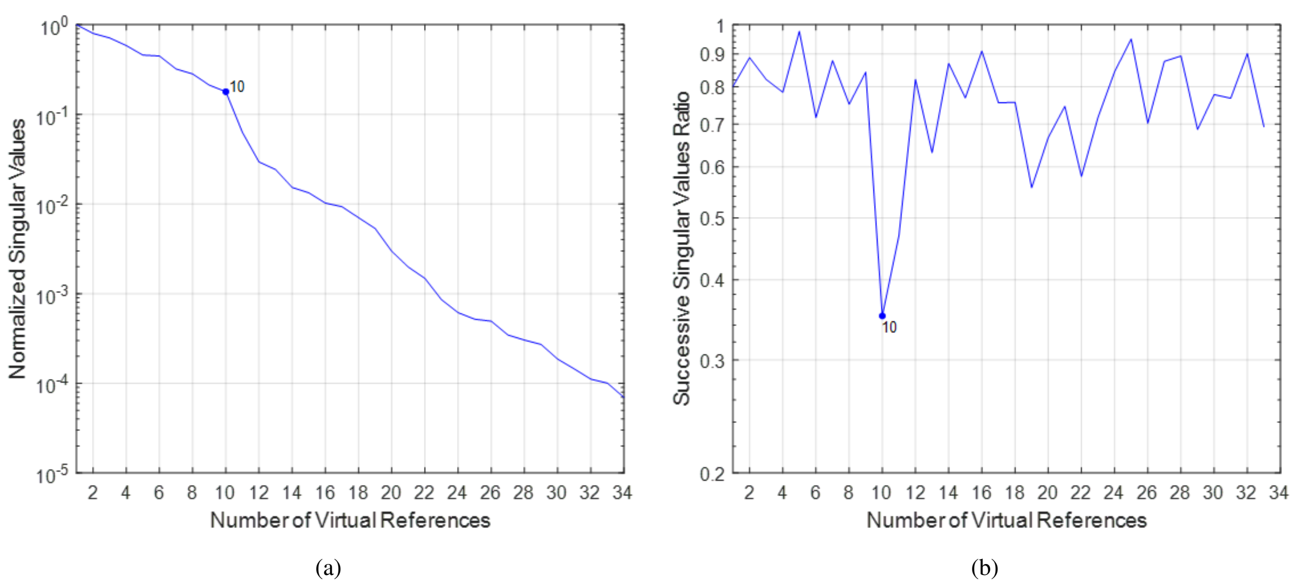


Fig. 7 (a) Normalized singular values of 23–59 Hz bandpass-filtered time response; (b) successive ratio of singular values

Fig. 7 shows the singular values of the time response that has been bandpass-filtered to the 23–59 Hz frequency band. In this case, there is a distinct knee in the normalized singular values plot in Fig. 7(a) at ten virtual references, which is confirmed by the successive ratio of the singular values in Fig. 7(b). Fig. 8 shows the CMIF of the cross spectra computed with ten virtual references, which compares very well with the CMIF of the original cross spectra with all the measured references. The deviation at the edges of the frequency band can be attributed to the roll-off of the filter.

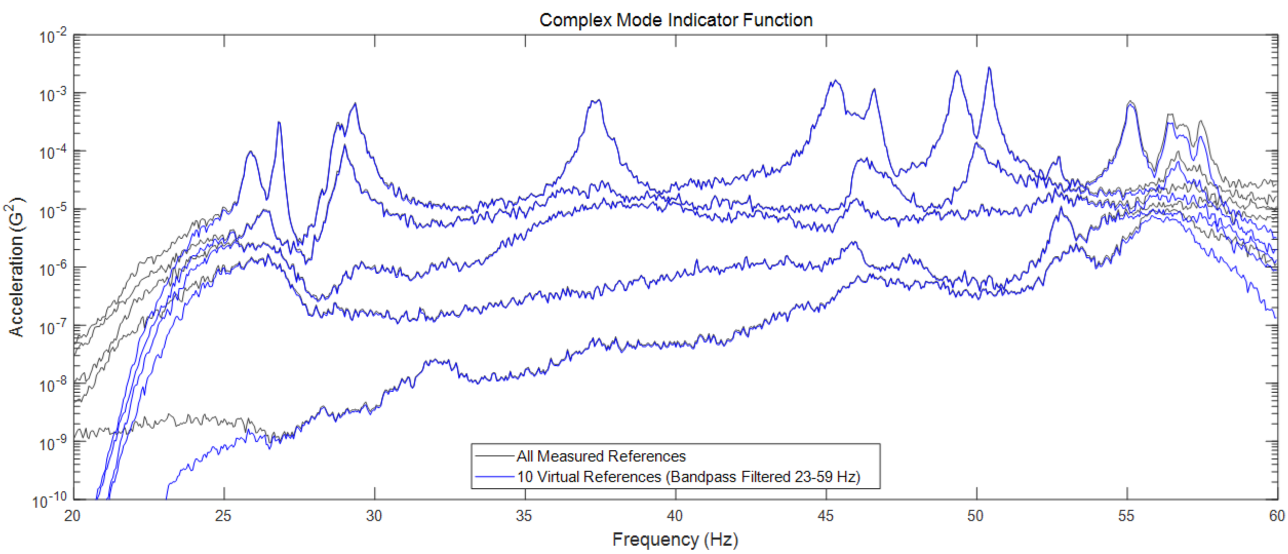


Fig. 8 CMIF of cross spectra for ten time-response virtual references for bandpass-filtered 23–59 Hz frequency band compared to CMIF of cross spectra for all measured references

Estimating Residues in Frequency Bands

When estimating residues, all of the spectral lines in the frequency range encompassing the selected poles are typically included, but as advocated in [6], sometimes it can be better to only use narrow frequency bands near the selected poles. This seems to be especially true for OMA with cross spectra, since the data in the valleys between the modes tend to be quite

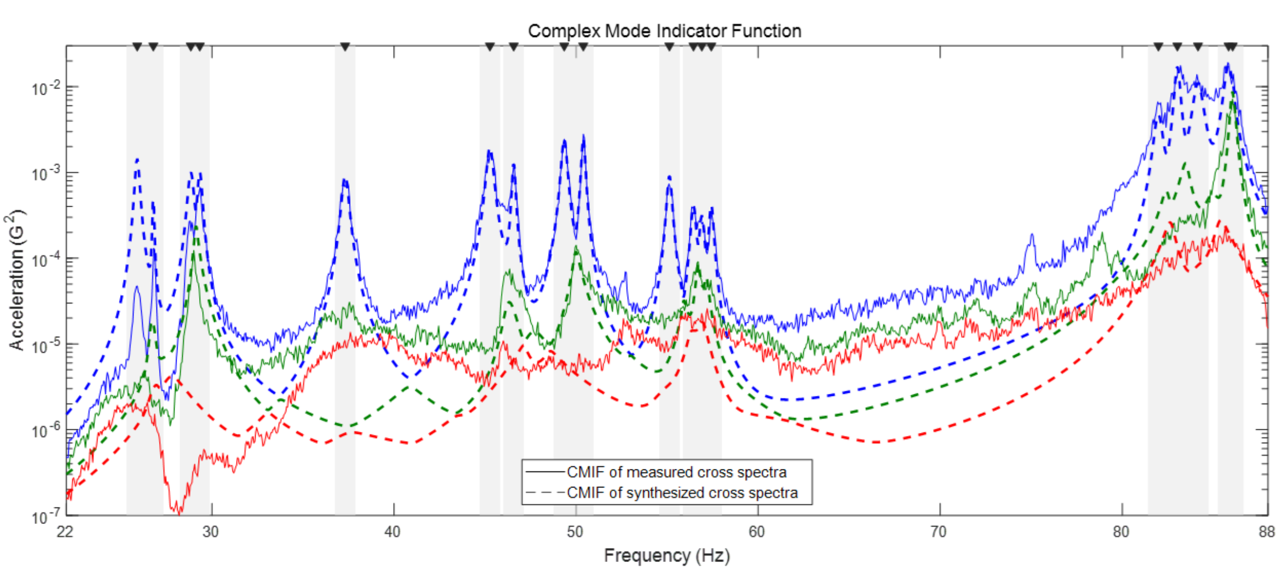


Fig. 9 CMIF for cross spectra synthesized from estimated modal parameters compared to CMIF for measured cross spectra when including all poles in residue solution

noisy. Additionally, we have found that including all the poles in the residues solution can sometimes adversely affect the estimates of some of them. In this case, better results can be obtained by estimating the residues as small groups in narrow frequency bands. An example of this occurrence is shown in Fig. 9, where the results for the four modes below 30 Hz are a cause for concern since the synthesized functions do not match the measured functions. For Fig. 10, the residues were estimated for the groups of poles in the vertical, gray bands, which greatly improved the results for those four modes. For both cases, the selected poles are designated by the triangular markers, and only the spectral lines in the vertical, gray bands were included in the residue estimation. While conclusive proof as to the cause of this phenomenon is not currently available, our prevailing conjecture is that the error in the residue estimation could be a result of violating a fundamental assumption of OMA—that the input spectrum is "broadband, smooth, and constant." For the DFAN test, the input spectra were controlled, but they were not completely flat over the entire frequency range and started to roll off below about 25 Hz (refer to Fig. 5 of [1]). Although this does invalidate the assumption, it's less invalid over small frequency spans, which might lessen the effect on the residue estimates.

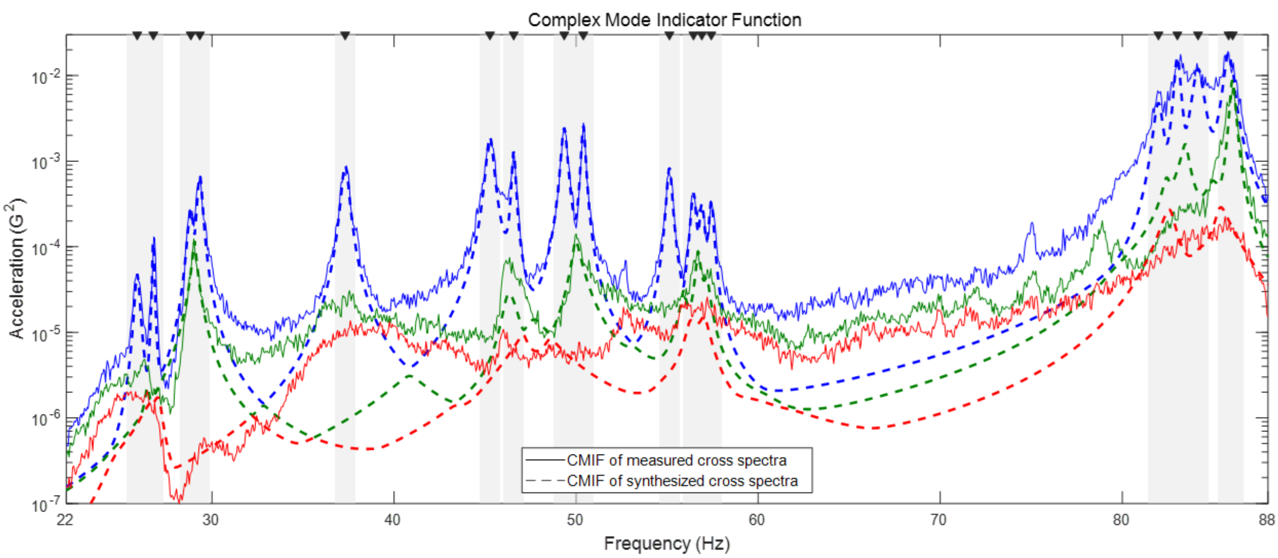


Fig. 10 CMIF for cross spectra synthesized from estimated modal parameters compared to CMIF for measured cross spectra when including narrow-frequency-band poles in residue estimation

Summary

When using cross spectra for OMA, the dataset is likely to have many more potential reference DOFs than are typically encountered when using FRFs for EMA. This paper has shown that the reference space can be reduced to something more amenable to a high-order algorithm by using virtual references instead of physical references. In the examples shown above, the transformation matrix to create the virtual references was derived from the SVD of the measured data from a frequency band (in either the frequency or time domain). However, other vectors, such as operating shapes or mode shapes from a finite element model, can also be used to create virtual references. By applying such a spatial filter, to either the cross spectra or the time response, with either a pseudo-inverse or mass-weighted-shape operation, those shapes can be emphasized in the frequency-domain functions. Doing so is advantageous when trying to extract a set of targets defined in a pretest analysis. Another obscure feature intended in the spirit of the "don't use data that doesn't help your cause" theme in [6] is to estimate the residues as small groups in narrow frequency bands if that improves your results.

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