Chapter 4
Shaping a Shock Response Spectrum via Metamaterial Topology Optimization

Brannon Boyd and Vijaya V. N. Sriram Malladi

Abstract This paper investigates the geometric effects of a metamaterial beam on shock response spectrum (SRS) amplitudes. We use a two-dimensional Timoshenko beam model, with a unit-cell sequence repeated 20 times along its length, and simulated its behavior using a finite element method (FEM) code. The model, consisting of N nodes and 2N degrees-of-freedom (DOF), was subjected to a shock load that propagated temporally through the structure. For comparison, a uniform, constant-thickness beam was used as a control case.

The shock response spectrum was computed for the deflection DOF at each node. An optimization study was then conducted using a Gaussian Process (GP) Bayesian optimization scheme. The parameter space consisted of the duty cycle, number of periods, and individual amplitudes with the objective function to minimize the area under the SRS curve within a specified frequency band. The resulting optimized metamaterial structure demonstrated substantial attenuation of a hypothetical single degree-of-freedom (SDOF) response, not only within the targeted frequency band of 1-2 kHz, but across the entire frequency range. Analysis of the transmission spectrum confirmed that the structural bandgaps were shifted to an ideal case, enabling the effective attenuation of the rapid transient input load. This shows the potential for further SRS shaping and tuning to achieve frequency-specific responses.

Keywords Metamaterials · Shock Response Spectrum · Bayesian Optimization · Timoshenko Beam · Bandgap

Glossary

DOF degrees-of-freedom.
EI expected improvement.
FEM finite element method.
GP gaussian process.
LR local resonance.
PVSS pseudo-velocity shock spectrum.
SDOF single degree-of-freedom.
SRS shock response spectrum.

Introduction

The analysis of beams is fundamental in understanding the underlying dynamics of systems in structural mechanics, with applications ranging from civil and aerospace engineering to biomedical engineering. More recently, metamaterials, which

Brannon Boyd · Vijaya V.N. Sriram Malladi
Vibrations, Intelligent Testing, & Active Learning of Structures (ViTALS) Group, Department of Mechanical and Aerospace Engineering, Michigan Technological University, Houghton, MI 49931
e-mail: boydbn@lanl.gov; smalladi@mtu.edu

Brannon Boyd
Los Alamos National Laboratory, Verification & Validation (V-14), Testing Development and Structural Analysis (TDSA) Team, Los Alamos National Laboratory, P.O. Box 1663, Los Alamos, NM 87545
e-mail: boydbn@lanl.gov

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are engineered materials with tailored properties achieved through periodic structural patterns, have shown great promise in controlling aspects of the dynamic behavior. This paper explores the design of a metamaterial beam engineered to mitigate shock response through strategic geometric shaping. The proposed methodology integrates FEM for discretizing and simulating the system, eigenvalue analysis to identify modal characteristics, a time integration scheme for simulating transient shock responses, and Gaussian process (GP) optimization to refine the beam's geometry for optimal performance within desired frequency bands.

Background

The control of mechanical waves, specifically for applications in vibration and shock isolation, is a central challenge in structural dynamics. Typical design methodologies use passive methods, such as modifying damping or increasing the gross structural rigidity to shift the fundamental frequency. However, these methods are often limited in their efficacy, especially when dealing with broadband transient inputs, and can lead to undesirably heavy or rigid structures and limit the ability to use joints.

Mechanical metamaterials have enabled new ways of understanding structural dynamics. Metamaterials are a class of engineered materials whose properties are derived from their intricate geometric design rather than their base material composition [5, 6]. A key feature of these materials are periodic structures, which enable the formation of phononic bandgaps—theoretical frequency ranges where elastic waves cannot propagate. This phenomenon, which can arise from Bragg scattering (due to periodicity) or local resonance (LR) (due to embedded oscillators), has been extensively studied for its potential to create highly effective vibration isolation systems [1, 2, 9]. A significant body of work has focused on designing these structures to create or shift bandgaps to targeted frequencies, with many studies demonstrating their effectiveness in attenuating steady-state vibrations [9].

More recently, the application of metamaterials has expanded to shock attenuation, which involves a rapid, broadband transient input. The SRS has become the standard metric for quantifying a structure's resilience to such loads [3, 4]. The metric involves the evaluation of a peak response, when modeled as a SDOF oscillator. This paper does not seek to evaluate the validity of the metric, but only use it for comparative purposes. Although several studies have proposed metamaterial designs for shock mitigation, most have focused on materials with negative stiffness or geometric nonlinearities to dissipate energy. Few studies have explored the purely geometric effects of periodic structures on broadband shock response, particularly the direct link between geometric parameters or added LRs, bandgap formation, and the resulting SRS. This gap in the literature represents a critical area for research, as it can lead to reusable, printable, all-elastic designs that do not rely on material dissipation mechanisms.

Furthermore, the design of these geometrically complex structures for a specific performance objective, such as a reduction in SRS amplitudes, constitutes the need for a high-dimensional optimization problem. Traditional gradient-based optimization methods are often ill-suited for this task due to the high computational cost of non-trivial geometries in finite element simulations and the often non-smooth nature of the objective function. Consequently, there is an interest in using more advanced optimization techniques that predict the exact solution with well-defined uncertainty. Bayesian optimization, which uses a GP to build a surrogate model of the objective function, is often employed due to its sample efficiency, making it ideal for problems with expensive black-box function evaluations [7, 10]. While Bayesian optimization has been successfully applied to other metamaterial design problems [8], its application to the specific challenge of minimizing an SRS is a novel contribution.

This paper fits into the existing literature by addressing the underexplored realm of geometric metamaterial design, shock response attenuation, and advanced optimization. We combine a first-order Timoshenko beam finite element methods model with a volume-constrained geometric parameterization and a GP-based Bayesian optimization scheme. This work demonstrates the potential feasibility of tailoring metamaterial beams for superior shock performance and also provides a systematic, computationally efficient methodology for their design.

Methods

The Control Beam

Most mechanical systems with non-trivial displacement are governed by the standard second-order linear dynamic equation of motion:

$$\mathbf{M}\ddot{x}(t) + \mathbf{C}\dot{x}(t) + \mathbf{K}x(t) = \mathrm{F}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the global mass, damping, and stiffness matrices, respectively. The vector $x(t)$ represents the nodal displacements (along with its derivatives $\dot{x}(t)$ and $\ddot{x}(t)$), and $\mathrm{F}(t)$ is the external force vector due to applied temporal load.

To quantify the dynamic performance improvements of the optimized metamaterial beam, a fundamental control case was established. This baseline model consisted of a uniform, constant-thickness first-order Timoshenko beam with a simple, homogeneous cross-section. Timoshenko beam theory is typically used for shorter beams as it accounts for both shear deformation and rotational inertia. This is a more accurate model for short, thick beams or for higher-frequency modes, where the Euler-Bernoulli assumption (plane sections remain plane and normal to the deformed axis) breaks down. However, the beam being analyzed herein is long and slender, but because of the periodic nature of a metamaterial, the wave propagation through the unit-cell and dispersion relation requires a Timoshenko beam to effectively model the physics.

The elemental mass matrix, $\mathbf{M}_e$, is a combination of the translational and rotational inertia:

$$\mathbf{M}_e = \mathbf{M}_{11} + \mathbf{M}_{22}$$

where

$$\mathbf{M}_{11} = \frac{\rho A h_e}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad \mathbf{M}_{22} = \frac{\rho I h_e}{6}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}.$$

Here, ρ is the density, A is the cross-sectional area, I is the second moment of area, and h_e is the element length. The first matrix, $\mathbf{M}_{11}$, represents the translational inertia, while the second, $\mathbf{M}_{22}$, accounts for the rotational inertia.

The elemental stiffness matrix, $\mathbf{K}_e$, includes contributions from both bending and shear deformation:

$$\mathbf{K}_e = \mathbf{K}_{11} + \mathbf{K}_{12} + \mathbf{K}_{21} + \mathbf{K}_{22}$$

where

$$\mathbf{K}_{11} = \frac{K_f GA}{h_e}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \qquad \mathbf{K}_{12} = -\frac{K_f GA}{2}\begin{bmatrix} 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{K}_{21} = -\frac{K_f GA}{2}\begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}; \quad \mathbf{K}_{22} = \frac{EI}{h_e}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} + \frac{K_f GA h_e}{4}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

Here, E is Young's modulus, G is the shear modulus, and K_f is the shear correction factor, $K_f = \left(\frac{0.87+1.12\,\nu}{1+\nu}\right)^2$. The matrices $\mathbf{K}_{11}$, $\mathbf{K}_{12}$, and $\mathbf{K}_{21}$ are related to the shear deformation, while $\mathbf{K}_{22}$ accounts for both the bending stiffness (EI/h_e) and an additional stiffness contribution from the shear deformation. The sum of these sub-matrices forms the full elemental stiffness matrix. The design of this control beam was crucial for ensuring a valid comparative analysis, isolating the effects of the metamaterial's geometric features from other variables. Rayleigh damping was used and modeled using the first two natural frequencies ω_1 and ω_2:

$$\alpha = \frac{2\zeta\omega_1\omega_2}{\omega_1+\omega_2}, \qquad \beta = \frac{2\zeta}{\omega_1+\omega_2}, \qquad C = \alpha\tilde{M} + \beta\tilde{K}, \tag{2}$$

where ζ is the target damping ratio and C is the damping matrix.

The control beam was modeled such that the metamaterial beam would have identical material properties, specifically the same Young's modulus (E), Poisson's ratio (ν), and density (ρ), as the control beam. Its total length, mass, and width were also kept consistent with the metamaterial counterpart. However, the most critical difference was the control beam maintained a constant thickness amplitude across the length. By maintaining this mass equivalence, any observed differences in the SRS amplitudes could be attributed solely to the geometric configuration of the beam, rather than to differences in total mass or nodal DOFs.

The control beam was subjected to the same temporal shock load as the metamaterial beam. A classical half-sine pulse was used for simplicity with an amplitude of 100 Gs and duration of 2.1 ms. The time waveform was propagated through the structure using a Newmark-beta integration technique due to its unconditional stability. Generally, the method updates the state vectors at each time step $i+1$ based on the values at the previous time step i, using the following relationships:

$$\dot{\mathbf{x}}_{i+1} = \dot{\mathbf{x}}_i + \Delta t\left[(1-\gamma)\ddot{\mathbf{x}}_i + \gamma\ddot{\mathbf{x}}_{i+1}\right]$$

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta t \dot{\mathbf{x}}_i + \Delta t^2 \left[\left(\frac{1}{2} - \beta \right) \ddot{\mathbf{x}}_i + \beta \ddot{\mathbf{x}}_{i+1} \right]$$

where Δt is the time step, and β and γ are Newmark-Beta parameters. Here we used the constant average acceleration method with $\beta = 0.25$ and $\gamma = 0.5$ due to not needing high-frequency algorithmic damping and desired unconditionally stability.

Finally, the shock response calculation was done with a Tustin bilinear filter using Smallwood's ramp invariant method. The selected objective metric for shock was the SRS. It is a plot of the maximum acceleration response of a set of SDOF systems to a transient input, as a function of their natural frequency. Each SDOF system represents a specific natural frequency and damping ratio. The SRS is a standard tool for quantifying the severity of a shock event on a structure. The transfer function for an SDOF system is given by:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ω_n here is the $n-th$ hypothetical natural frequency of the system when modeled as an SDOF oscillator subject to base excitation.

Design of 1D Metamaterial Beam

The metamaterial beam used all the same general methods as the control beam, with the exception that the beam's topology was periodically altered. From the Timoshenko beam equations, it can be seen that changing the topology of the beam *will* alter the mass, $\mathbf{M_e}$, and the stiffness $\mathbf{K_e}$ through the cross-sectional area, A, and the area moment of inertia, I. Therefore for a more one-to-one comparison the mass must be equated which can be accomplished by holding density fixed and conserving the volume. Thus to ensure mass (volume) conservation when applying topological optimization, we normalize the raw thickness array so that the average thickness equals a reference base thickness t_{base}. The unnormalized thickness values, t_i, where $i = 1, 2, \ldots, N$, and N is the total number of nodes, can be represented as:

$$\bar{t}_{\text{raw}} = \frac{1}{N} \sum_{i=1}^{N} t_i$$

To preserve total volume (assuming unit width), we scale the entire profile by the normalization factor $t_{\text{base}}/\bar{t}_{\text{raw}}$. Thus, the normalized thickness profile, t_i^{norm} is:

$$t_i^{\text{norm}} = t_i \cdot \left(\frac{t_{\text{base}}}{\bar{t}_{\text{raw}}} \right)$$

ensuring that $\frac{1}{N} \sum_{i=1}^{N} t_i^{\text{norm}} = t_{\text{base}}$ which means the beam with spatially varying thickness has the same average thickness (and hence the same total volume) as a beam of uniform thickness t_{base}. With this simple scaling, the only other adjustment that need be made is explicitly forming the cross-sectional area and area moment of inertia ($\frac{1}{12} bh^3$) with the thickness area. Representative geometry of the control beam and an example of a metamaterial beam can be found in Fig. 1.

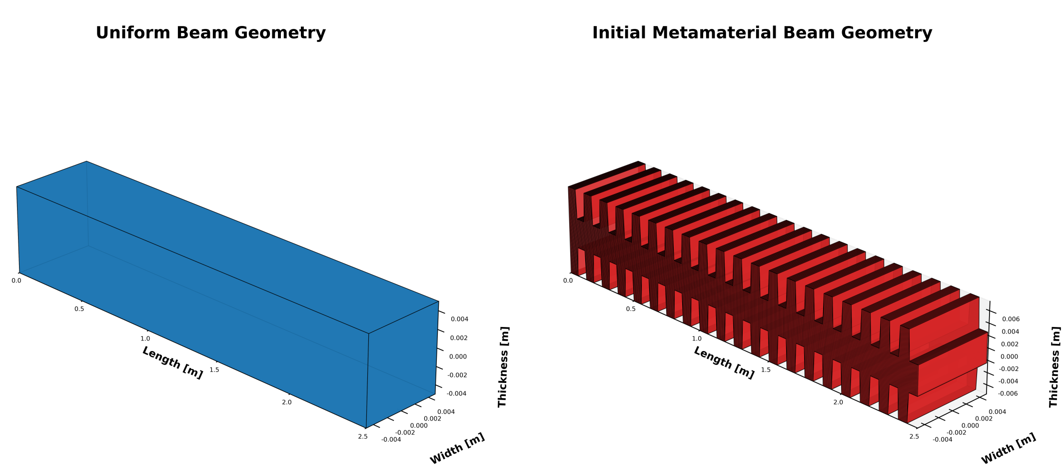

Fig. 1 Shown on the left is a 2.5 meter beam of uniform thickness, constant along the length. The beam depicted on the right is an initial metamaterial beam before geometrically optimizing via a Bayesian optimization technique. Both beams use the same material properties and have identical volume and are therefore mass-equivalent.

Optimization

We set out to optimize the initial metamaterial beam geometry using a Bayesian optimization approach. The parameter space consisted of the first section of a square wave when viewed from the side, along with the duty cycle and number of periods. The other amplitude for a full period was left unconstrained since it would need to be adjusted to conserve volume. The cost function was formed as a metric on the goodness of the metamaterial geometry under test in a simulated classical shock. Minimizing the cost function is the basis of a traditional optimization problem with the objective of reducing the beam's shock response within a specific frequency band.

With the system matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ fully defined for the next y (posterior) in the objective, a numerical simulation can be performed to determine the beam's response to the specified shock loading. By numerically integrating the equations of motion for the beam's DOFs, the full SRS for all nodes can be formed. Here a modified formulation of the SRS was used, the pseudo-velocity shock spectrum (PVSS), defined as: $\text{PVSS}(f) = \frac{\text{g}\cdot\text{SRS}(f)}{2\pi f}$ where g is the gravitational constant and f is the frequency in Hertz.

The cost function is the area under the PVSS curve within the specified frequency band, from a starting frequency $f_{start} = 1000\,\text{Hz}$ to an ending frequency $f_{end} = 2000\,\text{Hz}$. The objective function can thus be expressed as:

$$\text{Objective Value} = \int_{f_{start}}^{f_{end}} \text{PVSS}(f)df \tag{3}$$

This integral must be numerically solved; here, the trapezoidal method is used. By minimizing the objective value, the optimization process reduces the peak responses of the hypothetical SDOF oscillator responses of the beam across the specified frequency range. Using a standard GP with a Bayesian optimization scheme, future points can be selected from an expected improvement (EI) acquisition function. With a GP fitted to the priors, the acquisition function determines the next optimal point to evaluate (posterior). The EI function balances the trade-off between exploring unknown regions of the design space (high variance) and exploiting areas near the current best-known minima (low mean). The EI acquisition function, which computes the expected improvement over the current best observation value y_{best}, is defined as:

$$\text{EI}(x) = E\left[\max(0,\, y_{best} - Y(x))\right] \tag{4}$$

where $Y(x)$ is a random variable representing the objective value at x, with distribution given by the GP surrogate model. The next point to evaluate, x_{next}, is found by maximizing the acquisition function:

$$x_{next} = \arg\max x \in \mathcal{X}\ \text{EI}(x) \tag{5}$$

The Bayesian process requires an adequately sized distribution of priors in order to inform the next solution within the parameter space. The initial sampling can be accomplished using a Sobol statistics method of sampling. The Sobol sampling technique is a method used to generate low-discrepancy sequences, which are particularly useful in high-dimensional spaces for numerical integration and optimization problems. Sobol sequences are designed to fill a multi-dimensional space more uniformly than random sampling techniques such as a Latin Hypercube.

The Sobol sequence is generated using a recursive formula based on binary representations of integers. For a given dimension d, the n-th Sobol point s_n can be expressed as:

$$s_n = (s_{n,1}, s_{n,2}, \ldots, s_{n,d}) \tag{6}$$

where each component $s_{n,j}$ is computed using the following formula:

$$s_{n,j} = \sum_{i=0}^{\infty} \frac{b_{i,j}}{2^{i+1}} \cdot \text{mod}(n, 2^i) \tag{7}$$

where $b_{i,j}$ are the elements of a direction number matrix that defines the Sobol sequence for dimension j. A flowchart of the general process is shown in Fig. 2.

Discussion

The results of this study show the potential for SRS manipulation via geometric metamaterial design of a beam structure. The final optimized geometry can be seen in Fig. 3, where the optimized condition was a single period — always out-performing multiple periods slightly in the cost function for this specific formulation. A reduction in the SRS amplitude for

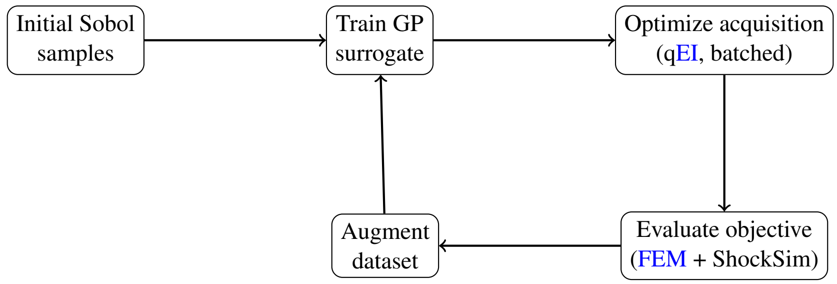

Fig. 2 Illustration of the closed-loop Bayesian optimization workflow used for design exploration: initial Sobol sampling to form the basis of priors, Gaussian process (GP) surrogate training, batched acquisition optimization using q-Expected Improvement (qEI), high-fidelity objective evaluation (FEM + ShockSim) of selected candidates, and dataset augmentation for the next optimization iteration (updated posterior from priors).

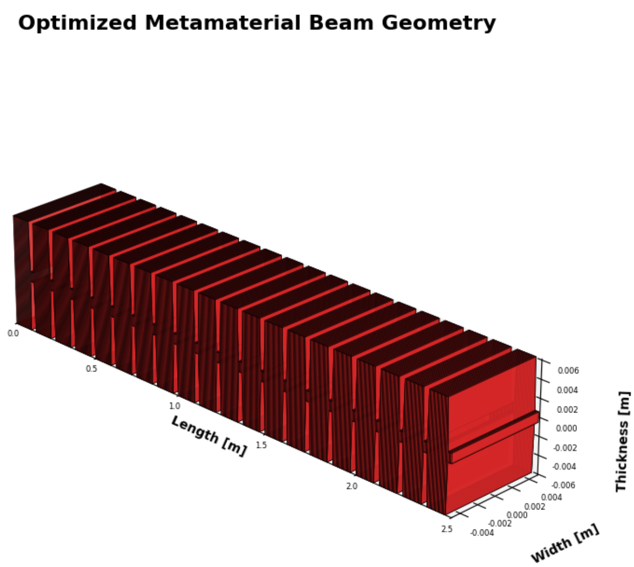

Fig. 3 Final optimized beam geometry with one period and an adjusted duty cycle and slight amplitude scaling which can be seen when comparing to Fig. 1. However, due to the single period, the optimized metamaterial is more similar to the initial metamaterial than it is different.

the optimized metamaterial beam, demonstrates the hypothesis that geometric patterning can be used to control transient mechanical waves. An exemplar case at 2.25 m response location along the length of the beam can be seen in Fig. 4 plotted on a tripartite pseudo-velocity plot. This improvement was not only to the targeted frequency band (1-2 kHz), but visible across the entire computed frequency range. The full spectral amplitude reduction suggests that the optimization process may yield an effective, broadband attenuation mechanism.

The initial metamaterial beam before optimization, as compared to the uniform beam transmission plot, is seen in Fig. 5. The transmission post optimization is also shown on the right, where it can be seen that the optimization function served to carve out the bandgap slightly more around the 1-2 kHz frequency range and shifted some of the dissipation away from the $\sim$ 4750 Hz frequency bandgap.

Understanding the mechanisms behind these mechanics lies in the analysis of the transmission spectrum. The optimization process, driven by the objective to minimize the area under the PVSS curve, modified the beam's geometric properties to shift and broaden the structural bandgaps in the desired frequency band. The transmission spectrum of the optimized beam, when compared to the uniform control beam, showed a clear and pronounced attenuation region corresponding to the targeted frequency band. This confirms that the observed shock attenuation is a direct consequence of the wave filtering properties of the periodic structure. While the fundamental frequency (and only frequency) of the shock pulse was at 2.99 kHz, this does not mean that the impulse energy will transmit through the structure at that energy, but may have other dominant frequencies as observed here ($\sim$ 1.5 kHz). A full SRS (traditional SRS) swept along the length of the beam is depicted

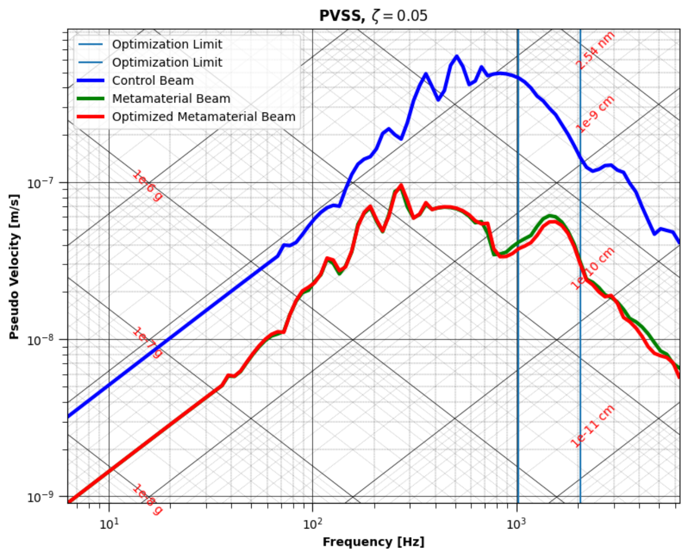

Fig. 4 Exemplar pseudo-velocity shock response spectrum for the control and different metamaterial beams at a response location of 2.25 m. The optimized metamaterial shows a slight reduction in the target frequency band at this specific location along the length of the beam. The initial metamaterial before optimization was quite close to the optimized structure — likely in part due to the simplicity of the structure and classical shock impulse.

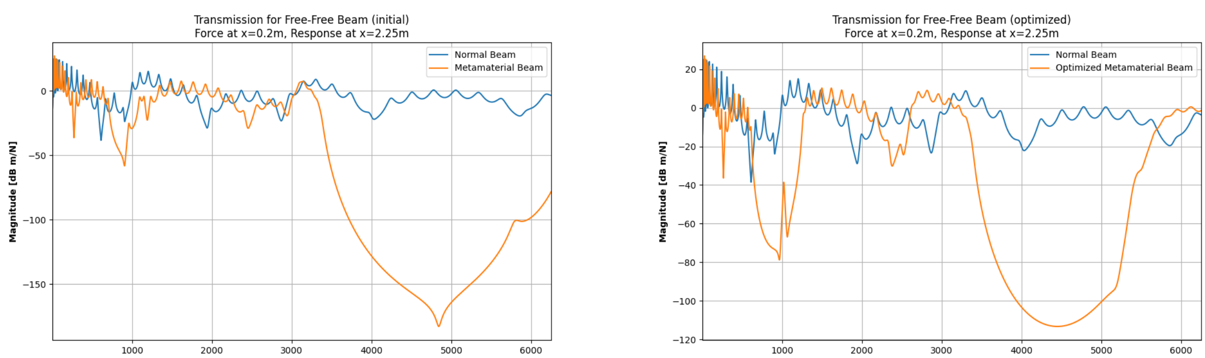

Fig. 5 The initial metamaterial beam dynamic transmission plotted on a linear abscissa in comparison to an equivalent mass control beam can be seen on the left. The right graph shows the transmission for the optimized case where the ~ 4750 Hz frequency bandgap can be seen to reduce in dissipation and shift some of the finite dissipation towards the 1-2 kHz bandgap that starts to form.

in Fig. 6. The control case is plotted above the optimization case. There is a clear reduction in the amplitudes along the entire length of the beam and especially in the peak region of 1-2 kHz.

The optimized geometry itself, a periodic pattern of varying beam thickness (a metamaterial), demonstrates that a Bayesian optimization approach is efficacious. This data-driven method efficiently explored the parameter space and, through a limited number of FEM simulations, identified a configuration that alters the beam's mass and stiffness matrices to create near-ideal bandgap behavior. The GP surrogate model functioned by balancing the exploration of new design regions with the exploitation of promising ones, thereby leading to an optimized final design with fewer trial cases than other schemes, such as neural networks.

Further work needs to explore the extension of this methodology into experimental spaces. The current study was limited to a two-dimensional simulated Timoshenko beam model; a logical next step would be to apply this technique to three-dimensional solid or shell model to capture more complex wave phenomena and predict physical systems. The optimization

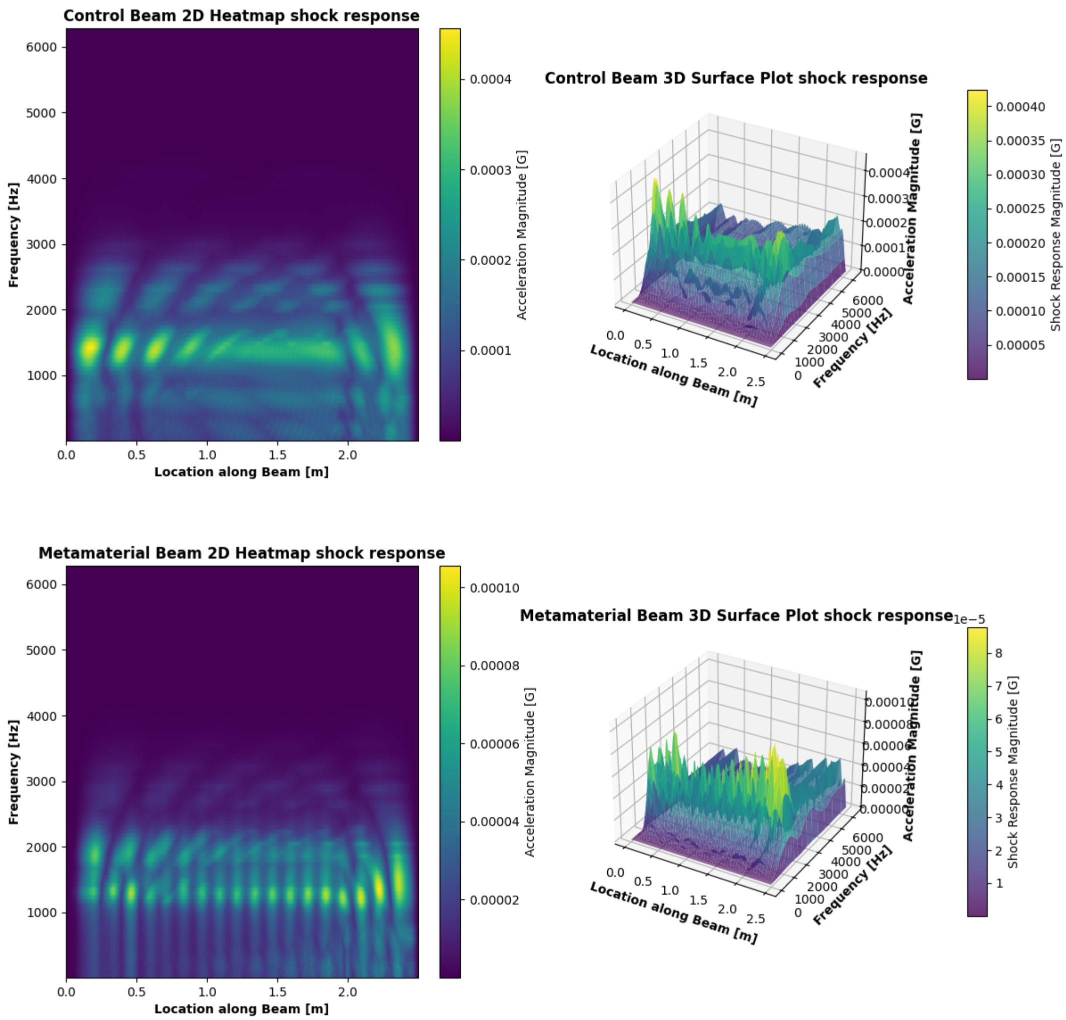

Fig. 6 SRS along the length of the beam for the displacement DOF. The optimized structure shows large reductions in the magnitude of the maximum accelerations for a hypothetical SDOF oscillator in the desired frequency band of 1-2 kHz. The amplitude reduction shows the potential for this scheme to be implemented to tune shock spectra for specific frequency transmission.

scheme can also be extended to a larger parameter space, for example, minimizing both the SRS and the overall structural mass, or by tuning for multiple shock inputs with different spectral characteristics.

Conclusion

This paper presented a novel methodology for shaping an SRS via a geometrically-driven metamaterial beam for enhanced shock response attenuation. By combining a two-dimensional first-order Timoshenko beam finite element model with a Bayesian optimization scheme, we demonstrated a systematic approach to manipulate a beam's geometry to achieve the specific objective of reducing the spectrum in the 1-2 kHz frequency band.

The results unequivocally showed that the optimized metamaterial beam suceeds in attenuating responses relative to a uniform control beam of equivalent mass. The attenuation of the resonant structures is directly attributed to the creation and shifting of structural bandgaps as seen in the transmission plots, which act as a mechanical wave-guide for propagation of structure-borne waves. The success of the optimization, driven by minimizing the area under the PVSS, highlights the efficacy of using a GP-based Bayesian optimization for computationally expensive, black-box problems in structural design.

In summary, this work provides initial simulation results for developing potential passive shock mitigation structures through the reduction of hypothetical SDOF oscillator peak responses.

References

1. Alya Alhammadi, Jin-You Lu, Mahra Almheiri, Fatima Alzaabi, Zineb Matouk, Mohamed Al Teneiji, Rashid K. Abu Al-Rub, Vincenzo Giannini, and Dong-Wook Lee. Numerical modelling and optimization of two-dimensional phononic band gaps in elastic metamaterials with square inclusions. *Applied Sciences*, 11(7), 2021.
2. Mahmoud I. Hussein, Matthew J. Leamy, and Massimo Ruzzene. Dynamics of phononic materials and structures: Historical origins, recent progress, and future outlook. *Applied Mechanics Reviews*, 66(4):040802, 2014.
3. Monty Kennedy. *Correlation of and Development of Procedure to Use a Resonant Plate With Mechanical Excitation for Shock Testing Small-to-Medium Size Spacecraft and Provide Aerospace Shock Analysis and Testing Guidelines*. PhD thesis, Michigan Technological University, 2023. Copyright - Database copyright ProQuest LLC; ProQuest does not claim copyright in the individual underlying works; Last updated - 2024-05-08.
4. William S. Larsen. *Analysis of the Shock Response Spectrum and Resonant Plate Testing Methods*. PhD thesis, Michigan Technological University, 2018.
5. B. Liu and C. Ji. Automated metamaterial design with computer model emulation and bayesian optimization. *Applied Mechanics and Materials*, 2014.
6. H. Luo, J. Xu, Y. Song, and et al. Machine intelligence in metamaterials design: a review. *Oxford Open Materials Science*, 4(1), 2024.
7. C. Sharpe, C. Seepersad, D. A. Tortorelli, and S. E. Watts. Design of mechanical metamaterials via constrained bayesian optimization. *OSTI*, 2018.
8. Zhichao Tian, Yang Yang, Sui Zhou, Tian Zhou, Ke Deng, Chunlin Ji, Yejun He, and Jun S. Liu. High-dimensional bayesian optimization for metamaterial design. *Materials Genome Engineering Advances*, 2(4):e79, 2024.
9. Thomas Vasileiadis, Jeena Varghese, Visnja Babacic, Jordi Gomis-Bresco, Daniel Navarro Urrios, and Bartlomiej Graczykowski. Progress and perspectives on phononic crystals. *Journal of Applied Physics*, 129(16):160901, 04 2021.
10. Y. Yang, C. Ji, and K. Deng. Rapid design of metamaterials via multitarget bayesian optimization. *The Annals of Applied Statistics*, 15(2), 2021.