Chapter 14
Data-Driven Aerodynamic Modeling for Flapping-Wing Robots

Costanza Speciale, Silvia Milana, Antonio Concilio, and Antonio Carcaterra

Abstract Flapping-Wing Aerial Vehicles (FWAVs), also known as ornithopters, draw inspiration from nature and use rapidly oscillating wings to generate sufficient lift and thrust to move forward while sustaining their weight. Their similarities to birds and compliance with the natural environment, combined with their lower noise levels compared to propeller-driven aerial vehicles, make them ideal tools for monitoring sensitive ecosystems with minimal disturbance.

Predicting the dynamic behavior of these bio-inspired robots is particularly challenging due to the difficulty in estimating the aerodynamic forces resulting from the complex, unsteady aerodynamic phenomena generated by the oscillating motion of the wing surfaces under variable wind conditions. This challenge is further compounded by the flexibility of their membrane wings, which typically undergo large shape changes under aerodynamic loads depending on factors such as flight speed, attitude and maneuvering.

A promising solution lies in data-driven reduced-order models. In this study, Volterra theory is explored as a framework to develop such a model for the unsteady aerodynamics of a flapping wing. Trained on experimental measurements or high-fidelity numerical simulations of an ornithopter wing operating under varying conditions, the resulting model can provide accurate and computationally efficient aerodynamic force predictions.

Keywords Flapping-wing aerial vehicles · Biomimetics · Aerodynamic modeling · Volterra theory · Robotics

Introduction

Researchers have long been fascinated by the unique flight capabilities of birds, as attested by the studies of Leonardo Da Vinci in the sixteenth century [1], who sketched machines intended to allow humans to fly inspired by avian flight (Figure 1a).

Despite pioneering attempts such as those of Otto Lilienthal, who designed and realized multiple manned, engine-powered flapping-wing machines other than gliders, the development of ornithopters was hindered for decades by their inherent complexity and by the success of fixed-wing aircraft [2].

With technological advancements, however, interest in natural fliers was renewed in the early 2000s, this time focusing on the development of Unmanned Aerial Vehicles (UAVs). Since then, several flapping-wing aerial vehicle (FWAV) prototypes have been realized, including Festo's SmartBird [3], the Dove [4], and the more recent RoboFalcon [5].

The growing interest in these platforms stems from their unique characteristics, which allow them to minimize disturbance to the surroundings, both acoustically and visually. These features makes FWAVs more suitable than other UAV types for a wide range of operations, such as wildlife monitoring in protected areas, forest surveillance, vehicle tracking and infrastructures inspection in urban environments with minimal impact on the local population.

Despite these advantages, several limitations still prevent their use in practical applications. These include strict payload restrictions, limited take-off and landing capabilities, and challenges in modeling and control, arising from the unsteady three-dimensional flow generated by the flapping-wing motion. The focus of this work lies in addressing this latter challenge.

There exist different approaches for unsteady aerodynamic modeling, which can be grouped into three main categories: theoretical modeling, computational modeling and data-driven modeling [6].

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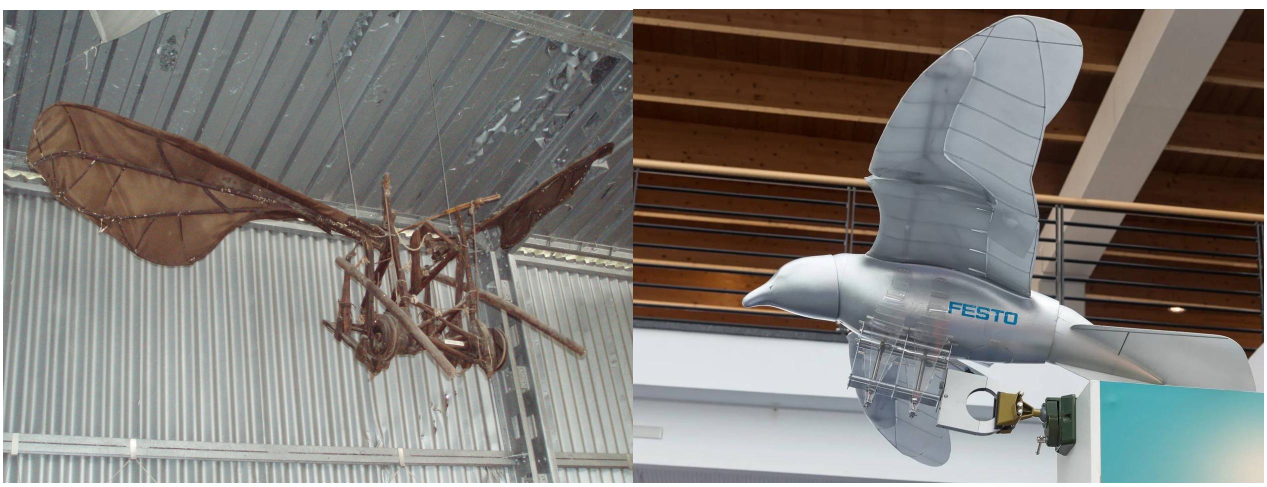
Kristin B. Zimmerman, PhD (eds.), *Structural Health Monitoring, Metamaterials, Metastructures and Additive Manufacturing and Dynamics of Biomedical and Bioinspired Systems, 2026, Vol. 10*, Conference Proceedings
of the Society for Experimental Mechanics Series, https://doi.org/10.1007/978-8-743-81449-8_14

Theoretical models, such as the classical analytical models of Wagner [7] and Theodorsen [8], remain valuable for certain applications but rely on the small perturbation assumption inherent in linear potential theory, which limits their accuracy when dealing with the large-amplitude motion typical of ornithopter wings.

Computational Fluid Dynamics (CFD) simulations, on the other hand, can predict aerodynamic loads with high accuracy, but their high computational cost makes them unsuitable for scenarios where fast predictions are required.

For real-time applications, rough estimates based on a flapping frequency of 1 Hz suggest a desired update rate around 20 Hz, which is incompatible with the computational demands of such simulations.

A promising alternative is the use of data-driven approaches, which leverage data from experiments or high-fidelity numerical simulations to construct Reduced Order Models (ROMs). These can provide both accurate and computationally efficient aerodynamic force estimations, which are fundamental, for example, for enabling real-time model-based control of ornithopters. This work focuses on a particular class of data-driven aerodynamic models based on system identification (i.e. the process of deriving mathematical models from observed system input-output data signals [9]) and specifically on those based on the Volterra theory [10]. Volterra-based models have been widely applied in both aerodynamic and aeroelastic modeling. The first to do so was Silva [11], who employed a second order Volterra series to model the transonic unsteady aerodynamic response of a pitching wing. Marzocca et al. [12] developed a Volterra-based aeroelastic ROM for a two-dimensional airfoil and used it to predict flutter instability. De Paula et al. [13] proposed a method to identify Volterra kernels for multiple-input, multiple-output systems using time-delay neural networks and applied it to the unsteady aerodynamic modeling of a heaving and pitching airfoil. This study is positioned within the context of ROMs for flapping wings and builds on the works of Liu et al. [14] and Ruiz et al. [15] by focusing on Volterra-series-based aerodynamic modeling.



(a) (b)

Fig. 1 (**a**) A full-sized replica of Leonardo da Vinci's ornithopter at the Museum of Aviation in Košice, Slovakia. Photo by ZemplinTemplar, licensed under CC BY-SA 4.0. (**b**) Festo's SmartBird [3]. Photo by Matti Blume, licensed under CC BY-SA 4.0

Volterra Theory

The Volterra series [10] is a mathematical tool widely used in many engineering applications to describe the behavior of nonlinear, time-invariant systems with fading memory (i.e. systems in which the output depends not only on the current input but also on its past values, with the influence of past states gradually decreasing over time [16]).

Considering a single-input, single-output system, its output $y(t)$ can be expressed using a Volterra series expansion as an infinite sum of multidimensional convolution integrals:

$$y(t) = k_0 + \sum_{n=1}^{\infty} \int_0^t \cdots \int_0^t k_n(\tau_1, \tau_2, \ldots, \tau_n) \prod_{i=1}^{n} u(t - \tau_i)\, d\tau_1 d\tau_2 \ldots d\tau_n \tag{1}$$

where $u(t)$ is the system input, k_0 is the zero-input response and $k_n(\tau_1, \tau_2, \ldots, \tau_n)$ is the n^{th}-order Volterra kernel, which represents the combined effect of n past input states on the present output.

The convolution structure of the series introduces memory effects into the system response, making it particularly suitable for applications such as unsteady aerodynamic modeling, where it can capture effects like the influence of a wing's vortex wake on the aerodynamic loads, which becomes significant when the characteristic timescale of the phenomenon is comparable to the convective timescale associated with the inflow speed.

The Volterra series can be also extended to the general case of multiple-input, multiple-output systems. The j^{th} output $q_j(t)$ can be expressed as:

$$\begin{aligned} q_j(t) = k_j^0 &+ \sum_{i=1}^{p} \int_0^\infty k_{j;i}^{(1)}(\tau_1)\, u_i(t-\tau_1)\, d\tau_1 \\ &+ \sum_{i_1=1}^{p} \sum_{i_2=1}^{p} \int_0^\infty \int_0^\infty k_{j;i_1,i_2}^{(2)}(\tau_1,\tau_2)\, u_{i_1}(t-\tau_1) u_{i_2}(t-\tau_2)\, d\tau_1 d\tau_2 \\ &+ \cdots \\ &+ \sum_{i_1=1}^{p} \cdots \sum_{i_n=1}^{p} \int_0^\infty \cdots \int_0^\infty k_{j;i_1,\ldots,i_n}^{(n)}(\tau_1,\ldots,\tau_n) \prod_{m=1}^{n} u_{i_m}(t-\tau_m)\, d\tau_1 \cdots d\tau_n + \cdots \end{aligned} \tag{2}$$

Here, p is the number of inputs, $u_i(\cdot)$ are the system inputs $(i = 1, ..., p)$ and $k_{j;i_1,\ldots,i_n}^{(n)}(\tau_1,\ldots,\tau_n)$ denotes the n^{th}-order multi-input Volterra kernel, where the subscript j identifies the system output and the indices following the semicolon indicate the inputs to which the kernel corresponds.

Methodology

The use of Volterra theory is explored to develop a model capable of accurately and efficiently estimating the aerodynamic forces of a flapping-wing.

Consider a flapping airfoil, representing a section of the ornithopter's wing, undergoing combined heaving and pitching motions, described by the heave displacement h and pitch angle θ, respectively. Focusing on the modeling of the lift force L (a similar procedure can be applied to drag and aerodynamic moment), it can be approximated by a Volterra series expansion truncated at the first order:

$$L(t) = k_l^0 + \int_0^t [k_{l;1}(\tau)u_1(t-\tau) + k_{l;2}(\tau)u_2(t-\tau) + \ldots k_{l;N}(\tau)u_N(t-\tau)]\, d\tau = k_l^0 + \mathbf{k} * \mathbf{u} \tag{3}$$

Where $\mathbf{k} = [k_{l;1}\ k_{l;2} \ldots k_{l;N}]$ is the vector of the first-order kernels, while $\mathbf{u} = [u_1\ u_2 \ldots u_N]^T$ is the input vector containing N variables that can be selected, using an appropriate criterion, from an initial set $\mathbf{u}_0$, that may include, for example, $h, \theta, \dot{h}$ and $\dot{\theta}$ [17].

To identify the unknown kernels $k_{l;1}\ k_{l;2} \ldots k_{l;N}$, a numerical or experimental training dataset containing inputs and output time histories obtained under varying conditions, such as different airspeeds, flapping frequencies and angles of attack, can be employed.

A possible technique for kernel identification from such datasets, which has shown promising results in flapping-wing applications [14, 15], is direct identification, which consists in identifying the Volterra kernels directly from input-output data without relying on intermediate transformations. For clarity, consider the single-input case, where the input may be, for instance, the time derivative of the effective angle of attack as in [15].

In discrete time, and assuming $k_l^0 = 0$, the first-order Volterra model for the lift force can be written as:

$$L[\tau] = \sum_{n=0}^{M} k_l[n] u[\tau - n] \Delta t \tag{4}$$

where τ and n are the time indices, M is the memory depth in time steps and Δt is the time step. The terms $k_l[n] = k_l(t)|_{t=n\Delta t}$ and $u[n] = u(t)|_{t=n\Delta t}$, are the discrete kernel and input values, respectively. Equation 4 can then be rearranged into matrix form, enabling the kernel values to be identified using least-squares or regularization techniques. In addition to direct identification, other methods for Volterra kernel identification can also be considered, these include the cross-correlation method [18], orthonormal basis functions [19], and time-delay neural networks [20].

Finally, once the model has been identified, its generalizability can be evaluated by validating it against data obtained under conditions not used during training.

Conclusions and further developments

Ornithopters are complex aerial vehicles that present several significant challenges. Among these, particularly critical are modeling and control, which are difficult due to the unsteady aerodynamics associated with their wing motion.

To address the challenges related to aerodynamic modeling, a promising approach is to develop a ROM based on Volterra theory. Such a model can combine computational efficiency with adequate accuracy, making it a particularly valuable tool for applications such as real-time control, where fast and reliable aerodynamic force prediction is a fundamental requirement. The resulting aerodynamic ROM can be indeed integrated into a dynamic model of the ornithopter, which, due to the structure of the Volterra series, would take the form of integro-differential equations. Advanced control strategies specifically developed for integro-differential systems, such as those proposed by Paifelman et al. [21], could then be applied to enable effective control of the ornithopter's flight.

Acknowledgments This research was funded by the European Union – NextGeneration EU, Mission 4 Component 2 Investment 3.3 CUP B53C24002450004.

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