Chapter 17
Analytical Modeling of Flexible Caudal Fins with Arbitrary Biomimetic Shapes

E. Paifelman, S. Milana, G. Costantino Muniz, A. Carcaterra, and E. Ciappi

Abstract This study presents an extension of an analytical model for the dynamic behavior of flexible bio-inspired fins, with the goal of enabling real-time predictions across arbitrary planform geometries. The work addresses the need for fast and versatile modeling tools to support the design of next-generation soft and smart fin technologies. The proposed fully analytical formulation captures the coupled fluid–structure interaction dynamics of elastic fins and is generalized to accommodate non-rectangular planforms, including trapezoidal and biomimetic shapes. This extension allows efficient exploration of geometric effects on propulsion without the computational cost of full numerical simulations. A systematic numerical study is performed to assess the influence of planform shape on key propulsion metrics such as thrust generation and efficiency. The results reveal clear trends in performance as a function of geometric features, providing valuable guidance for selecting optimal shapes in the design of adaptive and soft-actuated fins. The analytical model's real-time capability and generality make it a powerful tool for future integration with control strategies and smart actuation systems, paving the way for more efficient and versatile underwater robotic propulsion.

Keywords Fish-like · Bionspired robot · FSI model · Modal analysis · Biomimetic propulsion

Introduction

The design of efficient underwater propulsion systems increasingly draws inspiration from flexible bio-inspired fins, whose deformable nature can exploit fluid–structure interactions (FSI) to enhance thrust and propulsive efficiency. Foundational theories of aquatic locomotion established the role of unsteady lift, body–fin kinematics, and wake mechanics in thrust generation [6, 20, 19], while later studies clarified performance maps and optimality ranges (e.g., Strouhal range) for oscillating foils and flexible panels [17, 5]. Comprehensive reviews summarize undulatory and oscillatory swimming strategies and their scaling [18]. From an FSI standpoint, low-order analytical and semi-analytical formulations offer physical insight and computational efficiency, whereas high-fidelity CFD/FSI provide detailed wake resolution at higher computational cost [7]. The influence of flexibility, both distributed and localized, has been linked to resonance phenomena, thrust peaks, and efficiency trade-offs [4]. Geometry is likewise pivotal: trailing-edge shaping and planform effects modulate vortex dynamics and propulsive metrics [21]. In our previous publication, we presented a fluid–structure coupled framework for oscillatory fish-like propulsion in which only the added-mass contribution was derived in closed analytical form [8, 12]. That added-mass formulation was validated both against numerical simulations and via comparison with experimental data from the literature, confirming accuracy while maintaining a markedly low computational cost [12, 3]. Nevertheless, the prior model was limited to fins with a uniform rectangular planform (constant span along the chord), which restricts the exploration of span-varying, biomimetic geometries commonly observed in nature [2, 1]. The present work addresses these limitations by extending the framework to variable-span fins and non-rectangular planforms (e.g., trapezoidal and biomimetic/lunate) and,

E. Paifelman · G. Costantino Muniz · E. Ciappi
National Research Council, Institute of Marine Engineering, Via di Vallerano 139, 00128 Rome, Italy
e-mail: elena.paifelman@cnr.it; gabriele.costantinomuniz@uniroma1.it; elena.ciappi@cnr.it

S. Milana · G. Costantino Muniz · A. Carcaterra
Sapienza University of Rome, Department of Aerospace and Mechanical Engineering, Via Eudossiana 18, 00184 Rome, Italy
e-mail: silvia.milana@uniroma1.it; gabriele.costantinomuniz@uniroma1.it; antonio.carcaterra@uniroma1.it

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crucially, by promoting the formulation to a fully analytical model: all coupling coefficients that govern the FSI dynamics (including damping, stiffness-like terms, and vorticity-related operators) are now provided in closed form. This generalization preserves real-time or near-real-time capability across a broad geometric design space, enabling systematic interrogation of how planform variation influences thrust and efficiency without resorting to costly high-fidelity simulations for every candidate [3, 5]. The availability of a fully analytical reduced-order description paves the way for geometry-aware control and smart actuation strategies that explicitly exploit real-time prediction. In this context, our prior developments on optimal and feedback control for systems with memory and hydroelastic couplings provide a natural foundation for integrating the present modeling advances into closed-loop architectures [11, 10, 9, 16, 14, 15, 13].

Extended Analytical Model for Variable-Span Fins

The analytical framework previously introduced in [12] is here generalized to fins with non-rectangular planforms, in order to capture span variations typical of biomimetic morphologies. Three representative geometries are considered: a rectangular fin, a trapezoidal fin, and a biomimetic fin featuring a trailing-edge circular cut-out (see Figure 1).

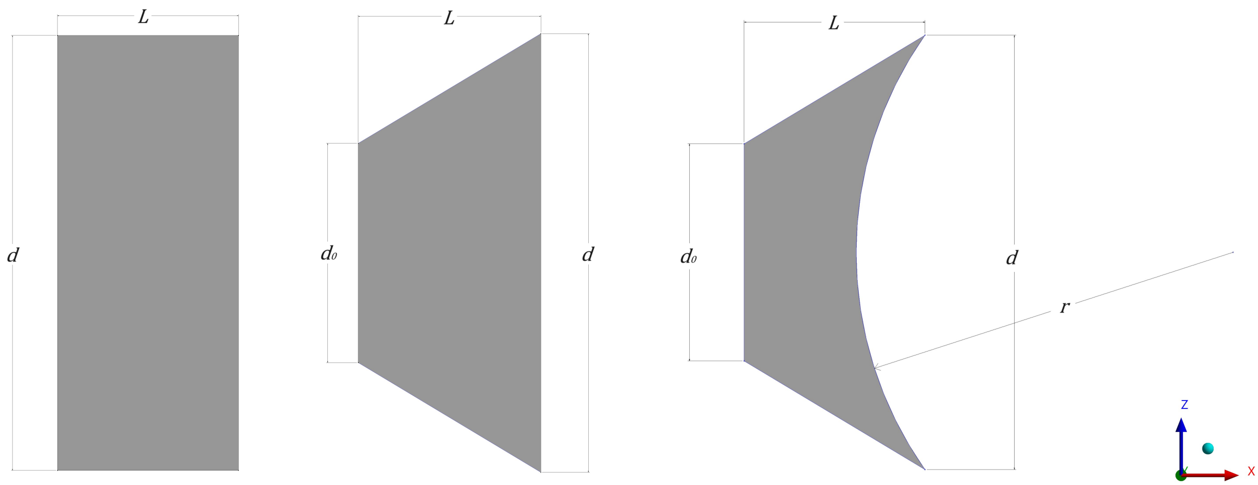


Fig. 1 Caudal fin geometries: rectangular, trapezoidal, and biomimetic/lunate

All configurations share the same material properties, namely polyethylene with Young's modulus $E_Y = 2 \cdot 10^9\,\mathrm{Pa}$ and density $\rho = 1300\,\mathrm{kg/m^3}$, the same thickness $h_b = 0.38\,\mathrm{mm}$, and the same aspect ratio $AR = 2.4$. In this way, all fins can be inscribed in the same bounding rectangle of chord L and maximum span d, thus ensuring comparability across planforms while maintaining consistency with available experimental data. The spanwise distribution is generalized from the constant value of the rectangular plate to more complex functional laws. In the trapezoidal case, the span varies linearly with the chordwise coordinate x, while in the biomimetic case it decreases nonlinearly due to the trailing-edge cut-out. In analytical terms, the span law is represented as $d(x) = \sum_{k=0}^{m} d_k\, x^k$, where the coefficients d_k encode the chosen geometry. The hydrodynamic response of the fin is modeled by coupling a linear potential flow formulation for the surrounding incompressible, inviscid fluid with the Euler–Bernoulli beam representation of the fin's mid-span section. For additional mathematical details the reader is referred to [12]. The dynamics of the fin in modal coordinates $\mathbf{q}(t)$ are governed by

$$\widetilde{\boldsymbol{M}}\,\ddot{\mathbf{q}}(t) + \widetilde{\boldsymbol{C}}\,\dot{\mathbf{q}}(t) + \widetilde{\boldsymbol{K}}\,\mathbf{q}(t) + \left[\widetilde{\boldsymbol{\Psi}}\,\ddot{\mathbf{q}}(t) + \widetilde{\boldsymbol{\Upsilon}}\,\dot{\mathbf{q}}(t)\right] * \varphi(t) = \widetilde{\boldsymbol{F}}(t), \tag{1}$$

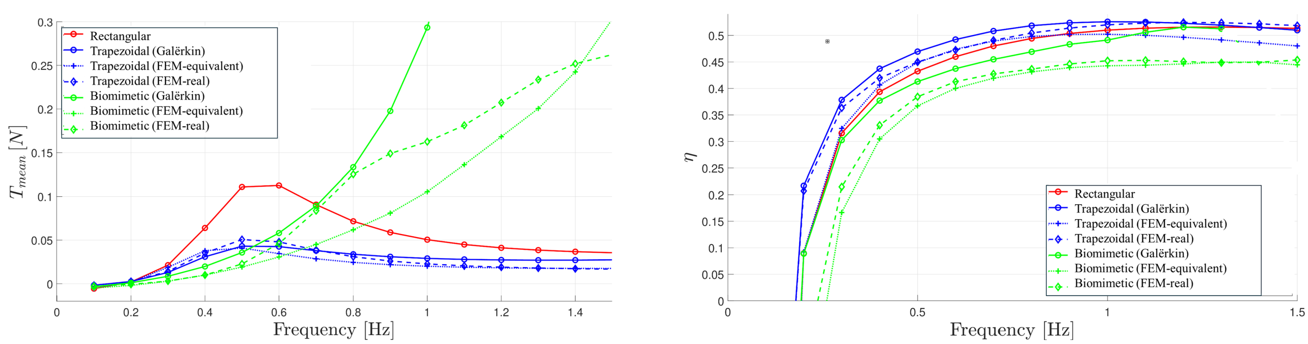
where $\widetilde{\boldsymbol{M}}$ is the generalized mass matrix, $\widetilde{\boldsymbol{C}}$ the damping matrix, $\widetilde{\boldsymbol{K}}$ the structural stiffness matrix, $\widetilde{\boldsymbol{\Psi}}$ and $\widetilde{\boldsymbol{\Upsilon}}$ the aerodynamic coupling operators, $\varphi(t)$ the bound circulation, and $\widetilde{\boldsymbol{F}}(t)$ the forcing vector due to heaving and pitching motions. All operators are expressed in closed form as functions of the span law coefficients d_k [12]. The application of the model requires suitable modal shapes. Two strategies were adopted. In the first, finite element (FE) simulations were employed to compute the natural frequencies and eigenmodes of each geometry, providing a high-fidelity reference. In the second, a Galerkin projection was used: the modal basis of a clamped uniform beam was chosen as reference, and the governing equations were projected onto this basis to derive approximate modal shapes. The resulting eigenvectors provide the coefficients of the modified modes, expressed as $\Phi_i^*(x) = \sum_{j=1}^{N} c_j \, \Phi_j(x)$, which approximate the true shapes with different levels of accuracy depending on the geometry. For trapezoidal fins, the Galerkin approach yields excellent agreement with FE predictions, while for the biomimetic geometry higher-order contributions become significant, leading to a frequency deviation of about 15% compared with FE.

Results and Discussion

The performance of the three fin geometries was evaluated in terms of thrust production and propulsive efficiency. Once the coupled fluid–structure dynamics were solved, the instantaneous hydrodynamic forces were obtained from the unsteady pressure distribution along the fin surface, reconstructed from the bound circulation predicted by the linear potential flow model. The thrust was computed by integrating the streamwise component of the pressure forces over the fin surface, while the propulsive efficiency was defined as the ratio between the useful propulsive power and the input mechanical power associated with the prescribed heave–pitch kinematics. Formally, these two indicators can be written as

$$T = \int_S p(\mathbf{x}, t)\, \mathbf{n} \cdot \hat{\mathbf{x}}\, dS, \qquad \eta = \frac{\overline{T}U}{\overline{P_{in}}}, \tag{2}$$

where T is the instantaneous thrust, $p(\mathbf{x}, t)$ the unsteady pressure field, $\mathbf{n}$ the outward surface normal, $\hat{\mathbf{x}}$ the streamwise unit vector, U the forward swimming speed, and P_{in} the input mechanical power. In Eq. (2), the overline denotes averaging over one oscillation period.



(a) Thrust comparison

(b) Propulsive efficiency

Fig. 2 Comparison across geometries and methods: (a) thrust and (b) efficiency

As shown in Figure 2 (a), the trapezoidal fin exhibits excellent agreement between the different modal identification methods across the full frequency range, confirming the robustness of the extended formulation for this geometry. For the biomimetic fin, the predictions remain consistent up to about 0.8 Hz, beyond which the Galerkin approximation begins to diverge from the finite element reference. Nevertheless, all approaches indicate that the biomimetic geometry is capable of producing larger thrust at higher frequencies, a consequence of the nonlinear span reduction and its effect on the dynamic response. The corresponding efficiencies in Figure 2 (b) show that the trapezoidal fin, although producing the lowest thrust, achieves the highest efficiency across the investigated frequency range, with a nearly perfect match between the different computational approaches. The rectangular fin occupies an intermediate position, balancing thrust and efficiency. The biomimetic fin, in contrast, is the least efficient configuration, even though it delivers the strongest thrust at high frequencies. This confirms that span variation introduces a trade-off between force production and energetic cost, with trapezoidal geometries favoring efficiency and biomimetic ones favoring thrust. Overall, these results demonstrate that the extended analytical model is able to capture the propulsive differences between variable-span fins in a consistent manner, while preserving the real-time capability required for design exploration and for potential integration with control strategies.

Future Remarks

Ongoing work is focused on extending the validation of the present framework through a dedicated experimental campaign aimed at building a comprehensive database of hydrodynamic and structural responses across a broad range of planform shapes. In parallel, an optimization study is being conducted on the basis of the proposed fully analytical model, exploiting its computational efficiency to explore large design spaces systematically. The ultimate objective is to provide designers with a real-time decision-support tool capable of identifying the most suitable fin geometry according to the specific operational requirements of biomimetic underwater vehicles.

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