Chapter 7
Experimental-numerical Hybrid Stress and Strain Analysis Based on the Principle of Superposition

Satoru Yoneyama

Abstract A simple hybrid method based on the principle of superposition is described. Using this method, for example, in photoelasticity, the distribution of stress components can be obtained by superimposing the solutions of the finite element method so as to obtain the same principal stress difference distribution as the measurement. Similarly, it is possible to evaluate strain distribution from the displacement distribution obtained using digital image correlation without the error increase caused by numerical differentiation. Furthermore, by extending this method, it becomes possible to obtain thermal stress in electronic components and residual stress in silicon wafers. As an example of the application of this method, the strain analysis around the fiber-matrix interface of CFRP cross section is shown. The proposed method has no difficulty for computing strains from displacements. Therefore, the proposed method is effective for obtaining strains from displacement distribution measurement results, and is expected to be applied to various problems in solid mechanics.

Keywords Photoelasticity · Digital image correlation · Finite element method · Hybrid method · Principle of superposition

Introduction

In general, experimental methods alone often fail to provide sufficient information. For example, full-field measurement methods such as optical interferometry and digital image correlation only provide surface displacement information. Therefore, it is necessary to differentiate the measured displacements to obtain strain distributions. However, numerical differentiation of the measurement data provides greater errors in their derivatives. Another example is photoelasticity, which provides only principal stress difference and principal direction, but does not directly obtain stress components or principal stresses. Therefore, obtaining the desired information from experimental measurements often requires various ingenuity, complex numerical calculations, and sometimes inverse problem analysis.

Several hybrid methods combining measurements and computational methods have been proposed to address this issue [1–6]. A finite element method can provide accurate results of stress and strain according to given conditions. However, it is often difficult to accurately specify various conditions, especially boundary conditions, in a finite element method. Therefore, if a finite element method can provide the same results as measurements without specifying boundary conditions, the stresses and strains obtained by the finite element method are considered to be correct.

In this study, based on the above idea, a simple hybrid method is proposed. This method is based on the principle of superposition. Therefore, the objects that can be studied are limited to linear elastic bodies or linear viscoelastic bodies. Using this method, for example, in the photoelasticity, the distribution of stress components can be obtained by superimposing the solutions of the finite element method so as to obtain the same principal stress difference distribution as the measurement. Similarly, it is possible to evaluate strain and stress distributions from the displacement distribution obtained using digital image correlation without the error increase caused by numerical differentiation. Furthermore, by extending this method, it becomes possible to obtain thermal stress in electronic components and residual stress in silicon wafers.

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Method

A hybrid method for obtaining strain distributions from measured full-field displacements is described below. Other hybrid methods for stress separation in photoelasticity, thermal strain analysis, and residual stress analysis of silicon wafers can be described in a similar manner. Consider a linear elastic body for which the in-plane displacement distributions in the analysis region are obtained by a measurement method. The displacements of some portions of the analysis region are fixed to prevent rigid body motion. The displacement components at points x_i and y_i inside the analysis object under a unit concentrated load $P_j = 1$ at a point on the boundary are represented as u'_{ij} and v'_{ij}. Here, i and j are indices to identify the data measurement point and the unit concentrated load applied to the boundary, respectively, and $i = 1$ to M and $j = 1$ to N. That is, the number of concentrated loads (load location and direction) applied to one point on the boundary is N, and the number of evaluation points for the displacements in the region is M. The displacement components u'_{ij} and v'_{ij} can be considered as compliances that represent the relationship between the load applied to a point on the boundary and the displacements at a point in the analysis region. If the load values actually acting on the boundary are F_j, the following equation holds according to the principle of superposition.

$$\begin{matrix} u_i = u'_{ij} F_j \\ v_i = v'_{ij} F_j \end{matrix} \quad (i = 1 \sim M,\ j = 1 \sim N) \tag{1}$$

where u_i and v_i are the displacement components at x_i and y_i. The displacement components u'_{ij} and v'_{ij} due to unit concentrated loads can be obtained by a finite element method. The displacement components u_i and v_i are the values obtained by a measurement. Therefore, if the number of the evaluation points M is greater than the number of the loads on the boundary N, the load values F_j acting on the boundary can be obtained by the method of least-squares as shown in the following equation.

$$F = \left(A^T A + \alpha A^T_{,xx} A_{,xx} + 2\alpha A^T_{,xy} A_{,xy} + \alpha A^T_{,yy} A_{,yy}\right)^{-1} A^T U \tag{2}$$

Here, **F**, **A**, and **U** are the nodal forces on the boundary, the displacements (compliances) obtained under unit concentrated loads, and the measured displacement values, respectively. The coefficient α is the regularization parameter. This minimizes the second-order derivatives of the displacements, i.e., the changes in strain, and reduces the amount of strain disturbance appearing at the boundaries. As a result, appropriate strain values can be obtained not only within the analysis region but also near the boundaries. If the load values F_j on the boundary can be obtained using Eq. (2), the values of strain and stress can be obtained using the principle of superposition as well as displacements. In this method, the boundary load values are determined so that the displacement distributions are close to the measured displacements. Furthermore, the method of least-squares is applied to account for measurement errors. Therefore, the obtained results are considered to be reliable.

Strain Measurement near the Fiber-matrix Interface of the CFRP Cross Section

The size of the image obtained by the laser microscope is 1164 pixels × 1164 pixels, the scale is 0.031 μm/pixel, and the analysis region is 800 × 800 pixels (24.8 × 24.8 μm) of a portion of the image obtained as shown in Fig. 1. The displacement distributions are obtained using digital image correlation from the images before and after deformation. The subset size is 21 pixels × 21 pixels, the shape function is linear, and the bicubic function is used for the gray level interpolation.

The displacement distributions of the analysis area obtained using two-dimensional digital image correlation are shown in Fig. 2. It can be seen that smooth displacement distributions are obtained by digital image correlation. Figure 3 shows the 2D finite element model used in the hybrid analysis. Several element sizes and numbers are examined to investigate the effect of differences in the number of elements and nodes on the results, but no significant differences are observed due to differences in the models, so only the results of the model shown in the figure are shown here. The size and position of the fibers are determined by careful observation of the image in Fig. 1. Second-order quadrature elements are used to express the strains in the strain concentration area. The number of elements is 1638, the number of nodes is 5401, the number of nodal forces N of the boundary to be determined is 555, and the number of data points M is 10201. Different material properties are input for the fiber and resin portions. The fiber portion has the elastic modulus $E = 220$ GPa and the Poisson's ratio $\nu = 0.2$, while the resin portion has the elastic modulus $E = 3.7$ GPa and the Poisson's ratio $\nu = 0.3$. The regularization coefficient is determined as in the previous section and set to $\alpha = 10^5$.

Using the displacement measurement results obtained by digital image correlation in Fig. 2 as input, the displacement distributions obtained by the proposed method are shown in Fig. 4, and the strain distributions are shown in Fig. 5. It can be observed that the displacement distributions in Fig. 4 are similar to the experimental displacement measurement results in Fig. 2. This means that the proposed method is working well. In the strain distributions obtained by the proposed method,

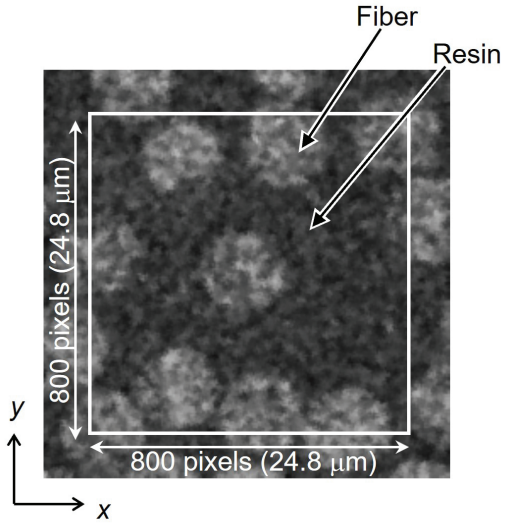

Fig. 1 Portion of microscopic image of CFRP cross section observed using laser microscope

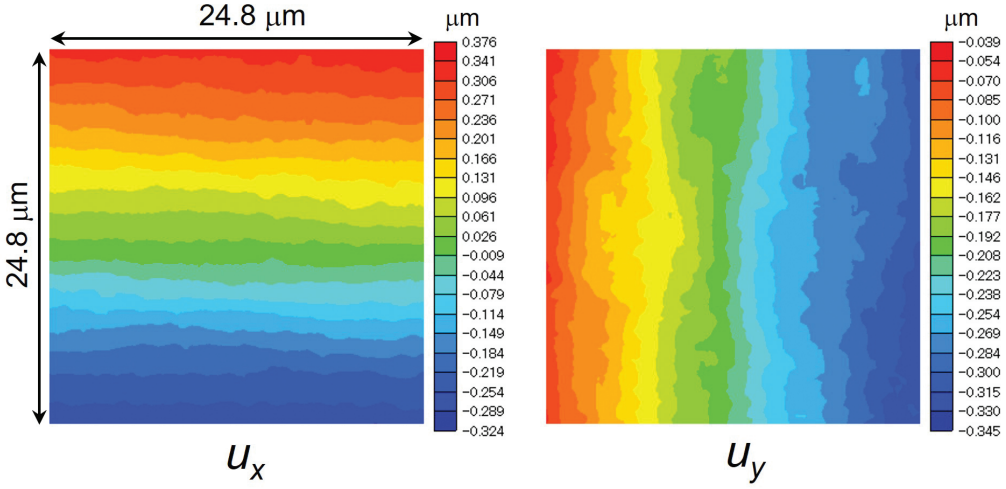

Fig. 2 Displacement distributions of CFRP cross section obtained using digital image correlation

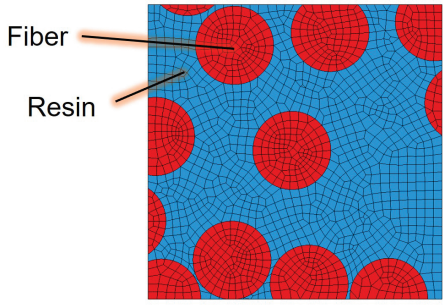

Fig. 3 Finite element model for hybrid analysis of CFRP cross section

it can be observed that the strains are concentrated in the resin portions between the fibers. On the other hand, the fiber portions are almost free from strain. This is due to the difference in the material properties between the fiber and resin. The modulus of elasticity of the fiber is about 56 times that of the resin part, and the fiber deforms very little compared to the

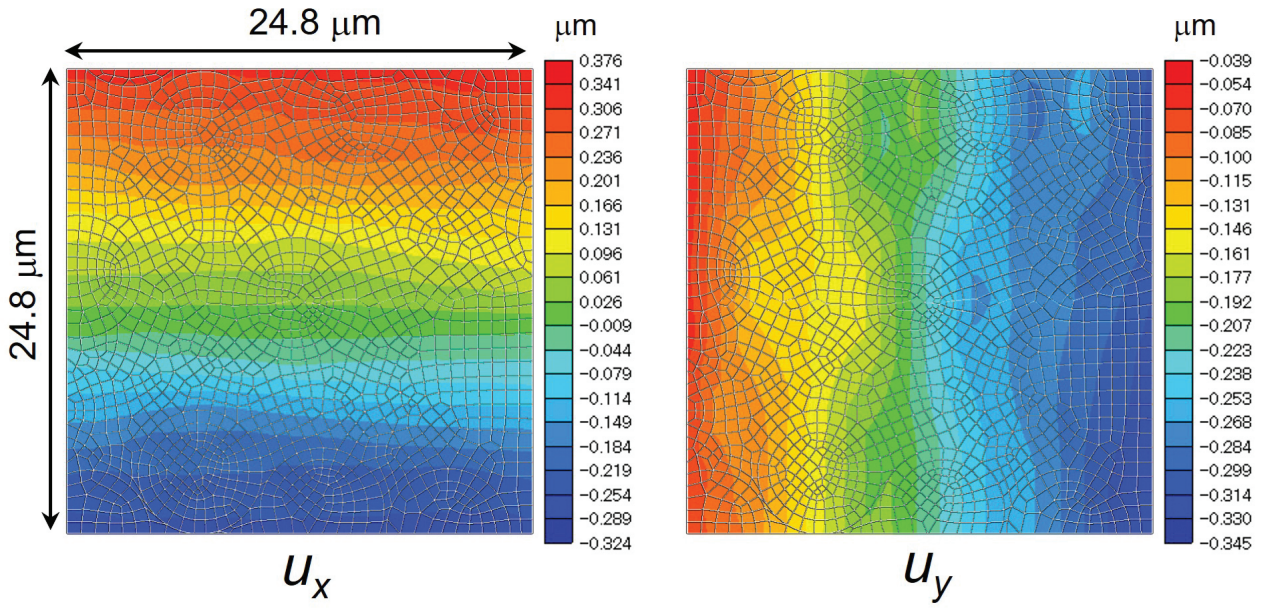

Fig. 4 Displacement distributions obtained using the proposed method

resin. Therefore, as can be seen in Fig. 5, the strain changes discontinuously at the interface between the fiber and the matrix. This is the cause of the strain concentration in the matrix portion.

In the proposed hybrid method, the displacement distributions obtained by the measurement using a full-field optical method are used as input, the locations of the fiber and the matrix are distinguished, and the strains are obtained considering the difference in material properties. The displacement measurement results themselves are not significantly affected by the image resolution, and reasonable results can be obtained even at low resolutions. However, it is difficult to obtain the strain appropriately from the displacement distributions. The proposed method can compensate for the low spatial resolution by distinguishing the locations of fibers and matrix from images of low-resolution conditions and the displacement distributions obtained from those images, and then computing strains by taking into account differences in material properties. Furthermore, calculating strains from displacements is generally difficult because it involves differentiating the measurement data. The proposed method does not present such difficulties. Therefore, the proposed method is effective for calculating strains from displacement distribution measurement results, and is expected to be applied to various problems, not limited to strain measurement near fibers and resins.

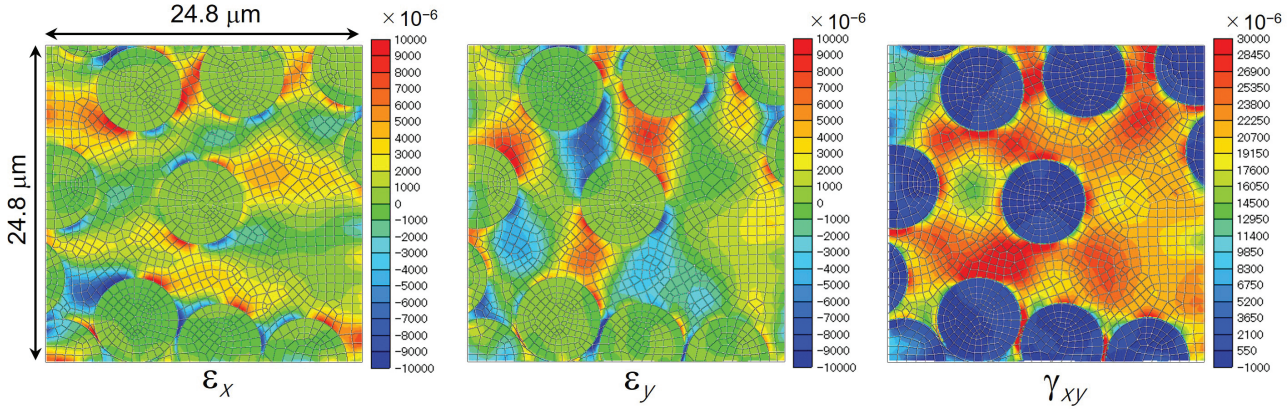

Fig. 5 Strain distributions obtained using the proposed method

Conclusions

A hybrid method that combines the displacement distribution measured using digital image correlation and analysis using a finite element method is described. In this method, the same displacement distributions as the measured displacement distributions are obtained based on the principle of superposition. The differences in strain values between the fiber and resin due to differences in the material properties and the discontinuities in the strain distributions can be observed. In general, calculating strain from displacement is difficult because it requires differentiation of measured data. The proposed method has no such difficulty. Therefore, the proposed method is effective for obtaining strains from displacement distribution measurement results, and is expected to be applied to various problems in solid mechanics.

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