Chapter 1
A Finite Element Framework for Thermo-hygro-viscoelastic Fracture in Wood

Loic Chrislin Nguedjio, Rostand Moutou Pitti, Benoit Blaysat, Pierre Kisito Talla, Fréderic Dubois, and Naman Recho

Abstract Fracture in orthotropic materials such as wood involves complex mechanisms, where crack propagation is influenced by both material anisotropy and time-dependent effects, including delayed viscoelasticity and thermo-hygro-mechanical couplings inducing shrinkage and swelling effects and hydric stresses. These factors critically affect fracture kinetics, complicating the prediction of crack initiation and growth. This study presents a numerical framework for modeling mixed-mode crack propagation in wood by integrating thermo-hygro-mechanical interactions with viscoelastic effects. The approach defines a spatial-temporal function capable of identifying critical times and environmental conditions that trigger crack initiation and evolution. To decouple fracture modes, the $A\theta$ invariant integral is employed to rigorously evaluate the corresponding fracture energies. A dedicated thermo-hygro-viscoelastic fracture algorithm is implemented within a finite element code, with delayed mechanical responses represented through rheological elements. This enables realistic simulation of crack initiation and propagation under complex loading, while maintaining separation of the distinct failure modes observed in wood. The proposed framework provides a unified approach to analyze the interplay between anisotropy, viscoelasticity, and environmental effects in wood fracture mechanics. It also offers insights for improving the reliability and durability assessment of wood structures subjected to variable mechanical and climatic conditions.

Keywords Fracture · moisture fields · orthotropic materials · viscoelasticity · finite element

Introduction

The computation of fracture parameters, particularly the stress intensity factors and the energy release rate through invariant integrals, has undergone significant developments in fracture mechanics. Initially introduced by Rice [1], the J-integral was one of the first invariant integrals allowing the evaluation of the energy release rate in isotropic materials for crack propagation under opening (Mode I) or in-plane shear (Mode II) loading. A three-dimensional formulation of this integral was later proposed by El Kabir *et al.* [2]. However, the classical J-integral formalism rapidly proved insufficient due to its limited applicability to simple fracture modes. Orthotropic materials such as wood commonly exhibit mixed-mode crack propagation, involving a combination of Mode I and Mode II, while an additional out-of-plane shear mode, referred to as

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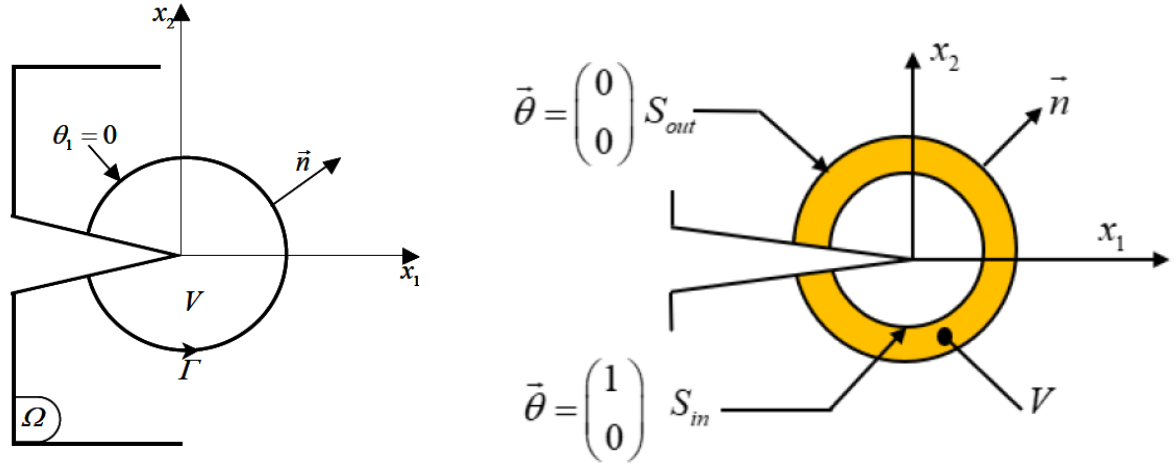
Mode III, may also occur [3]. A generalization of the conservation laws introduced by Chen [4] was developed by Moutou Pitti *et al.* [5, 6, 7, 8] through the invariant $M\theta$-integral, enabling the computation of fracture parameters under mixed-mode loading conditions, with applications to viscoelastic orthotropic materials. A three-dimensional extension of the M-integral was subsequently proposed by Afoutou *et al.* [9], although viscoelastic effects were not taken into account. For wood-based materials, it is essential to relate crack propagation to environmental conditions such as temperature and moisture variations. As demonstrated by Mézatio *et al.* [10], these environmental fluctuations significantly increase the random nature of the fracture process. Hamdi *et al.* [11] implemented the invariant $A\theta$-integral developed by Moutou Pitti *et al.* [12] to investigate the influence of temperature variations on the energy release rate. Their results showed that the energy release rate increases with temperature and that the $A\theta$-integral efficiently separates the different fracture modes.

The objective of the present work is to extend this framework further by incorporating local moisture effects at all levels of the formulation, particularly the influence of hygroscopic pressure acting on the crack faces.

Analytical Formulation of Invariant Integral

The analytical developments are based on Noether's theorem [13], extended to a quadrilinear formulation of the free energy of a cracked two-dimensional medium (see Fig. 1). The θ-field is continuous and differentiable. It is equal to unity inside the integration domain and vanishes outside it. The surfaces S_{int} and S_{out} correspond to the inner and outer contours, respectively.



(a) (b)

Fig. 1 2D fracture material : (a) Parameters around the crack tip; (b) $\vec{\theta}^{2D}$ field [14].

The formulation of the $A\theta$-integral, readily implementable in numerical computation codes and developed by Nguedjio *et al.* [15, 16, 17], is given by the following equation:

$$
\begin{aligned}
\mathbf{A}\,\theta = & \underbrace{\frac{1}{2}\int_{\Gamma}\left[\sigma^{(v)}_{ij,k}u_i - \sigma^{(u)}_{ij}v_{i,k} - \gamma_{ij}\left(\delta w_{,k}v_i + \delta w\, v_{i,k}\right)\right]\theta_{k,j}\,\mathrm{d}S}_{\text{Term 1 : stationary crack}} \\
& \underbrace{-\frac{1}{2}\int_{L}\left[\left(F_i - \gamma_{ij}\delta w_{ij}\right)v_{i,k}\right]\theta_k\,\mathrm{d}x}_{\text{Term 2 : pression on the crack lips}} \\
& \underbrace{+\frac{1}{2}\int_{S}\left[\begin{array}{l}\left(\sigma^{(v)}_{ij}u_{i,k}\right)_{,j} + \left(\sigma^{(u)}_{ij} - \gamma_{ij}\delta w\right)_{,j}v_{i,k} \\ -\sigma^{(v)}_{ij}(u_{i,j})_{,k} - \left(\sigma^{(u)}_{ij} - \gamma_{ij}\delta w\right)(v_{i,j})_{,k} \\ -\gamma_{ij}\left(\delta w_{,j}v_{i,k} - \delta w_{,k}v_{i,j}\right)\end{array}\right]\theta_k\,\mathrm{d}S.}_{\text{Terme 3 : propagation}}
\end{aligned}
\tag{1}
$$

This equation clearly highlights three main contributions: a stationary fracture energy evaluated through Term 1, the effect of pressure acting on the crack faces represented by Term 2, and Term 3, which ensures the separation of fracture modes during crack propagation. All parameters involved in this formulation are explicitly defined in our previous works [15, 16, 17] and δw represents the moisture content field.

To account for viscoelastic effects, the generalized Kelvin–Voigt rheological model (see Fig. 2), commonly used to simulate creep behavior in wood materials, is adopted in the present study [18]. k_{ijkl} and η_{ijkl} are respectively the stiffness of the spring and the viscous coefficient of the dashpot.

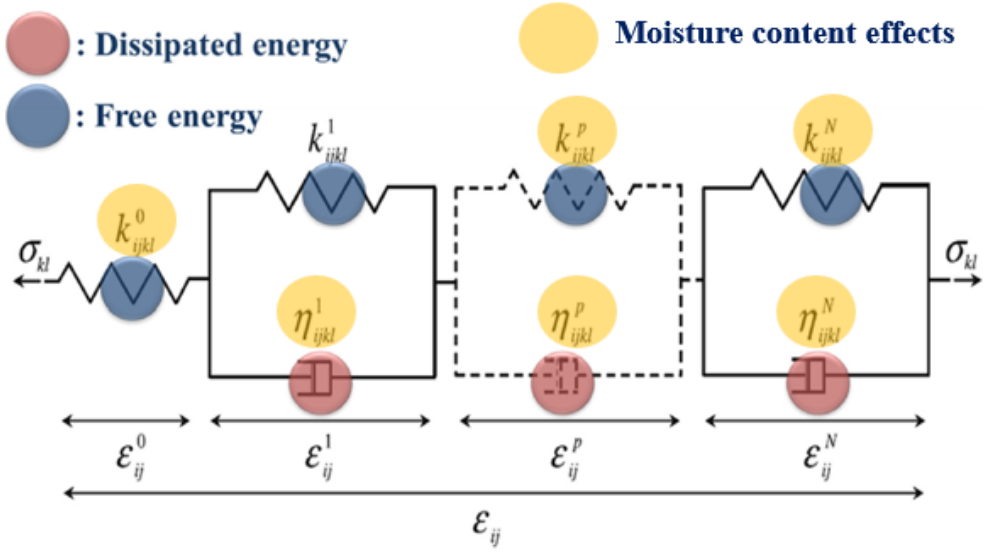


Fig. 2 Generalized Kelvin's rheological model.

The previously defined $A\theta$-integral, evaluated for a given spring in the rheological model, can be expressed as contributions from the opening mode and the shear mode to introduce the concept of mode separation, as illustrated by the following equation:

$$A\,\boldsymbol{\theta}^{(p)}(u,v) = \mathbf{C}_1^{(p)}\,\frac{{}^{u}K_I\,{}^{v}K_I}{8} + \mathbf{C}_2^{(p)}\,\frac{{}^{u}K_{II}\,{}^{v}K_{II}}{8}, \tag{2}$$

where K_I and K_{II} are the viscoelastic stress intensity factors taken respectively in the real and virtual configurations for (u,v), in opening and shear mode. $\mathbf{C}_1^{(p)}$ and $\mathbf{C}_2^{(p)}$ are the reduced orthotropic viscoelastic compliances (for a given moisture content) [17].

The energy release rate is then calculated in the following form, according to the details described in [17]:

$$\begin{aligned} {}^{(p)}G_I &= \frac{({}^{u}K_I)^2}{E_L} - \frac{4\,{}^{u}K_I}{3}\sqrt{\frac{2r_c}{\pi}}\,\alpha_L\,\delta w, \\ {}^{(p)}G_{II} &= \frac{({}^{u}K_{II})^2}{E_R} - \frac{4\,{}^{u}K_{II}}{3}\sqrt{\frac{2r_c}{\pi}}\,\alpha_R\,\delta w. \end{aligned} \tag{3}$$

r_c represents the process zone size [17]. E_L and E_R are respectively the Young modulus in longitudinal and radial orthotropic direction of wood.

Numerical Implementation, Results and Discussion

To compute the viscoelastic energy release rate with local moisture effects, the $A\theta$-integral is implemented in a numerical code using the finite element method. The material under study is a wood with reference moisture content of 12%, characterized by $E_L = 15000$ MPa, $E_R = 600$ MPa, the in-plane shear modulus $G_{LR} = 700$ MPa, and a Poisson's ratio $\nu = 0.4$. All these properties evolve throughout the simulations according to the laws proposed by Guitard [19]. Swelling and shrinkage phenomena due to moisture variations are accounted for through the in-plane coefficients α_L and α_R.

The meshed geometry corresponds to the Mixed Mode Crack Growth (MMCG) specimen [20], and all calculations are performed under plane stress assumptions. To trigger crack propagation, time increments of $\Delta t = 10$ s and moisture increments of $\delta w = 0.5$ were used, while a bisection algorithm was employed to determine the critical time and critical moisture content.

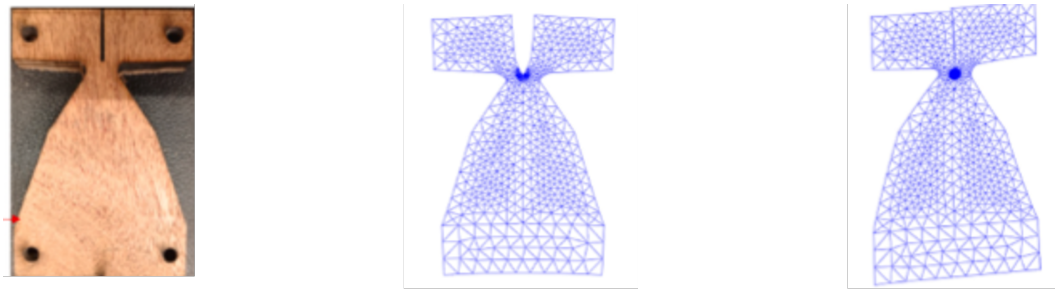


Fig. 3 (a) MMCG specimen; (b) Opening mode; (c) shear mode.

Once these critical values are reached, the crack propagates, as illustrated by the propagation kinetics curves showing the energy release rate as a function of time (see Fig. 4) and as a function of crack length (see Fig. 5).

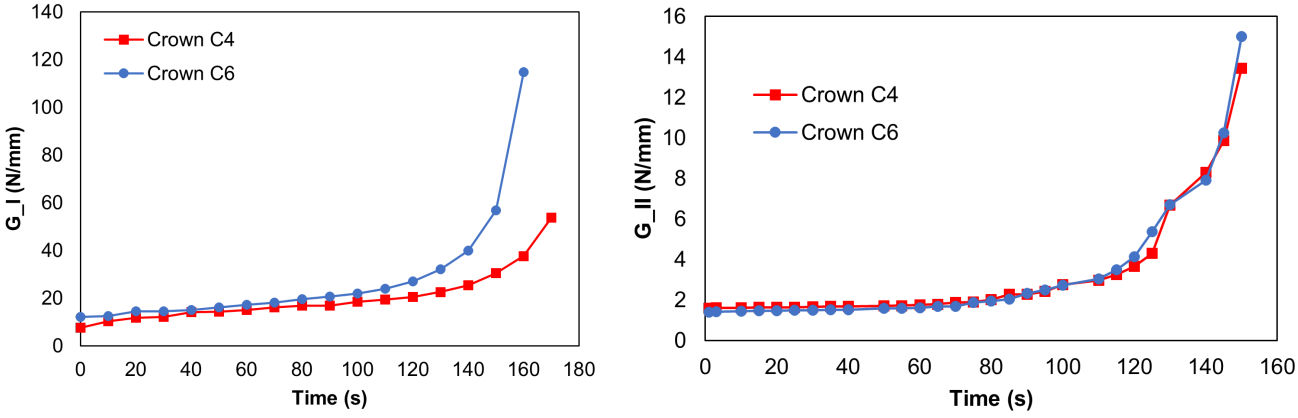


Fig. 4 Energy release rate as a function of time : (a) Mode I; (b) Mode II.

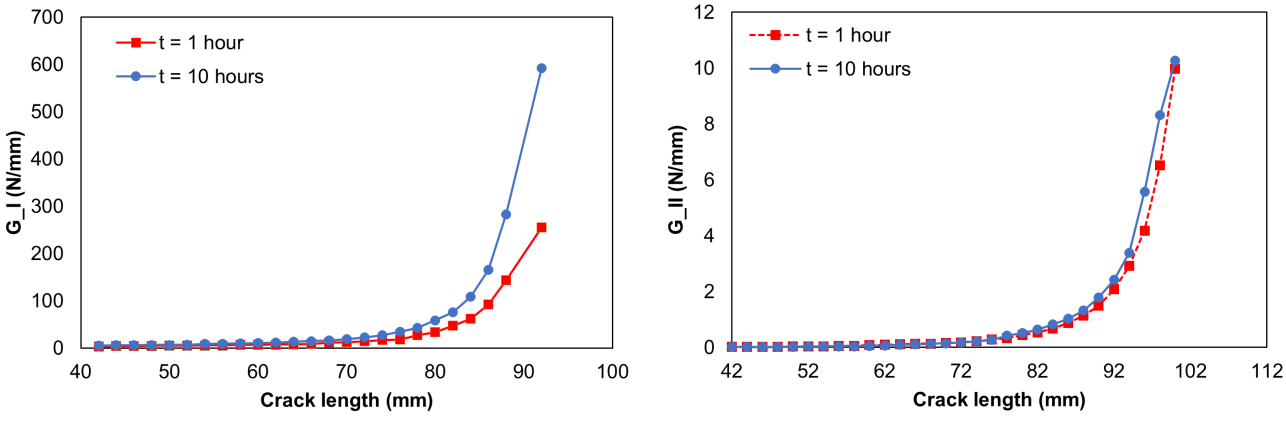


Fig. 5 Energy release rate as a function of crack length : (a) Mode I; (b) Mode II.

The curves in Figs. 4 and 5 respectively depict the evolution of the viscoelastic energy release rate as a function of time and crack length. These curves reflect the impact of delayed viscoelastic effects influenced by moisture variations, leading to the continuous decohesion of the process zone. In Fig. 4, it can be observed that the curves obtained for two distinct integration domain rings are very close, demonstrating the reliability of the method and serving as a verification of the integration domain independence. Fig. 5, in turn, further highlights the viscoelastic effects induced by relaxation.

Conclusion and Future Work

This study demonstrates the capability of invariant integrals to compute fracture parameters of orthotropic materials under the combined effects of viscoelasticity and moisture variations. The analytical formalism of the $A\theta$-integral used in this work, implemented in a finite element code, allowed us to plot the evolution of the energy release rate as a function of time and crack length. The results show that this invariant integral adequately separates the contributions of each fracture mode in mixed-mode loading. Moreover, the energy release rate increases with both time and crack length, demonstrating the good performance of the proposed model. Future work will focus on validating this model through experimental testing, and three-dimensional analyses will enable further advances in the modeling framework.

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