Chapter 5
Specimen Optimization for Characterizing Mixed-Mode Fracture in Wood Materials

Solenne Monzauge, Nathan J. Bechle, Loic Chrislin Nguedjio, and Rostand Moutou Pitti

Abstract Wood is a complex material that is not only highly heterogeneous but also exhibits anisotropic behavior and high variability accentuated by the diversity of species. In this case, the presence of a defect leads in most cases to the initiation and propagation of cracks that can cause structural failure. It is therefore crucial to assess the stability of the crack and identify the fracture parameters in order to correctly estimate the effect of long-term aging on wooden structures. A numerical investigation of different geometric shapes for the Mixed Mode Crack Growth (MMCG) specimen is conducted to simultaneously reduce the energy restitution rate during propagation in mode 1, mode 2, and mixed-modes 1 and 2. Next, experimental tests are carried out with the optimized test specimens to propose a stability range and better monitor crack propagation. Digital Image Correlation (DIC) method is used to synchronize deformations with applied loading and automate the crack detection process. The results should improve both scientific knowledge of mixed-mode cracking test specimens and the determination of fracture parameters under complex loads for highly utilized wood species.

Keywords Wood · MMGC specimen · Mixed-mode fracture · Numerical simulation · Digital image correlation

Introduction

Wood is a complex material whose behavior derives directly from its internal microstructure, producing its heterogeneous and anisotropic nature that dictates the mechanical response–with properties that vary significantly from one species to another. Natural features such as knots and growth irregularities disturb the continuity of the fibers, while harvested and processed wood is subject to machining operations, cutting processes, or handling which may introduce additional imperfections. This all makes wood both a particularly rich and a challenging material to investigate.

Stress concentrations arise near internal defects, which promote crack initiation and facilitate crack propagation. In the most critical configurations, propagation ultimately leads to structural failure. A rigorous assessment of crack stability therefore becomes essential. The identification of relevant fracture parameters such as stress intensity factors and energy release rate allows a reliable estimation of the influence of long-term aging on the mechanical integrity of wooden structures. The central research question concerns the choice of specimen geometry for experimental testing that can ensure

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sufficient stability of crack propagation, particularly under mixed-mode loading, which is very common in orthotropic materials such as wood. Although numerous specimen geometries have been proposed in the literature [1], only a limited number allow for the inclusion of crack propagation phases, as most are primarily designed to determine critical fracture parameters. This constitutes a major limitation, since such specimens are unable to monitor crack growth under viscoelastic conditions.

The first geometry explicitly incorporating stable crack propagation was introduced by Dubois *et al.* [2] through the use of a Double Cantilever Beam (DCB) specimen with variable inertia. However, this configuration is restricted to elementary fracture modes (mode I or mode II). This limitation was later addressed by Moutou Pitti *et al.* [3], who combined Richard's Compact Tension Shear (CTS) geometry [4] with the variable-inertia DCB concept to develop the MMCG specimen. This geometry has since been widely employed for studying mixed-mode crack propagation under both constant [5] and variable environmental conditions [6, 7]. Nevertheless, the experimental implementation remains highly challenging, particularly due to stress singularities introduced at the specimen fillets. These singularities, inherent to the geometric requirements for achieving variable inertia, often hinder crack propagation, especially in high-density wood species.

In this study, the fracture behavior of Sugar maple (*Acer saccharum*), a widely used hardwood species, is investigated under mixed-mode loading conditions. The primary objective of this work is to improve the MMCG specimen geometry through the optimization of several geometric parameters such as the radius of the connecting fillets, the fillet angle, and the overall fillet shape in order to achieve enhanced crack propagation stability. Particular emphasis is placed on reducing stress concentrations at the connecting fillets, with the aim of preventing premature brittle fracture of the specimens. The second objective is to improve the characterization of wood materials via the new Mixed-Mode Crack Growth (MMCG) specimens and the determination of fracture parameters under combined loading conditions. This introduction is followed by the presentation of the mathematical formalism of the M-integral used to evaluate fracture parameters, and subsequently by the first numerical results obtained using the newly proposed MMCG geometry.

Analytical formulation of M and M_θ integral

Starting from the J-integral, Chen and Shield [8] proposed the M-integral, defined along a curvilinear contour Γ_1 (See Fig. 1a), to achieve mixed-mode separation during elastic crack propagation. This approach enables the decomposition of fracture modes directly at the crack tip during the process.

For numerical implementation, however, a surface-based formulation proves more convenient. The curvilinear contour is therefore replaced by an integration domain defined through a continuous and differentiable vector field θ over Ω (See Fig. 1b) [9]. This leads to the following expression of the M_θ-integral [10]:

$$M_\theta = \frac{1}{2}\int_\Omega (\sigma_{ij}(u)v_{i,j} - \sigma_{ij,k}(v)u_i)\theta_{k,j}d\Omega \tag{1}$$

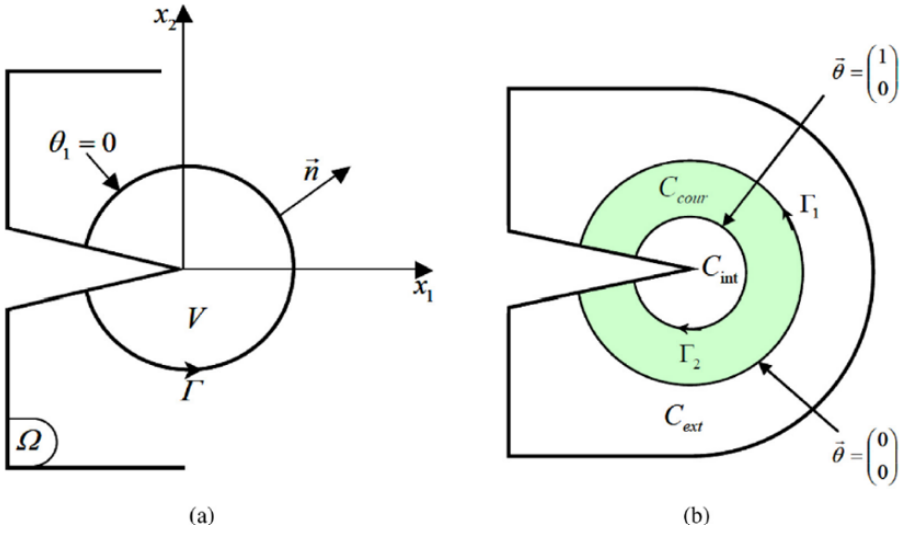

Fig. 1 (a) Parameters around the crack tip; (b) θ field [7]

In this formulation, $\sigma_{ij}(u)$ and $\sigma_{ij}(v)$ denote the actual and virtual stress tensors associated with the real displacement field u and the virtual displacement field v, respectively. The virtual fields are constructed from the singular asymptotic solutions introduced by Sih [11].

The real stress intensity factors K_I and K_{II}, corresponding to modes I and II, are determined by assigning suitable values to the virtual stress intensity factors $K_I^{(v)}$and $K_{II}^{(v)}$[10], namely:

$$K_I = \frac{8M_\theta(K_I^{(v)} = 1, K_{II}^{(v)} = 0)}{C_1}, K_{II} = \frac{8M_\theta(K_I^{(v)} = 0, K_{II}^{(v)} = 1)}{C_2} \tag{2}$$

The parameters C_1 and C_2 correspond to the elastic compliances associated with pure mode I and pure mode II loading, respectively. Finally, the energy release rate is obtained by summing the modal contributions:

$$G = G_1 + G_2 = \frac{C_1 K_I^2}{8} + \frac{C_2 K_{II}^2}{8} \tag{3}$$

G_1 and G_2 represents the energy release rate in opening and shear modes respectively. This formulation provides a consistent and numerically efficient framework for mixed-mode fracture analysis.

Numerical results

This numerical study aims to simulate the behavior of MMCG specimens in order to decouple fracture modes I and II and to evaluate the evolution of the energy release rate versus crack length in mixed mode configurations. The simulations were performed using the finite element software *Cast3M*, based on the numerical framework originally developed for the specimen proposed by Moutou Pitti [12]. An initial crack length of 20 mm was considered in all configurations. Nominal material properties for Sugar maple were utilized with the MMCG specimen in the radial-longitudinal plane and the crack along the longitudinal axis (RL orientation).

The geometry of the new specimen is based on the configuration established by Moutou Pitti [12]. The outside dimensions (height × width × thickness) were kept unchanged to ensure comparability and allow consistent experimental setups. The optimization focuses on the fillet regions at the thinnest and most mechanically sensitive areas of the specimen. In the initial configuration, these regions were identified as critical zones. In fact, during testing, systematic failure was observed at the connection radii. This failure is more systematic when approaching pure mode II, i.e., from a mix ratio of 60°.

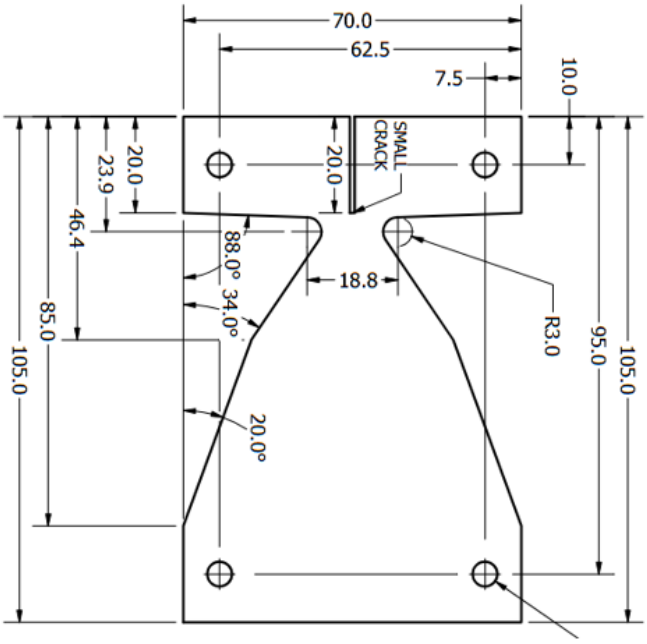

Fig. 2 Original MMCG specimen [12]

The proposed optimization will therefore consist of ensuring significant resistance around this mix ratio while guaranteeing the stability of the energy restitution rate during the propagation phase. In this case, the modification increases the radius of the fillets in order to strengthen this area and improve the global mechanical resistance of the specimen. The resulting improved geometry is shown in Figure 3.

To assess the relevance of the improved specimen, numerical simulations were carried out using *Cast3M* software, following the same computational approach as in previous studies [12]. This allows the evaluation of the energy release rate G versus crack length. In this study, the fracture modes are decoupled to separately analyze the contributions of mode I and

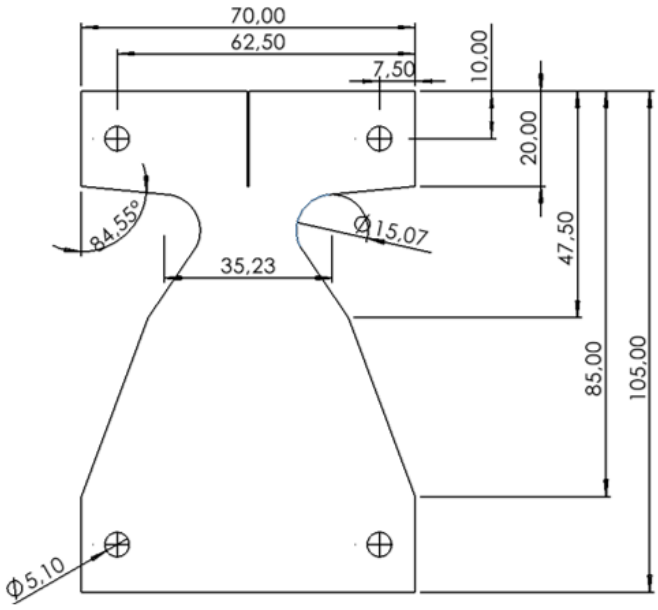

Fig. 3 Improved MMCG specimen

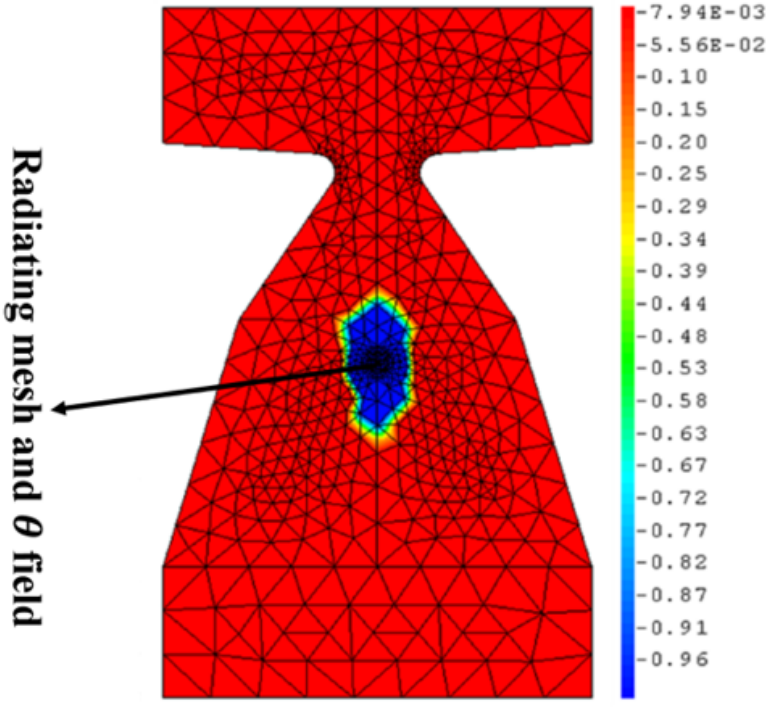

Fig. 4 θ field position during the crack growth.

mode II to the overall fracture process. The energy release rate G is divided into the part of G_1 and G_2 for the opening mode (I) and shear mode (II) in the mixed mode configuration, respectively.

The objective is to observe a stable range region where G remains relatively constant with increasing crack length, of G during crack growth under different loading conditions: mixed-mode (β ranging from 15° to 75°), pure opening ($\beta = 0°$), and pure shear ($\beta = 90°$). For the specimen to be considered valid for the study, the energy release rate should decrease as the crack begins to grow, just before reaching the stable range and it should then enter a stable plateau where G_1 and G_2 remain relatively constant despite further crack propagation. To reduce discrepancies in the numerical calculations, a dedicated radially structured mesh is implemented around the crack tip within the θ-field. At each increment of crack advancement, the crack tip is maintained inside the integration domain (see Fig. 4), which provides a practical guarantee of the accuracy and reliability of the numerical evaluation.

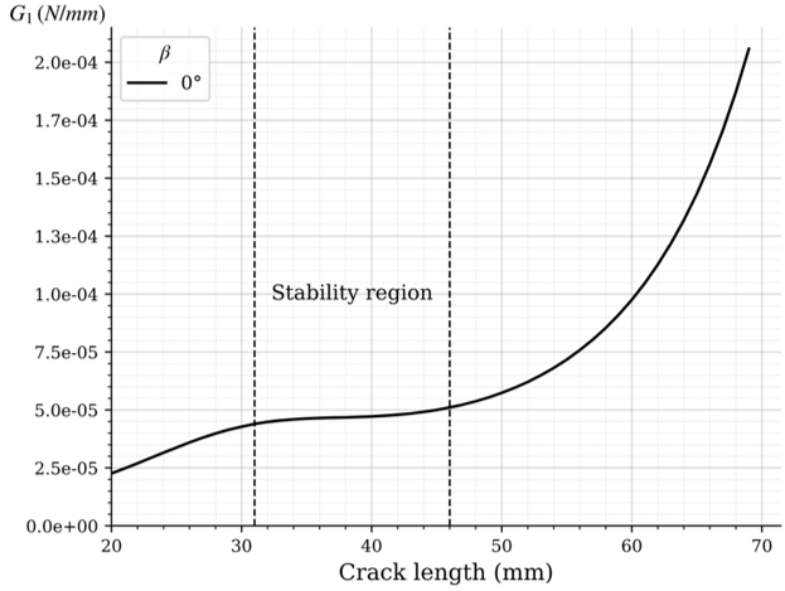

Fig. 5 Energy Release rate G_1 stability (Pure mode I)

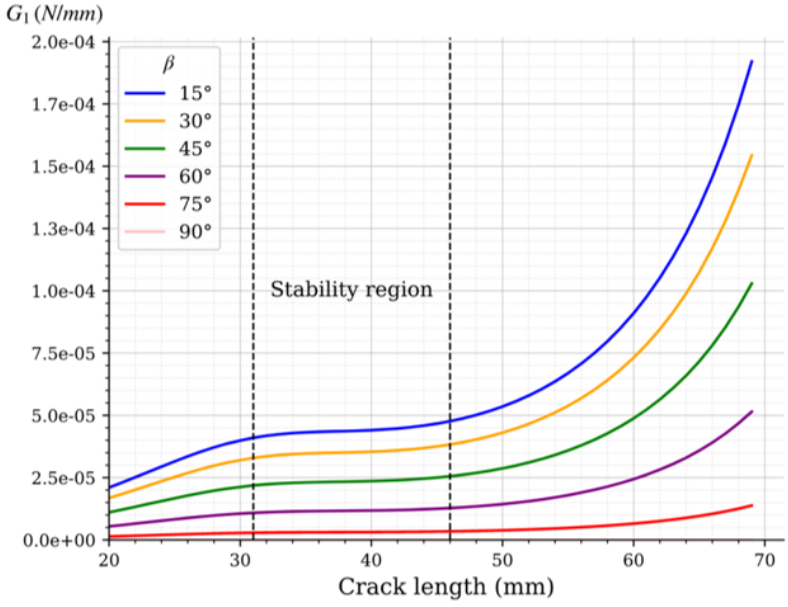

Fig. 6 Energy Release rate G_1 stability (Mixed mode)

Figures 5 to 8 show the evolution of the elastic energy release rates G_1 and G_2, calculated from equations (3), as a function of crack length under pure mode I, mixed-mode, and pure mode II loading.

It can be observed that, in the case of mode I, the stable region is located approximately between crack lengths 30mm and 45mm. In the case of mode II, between crack lengths 45mm and 60mm.

The numerical study reveals the presence of a stable region in the results, which is clearly visible in the G_1 plots and less pronounced in the G_2 plots. The low visibility of the stability zone in pure Mode II and for the Mode II portion in mixed mode strongly suggests that the test specimen still requires significant optimization in order to guarantee this stability in all load cases.

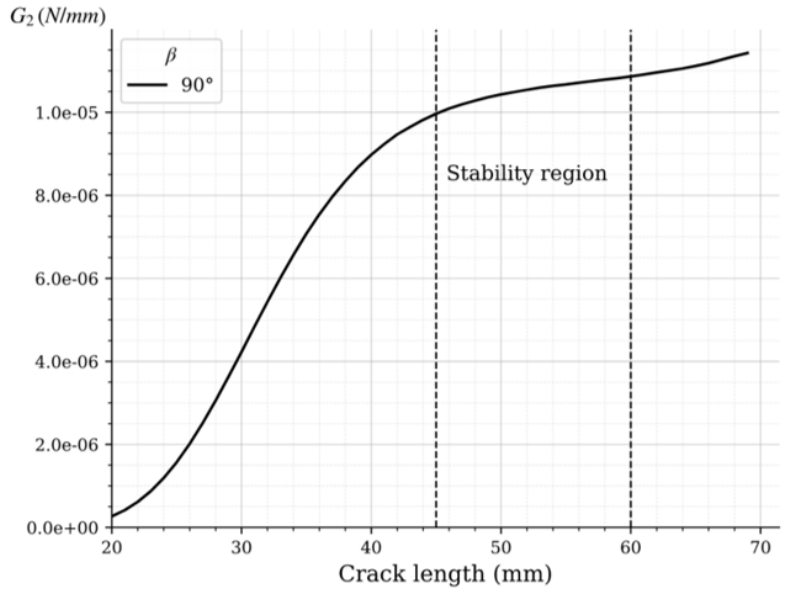

Fig. 7 Energy Release rate G_2 stability (Pure mode II)

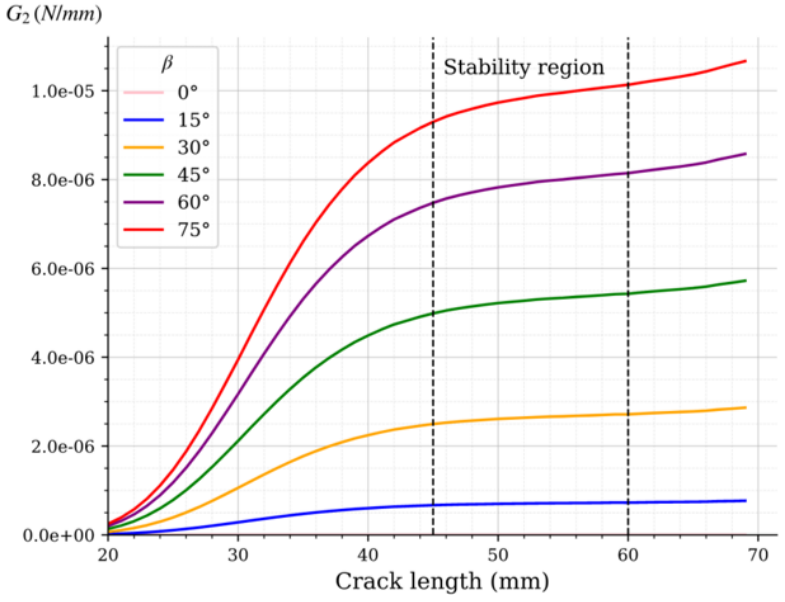

Fig. 8 Energy Release rate G_2 stability (Mixed mode)

Conclusion and Future Work

This work proposes an optimization of the MMCG test specimen, which is essential to ensure stable crack propagation in pure mode I and mode II, as well as for the selected mixed mode ratios and associated mechanical strength. The numerical results showed that a stable zone of mixed mode crack propagation was observed for Sugar maple. This zone will be further optimized, particularly in mode II, in order to extend its scope and ensure a better reduction in the critical energy release rate during crack evolution, regardless of the cracking mode. Subsequently, MMCG test specimens that comply with the new optimized configuration will be manufactured in order to perform tests in modes I and II incorporating the associated mixed fracture rates. Future work could include extending the study to other wood species in order to evaluate the versatility of the results and improve the characterization of the mixed fracture behavior of wood materials.

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