43. Discrete Fractional Fourier Transform based OFDMfor 5G Mobile Communication

Abhishek Kumar Singh, Vinay Kumar Trivedi, and Preetam Kumar
(1811ee01, vinay.pee14, and pkumar)@iitp.ac.in
Wireless Communication Research Lab, Electrical Engineering, Indian Institute of Technology Patna, India

ABSTRACT

State of the art radio access technologies (RAT) relies on the exclusive allotment of available time-slots and frequency bands or overlapped allotment using code and power domain multiplexing. Orthogonal frequency division multiple access (OFDMA) has been widely adopted to provide high data rate services for 5G mobile communication. OFDM is robust to frequency-selective wireless channels and enables a simplified mobile receiver design. Apart from the high peak to average power ratio (PAPR) problem that attracts significant research, the performance of OFDM systems is highly sensitive to synchronization errors and this problem will likely remain a significant challenge as higher frequency bands are utilized for next-generation wireless mobile applications. In this paper, the discrete fractional Fourier transform (DFRFT) based OFDM system is motivated in the presence of carrier frequency offset (CFO) for next-generation wireless mobile applications. Preliminary parametric results are presented to emphasize the performance gain with DFRFT based OFDM system and its comparison with conventional DFT based OFDM system. Some major implementation challenges for DFRFT based OFDM system is also indicated as future works.

Index Terms— *5G, OFDM, CFO, ICI, and DFRFT.*

INTRODUCTION

The next generation of the wireless mobile communication system is designed to deliver significantly increased operational performance in terms of high data rates, high spectral efficiency and low latency achieved with significantly low implementation complexity. Moreover, the high data rate wireless link should support the performance with high mobility and in the presence of synchronization error at the receiver. Orthogonal frequency division multiplexing (OFDM) is the baseline physical layer technique for 4G and LTE. Apart from being spectrally efficient, the OFDM system enables a very simple receiver implementation [1]. The third generation partnership project (3GPP) in its recent release [2] still agrees to OFDM and DFT spread OFDM as 5G new radio (NR) waveform for downlink and uplink respectively. Other more advanced and sophisticated waveform options are postponed mainly because of high implementation complexity or low backward compatibility. The reliable performance of the OFDM system mainly relies on orthogonal sub-carriers, and hence a OFDM system is highly sensitive to the presence of carrier frequency offset (CFO) [3], [4]. It violates the sub-carrier orthogonality that creates inter-carrier interference directly degrading the error rate performance. OFDM system also has a problem with high PAPR that results in low amplifier efficiency and thus reduced battery power and coverage. Among other solutions, precoding is studied as a simple technique for PAPR reduction without any increase in complexity or feedback [5]. DFT precoded OFDM (or) multiple access version of which is single-carrier FDMA (SC-FDMA) is used in the uplink for 4G and LTE.

DESIGN AND ANALYSIS

In Fig. 42-1 hexagonal solid core PCF with inner ring filled with methanol is shown. The background material is chosen as silica having the refractive index of 1.45. The diameter of air holes is same throughout the structure for ease in fabrication. This inner ring is filled with the liquid methanol having refractive index of 1.317. The distance between two air holes, which is known as pitch (Λ) is considered same in proposed structure i.e. 3μm.

The proposed work consists of micro-structured fiber whose air holes are of the same size. Also, the diameter of air holes of cladding region to has been varied from 1.5μm to 2.1μm and keeping the pitch (Λ) constant at 3μm throughout to compare and analyzed the results of the effect of air holes size. The proposed work is stimulated using FEMSIM module of R-Soft software and further all the requisite optical parameters are calculated and performance analysis has been done.

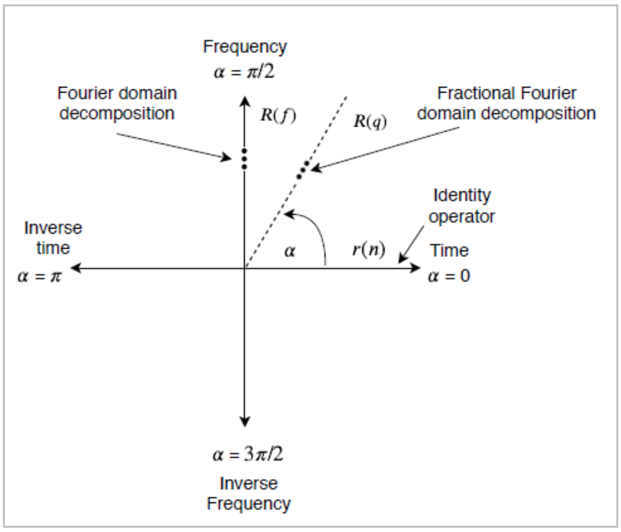

Figure 43-1 Represenation of time-frequency plane and corresponding fractional Fourier domain representation rotated by an angle of α in counter-clockwise direction

The fundamental task of multicarrier communication is to select the transmission basis such that the projection of the received signal to an identical basis at the receiver provides the estimates of signal that was transmitted. Therefore, the transform used for multicarrier communication must exhibit reversibility property. In this regard, several alternate orthogonal trigonometric transforms like discrete cosine transform (DCT) are also studied and analyzed for OFDM application [5]. The conventional DFT based or trigonometric transformbased OFDM system relies on interference compensation at the receiver to preserve the interference-free performance in the presence of CFO. This is because the channel partitioning established by Fourier transform loses its optimal decomposition to time-frequency localization. The optimal solution should now be able to project non-stationary received signals into basis waveform with time-varying sub-carrier frequencies. For this purpose, discrete fractional Fourier transform (DFRFT) employing chirp harmonics basis function instead of the complex exponential basis of DFT is studied as a potential multicarrier transform. FRFT is a rotation operator in a time-frequency plane as shown in Fig. 43-1 in counter-clockwise direction [6]. DFRFT at $\alpha=\pi/2$ is essentially a DFT (or) 90° rotation of signal representation in time-frequency plane. DFRFT based OFDM system is similar to a conventional DFT based OFDM system with IFFT and FFT replaced by IDFRFT and DFRFT at transmitter and receiver respectively [7]. By using the sampling-based DFRFT [8], the implementation cost can be brought similar to FFT i.e. $O(\frac{N}{2}\log_2 N)$. This is another important motivation apart from performance gain to use DFRFT based OFDM system for wireless mobile communication with CFO.

DFRFT based OFDM system is studied by many researchers in [7]- [12] for both multicarrier and single-carrier communication to provide robust error rate performance over multipath fading channel in presence of residual CFO. The ICI and signal to interference ratio (SIR) for the DFRFT-OFDM system is studied by authors in [9] which show that the ICI always degrades the performance no matter which transform is utilized for the purpose. The performance degradation from ICI is comparatively very less for DFRFT based OFDM system. Authors in [10] analytically established an optimum DFRFT angle rather than performing an exhaustive search.

In this paper, a consolidated performance evaluation of the DFRFT-OFDM system is considered and the system configuration is motivated as a potential candidate for next-generation mobile communication with CFO. The preliminary parametric result is presented and a detailed comparison is made with conventional and state of the art DFT based OFDM system. Through the results presented in this paper some possible future work is also indicated considering DFRFT based OFDM system.

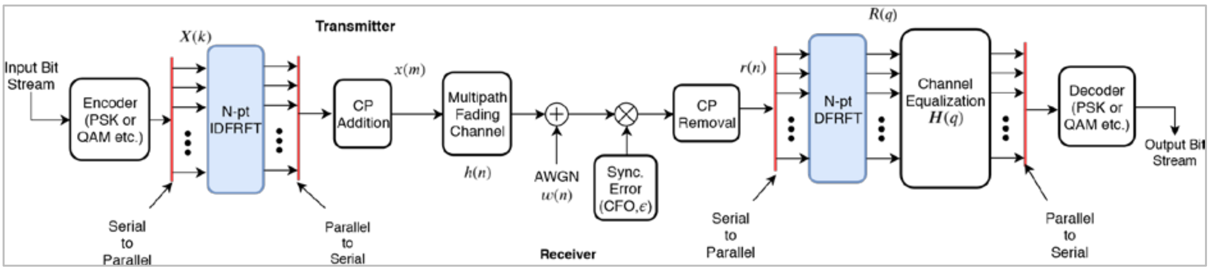

Figure 43-2 System model for the DFRFT based OFDM system

SYSTEM AND SIGNAL MODEL

In this section, the system and signal model of the DFRFTOFDM system over multipath fading in the presence of CFO is outlined with definitions of the IDFRFT and DFRFT kernels. Figure 43-2 presents the schematic of both the transmitter and receiver for the DFRFT-OFDM system. A set of N independent and identically distributed (*iid*) modulated symbols $X(k)$ generated using phase shift keying (PSK) are first processed with N-point inverse DFRFT (IDFRFT) block. A cyclic prefix (CP) is then appended to every block of IDFRFT processed symbols before transmitting over a multipath wireless channel. The m^{th} sample of this resulting output sequence from the transmitter is $N-1$,

$$x(m) = {}^{X} X(k)F_{-\alpha}(m,k), -N_{cp}\ 6\ m\ 6\ N-1 \qquad (1)$$

$k=0$

where $F_{-\alpha}(m,k)$ is the sampling based IDFRFT kernel that can be found in [8] as

$$F_{-\alpha}(m,k) = \sqrt{\frac{sin\alpha + jcos\alpha}{N}} e^{-\frac{j}{2}m^2T_s^2cot\alpha} e^{-\frac{j}{2}k^2u^2cot\alpha} e^{\frac{j2\pi mk}{N}} \qquad (2)$$

The angle between the time domain and fractional Fourier domain is represented as $\alpha = p\pi/2$ where 0 6 ($p \in$ R) 6 2 and the chirp rate is $cot\alpha$. It can be checked that at $\alpha = \pi/2$, the above kernel is nothing but IDFT. The sampling interval in time domain (i.e T_s) and in fractional Fourier domain (i.e., u) are now related as $u \times T_s = 2\pi |sin\alpha|/N$. The n^{th} sample of the received OFDM symbol over the multipath wireless channel and in the presence of normalized CFO () is

$$r(n) = e^{\frac{j2\pi\epsilon n}{N}} \sum_{m=-N_{cp}}^{N-1} x(m)h(n,n-m) + w(n) \qquad (3)$$

where $h(n,l)$ is the l^{th} coefficient of multipath channel impulse response (CIR) with L number of paths at time nT_s, $w(n)$ is additive white Gaussian noise (AWGN) at the receiver and is the CFO normalized to the subcarrier spacing. We have assumed the block fading channel model where the coefficients don't change significantly during one OFDM symbol period, i.e., $h(n,l) \approx h(l)$. At the receiver, after removing the CP, the DFRFT is performed to the received block of symbols, the q^{th} output sample of which in fractional Fourier domain is

$N-1$

$$R(q) = {}^{X} r(n)F_{\alpha}(q,n) + W(q) \qquad (4)$$

$n=0$

where $W(q)$ is the DFRFT of the AWGN samples and

$F_{\alpha}(q,n)$ is the DFRFT kernel defined in [8] as

$$F_{\alpha}(q,n) = \sqrt{\frac{sin\alpha - jcos\alpha}{N}} e^{\frac{j}{2}n^2T_s^2cot\alpha} e^{\frac{j}{2}q^2u^2cot\alpha} e^{-\frac{j2\pi nq}{N}} \qquad (5)$$

The DFRFT at the receiver is followed by OFDM per subcarrier conventional channel equalization in fractional Fourier domain with channel response $H(q)$ at the q^{th} subcarrier obtained from the DFRFT of an $(N-L)$ zero padded channel filter, i.e., $[h(0),h(1),...,h(L-1),0,...,0]$.

PERFORMANCE EVALUATION

Symbol error rate (SER) is an important end to end performance measure for the OFDM system in the presence of CFO and is also considered for performance evaluation here. Uncoded PSK modulated symbols are transmitted over Rayleigh multipath fading channel with CFO at the receiver for Monte Carlo simulation of SER. The DFRFT based OFDM system is interpreted as a modified OFDM configuration with IFFT and FFT in conventional DFT based OFDM system replaced by IDFRFT and DFRFT at transmitter and receiver for multicarrier modulation/demodulation respectively with channel equalization in fractional Fourier domain. The DFRFT angle at which minimum SER is achieved is termed as optimum DFRFT angle (α_{opt}).

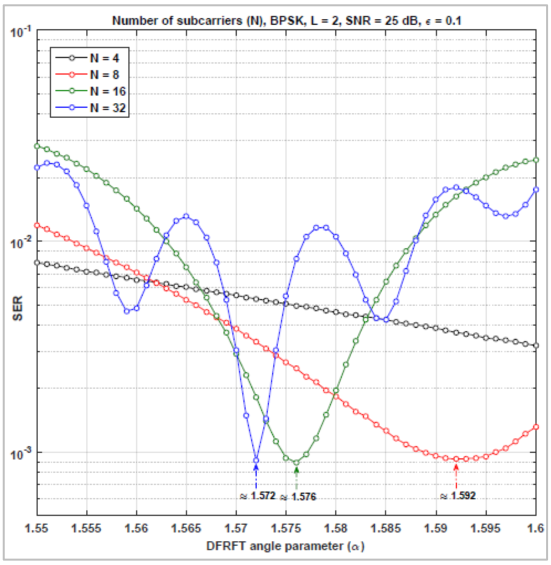

Figure 43-3 SER vs DFRFT angle (α) for different number of subcarriers

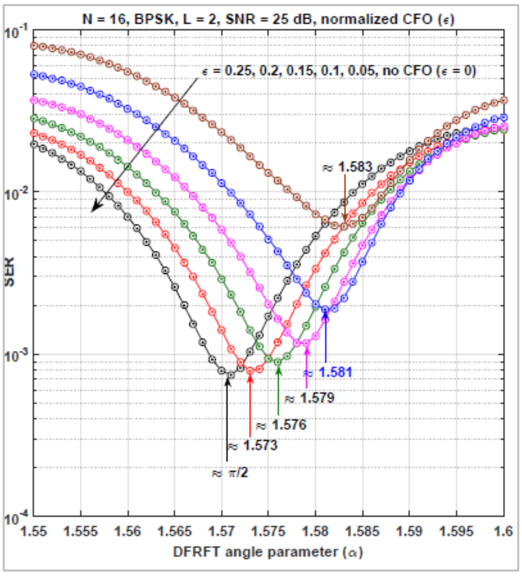

Figure 43-4 SER vs DFRFT angle (α) for different value of normalised CFO (ϵ)

Figure 43-3 shows the variation of SER with DFRFT angle(α) when transmitting BPSK modulated symbols using different number of OFDM subcarriers (N) over two tap Rayleigh multipath fading channel with normalized CFO value of 0.1 at an SNR of 25 dB. It can be observed that for a fixed value of CFO, optimum DFRFT angle (α_{opt}) is different for different number of OFDM subcarriers. With increasing number of subcarriers, α_{opt} shifts closer to $\pi/2$. For a high number of subcarriers, the accurate value of α_{opt} should be evaluated with increased decimal places in α and hence we have considered only 16 subcarriers or $N = 16$ for presenting remaining results.

Figure 43-4 shows the variation of SER with DFRFT angle α for different values of normalized CFO over two tap Rayleigh multipath fading channel given $N = 16$ at an SNR of 25 dB. It is clear that when no CFO is present, the optimum performance is at $\alpha = \pi/2$ or DFT based OFDM system. In the presence of CFO, performance at $\alpha = \pi/2$ is not optimal and minimum SER is achieved at $\alpha = \alpha_{opt}$, i.e., indicated for different values of normalized CFO. With increasing value of CFO, α_{opt} shifts away from $\pi/2$. It can be observed that at high values of CFO, the minimum SER obtained at α_{opt} with DFRFT based OFDM system is also degraded. This implies that although the DFRFT based OFDM system achieves interference-free performance at low values of CFO, this is not the case in the presence of high values of CFO.

Figure 43-5 shows the variation of SER with DFRFT angle α for different number of channel taps L when transmitting BPSK modulated symbols in the presence of normalized CFO of 0.1. For flat fading channel or $L = 1$, the performance is nearly independent of α. It can be also observed from the plot that the optimum value of

DFRFT angle (α_{opt}) to achieve minimum SER remains unchanged with the different number of channel taps (L) for a given value of normalized CFO (). Moreover, the SER performance at α_{opt} is nearly independent of the number of multipath components. This implies that the DFRFT based OFDM systems nearly achieve multipath free performance.

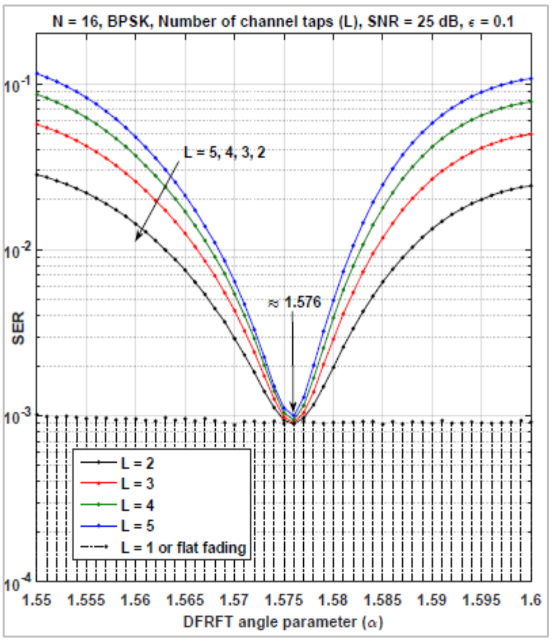

Figure 43-5 SER vs DFRFT angle (α) for different number of channel taps (L)

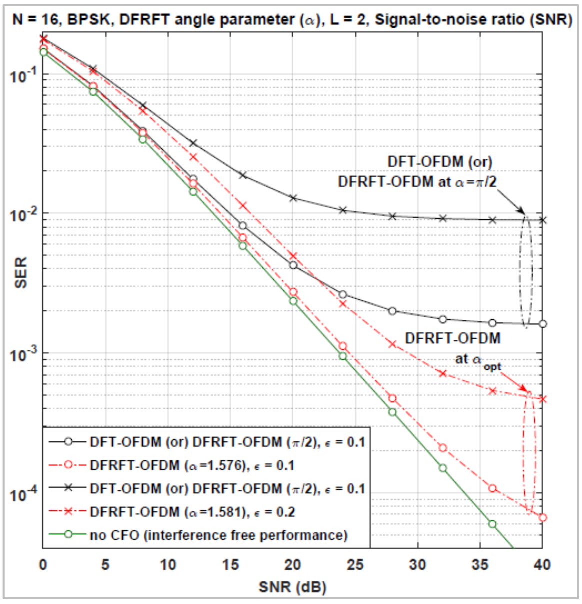

Figure 43-6 SER vs SNR (dB) with BPSK modulation over two tap Rayleigh fading channel with normalized CFO () of 0.1 and 0.2 for both DFT and DFRFT based OFDM systems

The DFRFT based OFDM system at α_{opt} over multipath fading channel nearly retains the flat fading performance corresponding to different values of CFO. It is important to note that the DFRFT based OFDM system needs not to aligned to distinct values of α_{opt} every time the CFO values changes. Any close value can always provide significant performance gain like $\alpha_{opt} = 1.573$ can efficiently serve the CFO values between 0.01 to 0.07, $\alpha_{opt} = 1.576$ between 0.08 to 0.13 , $\alpha_{opt} = 1.579$ from 0.14 to 0.17 and so on.

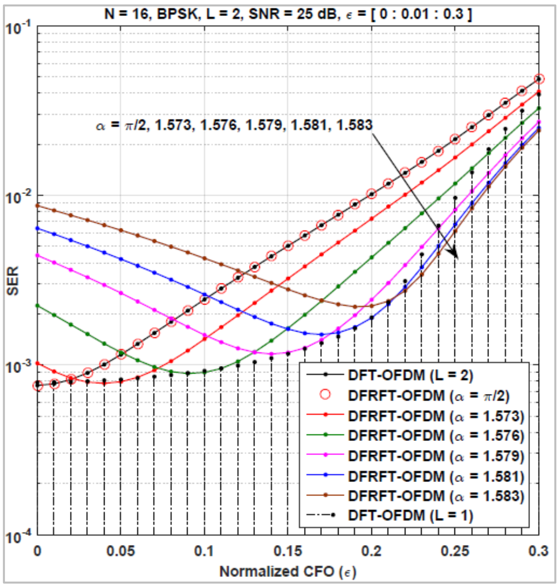

Figure 43-7 SER vs CFO for different values of αopt obtained in Fig. 43-4

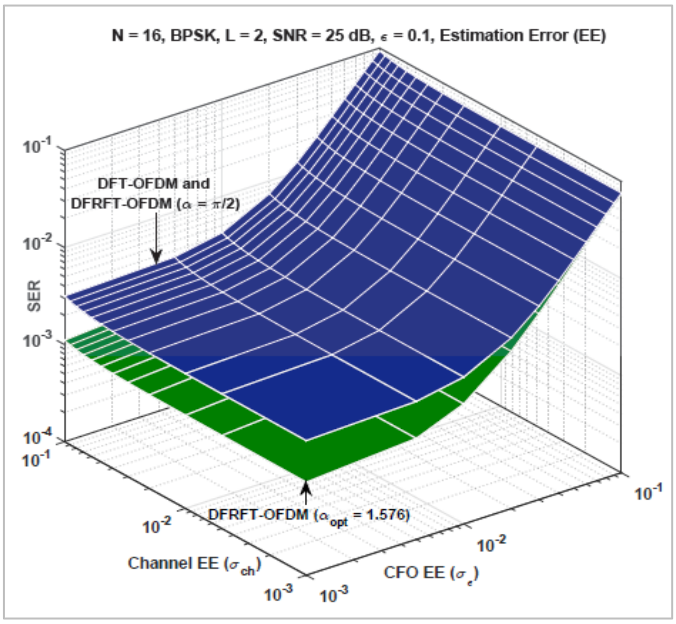

Figure 43-8 SER vs simultaneous channel and CFO estimation error for both DFT and DFRFT based OFDM system with normalized CFO (ϵ) equals 0.1

Perfect estimation of both the multipath channel and CFO is utilized to generate all the previous results. Moreover, estimation errors arising out of incorrect or approximate estimator leads to performance degradation. Estimation error also arises in a fast varying channel, where the estimator is unable to track the fast-changing channel. In Fig. 43-8, the performance of DFT and DFRFT based OFDM system are compared in the presence of both channel and CFO estimation errors at the receiver. The performance gain achieved by using DFRFT based OFDM system reduces as the CFO estimation error increases. Fortunately, the range of tolerable estimation error is well within the conventional estimator performance that motivates the performance gain with DFRFT based OFDM system.

CONCLUSION AND FUTURE WORK

DFRFT based OFDM system outperforms the conventional DFT based OFDM system in the presence of CFO. The SER performance gain is evaluated for different values of CFO, OFDM subcarriers, and multipath channel taps. The variation/dynamics of optimum DFRFT angle are observed with different OFDM parameters assuming both perfect estimations and in the presence of estimation errors. Preliminary results strongly motivate the potential use of DFRFT based multicarrier modulation/demodulation and equalization in the fractional Fourier domain for next-generation wireless mobile communication with CFO. Moreover, it is important to note that DFRFT and IDFRFT can be implemented using similar computational complexity as FFT and IFFT that justifies the immediate advantage of employing DFRFT based OFDM system.

However, DFRFT based OFDM system involves the use of IDFRFT for multicarrier modulation that requires the chirp rate (α) information to be relayed by the receiver to the transmitter through a feedback channel. Apart from occupying feedback bandwidth, this can be more problematic with fast changes in CFO that needs to update the DFRFT angle (α) at the transmitter. DFRFT based OFDM system is shown to achieve approximately the interference-free performance at low values of CFO. However, with high CFO and high modulation order, residual interference compensation is still needed and has to be figured out for envisioning such systems. Moreover, DFRFT based OFDM system involves the equalization in the fractional Fourier domain after DFRFT at the receiver. Also, the DFRFT at the receiver doesn't follow the circular convolution property as DFT. Therefore, conventional channel estimation algorithms are required to be reconfigured for DFRFT based OFDM system.

ACKNOWLEDGEMENT

This work is supported by Ministry of Electronics and Information Technology (MeitY), Government of India, Project no: 0429.

REFERENCES

[1] R. van Nee and R. Prasad,”OFDM for Wireless Multimedia Communications,” Boston, MA: Artech House, 2000.

[2] 3rd Generation Partnership Project, 3GPP TS 38.211 V15.4.0,”Technical Specification Group Radio Access Network; NR; Physical channels and modulation (Release 15),” Dec. 2018.

[3] Jungwon Lee, Hui-Ling Lou, D. Toumpakaris and J. M. Cioffi, ”Effect of carrier frequency offset on OFDM systems for multipath fading channels,” IEEE Global Telecommunications Conference, 2004. GLOBECOM ’04., Dallas, TX, 2004, pp. 3721-3725 Vol.6.

[4] A. Omri, M. Shaqfeh, A. Ali and H. Alnuweiri, ”Synchronization Procedure in 5G NR Systems,” in IEEE Access, vol. 7, pp. 41286-41295, 2019.

[5] V. K. Trivedi, K. Ramadan, P. Kumar, M. I. Dessouky, and F. E. Abd ElSamie, “Trigonometric Transforms and Precoding Strategies for OFDMbased Uplink Hybrid Multi-carrier Non Orthogonal Multiple Access,” Transactions on Emerging Telecommunications Technologies, 2019.

[6] L. B. Almeida, ”The fractional Fourier transform and time-frequency representations,” in IEEE Transactions on Signal Processing, vol. 42, no. 11, pp. 3084-3091, Nov. 1994.

[7] M. Martone, ”A multicarrier system based on the fractional Fourier transform for time-frequency-selective channels,” in IEEE Transactions on Communications, vol. 49, no. 6, pp. 1011-1020, June 2001.

[8] Soo-Chang Pei and Jian-Jiun Ding, ”Closed-form discrete fractional and affine Fourier transforms,” in IEEE Transactions on Signal Processing, vol. 48, no. 5, pp. 1338-1353, May 2000.

[9] J. Zheng and Z. Wang, ”ICI Analysis for FRFT-OFDM Systems to Frequency Offset in Time-Frequency Selective Fading Channels,” IEEE Commun. Lett., vol. 14, no. 10, pp. 888–890, Oct. 2010.

[10] Z. Mokhtari and M. Sabbaghian, ”Near-Optimal Angle of Transform in FrFT-OFDM Systems Based on ICI Analysis,” in IEEE Transactions on Vehicular Technology, vol. 65, no. 7, pp. 5777-5783, July 2016.

[11] V. K. Trivedi, S. Kumari and P. Kumar, ”Generalised error analysis of FRFT-OFDM over Nakagami-m fading channel with arbitrary m,” in IET Communications, vol. 11, no. 9, pp. 1497-1502, 2017.

[12] KW. Huang, R. Tao, Y. Wang, ”Fractional Fourier domain equalization for single carrier broadband wireless systems”, Sci. China, 2012.