# Review of Isogeometric Analysis based Shape Optimization Methods and Applications

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## Abstract

Shape optimization is an important technique to improve the efficiency of engineered components by achieving the optimized geometry of the component. In the optimization process, three steps are involved, namely design, analysis, and optimization. Different numerical methods can be used during the analysis stage. One of the popular numerical methods is classical Finite element analysis. This study examines how the traditional Finite element method is used to solve shape optimization problems covering structural problems, thermal problems, fluid-structure interactions, and contact problems. The key issues raised by researchers in the field of FEM based shape optimization are also examined in this study. Recently, Isogeometric analysis has evolved as a powerful tool for solving shape optimization problems. The purpose of this study is to see how successfully IGA can tackle shape optimization problems and how it performs in comparison to standard FEM. An overview of the methods based on finite element analysis and current Isogeometric analysis are discussed with their applications to shape optimization.

**Keywords**: Isogeometric analysis, Shape optimisation, FEM

## 1. Introduction and Background

Shape optimization is the process of designing a structure for better performance. It has been applied in a variety of domains, including structural analysis, fluid analysis, acoustics, nano-photonics, and micro-scale optimisation. The shape optimisation process relies heavily on geometric parameterization and a correct boundary description. In shape optimization, proper boundary representation and selection of design variables are essential to the success of the optimization. The coordinates of the nodes of elements are used as design variables in finite element based approaches [1]. Because of discrete representation and further changes in nodal coordinates, optimized designs are often irregular in shape thereby making the manufacturing of the design difficult [2]. Some engineering structures shapes are very complex. To represent the geometry more precisely, a very finer mesh is required, thus increasing the computational time as the number of design variables will be very high. In Iso-parametric formulation of FEM, for geometry representation and field variable, a Lagrange polynomial with $c^0$ continuity across the elements is used. Hence, the first order derivatives of field variable are discontinuous across the element [3, 4]. Therefore, it becomes difficult to perform a sensitivity analysis. As a result, the gradient-based optimization methods do not succeed, and the most sensitive studies are being performed using other techniques. Using higher-order Lagrange polynomials is one of the techniques to deal with this issue [5]. However, the robustness of the method and efficiency are compromised. The drawbacks can be addresses by using a different set of basis functions such as B-spline, NURBS(Non Uniform Rational B spline), and T-spline to precisely describe the geometry and also field variable. A recent analysis approach, Isogeometric analysis (IGA) has brought a new direction to numerical analysis. This study is yielding fresh insights into shape optimization. Several researchers have focused on studies relating to the benefits of Isogeometric analysis and have achieved significant improvements. The fundamental benefit of IGA is that it uses the same NURBS basis function for representing geometry and field variable. The function variable, shape design, and analysis all share the same domain space in the context of shape optimization, making it easier to do sensitivity analysis. This reduces communication with the CAD in every phase of the optimization process, which saves time and enables faster results. Moreover, the IGA based methods results in a smooth structure. The resulting structures are manufacturable. Classical FEM boundaries are not properly captured, which leads to various inaccuracies in the final results. With splines in IGA, boundaries are captured accurately.

In this paper first, an overview of FEM-based shape optimization methods is given, followed by a discussion of the shortcomings. A brief overview of the IGA-based shape optimization is discussed in the later part. The overall paper is divided into the following sections 2) Process of shape optimisation 3) Shape optimisation using Finite element technique 4) Shape optimisation with IGA 5) Comments and future scope 6) Conclusion and discussion

## 2. PROCESS OF SHAPE OPTIMISATION

In general, shape optimization involves either minimizing or maximizing an objective function that is constrained in some way. Mathematically the shape optimisation equation can be written as below

$$\text{Min } C(u)$$

$$\text{Subjected to } \sigma \leq \sigma_{max}$$

where, C is the compliance, u is the design variable, $and\ \sigma_{max}$ is the max stress induced in the component. The objective could be to reduce the amount of weight or to reduce the amount of stress. Design variables are bounded with some upper and lower limits. Constraints can sometimes be either equality or inequality. In certain instances, objective functions are linear, while in others, they are nonlinear, based on the scale of complexity involved.

There are several optimization algorithms in the literature, which are divided into two types: gradient-based optimization and gradient-free optimization. Sensitivity information is required for gradient-based optimization, while it is not required for gradient-free optimization. The techniques used so far are mentioned in the next section. Figure 1 represents a flow chart to understand this shape optimisation process.

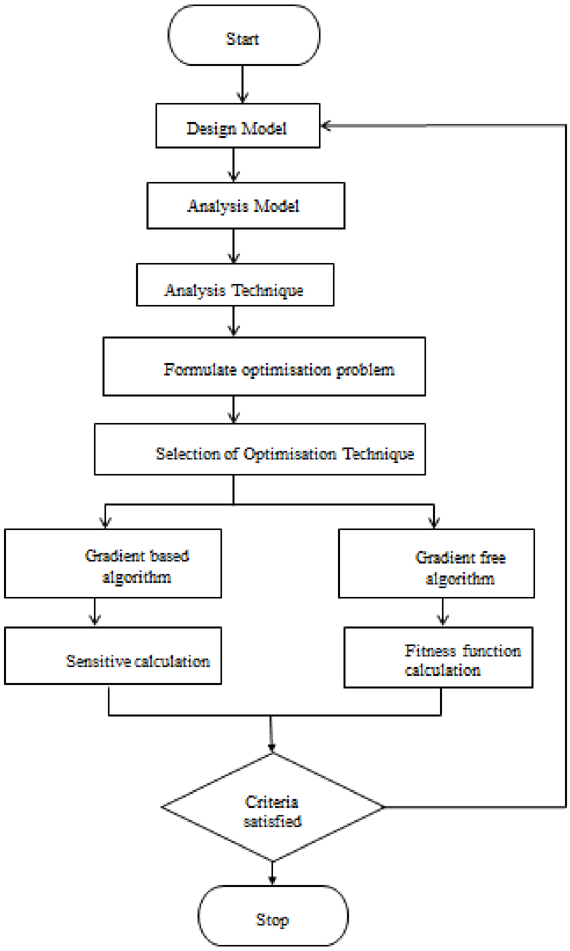


Figure 1. Shape optimisation process

## 3. SHAPE OPTIMISATION USING FINITE ELEMENT TECHNIQUE

The subject of structural optimization arose at the same time as finite element analysis technology. However, due to the fact that it has to undergo repetitive analysis the development of structural optimization lags behind. Several commercial analysis softwares e.g., Ansys , Abaqus, Nastran offer separate module for structural optimisation. Although the finite element technique is a well-developed numerical technique, the utility of this method in shape optimization has some issues like re-meshing of the model in each optimization phase; Geometry model and analysis model have different parameterization schemes. Following section deals with research papers focussed on fem technique especially in application to structural problems.

### *3.1. Finite element analysis steps*

In finite element method the domain is discretised into a collection of preselected finite elements. It mainly consists of three features those are given below

- The entire domain is divided into small shapes
- Lagrange basis functions are used over each element depending on the dimension and location of nodes (Ex 1D element, Triangular element, rectangular element , Hex element)
- Assembly of elements are done by ensuring continuity of field variables across each element.

They are mainly three types of errors present in the finite element method based on above process

- Domain approximation errors occur in complex geometry where curvature are present
- Approximation error which is due to the approximation of the solution by piecewise polynomials
- Computational error occurred due to inexact evaluation of stiffness and force matrix

### *3.2. Applications*

Several review articles [6, 7, 31] are published on FEM based shape optimisation. Hence, focus of this paper mainly on shape optimisation using IGA. Although shape optimisation is used in variety of fields like structural, fluid, and aerodynamic. This paper is focused mainly on structural optimisation.

In early shape optimisation methods based on FEM, boundary nodes of mesh are used as design variables [9]. This process offers more design options, but increases computational effort and generates an infeasible geometric shape [2]. Due to random deformation of the elements, this leads to inaccurate stresses in the final design (Figures 2 and 3).

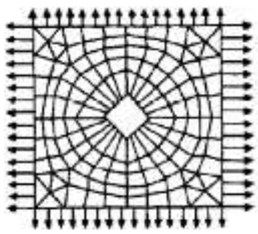

Figure 2. Initial design [2]

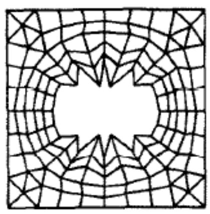

Figure 3. Final design [2]

Separation of finite element mesh and design variable could be one solution. Polynomials are also explored as an alternative to represent the boundaries [11]. The idea was to define the boundary as a linear combination of shape function with coefficients. The coefficients will act as design variable. Thus, Kristensen [5] has used linear combination of orthogonal function to represent the boundary and its coefficient are design variable. Dems [12] has solved by considering simple linear boundaries. Most approaches are restricted to solving linear boundaries since; with higher order polynomials suffers from oscillatory behaviour. This is a well-recognized issue which necessitates a better approach for representing the boundary. This issue can be tackled by representing boundary with Splines. Splines can have high order smoothness with lower order polynomial. So it becomes another alternative for boundary representation (e.g., [5]). Yang and Choi showed that the spline representation has better sensitivity accuracy than a piecewise linear representation of the boundary. Briabant [2] has used Bezier and B-spline blending functions to describe design element boundaries. With the B-spline formulation, boundary regularity requirements are automatically taken into consideration and also an analytical formulation of the sensitivity derivatives can be established.

Shape optimization has proven to be very effective in the area of computational fluid dynamics. Chan et al [13] have enhanced the power coefficient of the wind turbine blade using shape optimisation. To improve the power coefficient, the geometry of the semi-circular blade is optimized using an evolutionary-based genetic algorithm. It is solved through the ANSYS Fluent software. Additive manufacturing may be used to manufacture the optimised blade shape. Artificial intelligence is progressing, and deep learning-based algorithms are increasingly being applied in a variety of fields. Shape optimisation is one such area. Jichao et al. [14] has verified the abnormality in aerofoil wings using a surrogate-based optimization approach. The standard neural network is first trained using 20,000 existing data and then coupled to a surrogate-based optimization framework. The algorithm produced timely and accurate findings. Ramadan et al.[15] have developed an optimised vertical axis wind turbine blade using a genetic algorithm method. FEA software, Ansys fluent is used for the CFD analysis. The power coefficient is increased about four times for the optimized blade shape and validated experimentally.

Tada et al.[16] used FEM to optimize the contact forces between two elastic bodies. Contact forces between the bodies are optimized. Butt et al.[17] developed the material derivation method for shape optimization of contact problems. Weil et al. (2001) worked on the uniform distribution of contact stresses between two- and three-dimensional elastic bodies. In this work, evolutionary techniques are used in combination with the finite element method.

In the work of Daniel Hilding et al. [18], shape optimization software was developed based on the following four blocks- namely, account analytic sensitivity analysis, adaptive finite element method, contact solver, and sequence convex programming. Ou et al. [10] focused on reducing boundary stress and contact pressure between multi-body contact systems. He has proposed a novel way that does not require sensitive studies.

### *3.3. Limitations of FE shape optimisation*

The limitations of the numerical analysis techniques also prevent a successful structural shape optimization, as the solution accuracy and the computation time for the shape optimization strongly depend on them. Because FEM is widely used for structural analysis, the resulting framework features FEM-related difficulties such as mesh distortion and subsequent re-meshing, discontinuous stresses across element borders due to linear approximation field function, and so on. Despite its success in several domains, the FEM approach still has limitations, some of which are listed below.

1) Different basis functions used to describe geometry in design and analysis models are the crucial bottleneck in classic FEM-based shape optimization [19]
2) To reduce computational cost, it is desirable to use as few design variables as possible. But, it is not possible in FEM for accurate results.
3) Each iteration of the optimization process necessitates back-and-forth communication with CAD. As a result, it takes a long time to compute.
4) Sensitive analysis needs to carry out in gradient-based optimisation methods and it is difficult with classical FEM.
5) Traditional FEM has discontinuous stresses across the elements because it approximates linear interpolation for design variables. The first derivative of stresses concerning the design variable is necessary for sensitive analysis. So higher-order representation of the field variable is required.
6) In FEM-based form optimization, a wavy or uneven shape will emerge, which is unsuitable for manufacturing.

## 4. SHAPE OPTIMISATION WITH IGA

Many commercial software packages employ an Iso-parametric formulation in their FEM code, which means that the field variable and geometry descriptions share the same basis. Those are Lagrange basis with $c^0$ continuity. NURBS are used in CAD modelling software to describe geometry. IGA approach uses the same NURBS basis function for the filed variable also. This new concept has opened the opportunity for a better type of analysis known as Isogeometric analysis. They have successfully implemented these new improvements in a variety of fields. Those publications that employed IGA shape optimization are discussed in the following section.

### *4.1. Isogeoemtric analysis steps*

CAD geometry is represented using NURBS from past two decades because of their inherent properties. In Isoparametric FEM the field variable is discretised first and then the same basis discretisation is used for domain. In IGA the domain is discretised by NURBS and same is used for field variable.

In B-spline curve is represented with a set of basis function combines n+1 control points.

$$P(u)=\sum_{i=0}^{n} P_i \quad N_{i,k}(u)$$

Where k is the order of B-spline and k-1 is the degree of the B-spline. It is independent of number of control points

The NURBS are recursively defined by the following

$$N_{i,1}(u)= 1 \quad \text{if } t_i \leq u \leq t_{i+1}$$

$$=0 \quad \text{otherwise}$$

and

$$N_{i,k}(u) = \frac{(u - t_i)N_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)N_{i+k,k-1}(u)}{t_{i+k} - t_{i-1}}$$

k value ranges from 2,…,k where k is the order of the B-spline and controls the degree (k-1) of the resulting polynomial in u and also controls the continuity of the curve.. The $t_i$ are called knot values, and set of knot values comprise a knot vector. They relate the parametric variable u to the $P_i$ control points where i=0,…,n. For an open uniform curve the $t_i$ are calculated once using k:

$$t_j = 0 \quad \text{if} \quad j<k$$

$$t_j = j-k+1 \quad \text{if} \quad k \le j \le n$$

$$t_j = n+k+2 \quad \text{if} \quad i>n$$

Using above knot values the basis functions can be generated. B spline surface is generates using tensor product of bi-variant and Tri-variant B splines. Third order and fourth order B-spline curves generated using MATLAB code are shown below

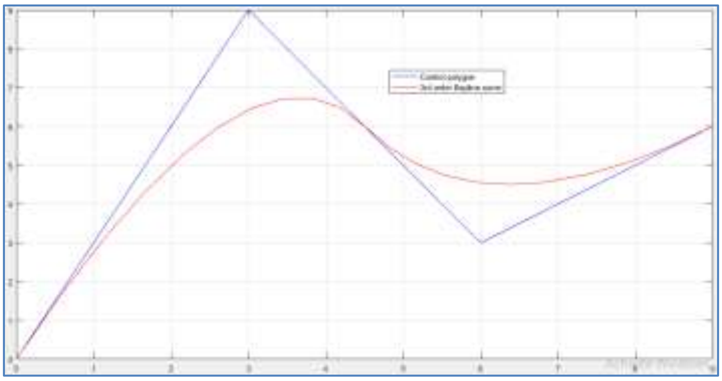

Figure 4. Third order B-spline with four control points

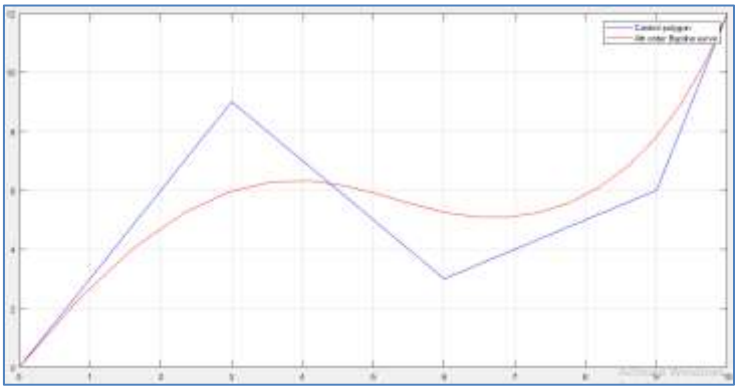

Figure 5. Fourth order B-spline with five control points

### *4.2. Applications*

Due to its precise shape representation, NURBS-based geometry representation analysis has become increasingly popular in shape optimization applications in recent years. Because the approach uses a higher-order foundation, sensitive studies are simple to perform. Wall et al.[20] worked on shape optimization of structures with IGA. The effectiveness of linking the design and analysis model is intensified. Simple 2D problems like a plate with a hole and wrench are solved using NURBS. The sensitive analysis is performed using a gradient-based optimization method. The majority of studies have used NURBS control points as a decision variable in their sensitivity analysis. For the sensitivity analysis, Qian et al. [27] employed both control points and weights. In his study, he used a gradient-based optimization approach. According to this approach, weights and control points as design variables help achieve optimal shapes.

Another area of application is the optimization of shell structures. Boilers, roof structures, and the automotive and aerospace industries use shells. In all these cases, optimizing the shell structure is very important. Keindl et al. [22] has pointed out that when representing geometry using splines, the rotational degree of freedom need not be a field variable due to their higher-order continuity. In addition, the flow structure of the rotor blade interaction of the wind turbine is optimized. It is mentioned that the flexibility of this IGA-based representation has many advantages and reduces the computational time. The 2D plate and shell problems are optimised using Iso-geometric analysis in the work of Yudeok Seo et al. [23]. The optimization of vibrating membranes with IGA is performed by Nguyen et al [24]. Two methods, namely the quasi-conformal mapping and the spring-based mesh method, are proposed to map the boundary of the domain to its interior. IGA goes well with shape optimization of vibration problems. The IGA has also been used for topology optimisation using the trimming technique. For topology optimization, trimmed surface analysis is used, which was recently proposed to analyse any complex topology problem. Some benchmark problems are solved in shape and topology optimisation. Li et al. [25] have studied shape optimisation using the Iso-geometric boundary integral method. The author has performed an h-p-k refinement and concluded that the NURBS technique offers better performance. The sensitive analysis can be done either with NURBS control points or its weights. In shape optimization, a sensitivity analysis is much more important to see how sensitive the objective functions are to design variables. Hassani et al. [26] have solved two and three-dimensional simple problems using the NURBS basis function and obtained a very smooth optimised surface. The boundary variable and field variable are approximated with NURBS basis in the Iso-geometric boundary integral method.

Xiaoping Qian et al. [21] has used multi patch coons to generate the complex geometry. It allows to user to design boundary shapes without specifying the internal control points. The specification of internal nodes can be avoided in this process. It is applied to maximise the band gap in photo crystal design. The internal nodal data is embedded in the NURBS formulation itself. Analytical and semi-analytical techniques have been used for performing sensitivity analysis.

An optimised location quadrature point has been proposed by Zhen Lei et al. [28], which addresses the locking issue in shell formulation. A mixed grid Reissner-Mindlin shell formulation is used in this work. The classic modal synthesis method and the Craig-Brampton fixed interface method are used for shell-patch coupling. The analysis time decreases more in the modal synthesis method. Ummidivarapu et al. [29] has optimized the acoustic horn with TLBO (Teaching Learning-based algorithm). The acoustic field is modelled and analysed in the IGA framework. The optimal shape of the horn speaker has resulted in reduced back reflection. This IGA analysis has shown significant improvement and surpassed the FEM results. The shape of the horn is also easy to manufacture. Recently, Qin et al. [31] worked on the shape and material optimization of functionally graded material (FGM). MMA method and SQP are used for shape and material optimization respectively. The optimal shape of the stiffener is obtained to minimize the flexibility of the plate subjected to a volume constraint.

Ummidivarupu et al. [32] has used IGA on cantilever beams and square plates with a circular hole to achieve an optimal area under a given load condition. Genetic algorithm and Nelder and Mead simplex algorithm were used to solve the problems. Lopez et al. [4] performed an automated sensitive analysis using a differential toolbox. Automatic differentiation can be used to perform a forward and reverse mode sensitive analysis. This AD (automatic differentiation) is far superior to analytical and semi-analytical sensitive techniques, as demonstrated by the practical application.

The analysis of shells required a curvilinear representation of the surface. IGA and NURBS are very suitable for shell analysis, as they reproduce the geometry very smoothly. Hirschler et al. [34] optimized the shape of solid shell and Kirchhoff Love Shells. The author observed that both methods gave similar results. The size and shape optimisation are integrated by the author and applied to the cylinder.

The majority of engineering components are subjected to thermal conditions. As a result, one of the most important areas to focus on is heat exchange. Components are optimised either to increase or decrease the heat exchange. The shape and material selection plays a key role in efficient performance. Wang et al. [28] have worked on optimised shapes subjected to steady-state heat conduction. Active control of heat is not feasible in fluctuating thermal conditions. Shape optimization is a more effective method that acts as a passive control for thermal conditions.

Any machine consists of more elements under relative motion. Because of their relative motion, engineering components are prone to wear. Engineers are always concerned with wear and stress distribution between contact surfaces. Shape optimization can be applied to control these parameters. The contact problems are nonlinear since the point before contact is unknown. These boundary conditions are nonlinear. It is difficult to perform a sensitivity analysis using a gradient-based method since most contact equations are non-differentiable. So gradient-free optimisation techniques are adopted by most cases in contact problems.

Li w et al. [35] solved the optimal distance between contact bodies to limit the contact stress between multi-body systems. An evolutionary optimization method is used to solve the problem. In this multiple contact problems are solved by adopting individual criteria and the unified criteria method. Nam Ho Kim et al. [36] used a material-derivative approach for sensitive analysis of the three-dimensional contact problem. Mesh-free methods are used to solve the design sensitivity equation. Not much of the work is reported on shape optimization with IGA for contact problems.

### *4.3. Comments and future scope*

Iso-geometric analysis can be more potential in shape optimisation problems, shell structure analysis. The domain discretisation error present in FEM can be avoided in Iso geometric analysis. Contact problems is another important where it require to represent the geometry exactly. Recently Functionally graded materials (FGM) have been analysing for estimating the strength and stability. FGM requires higher order basis for better analysis. In these problems IGA make the process more robust and computationally more efficient.

## 5. CONCLUSION AND DISCUSSION

The structural shape optimisation topic is widespread. It is difficult to describe all of the interesting aspects in one paper. An attempt is made to review of literature related to shape optimisation. The scope of the present article is restricted to only FEM and IGA based shape optimisation. One particular advantage of IGA is using the same basis for analysis and design models. The researchers have focussed on thermal, structural, fluid-structure interaction, and contact problems. Due to its attractive features, IGA is expected accelerate the shape optimization research further. In brief, the advantages of IGA-SO are listed below.

- complex geometry can be represented accurately
- Analysis and geometry model share the same basis.
- Number of design variables are less in IGA based shape optimization
- Stresses are continuous across the elements so sensitive analysis is easy.
- CAD and analysis communication can be avoided
- Re-meshing tasks can be avoided in every iteration process

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