ROLE OF LINEAR ALGEBRA IN IMAGE COMPRESSION

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Abstract.

A comparison will be made between Singular Value Decomposition (SVD) and Block Truncation Coding (BTC) in this paper, as well as analyse how Algebra contributes to image compression and Block Truncation Coding (BTC). We will also examine how Algebra influences image compression. The input picture will be compressed first using the SVD method to reduce the image matrix rank, BTC will then be used to compress the matrix produced. The suggested approach improves the JPEG compression process by adding lossless compression, resulting in a compression rate of over 99 percent.

Keywords. Linear Algebra, BTC, SVD & PSNR.

1. INTRODUCTION

1.1. Image Processing

Basically, it's a way of processing images by extracting their characteristics or altering their inputs. Image processing is one of the most quickly evolving technology in today's world. It is also an important research field in computer science and engineering.

The following phases that makeup the image processing is:

- Input of the image data via software.
- Analysing and changing the input.
- And generating an output that could be a changed image or a report based on analysis.

When using digital techniques, all sorts of data must go through three general processing steps: pre-processing, enhancement, and display. Information extraction is the last of these steps.

1.2 Digital image processing (DIP)

The process of converting digital photos into digital images is called DIP. Analogue image processing can be compared to DIP as a subset of digital signal processing. It offers many procedures to choose from and apply to the input image, as well as discarding some issues such as noise and or unwanted distortion during the process. Image processing can be viewed as a multidimensional system because images can be displayed in 2D or more.

1.2. Image Compression

Digital image processing is a tool to process digital photos using the digital computer A procedure is used to process digital photos electronically with the help of a digital computer. processing. It offers many algorithms to choose from Image compression can be accomplished in a variety of ways. Internet users primarily use GIF and JPEG compression for graphic images. GIF is most often used for line art and graphics with simple geometric patterns, while JPEG is more often used for photos.

1.3. Linear Algebra

We all underestimated Linear Algebra's potential. It is composed of algorithms and approaches that are extremely useful in the real world, particularly in image analysis and manipulation. Images are one of the most widely used forms of communication in today's digital and social environments.
The two fundamental elements of linear algebra are the vector and matrix. A matrix is a linear mapping that converts vectors from one space to another, whereas a vector is a Euclidean space point (both the spaces could be of the same or different dimensions).

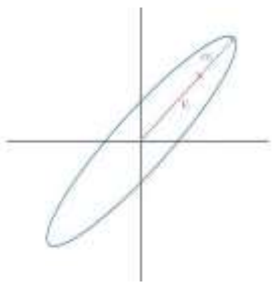
1.3.1. Eigenvalue and Eigenvector λ

An equation as simple as Av = λv could be so significant. A matrix's eigenvalues and eigenvectors can be used to solve many problems, from machine learning to quantum computing. In other words, λ is the eigenvalue of A, and v is the eigenvector,

if

$$Av = \lambda v$$

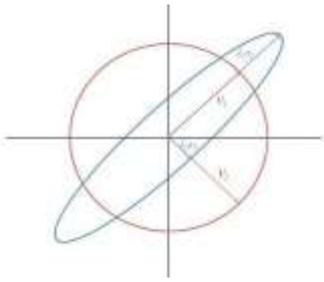
From a visual point of view, Av and the eigenvector v appear to be on the same plane.



It doesn't always follow that x equals Ax. Only a few exceptional vectors meet the criteria. Here is an instance of eigenvectors.

$$\begin{matrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{matrix} \quad \begin{matrix} 1/2 \\ 1/2 \\ 1 \end{matrix} \quad = \quad 4 \quad \begin{matrix} 1/2 \\ 1/2 \\ 1 \end{matrix}$$

The related Av_i will grow if the eigenvalue is bigger than one. It will shrink if it is less than one.



2. SINGLE VALUE DECOMPOSITION

Single Value Decomposition (SVD). SVD can decompose any matrix into three matrices, unlike other decompositions that need a square matrix to be decomposed, SVD allows you to decompose a rectangle matrix (a matrix that has different numbers of rows and olumns).

Q^T, , and T are Z's decomposed matrices. As a result, any linear map may be deconstructed into these three fundamental transformations, this process known as Singular Value decomposition (SVD).

$$B = TQZ^t \qquad \ldots (1)$$

Figure 1: Example of Factorisation of *B* to *TQZ*^{t}

Where T is an $a \times a$ orthogonal matrix

$$T = [t_1, t_2, \ldots t_r, t_{r+1}, \ldots, t_m] \qquad \ldots\ldots(2)$$

column vectors $\mathbf{t_i}$, for $i = 1, 2, \ldots, m$, form an orthonormal set:

And Z is an $b \times b$ rectangular matrix

$$Z = [\, z_1, z_2, \ldots z_r, z_{r+1}, \ldots, z_n] \qquad \ldots (3)$$

column vectors $\mathbf{v}i$ for i = 1, 2, …, n, form an orthonormal set:

Here, Q is an $a \times b$ diagonal matrix with

singular values (SV) on the diagonal. The matrix Q can be as shown :

Q = σ1 0 --- 0 0 ---- 0

0 σ2 --- 0 0 --- 0

- - - - - - - -

- - - - - - - -

0 0 --- σr 0 ----- 0

0 0 --- 0 σ(r+1) ----- 0

- - --- - ------ ----- -

0 0 --- 0 0 ----- σr

0 0 --- 0 0 ----- 0

For $i = 1, 2, \ldots, n$, σ_i are called Singular Values (SV) of matrix B. It can be proved that

$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq 0$, and

$$\sigma_{r+1} = \sigma_{r+2} = \ldots = \sigma_n = 0 \qquad \ldots (5)$$

For $i = 1, 2, \ldots, n$, is called Singular Values (SVs) of B. The $\mathbf{v}i$'s and $\mathbf{t}i$'s are called right and left singular vectors of B [1].

Digital images can be compressed to reduce the quantity of information they require. Compression is attained by reducing three essential data severances: 1) coding redundancy, caused by poor coding quality; 2) interpixel redundancy, caused by pixel correlations; and 3) psychovisual redundancy, caused by data disregard by the System of visual perception in humans.

Its singular values decrease rapidly. Based on the rank of an asymmetric matrix increases. This property can be useful for reducing noise and compressing matrix data by removing single values or higher ranks.

We present detailed steps to demonstrate the SVD image compression process:

$$B = TQZ^t$$

i.e. *B* can be epitomized by the external as BB^T

product expansion:

$$BB^T = TQZ^T ZDT^T$$

By truncating the sums after the first k terms, the closest matrix of rank k is obtained:

$$Bk = \sigma_1 t_1 z_1 + \sigma\ 2t\ 2z\ 2t + \text{---------} + \sigma_{K1} t_{K1} z_{K1}$$

The total storage for Bk will be

k(a + b+1)

A digital picture that corresponds to Bk will remain substantially similar to the original image even if the integer k is less than n. On the other hand, the remaining k will have a diverse storage and picture. With typical k selections, A_k will require less than 20% of storage.

Compressed Images at different values of K.

Original Image

- Result of Experimentations for Image Compression

3. BLOCK TRUNCATION CODING

As a result, moments for each picture block are preserved. It is known as moment-preserving block truncation. In order to implement the BTC algorithm, following steps must be taken:

The first step is to divide the image into rectangular parts that don't overlap with each other. In order to simplify the process, we decided to make the blocks squares measuring m x m.

In the second step, each pixel in the block is quantized into two brightness values using a two-level quantizer (1 bit). The mean x and the standard deviation σ are these values.

$$\underline{\square} = 1/\square \ \sum_{\square=1}^{\square} \square\square$$

$$\sigma = \sqrt{1/\square} \sum_{\square=1}^{\square} (\square\square - \underline{\square}i)^2$$

Step 3: The two values x and σ are referred to as BTC quantizers. Two-level bit planes are generated by comparing each pixel value xi to the threshold value x.

$$\square = 1\square\square \geq \underline{\square}$$

$$0 \ \square\square < \underline{\square}$$

Each block is converted to a bit plane using this method. A block of 4 × 4 pixels, for example, will yield 32-bit compressed data, or 2 bits per pixel (bpp).

The fourth step involves rebuilding an image block in the decoder by replacing "1"s in the bit plane with "H", and "0"s with "L", as shown in the following equations:

$$H = \underline{\square} + \sigma\sqrt{\square/\square}$$

$$L = \underline{\square} + \square\sqrt{\square/\square}$$

The number of 0’s and 1’s in the compressed bit plane is denoted by p and q, accordingly.

Original Compressed Image

- **Result of Experimentations for Image Compression**

4. PSNR

Two images are compared to calculate their peak signal-to-noise ratios (in decibels). In this ratio, original and compressed images are compared for quality. With increasing PSNR, the quality of the compressed or rebuilt image improves.
In this ratio, the original and compressed image quality are compared. In PSNR, the peak error is represented by the PSNR, whereas in MSE, By MSE, we can measure the squared error between the original and compressed images. TUsing MSE as a measure of error is in inverse relationship with the error. PSNR is calculated in two stages: calculating mean square error and calculating PSNR.

$$MSE = \frac{1}{M * N}\{[f(a,b) - f'(a,b)^2]\}$$

M and N in the above equation stand for the input pictures' respective rows and columns. The block then uses the following calculation to get the PSNR:

$$PSNR = 10log10\frac{R^2}{MSE}$$

A picture's input data type has the greatest variation when viewed in the previous equation. The R value is 1, for instance, if the key picture uses floating (double-precision) points. A data type that is 8 bits unsigned has R = 255, for example.

5. CONCLUSION

When the singular value of an SVD increases, the quality of compressed image is enhanced significantly but the size also increases. The image's visual quality degrades as the block size increases, and the compression size does not shrink as much.
The block truncation coding (BTC) scheme is effective in terms of higher compression ratios

as inferred from our findings. When data loss is unacceptable, Singular Value Decomposition (SVD) may be used for passwords, financial information, and confidential papers; however, BTC can be used when the focus is more on transmission than what information is included in the picture.

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Biographies

Dhruv Sharma currently pursuing B,Tech (CSE) from ICFAI University, Jaipur.

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