## 6

## A Switching Current-Source Oscillator

Ultra-low-power (ULP) transceivers underpin short-range communications for wireless Internet-of-Things (IoT) applications. However, their system lifetime is extremely limited by the transceiver power consumption and available battery technology. On the other hand, energy harvesting technologies typically deliver supply voltages that are much lower than the standard supply of CMOS circuits; e.g., on-chip solar cells can supply only $200-800 \mathrm{mV}$. Although boost converters can bring the level up to the required $\sim 1 \mathrm{~V}$, their poor efficiency ( $\leq 80 \%$ ) wastes the harvested energy. Consequently, RF oscillators, as one of the transceiver's most power hungry circuitry, must be very power efficient and preferably operate directly at the energy harvester output. In this chapter, we analyze in depth design of an oscillator topology to address the aforementioned constraints without sacrificing manufacturability and phase purity.

### 6.1 Introduction

Ultra-low-power (ULP) radios underpin short-range communications for wireless Internet-of-Things (IoT). Since RF transmitters (TX) have consumed a significant portion, if not the majority, of the radio's power, the IoT system lifetime tends to be severely limited by the TX power consumption and available battery technology.

Figure 6.1 shows the system lifetime for various battery choices as a function of current consumption. State-of-the-art Bluetooth Low Energy (BLE) radios [1,2] consume $\sim 7 \mathrm{~mW}$ and thus can continuously operate no more than 40 hours on an SR44 battery, which has a comparable dimension to the radio module. This directly causes inconvenient battery replacements at least every few months, which limits their attractiveness from the market


Figure 6.1 BLE system lifetime versus radio current consumption for various battery types.


Figure 6.2 Delivered voltage and power density for various harvester types.
perspective. The lifetime can be easily increased by employing larger batteries, but that comes at a price of increased weight and dimensions and it is clearly against the miniaturization vision of IoT. This has motivated an intensive research leading to miniaturized transceivers with a high power efficiency [1-12].

Energy harvesting from a surrounding environment can enable and further spur the IoT applications by significantly extending their lifetime. The delivered voltage versus power density of different harvesting methodologies is depicted in Figure 6.2 [10, 13]. Solar cells offer the highest harvested power per area in both indoor and outdoor conditions. However, they provide lower voltages $(0.25-0.75 \mathrm{~V})$ than the expected deep-nanoscale CMOS supply of $\sim 1 \mathrm{~V}$. Hence, boost converters are typically used to bring the supply level up to the required $\sim 1 \mathrm{~V}$. As can be gathered from Table 6.1, the relatively poor efficiency ( $\leq 80 \%$ ) of boost converters wastes the harvested energy, thus worsening the system-level efficiency, in addition to increasing

Table 6.1 Performance summary of state-of-the-art boost converters

|  | $[14]$ <br> ISSCC' 12 | $[15]$ <br> ISSCC' 14 | $[16]$ <br> ISSCC' 15 |
| :--- | :--- | :--- | :--- |
| Technology | N/A | 65 nm CMOS | $0.18 \mu \mathrm{~m}$ CMOS |
| Input voltage range | $0.1-2.9 \mathrm{~V}$ | $0.15-0.5 \mathrm{~V}$ | $0.45-3 \mathrm{~V}$ |
| Output voltage range | 3 V | $0.5-0.6 \mathrm{~V}$ | 3.3 V |
| Efficiency @ $V_{\text {in }}=0.5 \mathrm{~V}$ | $\leq 80 \%$ | $\leq 72.5 \%$ | $\leq 78.5 \%$ |

the hardware complexity coupled with issues of switching ripples. Consequently, it would be highly desirable for the ULP transceivers to operate directly from the harvested voltage.

The rest of this chapter is dedicated to introduce and analyze a switching current-source oscillator $[17,18]$ which is optimized for 28-nm CMOS, can operate directly at the low voltage of harvesters, and reduces power and supply voltage without compromising the robustness of the oscillator start-up or loading its tank quality factor.

### 6.2 Oscillator Power Consumption Trade-offs

The oscillator phase noise (PN) requirements can be calculated by considering the toughest BLE blocking profile:

$$
\begin{equation*}
\mathcal{L}(\Delta \omega)=P_{\text {signal }}-P_{\text {blocker }}-S N R_{\min }-10 \log _{10}(B W) \tag{6.1}
\end{equation*}
$$

The received packet error rate (PER) must be better than $30.8 \%$ while the wanted signal is just 3 dB above the reference sensitivity level of -70 dBm and in face of an in-band blocker of -40 dBm located at $3-\mathrm{MHz}$ offset from the desired channel. Furthermore, the required signal-to-noise ratio (SNR) should be better than 15 dB to support such PER for a GFSK signal with a modulation index $m=0.5$ [20]. By replacing the aforementioned values in (6.1), PN shall be better than $-105 \mathrm{dBc} / \mathrm{Hz}$ at $\Delta \omega=2 \pi \cdot 3 \mathrm{MHz}$. Hence, the PN requirements are quite trivial for IoT applications and can be easily met by LC oscillators as long as Barkhausen start-up criterion is satisfied over process, voltage, and temperature (PVT) variations. ${ }^{1}$ Consequently, reducing oscillator power consumption, $P_{D C}$, is the ultimate goal in IoT applications.

[^0]The PN of any class of an RF oscillator (i.e., class-B) at an offset frequency $\Delta \omega$ from its resonating frequency $\omega_{0}$ can be expressed as

$$
\begin{equation*}
\mathcal{L}(\Delta \omega)=10 \log _{10}\left(\frac{K T}{2 Q_{t}^{2} \alpha_{I} \alpha_{V} P_{D C}} \cdot F \cdot\left(\frac{\omega_{0}}{\Delta \omega}\right)^{2}\right) \tag{6.2}
\end{equation*}
$$

where $K$ is the Boltzmann's constant; $T$ is the absolute temperature; $Q_{t}$ is the LC-tank quality factor; $\alpha_{I}$ is the current efficiency, defined as ratio of the fundamental current harmonic $I_{\omega_{0}}$ over the oscillator DC current $I_{D C}$; and $\alpha_{V}$ is the voltage efficiency, defined as a ratio of the single-ended oscillation amplitude, $V_{o s c} / 2$, over the supply voltage $V_{D D}$ [21,22]. Furthermore, $F$ is the effective noise factor of the oscillator.

Equation (6.2) clearly demonstrates a trade-off between the oscillator's $P_{D C}$ and PN. Furthermore, the oscillator's FoM normalizes the PN performance to the oscillation frequency and power consumption, yielding

$$
\begin{equation*}
F o M=10 \log _{10}\left(\frac{10^{3} K T}{2 Q_{t}^{2} \alpha_{I} \alpha_{V}} \cdot F\right) \tag{6.3}
\end{equation*}
$$

The effective noise factor $F$ is expressed by [23,24]

$$
\begin{equation*}
F=\frac{R_{i n}}{2 K T} \cdot \sum_{i} \frac{1}{2 \pi} \int_{0}^{2 \pi} \overline{i_{n, i}^{2}(\phi)} \cdot \Gamma_{i}^{2}(\phi) d \phi \tag{6.4}
\end{equation*}
$$

where $\phi=\omega_{0} t, \overline{i_{n, i}^{2}(\phi)}$ is the white current noise power density of the $i$ th noise source and $\Gamma_{i}$ is its relevant ISF function from the corresponding $i$ th device noise [25]. Finally, $R_{i n}$ is an equivalent differential input parallel resistance of the tank's losses. The oscillator $I_{D C}$ may be estimated by one of the following equations:
$I_{D C}=\frac{I_{\omega_{0}}}{\alpha_{I}} \xrightarrow{I_{\omega_{0}}=\frac{V_{o s c}}{R_{i n}}} I_{D C}=\frac{V_{o s c}}{R_{i n}} \cdot \frac{1}{\alpha_{I}} \xrightarrow{V_{o s c}=2 \alpha_{V} V_{D D}} I_{D C}=\frac{2 V_{D D}}{R_{i n}} \cdot \frac{\alpha_{V}}{\alpha_{I}}$.

As a result, the RF oscillator's $P_{D C}$ is derived by

$$
\begin{equation*}
P_{D C}=\frac{V_{D D}^{2}}{R_{i n}} \cdot \frac{\alpha_{V}}{\alpha_{I}} \tag{6.6}
\end{equation*}
$$

Equation (6.6) indicates that the minimum achievable $P_{D C}$ can be expressed in terms of a set of optimization parameters, such as $R_{i n}$, and
a set of topology-dependent parameters, such as minimum supply voltage ( $V_{D D, \text { min }}$ ), current and voltage efficiencies.

Lower $P_{D C}$ is typically achieved by scaling up $R_{i n}=L_{p} \omega_{0} Q_{t}$ simply via a large multi-turn inductor, as in [26]. For example, while maintaining a constant $Q_{t}$, doubling $L_{P}$ would theoretically double $R_{i n}$, which would reduce $P_{D C}$ by half but at a cost of a $3-\mathrm{dB}$ PN degradation. However, at some point, that trade-off stops due to a dramatic drop in the inductor's selfresonant frequency and Q-factor. Figure 6.3(a) shows the simulated Q-factor of several multi-turn inductors in $28-\mathrm{nm}$ CMOS versus their inductance values. As the inductor enlarges, the magnetic and capacitive coupling to the low-resistivity substrate increases such that the tank Q-factor drops almost linearly with $L_{P}$. As can be gathered from Figure 6.3(b), this constraint sets an upper limit on maximum $R_{i n}=L_{p} \omega_{0} Q_{t}$, which is chiefly a function of the technology node. Note that the inductor's value is largely dependent on its physical dimensions, rather than on the technology. However, the tank Q-factor is a bit degraded in the most recent process nodes (i.e., 28 nm ) mainly due to more stringent minimum metal density rules, closer separation between the top-metal and substrate, as well as thinner lower-level metals that are used in metal-oxide-metal (MoM) capacitors. As a consequence, it is expected that $R_{\text {in( } \max )}$ slightly reduces by migrating to finer CMOS technologies.

Parasitic capacitance of inductor windings, gm-devices, switchable capacitors, and oscillator routings determines a minimum floor of the tank's capacitance, which appears to be $\sim 250 \mathrm{fF}$ at $f_{0}=4.8 \mathrm{GHz}$. It puts another restriction on $L_{p}$ and $R_{\text {in( } \max )}$ to $\sim 4.5 \mathrm{nH}$ and $\sim 1.3 \mathrm{k} \Omega$ and sets a lower limit on $P_{D C}$ of each oscillator structure. Under this condition, the tank's Q-factor drops to $\leq 9$. This explains the poor FoM of RF oscillators in modern BLE transceivers.


Figure 6.3 Dependency of various inductor parameters in $28-\mathrm{nm}$ LP CMOS across inductance value: (a) inductor and tank Q -factor; (b) equivalent differential input resistance of the tank; and (c) required tank capacitance at $4.8-\mathrm{GHz}$ resonance. Note that at this point the inductors are without dummy metal fills.

The topology-dependent parameters also play an important role in trying to reduce $P_{D C}$. Equation (6.6) favors structures that offer higher $\alpha_{I}$ or can sustain oscillation with smaller $V_{D D}$ and $\alpha_{V}$. On the other hand, $\alpha_{V} \cdot \alpha_{I}$ should be maximized to avoid any penalty on FoM [22, 27], as evident from (6.2). Consequently, to efficiently reduce $P_{D C}$ without disproportionately worsening the FoM, it is desired to employ structures with a higher $\alpha_{I}$ and a lower minimum $V_{D D}$. To get a better insight, Figure 6.4 shows such effects for the traditional cross-coupled NMOS-only $\left(\mathrm{OSC}_{\mathrm{N}}\right)$ and complementary push-pull $\left(\mathrm{OSC}_{\mathrm{NP}}\right)$ structures [28, 29]. Due to the less stacking of transistors, the $V_{D D, \min }$ of $\mathrm{OSC}_{\mathrm{N}}$ can go $40 \%$ lower than that of $\mathrm{OSC}_{\mathrm{NP}}$. However, $\alpha_{I}$ of $\mathrm{OSC}_{\mathrm{NP}}$ is doubled due to the switching of tank current direction every half period. Its oscillation swing, and thus $\alpha_{V}$, is also $50 \%$ smaller. Hence, $\mathrm{OSC}_{\mathrm{NP}}$ offers $\sim 3 \times$ lower $\alpha_{V} / \alpha_{I}$. However, both structures demonstrate similar $\alpha_{V} \cdot \alpha_{I}$ product [30]. Consequently, each of them has its own set of advantages and drawbacks such that the minimum achievable $P_{D C}$ and FoM is almost identical, as shown in Table 6.2. Note that applying a tail filtering technique to a class-B oscillator increases its $\alpha_{V}$ [22,31], which is in line with the FoM optimization but against the $P_{D C}$ reduction, as evident from (6.2) and (6.6). Furthermore, while maintaining the same $R_{\text {in }}$, a class$\mathrm{F}_{3}$ operation does not reduce $P_{D C}$ of traditional oscillators since its minimum $V_{D D}, \alpha_{V}$ and $\alpha_{I}$ are identical to $\mathrm{OSC}_{\mathrm{N}}$ (see Chapter 3).

A push-pull class-C oscillator appears as an excellent choice for ULP applications due to its largest $\alpha_{I}$ and smallest $\alpha_{V}$ [32], as per Table 6.2. However, it needs an additional complex biasing circuitry (e.g., an opamp)


Figure 6.4 $V_{D D, \text { min }}, \alpha_{I}$ and $\alpha_{V}$ parameters for: (a) cross-coupled NMOS and (b) complementary push-pull oscillators.

Table 6.2 Minimum $\mathrm{P}_{\mathrm{DC}}$ for different RF oscillator topologies

| Topology | $V_{D D, \text { min }}{ }^{\dagger}$ | $\alpha_{V^{\ddagger}}{ }^{\ddagger}$ | $\alpha_{I}{ }^{*}$ | $P_{D C \min }$ | $\alpha_{V} \cdot \alpha_{I}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{OSC}_{\mathrm{N}}$ | $V_{t}+V_{O D} \approx 1.5 V_{t}$ | 0.66 | $2 / \pi$ | $4.66 V_{t}^{2} / R_{i n}$ | 0.42 |
| $\mathrm{OSC}_{\mathrm{NP}}$ | $2 V_{t}+V_{O D} \approx 2.5 V_{t}$ | 0.4 | $4 / \pi$ | $3.92 V_{t}^{2} / R_{i n}$ | 0.51 |
| $\mathrm{OSC}_{\mathrm{NP}}$ with <br> tail filter | $2 V_{t}+V_{O D} \approx 2.5 V_{t}$ | 0.63 | $4 / \pi$ | $6.2 V_{t}^{2} / R_{i n}$ | 0.8 |
| Class-C |  |  |  |  |  |
| Class-D | $2 V_{t}+V_{O D} \approx 2.5 V_{t}$ | 0.25 | 2 | $0.15 \mathrm{~mW}+1.56 V_{t}^{2} / R_{i n}$ | 0.5 |
| Class-F 3 | $\approx V_{t}$ | 1.635 | 0.5 | $6.54 V_{t}^{2} / R_{i n}$ | 0.82 |
| This work | $V_{t}+V_{O D} \approx 1.5 V_{\mathrm{t}}$ | 0.66 | $2 / \pi$ | $4.7 V_{t}^{2} / R_{i n}$ | 0.42 |

[^1]to guarantee the proper oscillator start-up and to keep the transistors in saturation during the on-state. There are also strong mutual trade-offs between the biasing circuit's $P_{D C}$, oscillator's amplitude stability and PN, much intensified in ULP applications where the tank capacitance tends to be smaller [33]. As a consequence, the biasing circuitry can end up consuming comparable power as the ULP oscillator itself. On the other hand, $V_{D D}$ of class-D oscillators can go below a threshold voltage, $V_{t}$. However, due to hard switching of core transistors, its $\alpha_{V}$ and $\alpha_{I}$ are, respectively, higher and lower than other structures [34], as shown in Table 6.2. According to (6.6), this trend is against the $P_{D C}$ reduction. Consequently, the current oscillator structures have issues with reaching simultaneous ultra-low power and voltage operation.

In this chapter, we disclose how to convert the fixed current source of the traditional NMOS topology into a structure with alternating current sources such that the tank current direction can change every half period. Consequently, the benefits of low supply of the $\mathrm{OSC}_{\mathrm{N}}$ topology and higher $\alpha_{I}$ of $\mathrm{OSC}_{\mathrm{NP}}$ structure are combined to reduce power consumption further than practically possible in the traditional oscillators.

### 6.3 Switching Current-Source Oscillator

Figure 6.5 shows an evolution towards the switching current-source oscillator. The $\mathrm{OSC}_{\mathrm{N}}$ topology is chosen as a starting point due to its low $V_{D D}$ capability. To reduce $P_{D C}$ further, it is desired to switch the direction of the

(a)

(b)

(c)

(d)

Figure 6.5 Evolution towards the switching current-source oscillator.

LC-tank current in each half period, which will double $\alpha_{I}$. Consequently, we beneficially split the fixed current source $\mathrm{M}_{1}$ in Figure 6.5(a) into two switchable current sources, $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$, as suggested in Figure 6.5(b). This allows for the tank to be disconnected from the $\mathrm{V}_{D D}$ feed and be moved inbetween the upper and lower NMOS transistor pairs to give rise to an H -bridge configuration. In the next step, the passive voltage gain blocks, $A_{0}$, are added to the NMOS gates, as shown in Figure 6.5(c). Both upper and lower NMOS pairs should each independently demonstrate synchronized positive feedback to realize the switching of the tank current direction. The "master" positive feedback enforces the differential-mode operation and is realized by the lower-pair transistors configured in a conventional crosscoupled manner. Its negative conductance seen by the tank may be estimated by (see Section 6.4 for detailed calculations)

$$
\begin{equation*}
G_{\text {down }}=\frac{-A_{0}}{4} \cdot\left(g_{m 1}(\phi)+g_{m 2}(\phi)\right) \tag{6.7}
\end{equation*}
$$

On the other upper side, the differential-mode oscillation of the tank is reinforced by the $\mathrm{M}_{3,4}$ devices which realize the second positive feedback. ${ }^{2}$ The negative conductance seen by the tank into the upper pair can be calculated by (see Section 6.4 for detailed calculations)

$$
\begin{equation*}
G_{u p}=G_{\text {down }}=\frac{-\left(A_{0}-1\right)}{4} \cdot\left(g_{m 3}(\phi)+g_{m 4}(\phi)\right) . \tag{6.8}
\end{equation*}
$$

Equation (6.8) clearly indicates that the voltage gain block is necessary and $A_{0}$ must be safely larger than 1 to be able to present a negative conductance to the tank, thus enabling the H -bridge switching. By merging

[^2]the redundant voltage gain blocks, the disclosed switching current-source oscillator is shown in Figure 6.5(d). Note that the tank with an implicit voltage gain can be realized by using a capacitive divider, autotransformer, or step-up transformer, as illustrated in Figure 6.6. The transformer-based tank is chosen in this work due to its simplicity.

Figures 6.7 and 6.8 illustrate the novel oscillator schematic as well as waveforms and various operational regions of $\mathrm{M}_{1-4}$ transistors across the oscillation period. The two-port resonator consists of a step-up 1:2 transformer and tuning capacitors, $C_{1}, C_{2}$, at its primary and secondary windings.


Figure 6.6 Various options of a tank providing voltage gain.


Figure 6.7 Schematic of the switching current-source oscillator.


Figure 6.8 Waveforms and various operational regions of $\mathrm{M}_{1-4}$ transistors across the oscillation period.

The transistors $\mathrm{M}_{1,2}$ set the oscillator's DC current, while the $\mathrm{M}_{3,4}$ pair acts as a switching current source. Both $\mathbf{M}_{1,2}$ and $\mathbf{M}_{3-4}$ pairs play an equally vital role of switching the tank current direction. As can be gathered from Figure $6.8, \mathrm{G}_{\mathrm{B}}$ oscillation voltage is high within the first half period. Hence, only $\mathrm{M}_{2}$ and $\mathrm{M}_{3}$ transistors are on and the current flows from left to right side of the tank. However, $\mathrm{M}_{1}$ and $\mathrm{M}_{4}$ are turned on for the second half period and the tank's current direction is reversed. Consequently, just like in the push-pull structure, the tank current flow is reversed every half period, thus doubling the oscillator's $\alpha_{I}$ to $4 / \pi$.

The $V_{D D}$ of the new oscillator can be as low as $V_{O D 1}+V_{O D 3} \approx V_{t}$, which is extremely small for an oscillator with a capability of switching the tank current direction. This makes it suitable for a direct connection to solar cells. Note that the oscillation swing cannot go further than $V_{O D 1,2}$ at DA/DB nodes, which is chosen $\sim 150 \mathrm{mV}$ to satisfy the system's phase noise requirement by a few dB margin. However, the maximum required voltage of the circuit is determined by the bias voltage $V_{B}$.

$$
\begin{equation*}
V_{B} \approx V_{O D 1}+V_{g s 3} \tag{6.9}
\end{equation*}
$$

Equation (6.9) implies that $\mathrm{M}_{3,4}$ should work in weak inversion keeping $V_{g s 3}<V_{t}$ to achieve lower $V_{D D, \min }$. However, the transistor's cut-off frequency $f_{\max }$ drops dramatically in the subthreshold operation. Note that $f_{\max }$ should be at least $4 \times$ higher than the operating frequency $f_{0}=4.8 \mathrm{GHz}$


Figure $6.9 f_{\text {max }}$ of low- $V_{t} 28 \mathrm{~nm}$ transistor versus $V_{D S}$ for different $V_{G S}$.
to guarantee the oscillator start-up over PVT variations. This constraint limits $V_{g s 3} \approx 0.32 \mathrm{~V}$ for $V_{O D 3} \approx 150 \mathrm{mV}$, as can be gathered from Figure 6.9 . Consequently, even by considering only the tougher $V_{B}$ requirement, the new structure can operate at $V_{D D}$ as low as 0.5 V , on par with $\mathrm{OSC}_{\mathrm{N}}$.

Such a low $V_{D D}$ and oscillation swing can easily lead to start-up problems in the traditional structures. It will certainly increase power consumption, $P_{b u f}$, of the following buffer, which would require more gain in order to provide a rail-to-rail swing at its output that is interpreted as a local oscillator (LO) clock. Fortunately, the transformer gain enhances the oscillation swing at $\mathrm{M}_{1,2}$ gates to even beyond $V_{D D}$, guaranteeing the oscillator start-up and reduction of $P_{b u f}$. Consequently, the oscillator buffer is connected to the secondary winding of the transformer in this design.

As can be gathered from Figure 6.8 , the $\mathrm{M}_{3,4}$ switching current-source transistors operate in a class-C manner as in a Colpitts oscillator, meaning that they deliver more or less narrow-and-tall current pulses. However, their non-zero conduction angle is quite wide, $\sim \pi$, due to the low overdrive voltage in the subthreshold operation. On the other hand, $\mathrm{M}_{1,2}$ operate in a class-B manner like cross-coupled oscillators, meaning that they deliver square-shaped current pulses. Hence, the shapes of drain currents are quite different for the lower and upper pairs. However, their fundamental components demonstrate the same amplitude ( $\alpha_{I} \approx 2 / \pi$ ) and phase to realize the constructive oscillation voltage across the tank. The higher drain harmonics obviously show different characteristics. However, they are filtered out by the tank's selectivity characteristic. Note that the current through a transistor of the upper pair will have two paths to ground: through the corresponding transistor of the lower pair and through the single-ended capacitors. Consequently, the single-ended capacitors sink the higher current harmonics of $\mathrm{M}_{3,4}$ transistors.

### 6.4 Thermal Noise Upconversion

To calculate a closed-form PN equation, the oscillator model is simplified in Figure 6.10. At the resonant frequency, the transformer-based tank can be modeled by an equivalent LC tank of elements $L_{e q}, C_{e q}$, and $R_{i n} .{ }^{3}$ On the other hand, $\mathrm{M}_{1-4}$ transistors with passive voltage gain of the transformer are decomposed into two nonlinear time-variant conductances. Note that the active elements in the circuit may add to the resonator loss, particularly at the extremes of large oscillation waveforms which may push transistors into their triode regions. Consequently, the nonlinearity is decomposed into two nonlinear resistances: one that is always positive, $G_{d s}(\phi)$, and one that is always negative, $G_{n}(\phi)$, where $\phi=\omega_{0} t$. Further, to get a better insight, the effects of noise on the oscillator phase noise due to channel conductance $\left(\overline{i_{n, G d s}^{2}(\phi)}=4 K T G_{d s}(\phi)\right)$ and transconductance gain $\left(\overline{i_{n, G m}^{2}(\phi)}\right)$ of $\mathrm{M}_{1-4}$ transistors are separately modeled in Figure 6.10. All circuit variables in this generic model will be obtained in the following sections.

### 6.4.1 Calculating the Effective Noise Due to Transconductance Gain of $\mathbf{M}_{1-4}$ Transistors ( $\overline{\boldsymbol{i}_{n, G m}^{2}(\phi)}$ )

It is clear that the lower pair is a voltage-biased circuit. Consequently, the noise sources of $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ are uncorrelated for the entire oscillation period. However, the situation is more complicated for the upper pair. For a short time around zero-crossings, both transistors of the upper pair work in the subthreshold region, while elsewhere one of them is off and the other device will be driven into saturation. In this situation, current through the upper NMOS transistor will have no path to ground other than through the corresponding


Figure 6.10 Generic noise circuit model of the disclosed oscillator.

[^3]NMOS transistor of the lower pair if there were no single-ended capacitance at the tank. This phenomenon creates a common-mode oscillation across the tank, which ensures that the drain currents of both lower and upper NMOS transistor are the same.

However, if the tank includes some single-ended capacitors connected to ground, the oscillator will behave very differently (we also use this single-ended capacitors to create a common-mode resonant frequency at the second harmonic of the fundamental frequency to reduce $1 / \mathrm{f}$ noise upconversion). Note that one cannot avoid the presence of single-ended capacitors in the tank due to drain-bulk and drain-source parasitic capacitance of lower-pair transistors, source-bulk and drain-source parasitic capacitance of upper pair transistors, and parasitic capacitance of the transformer's primary winding. In this situation, the current through transistors of upper pair will have two paths to the ground: through the corresponding NMOS transistor of the lower pair and through the single-ended capacitors. Consequently, single-ended capacitors suppress the common-mode oscillation voltage across the tank. In this instance, the upper pair is more appropriately viewed as a voltage-biased circuit. Consequently, noise sources due to transconductance gain of $\mathrm{M}_{1-4}$ transistors are absolutely uncorrelated.

By using the same approach as [36], we are going to replace all noise sources with an equivalent noise source across the tank. By writing KCL at DA and DB nodes, it is straightforward to show that the instantaneous equivalent current can be calculated by (see Figure 6.11)

$$
\left.\begin{array}{r}
I_{e q}=I_{M 1}-I_{M 3} \\
I_{e q}=I_{M 4}-I_{M 2}, \tag{6.10}
\end{array}\right\} \rightarrow I_{e q}=\frac{1}{2}\left(\left(I_{M 1}+I_{M 4}\right)-\left(I_{M 1}+I_{M 4}\right)\right) \rightarrow, ~(6) ~ I_{e q}=\left(I_{M 1}-I_{M 2}\right)+\left(I_{M 3}-I_{M 4}\right) .
$$

As a consequence, the resulting differential noise current through the tank is

$$
\begin{align*}
& \overline{i_{n, G m}^{2}(\phi)}=\frac{1}{4}\left(\overline{i_{n, g m 1}^{2}(\phi)}+\overline{i_{n, g m 2}^{2}(\phi)}+\overline{i_{n, g m 3}^{2}(\phi)}+\overline{i_{n, g m 4}^{2}(\phi)}\right) \rightarrow \\
& \overline{i_{n, G m}^{2}(\phi)}=K T\left(\gamma_{1}\left(g_{m 1}(\phi)+g_{m 2}(\phi)\right)+\gamma_{3}\left(g_{m 3}(\phi)+g_{m 4}(\phi)\right)\right) \tag{6.11}
\end{align*}
$$



Figure 6.11 Simplified schematic of the switching current-source oscillator.


Figure 6.12 Simplified schematic of the lower pair of the oscillator.

### 6.4.2 Calculating the Negative Conductance of the Oscillator ( $G_{n}(\phi)$ )

The negative conductance of lower and upper pairs will be calculated separately in the following sections. The upper and lower negative conductances are in parallel. Hence, the total negative conductance is calculated by adding the negative conductance of lower and upper pairs.

The gate-source voltage of $\mathrm{M}_{1}$ is calculated by (see Figure 6.12)

$$
\begin{equation*}
V_{G S 1}(\phi)=V_{B}-\frac{A_{0} V_{\text {out }}(\phi)}{2} . \tag{6.12}
\end{equation*}
$$

As a result, the derivative of gate-source voltage of $\mathrm{M}_{1}$ is calculated by

$$
\begin{equation*}
\frac{d V_{G S 1}(\phi)}{d \phi}=-\frac{A_{0}}{2} \cdot \frac{d V_{o u t}(\phi)}{d \phi} \tag{6.13}
\end{equation*}
$$

The transconductance gain of $\mathrm{M}_{1}$ transistor may be estimated by

$$
\begin{equation*}
g_{m 1}(\phi)=\frac{d I_{1}(\phi)}{d V_{G S 1}}=\frac{d I_{1}(\phi) / d \phi}{d V_{G S 1} / d \phi}=\frac{d I_{1}(\phi) / d \phi}{\frac{-A_{0}}{2} d V_{\text {out }} / d \phi}=-\frac{2}{A_{0}} \cdot \frac{d I_{1}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.14}
\end{equation*}
$$

We can rewrite Equation (6.14) as

$$
\begin{equation*}
\frac{d I_{1}(\phi)}{d V_{\text {out }}(\phi)}=-\frac{2}{A_{0}} \cdot g_{m 1}(\phi) \tag{6.15}
\end{equation*}
$$

On the other hand, the gate-source voltage of $\mathrm{M}_{2}$ is calculated by

$$
\begin{equation*}
g_{m 2}(\phi)=\frac{d I_{2}(\phi)}{d V_{G S 2}}=\frac{d I_{2}(\phi) / d \phi}{d V_{G S 2} / d \phi}=\frac{d I_{2}(\phi) / d \phi}{\frac{A_{0}}{2} d V_{\text {out }} / d \phi}=\frac{2}{A_{0}} \cdot \frac{d I_{2}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.16}
\end{equation*}
$$

And again, we can rewrite Equation (6.16) as

$$
\begin{equation*}
\frac{d I_{2}(\phi)}{d V_{\text {out }}(\phi)}=\frac{2}{A_{0}} \cdot g_{m 2}(\phi) \tag{6.17}
\end{equation*}
$$

The effective negative conductance of lower pair,

$$
\begin{equation*}
G_{n d}(\phi)=\frac{d I_{\text {eq } 2}(\phi)}{d V_{\text {out }}(\phi)}=\frac{1}{2} \cdot \frac{d I_{1}(\phi)-d I_{2}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.18}
\end{equation*}
$$

By using (6.15) and (6.17), the above equation can be rewritten by

$$
\begin{equation*}
G_{n d}(\phi)=-\frac{1}{4} \cdot A_{0} \cdot\left(g_{m 1}(\phi)+g_{m 2}(\phi)\right) \tag{6.19}
\end{equation*}
$$

The same calculations can be done for $\mathrm{M}_{3}$ and $\mathrm{M}_{4}$.
The gate-source voltage of $\mathrm{M}_{3}$ is calculated by (see Figure 6.13)

$$
\begin{equation*}
V_{G S 3}(\phi)=V_{B}-V_{0}+0.5\left(A_{0}-1\right) V_{\text {out }}(\phi) \tag{6.20}
\end{equation*}
$$

As a result, the derivative of gate-source voltage of $\mathrm{M}_{3}$ is calculated by

$$
\begin{equation*}
\frac{d V_{G S 3}(\phi)}{d \phi}=0.5\left(A_{0}-1\right) \cdot \frac{d V_{o u t}(\phi)}{d \phi} \tag{6.21}
\end{equation*}
$$



Figure 6.13 Simplified schematic of the upper pair of the oscillator.

The transconductance gain of $\mathbf{M}_{3}$ transistor may be estimated by

$$
\begin{align*}
g_{m 3}(\phi) & =\frac{d I_{3}(\phi)}{d V_{G S 3}}=\frac{d I_{3}(\phi) / d \phi}{d V_{G S 3} / d \phi}=\frac{d I_{3}(\phi) / d \phi}{0.5\left(A_{0}-1\right) \frac{d V_{\text {out }}}{d \phi}} \\
& =\frac{2}{\left(A_{0}-1\right)} \cdot \frac{d I_{3}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.22}
\end{align*}
$$

We can rewrite the above equation by

$$
\begin{equation*}
\frac{d I_{3}(\phi)}{d V_{\text {out }}(\phi)}=\frac{A_{0}-1}{2} \cdot g_{m 3}(\phi) \tag{6.23}
\end{equation*}
$$

On the other hand, the gate-source voltage of $\mathrm{M}_{4}$ is calculated by

$$
\begin{equation*}
V_{G S 4}(\phi)=V_{B}-V_{0}-\frac{\left(A_{0}-1\right)}{2} V_{o u t}(\phi) \tag{6.24}
\end{equation*}
$$

As a result, the derivative of gate-source voltage of $\mathrm{M}_{4}$ is calculated by

$$
\begin{equation*}
\frac{d V_{G S 4}(\phi)}{d \phi}=\frac{\left(A_{0}-1\right)}{2} \cdot \frac{d V_{\text {out }}(\phi)}{d \phi} \tag{6.25}
\end{equation*}
$$

The transconductance gain of $\mathrm{M}_{4}$ transistor then is estimated by

$$
\begin{align*}
g_{m 4}(\phi) & =\frac{d I_{4}(\phi)}{d V_{G S 4}}=\frac{d I_{4}(\phi) / d \phi}{d V_{G S 4} / d \phi}=-\frac{d I_{4}(\phi) / d \phi}{0.5\left(A_{0}-1\right) \frac{d V_{\text {out }}}{d \phi}} \\
& =-\frac{2}{\left(A_{0}-1\right)} \cdot \frac{d I_{4}(\phi)}{d V_{\text {out }}(\phi)} . \tag{6.26}
\end{align*}
$$

We can rewrite the above equation by

$$
\begin{equation*}
\frac{d I_{4}(\phi)}{d V_{o u t}(\phi)}=-\frac{A_{0}-1}{2} \cdot g_{m 4}(\phi) \tag{6.27}
\end{equation*}
$$

The negative conductance of upper pair

$$
\begin{equation*}
G_{n u}(\phi)=\frac{d I_{\text {equ }}(\phi)}{d V_{\text {out }}(\phi)}=\frac{1}{2} \cdot \frac{d I_{4}(\phi)-d I_{3}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.28}
\end{equation*}
$$

By using (6.23) and (6.27), the above equation can be rewritten by

$$
\begin{equation*}
G_{n u}(\phi)=-\frac{1}{4} \cdot\left(A_{0}-1\right) \cdot\left(g_{m 3}(\phi)+g_{m 4}(\phi)\right) \tag{6.29}
\end{equation*}
$$

The upper and lower negative conductance are in parallel. Hence, the total negative conductance is calculated by

$$
\begin{align*}
G_{n}(\phi) & =G_{n d}(\phi)+G_{n u}(\phi) \\
& =\frac{1}{4} \cdot\left[A_{0} \cdot\left(g_{m 1}(\phi)+g_{m 2}(\phi)\right)+\left(A_{0}-1\right) \cdot\left(g_{m 3}(\phi)+g_{m 4}(\phi)\right)\right] \tag{6.30}
\end{align*}
$$

### 6.4.3 Calculating the Positive Conductance of the Oscillator ( $\left.G_{D S}(\phi)\right)$

The positive conductance of lower and upper pairs will be calculated separately in the following sections. The upper and lower positive conductance are in parallel. Hence, the total positive conductance is calculated by adding the positive conductance of lower and upper pairs.

The drain-source voltage of $\mathrm{M}_{1}$ is calculated by (see Figure 6.12)

$$
\begin{equation*}
V_{d s 1}(\phi)=V_{0}+\frac{V_{o u t}(\phi)}{2} \tag{6.31}
\end{equation*}
$$

As a result, the derivative of drain-source voltage of $\mathrm{M}_{1}$ is calculated by

$$
\begin{equation*}
\frac{d V_{d s 1}(\phi)}{d \phi}=\frac{1}{2} \cdot \frac{d V_{\text {out }}(\phi)}{d \phi} \tag{6.32}
\end{equation*}
$$

The drain-source conductance of $\mathrm{M}_{1}$ transistor may be estimated by

$$
\begin{equation*}
g_{d s 1}(\phi)=\frac{d I_{1}(\phi)}{d V_{d s 1}}=\frac{d I_{1}(\phi) / d \phi}{d V_{d s 1} / d \phi}=-\frac{d I_{1}(\phi) / d \phi}{0.5 \frac{d V_{\text {out }}}{d \phi}}=2 \cdot \frac{d I_{1}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.33}
\end{equation*}
$$

We can rewrite the above equation by

$$
\begin{equation*}
\frac{d I_{1}(\phi)}{d V_{\text {out }}(\phi)}=\frac{1}{2} \cdot g_{d s 1}(\phi) . \tag{6.34}
\end{equation*}
$$

On the other hand, the drain-source voltage of $\mathrm{M}_{2}$ is calculated by

$$
\begin{equation*}
V_{d s 2}(\phi)=V_{0}-\frac{V_{\text {out }}(\phi)}{2} . \tag{6.35}
\end{equation*}
$$

As a result, the derivative of drain-source voltage of $\mathrm{M}_{2}$ is calculated by

$$
\begin{equation*}
\frac{d V_{d s 2}(\phi)}{d \phi}=-\frac{1}{2} \cdot \frac{d V_{\text {out }}(\phi)}{d \phi} . \tag{6.36}
\end{equation*}
$$

The drain-source conductance gain of $\mathrm{M}_{2}$ transistor may be estimated by

$$
\begin{equation*}
g_{d s 2}(\phi)=\frac{d I_{2}(\phi)}{d V_{d s 2}}=\frac{d I_{2}(\phi) / d \phi}{d V_{d s 2} / d \phi}=-\frac{d I_{2}(\phi) / d \phi}{-0.5 \frac{d V_{\text {out }}}{d \phi}}=-2 \cdot \frac{d I_{2}(\phi)}{d V_{\text {out }}(\phi)} . \tag{6.37}
\end{equation*}
$$

We can rewrite the above equation as

$$
\begin{equation*}
\frac{d I_{2}(\phi)}{d V_{\text {out }}(\phi)}=-\frac{1}{2} \cdot g_{d s 2}(\phi) \tag{6.38}
\end{equation*}
$$

The positive conductance of lower pair then will be

$$
\begin{equation*}
G_{d s-\text { down }}(\phi)=\frac{1}{2} \cdot \frac{d I_{1}(\phi)-d I_{2}(\phi)}{d V_{\text {out }}(\phi)} . \tag{6.39}
\end{equation*}
$$

Using (6.34) and (6.38), we can conclude:

$$
\begin{equation*}
G_{d s-d o w n}(\phi)=\frac{1}{4}\left(g_{d s 1}(\phi)+g_{d s 2}(\phi)\right) . \tag{6.40}
\end{equation*}
$$

The drain-source voltage of $\mathrm{M}_{3}$ is calculated by (see Figure 6.13)

$$
\begin{equation*}
V_{d s 3}(\phi)=V_{D D}-V_{0}-\frac{V_{\text {out }}(\phi)}{2} . \tag{6.41}
\end{equation*}
$$

As a result,

$$
\begin{equation*}
\frac{d V_{d s 3}(\phi)}{d \phi}=-\frac{1}{2} \cdot \frac{d V_{\text {out }}(\phi)}{d \phi} . \tag{6.42}
\end{equation*}
$$

The drain-source conductance gain of $\mathrm{M}_{3}$ transistor then is estimated by

$$
\begin{equation*}
g_{d s 3}(\phi)=\frac{d I_{3}(\phi)}{d V_{d s 3}}=\frac{d I_{3}(\phi) / d \phi}{d V_{d s 3} / d \phi}=\frac{d I_{3}(\phi) / d \phi}{-0.5 \frac{d V_{\text {out }}}{d \phi}}=-2 \cdot \frac{d I_{3}(\phi)}{d V_{o u t}(\phi)} \tag{6.43}
\end{equation*}
$$

We can rewrite the above equation as

$$
\begin{equation*}
\frac{d I_{3}(\phi)}{d V_{\text {out }}(\phi)}=-\frac{1}{2} \cdot g_{d s 3}(\phi) \tag{6.44}
\end{equation*}
$$

On the other hand, the drain-source voltage of $\mathrm{M}_{4}$ is calculated by

$$
\begin{equation*}
V_{d s 4}(\phi)=V_{D D}-V_{0}+\frac{V_{o u t}(\phi)}{2} \tag{6.45}
\end{equation*}
$$

As a result,

$$
\begin{equation*}
\frac{d V_{d s 4}(\phi)}{d \phi}=+\frac{1}{2} \cdot \frac{d V_{\text {out }}(\phi)}{d \phi} \tag{6.46}
\end{equation*}
$$

and

$$
\begin{equation*}
g_{d s 4}(\phi)=\frac{d I_{4}(\phi)}{d V_{d s 4}}=\frac{d I_{4}(\phi) / d \phi}{d V_{d s 43} / d \phi}=\frac{d I_{4}(\phi) / d \phi}{0.5 \frac{d V_{\text {out }}}{d \phi}}=2 \cdot \frac{d I_{4}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.47}
\end{equation*}
$$

We can rewrite the above equation as

$$
\begin{equation*}
\frac{d I_{4}(\phi)}{d V_{\text {out }}(\phi)}=\frac{1}{2} \cdot g_{d s 4}(\phi) \tag{6.48}
\end{equation*}
$$

The positive conductance of upper pair then will be

$$
\begin{equation*}
G_{d s-u p}(\phi)=\frac{1}{2} \cdot \frac{d I_{4}(\phi)-d I_{3}(\phi)}{d V_{\text {out }}(\phi)} \tag{6.49}
\end{equation*}
$$

Using (6.44) and (6.48), we can conclude

$$
\begin{equation*}
G_{d s-u p}(\phi)=\frac{1}{4}\left(g_{d s 3}(\phi)+g_{d s 4}(\phi)\right) \tag{6.50}
\end{equation*}
$$

The upper and lower positive conductance are in parallel. Hence, the total negative conductance is calculated by

$$
\begin{equation*}
G_{d s}(\phi)=G_{d s-u p}(\phi)+G_{d s-d o w n}=\frac{1}{4}\left[g_{d s 1}+g_{d s 2}+g_{d s 3}(\phi)+g_{d s 4}(\phi)\right] \tag{6.51}
\end{equation*}
$$

### 6.4.4 Satisfying Barkhausen Criterion

To sustain oscillation, the average power dissipated in the tank ( $\mathrm{R}_{i n}$ ) and positive conductance of active devices $\left(\mathrm{G}_{d s}(\phi)\right)$ must equal the average power delivered by the negative conductance of nonlinearity $\left(\mathrm{G}_{n}(\phi)\right)$ [36]. Hence,

$$
\begin{equation*}
P_{R_{i n}}+P_{G_{d s}}=-P_{G_{n}} \tag{6.52}
\end{equation*}
$$

Assuming $V_{\text {out }}=A_{c} \cos \left(\omega_{0} t\right)$, the average power dissipated in the tank can be calculated by

$$
\begin{equation*}
P_{R_{i n}}=\frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{\left(A_{c} \cos \left(\omega_{0} t\right)\right)^{2}}{R_{p}} d t=\frac{A_{c}^{2}}{2 R_{p}} \tag{6.53}
\end{equation*}
$$

The current drawn by the positive conductance of nonlinearity can be described as

$$
\begin{align*}
I_{G_{d s}}(t) & =I_{G_{d s-D C}}+\int_{-\infty}^{t} d\left(I_{G_{d s}}(\tau)\right) d \tau \\
& =I_{G_{d s-D C}}+\int_{-\infty}^{t} G_{d s}(\tau) d\left(V_{\text {out }}(\tau)\right) d \tau \\
& \left.=I_{G_{d s-D C}}-A_{c} \omega_{0} \int_{-\infty}^{t} G_{d s}(\tau) \cdot \sin \left(\omega_{0} \tau\right)\right) d \tau \tag{6.54}
\end{align*}
$$

and the average power dissipated by the positive conductance of the nonlinearity is

$$
\begin{align*}
P_{G_{d s}}(t)= & \frac{1}{T} \int_{-T / 2}^{T / 2} V_{\text {out }}(t) \cdot I_{G_{D S}}(t) d t \\
= & \frac{1}{T} \int_{-T / 2}^{T / 2} A_{c} \cos \left(\omega_{0} t\right) \cdot\left[I_{G_{D S-D C}}-A_{c} \omega_{0}\right. \\
& \left.\int_{-\infty}^{t} G_{d s}(\tau) \cdot \sin \left(\omega_{0} \tau\right) \cdot d \tau\right] \cdot d t \\
= & \frac{1}{T} \int_{-T / 2}^{T / 2} A_{c} \cos \left(\omega_{0} t\right) \cdot I_{G_{D S-D C}} \cdot d t \\
= & -\frac{A_{c}^{2} \omega_{0}}{T} \int_{-T / 2}^{T / 2} \cos \left(\omega_{0} t\right)\left[\int_{-\infty}^{t} G_{d s}(\tau) \cdot \sin \left(\omega_{0} \tau\right) d \tau\right] \cdot d t \tag{6.55}
\end{align*}
$$

If we switch the order of the integrals, we may write

$$
\begin{align*}
P_{G_{d s}}(t) & =-\frac{A_{c}^{2} \omega_{0}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{\tau}^{\frac{T}{2}} G_{d s}(\tau) \cdot \sin \left(\omega_{0} \tau\right) \cdot \cos \left(\omega_{0} t\right) \cdot d t \cdot d \tau \\
& =-\left.\frac{A_{c}^{2}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} G_{d s}(\tau) \cdot \sin \left(\omega_{0} \tau\right) \cdot d \tau \cdot \sin \left(\omega_{0} t\right)\right|_{\tau} ^{\frac{T}{2}} \\
& =+\frac{A_{c}^{2}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} G_{d s}(\tau) \cdot\left(\sin \left(\omega_{0} \tau\right)\right)^{2} \cdot d \tau \\
& =+\frac{A_{c}^{2}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} G_{d s}(\tau) \cdot\left(1-\cos \left(2 \omega_{0} \tau\right)\right) \cdot d \tau \tag{6.56}
\end{align*}
$$

We know

$$
\begin{equation*}
G_{d s}[0]=\frac{1}{T} \int_{-T / 2}^{T / 2} G_{D S}(\tau) \cdot d \tau=\frac{1}{2 \pi} \int_{-\pi}^{\pi} G_{D S}(\phi) \cdot d \phi \tag{6.57}
\end{equation*}
$$

and

$$
\begin{align*}
G_{d s}[2] & =\frac{1}{T} \int_{-T / 2}^{T / 2} G_{D S}(\tau) \cdot \cos \left(2 \omega_{0} \tau\right) \cdot d \tau \\
& =\frac{1}{2 \pi} \int_{-\pi}^{\pi} G_{D S}(\phi) \cdot \cos (2 \phi) \cdot d \phi \tag{6.58}
\end{align*}
$$

where $\mathrm{G}_{d s}[k]$ describes the Fourier series coefficients of the instantaneous positive conductance of nonlinearity $\mathrm{G}_{d s}(t)$. By replacing (6.57) and (6.58) in (6.56)

$$
\begin{equation*}
P_{G_{d s}}=\frac{A_{c}^{2}}{2} \cdot\left(G_{D S}[0]-G_{D S}[2]\right) \tag{6.59}
\end{equation*}
$$

We also define,

$$
\begin{equation*}
G_{D S E F}=G_{D S}[0]-G_{D S}[2] \tag{6.60}
\end{equation*}
$$

By replacing (6.60) in (6.59),

$$
\begin{equation*}
P_{G_{d s}}=\frac{A_{c}^{2}}{2} \cdot\left(G_{D S E F}\right) \tag{6.61}
\end{equation*}
$$

Now let us calculate the average power delivered $\mathrm{G}_{n}(\phi)$. The current drawn by the $\mathrm{G}_{n}(\phi)$ can be described as

$$
\begin{align*}
I_{G_{n}}(t) & =I_{G_{n-D C}}+\int_{-\infty}^{t} d\left(I_{G_{n}}(\tau)\right) d \tau \\
& =I_{G_{n-D C}}+\int_{-\infty}^{t} G_{n}(\tau) d\left(V_{\text {out }}(\tau)\right) d \tau \\
& \left.=I_{G_{n-D C}}-A_{c} \omega_{0} \int_{-\infty}^{t} G_{n}(\tau) \cdot \sin \left(\omega_{0} \tau\right)\right) d \tau \tag{6.62}
\end{align*}
$$

and the average power dissipated by the positive conductance of the nonlinearity is

$$
\begin{align*}
P_{G_{n}}(t)= & \frac{1}{T} \int_{-T / 2}^{T / 2} V_{\text {out }}(t) \cdot I_{G_{n}}(t) d t \\
= & \frac{1}{T} \int_{-T / 2}^{T / 2} A_{c} \cos \left(\omega_{0} t\right) \cdot\left[I_{G_{D S-D C}}\right. \\
& \left.-A_{c} \omega_{0} \int_{-\infty}^{t} G_{n}(\tau) \cdot \sin \left(\omega_{0} \tau\right) \cdot d \tau\right] \cdot d t \\
= & \frac{1}{T} \int_{-T / 2}^{T / 2} A_{c} \cos \left(\omega_{0} t\right) \cdot I_{G_{n-D C}} \cdot-\frac{A_{c}^{2} \omega_{0}}{T} \int_{-T / 2}^{T / 2} \cos \left(\omega_{0} t\right) \\
& {\left[\int_{-\infty}^{t} G_{n}(\tau) \cdot \sin \left(\omega_{0} \tau\right) d \tau\right] \cdot d t . } \tag{6.63}
\end{align*}
$$

If we switch the order of the integrals, we may write

$$
\begin{align*}
P_{G_{n}}(t) & =-\frac{A_{c}^{2} \omega_{0}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \int_{\tau}^{\frac{T}{2}} G_{n}(\tau) \cdot \sin \left(\omega_{0} \tau\right) \cdot \cos \left(\omega_{0} t\right) \cdot d t \cdot d \tau \\
& =-\left.\frac{A_{c}^{2}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} G_{n}(\tau) \cdot \sin \left(\omega_{0} \tau\right) \cdot d \tau \cdot \sin \left(\omega_{0} t\right)\right|_{\tau} ^{\frac{T}{2}} \\
& =+\frac{A_{c}^{2}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} G_{n}(\tau) \cdot\left(\sin \left(\omega_{0} \tau\right)\right)^{2} \cdot d \tau \\
& =+\frac{A_{c}^{2}}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} G_{n}(\tau) \cdot\left(1-\cos \left(2 \omega_{0} \tau\right)\right) \cdot d \tau \tag{6.64}
\end{align*}
$$

Consequently,

$$
\begin{equation*}
P_{G_{n}}=\frac{A_{c}^{2}}{2} \cdot\left(G_{N S E F}\right) . \tag{6.65}
\end{equation*}
$$

To sustain oscillation, the average power dissipated in the tank $\left(\mathrm{R}_{\text {in }}\right)$ and positive conductance of active devices $\left(\mathrm{G}_{d s}\right)$ must equal the average power delivered by the negative conductance of nonlinearity $\left(\mathrm{G}_{n}\right)$. By replacing (6.53), (6.61) and (6.65) in (6.52),

$$
\begin{equation*}
\frac{A_{c}^{2}}{2 R_{i n}}+\frac{A_{c}^{2}}{2} \cdot\left(G_{D S E F}\right)=-\frac{A_{c}^{2}}{2} \cdot\left(G_{N E F}\right) \tag{6.66}
\end{equation*}
$$

Consequently,

$$
\begin{equation*}
G_{N E F}=-\frac{1+R_{i n} \cdot G_{D S E F}}{R_{i n}} \tag{6.67}
\end{equation*}
$$

On the other hand, the total effective negative conductance can be rewritten as sum of the effective negative conductance of lower and upper pairs

$$
\begin{equation*}
G_{N D E F}+G_{N U E F}=-\frac{1+R_{i n} \cdot G_{D S E F}}{R_{i n}} \tag{6.68}
\end{equation*}
$$

Note that both upper and lower NMOS pairs should each individually demonstrate synchronized positive feedback to realize the switching of the tank current direction. Consequently, as with traditional complimentary oscillator, each pair should roughly compensate half of the oscillator losses.

$$
\begin{equation*}
G_{N D E F}=G_{N U E F}=-\frac{1}{2} \cdot \frac{1+R_{i n} \cdot G_{D S E F}}{R_{i n}} \tag{6.69}
\end{equation*}
$$

On the other hand, $\mathrm{G}_{N D E F}$ and $\mathrm{G}_{N U E F}$ can be, respectively, calculated by

$$
\begin{equation*}
G_{N D E F}=-\frac{1}{4} \cdot A_{0} \cdot\left[G_{M 1 E F}+G_{M 2 E F}\right] \tag{6.70}
\end{equation*}
$$

and

$$
\begin{equation*}
G_{N U E F}=-\frac{1}{4} \cdot\left(A_{0}-1\right) \cdot\left[G_{M 3 E F}+G_{M 4 E F}\right] \tag{6.71}
\end{equation*}
$$

By merging (6.70), (6.71) and (6.69), we have

$$
\begin{align*}
G_{M 1 E F}+G_{M 2 E F} & =\frac{2}{\left(A_{0}\right)} \cdot \frac{1+R_{i n} \cdot G_{D S E F}}{R_{i n}}  \tag{6.72}\\
G_{M 3 E F}+G_{M 4 E F} & =\frac{2}{\left(A_{0}-1\right)} \cdot \frac{1+R_{i n} \cdot G_{D S E F}}{R_{i n}} \tag{6.73}
\end{align*}
$$

Since the oscillator is a symmetric circuit, the effective transconductance of $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ (also, $\mathrm{M}_{3}$ and $\mathrm{M}_{4}$ ) are the same. Hence, we can rewrite the above equation by

$$
\begin{equation*}
G_{M 1 E F}=\frac{1}{\left(A_{0}\right)} \cdot \frac{1+R_{i n} \cdot G_{D S E F}}{R_{i n}} \tag{6.74}
\end{equation*}
$$

and

$$
\begin{equation*}
G_{M 4 E F}=\frac{1}{\left(A_{0}-1\right)} \cdot \frac{1+R_{\text {in }} \cdot G_{D S E F}}{R_{i n}} . \tag{6.75}
\end{equation*}
$$

We will use (6.74) and (6.75) later for calculating a closed-form equation of this oscillator.

### 6.4.5 Phase Noise Equation

It is well known that the phase noise and FoM of any RF oscillator at an offset frequency $\omega_{0}$ from its resonating frequency $\omega_{0}=2 \pi \mathrm{f}_{0}$ can be expressed by

$$
\begin{equation*}
L(\Delta \omega)=10 \log _{10}\left(\frac{K T}{2 \cdot Q_{t}^{2} \cdot \alpha_{I} \cdot \alpha_{V} \cdot P_{D C}} \cdot\left(\frac{\omega_{0}}{\Delta \omega}\right)^{2}\right) \tag{6.76}
\end{equation*}
$$

and,

$$
\begin{equation*}
F o M=10 \log _{10}\left(\frac{10^{3} \cdot K \cdot T}{2 \cdot Q_{t}^{2} \cdot \alpha_{I} \cdot \alpha_{V}} \cdot F\right), \tag{6.77}
\end{equation*}
$$

where $K$ is the Boltzmann's constant; $T$ is the absolute temperature; $\mathrm{Q}_{t}$ is the LC-tank quality factor; $\alpha_{I}$ is the current efficiency, defined as ratio of the fundamental current harmonic $\mathrm{I}_{\omega_{0}}$ over the oscillator DC current $\mathrm{I}_{D C}$; and $\alpha_{V}$ is the voltage efficiency, defined as ratio of single-ended oscillation amplitude $\mathrm{V}_{o s c} / 2$ over the supply voltage $\mathrm{V}_{D D} . F$ is the oscillator's effective noise factor and estimated by

$$
\begin{equation*}
F=\frac{R_{i n}}{2 K T} \cdot \sum_{k} \frac{1}{2 \pi} \int_{0}^{2 \pi} \overline{i_{n, k}^{2}(\phi)} \cdot \Gamma_{k}^{2}(\phi) d \phi \tag{6.78}
\end{equation*}
$$

Let us now calculate the contribution of the losses and active devices. The white current noise power density of the resistive loss of the oscillator is given by

$$
\begin{equation*}
\overline{i_{n, l o s s}^{2}(\phi)}=\overline{i_{n, t a n k}^{2}(\phi)}+\overline{i_{n, G_{d s}}^{2}(\phi)}=4 K T\left(\frac{1}{R_{i n}}+G_{d s}(\phi)\right) . \tag{6.79}
\end{equation*}
$$

The relevant impulse sensitivity function of noise sources associated with a sinusoidal waveform oscillator, $V_{o s c} \cdot \cos (\phi)$, may be estimated by $\Gamma=$ $\sin (\phi)[25,28]$. By exploiting (6.4), the effective noise factor due to resistive losses of the oscillator becomes

$$
\begin{align*}
F_{l o s s}= & \frac{R_{\text {in }}}{2 K T} \cdot \frac{1}{2 \pi} \int_{0}^{2 \pi} \overline{i_{n, l o s s}^{2}(\phi)} \Gamma_{l o s s}^{2}(\phi) d \phi \\
= & \frac{R_{\text {in }}}{2 K T} \cdot \frac{1}{2 \pi} \int_{0}^{2 \pi} 4 K T\left(\frac{1}{R_{i n}}+G_{d s}(\phi)\right) \cdot \sin ^{2}(\phi) \cdot d \phi \\
= & \frac{1}{2 \pi} \int_{0}^{2 \pi} 2 \sin ^{2}(\phi) d \phi+R_{i n}\left(\frac{1}{2 \pi} \int_{0}^{2 \pi} G_{d s}(\phi) \cdot d \phi\right. \\
& \left.-\frac{1}{2 \pi} \int_{0}^{2 \pi} G_{d s}(\phi) \cdot \cos (2 \phi) \cdot d \phi\right) \rightarrow F_{\text {loss }} \\
= & 1+R_{\text {in }}\left(G_{D S}[0]-G_{D S}[2]\right)=1+R_{i n} G_{D S E F}, \tag{6.80}
\end{align*}
$$

where $G_{D S}[k]$ describes the $k$ th Fourier coefficient of the instantaneous $G_{d s}(\phi)$. From (6.51) and since the oscillator is a symmetric circuit, $G_{D S E F}=\frac{1}{4} \cdot\left[G_{D S E F 1}+G_{D S E F 2}+G_{D S E F 3}+G_{D S E F 4}\right]=\frac{1}{2}$. $\left[G_{D S E F 1}+G_{D S E F 4}\right]$. Consequently, we can rewrite (6.80) as

$$
\begin{equation*}
F_{l o s s}=1+\frac{R_{i n}}{2}\left(G_{D S 1 E F}+G_{D S 4 E F}\right) \tag{6.81}
\end{equation*}
$$

To get a better insight, different components of the above equation are graphically illustrated in Figure 6.14(a)-(c). The literature interprets $R_{\text {in }} G_{D S E F}$ term in (6.81) as the tank loading effect. In our design, $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ alternatively enter the triode region for part of the oscillation period and exhibit a large channel conductance. As shown in Figure 6.14(a), simulated $0.5 R_{\text {in }} G_{D S 1 E F}$ can be as large as 0.6 for the lower-pair transistors. However, $\mathrm{M}_{3,4}$ work only in saturation and demonstrate small channel conductance for their entire on-state operation, as evident from Figure 6.14(a). Hence, the simulated value of $0.5 R_{i n} G_{D S 4 E F}$ is as low as 0.15 for upper pair transistors. Note that both NMOS and PMOS pairs of the OSC $\mathrm{O}_{\mathrm{NP}}$ structure simultaneously enter the triode region for part of the oscillation period and load the tank from both sides. In this structure, however, only one side of the tank is connected to the $A C$ ground when either $\mathrm{M}_{1} / \mathrm{M}_{2}$ is in triode while the other side sees high impedance. Hence, this structure at least preserves the charge of differential capacitors over the entire oscillation period. Consequently, compared to the traditional oscillators, the tank loading effect is somewhat reduced here.


Figure 6.14 Circuit-to-phase-noise conversion across the oscillation period in the switching current-source oscillator. Simulated (a) channel conductance of $M_{1-4}$; (b) conductance due to resistive losses; (c) noise factor due to losses; (d) transconductance of $M_{1-4}$; (e) effective noise factor due to transconductance gain; (f) effective noise factors due to different oscillator's components.

The effective noise factor due to transconductance gain can be calculated by

$$
\begin{equation*}
F_{\text {active }}=\frac{R_{i n}}{2 K T} \cdot \frac{1}{2 \pi} \int_{0}^{2 \pi} \overline{i_{G m}^{2}(\phi)} \Gamma_{G m}^{2}(\phi) d \phi . \tag{6.82}
\end{equation*}
$$

Replacing (6.11) in (6.82),

$$
\begin{align*}
F_{\text {active }}= & \frac{R_{i n}}{2 K T} \cdot \frac{1}{2 \pi} \int_{0}^{2 \pi} K T\left(\gamma_{1}\left(g_{m 1}(\phi)+g_{m 2}(\phi)\right)+\gamma_{3}\left(g_{m 3}(\phi)\right.\right. \\
& \left.\left.+g_{m 4}(\phi)\right)\right) \cdot \sin ^{2}(\phi) \cdot d \phi \\
= & \frac{R_{i n}}{4 \pi} \int_{0}^{2 \pi} 2 \sin ^{2}(\phi)\left(\gamma_{1}\left(g_{m 1}(\phi)+g_{m 2}(\phi)\right)\right. \\
& \left.+\gamma_{3}\left(g_{m 3}(\phi)+g_{m 4}(\phi)\right)\right) \cdot\left(\frac{1}{2}-\frac{1}{2} \cos (2 \phi)\right) \\
= & \frac{R_{i n}}{4}\left[\gamma_{1}\left(G_{M 1}[0]-G_{M 1}[2]+G_{M 2}[0]-G_{M 2}[2]\right)\right. \\
& \left.+\gamma_{4}\left(G_{M 4}[0]-G_{M 4}[2]+G_{M 4}[0]-G_{M 4}[2]\right)\right] \\
= & \frac{R_{i n}}{4}\left[\gamma_{1}\left(G_{M 1 E F}+G_{M 2 E F}\right)+\gamma_{4}\left(G_{M 3 E F}+G_{M 4 E F}\right)\right] \tag{6.83}
\end{align*}
$$

By replacing (6.72) and (6.73) in (6.83),

$$
\begin{equation*}
F_{\text {active }}=\left(1+R_{i n} G_{D S E F}\right) \cdot\left(\frac{\gamma_{1}}{2 A_{0}}+\frac{\gamma_{4}}{2\left(A_{0}-1\right)}\right) \tag{6.84}
\end{equation*}
$$

To get a better insight, different components of the above equation are graphically illustrated in Figure 6.14(d-e).

As discussed in conjunction with Figure 6.5(c), the transformer's passive voltage gain, $A_{0}$, covers a significant part of the required loop gain of the lower positive feedback. Hence, the lower-pair transistors have to compensate only $1 /\left(2 A_{0}\right)$ of the circuit losses. For the upper positive feedback, however, $A_{0}$ covers a smaller part of the required loop gain. Consequently, the upper transistors should work harder and compensate $1 /\left(2\left(A_{0}-1\right)\right)$ of the oscillator loss. Consequently, as (6.84) indicates, the $G_{M}$ noise contribution by the lower pair is smaller. However, its effect on $F_{\text {loss }}$ is larger such that both pairs demonstrate more or less the same contribution to the oscillator PN (see Figure 6.14(f)). Finally, the total oscillator effective noise factor is given by

$$
\begin{equation*}
F=F_{\text {loss }}+F_{\text {active }}=\left(1+R_{i n} G_{D S E F}\right) \cdot\left(1+\frac{\gamma_{1}}{2 A_{0}}+\frac{\gamma_{4}}{2\left(A_{0}-1\right)}\right) \tag{6.85}
\end{equation*}
$$

To obtain the oscillator phase noise, $G_{D S 1 E F}$ and $G_{D S 4 E F}$ should also be calculated or simulated. Since transistor size and oscillation waveforms are known, it is pretty straight-forward to calculate a closed-form equation for them. However, the final equation will be huge and these parameters are calculated numerically here.

When $M_{3,4}$ are not turned off, they work only in saturation and thus their channel conductance and $G_{D S 4 E F}$ are negligible. However, as shown in the manuscript, precise simulations show that $\frac{R_{i n}}{2} \cdot G_{D S 4 E F}$ can be as large as 0.15 even if the transistor works only in the saturation. It translates to 0.6 dB higher noise factor for this oscillator due to channel conductance noise of $M_{3,4}$ transistors. On the other hand, $\mathrm{M}_{1}$ and $\mathrm{M}_{2}$ alternatively enter the triode region for part of the oscillation period. Hence, their effective conductance $G_{D S 1 E F}$ is larger. Simulations show that $\frac{R_{i n}}{2} \cdot G_{D S 1 E F}$ is about 0.6 in this oscillator. We will also show later that the excess noise factor of NMOS
transistors is 1.4 . The voltage gain is 2.16 . By replacing those numbers in the nose factor equation, we have

$$
\begin{equation*}
F=(1+0.6+0.15) \cdot\left[1+\left(\frac{1.4}{2 \cdot 2.15}+\frac{1.4}{2 \cdot 1.15}\right)\right] \approx 5.3 d B \tag{6.86}
\end{equation*}
$$

the noise factor is just 1.5 dB higher than the ideal value of $(1+\gamma)$, despite the aforementioned practical issues of designing ulta-low voltage and power oscillators. The phase noise and FoM of this oscillator can be calculated by replacing (6.85) in (6.2).

### 6.5 1/f Noise Upconversion

Several techniques have been exploited to improve the oscillator's $1 / \mathrm{f}$ noise upconversion. First, dynamically switching the bias-setting devices $\mathrm{M}_{1,2}$ will reduce their flicker noise, as also demonstrated in [37]. It also lowers a DC component of their effective impulse sensitivity function. Second, as discussed in Chapter 5, [38,39], a second-order harmonic of the gm-devices' drain current flows into the capacitive part of the tank due to its lower impedance and creates asymmetric rise and fall times for the oscillation waveform. It directly increases a DC value of the oscillator ISF and thus its $1 / \mathrm{f}^{3} \mathrm{PN}$ corner. This phenomenon can be alleviated by realizing an auxiliary resonance at $2 \omega_{0}$ such that the second harmonic current flows into equivalent resistance of the tank in order to avoid disturbing the rise and fall time symmetry of the oscillation voltage. Since common-mode signals, such as a second harmonic of the drain current, cannot see the tuning capacitance at the transformer secondary winding [21], the auxiliary $2 \omega_{0}$ can be realized without die area penalty and by adjusting the single-ended capacitance at the transformer primary winding [39].

The last source of the $1 / \mathrm{f}$ noise is $\mathrm{M}_{\mathrm{B} 1}$ in the biasing circuitry. By exploiting long channel device for biasing, its power consumption becomes negligible compared to the oscillator core while $\mathrm{M}_{\mathrm{B} 1 / \mathrm{B} 2}$ occupy larger area and thus generate lower 1/f noise. Consequently, based on aforementioned techniques, a lower $1 / \mathrm{f}^{3} \mathrm{PN}$ corner is expected than in the traditional oscillators.

### 6.6 Optimizing Transformer-Based Tank

The transformer-based tank's input equivalent resistance, $R_{\text {in }}$, and voltage gain, $A_{0}$, should be maximized for the best system efficiency. Both optimization parameters are a strong function of $\zeta=L_{2} C_{2} / L_{1} C_{1}$ [35], as shown in


Figure 6.15 Transformer-based tank: (a) schematic; (b) input parallel resistance; (c) voltage gain; and (d) $R_{21}$ versus $\zeta$-factor.

Figure 6.15. $R_{i n}$ may be estimated by

$$
\begin{equation*}
R_{i n}=L_{1} \omega_{0} Q_{1} \cdot \frac{\left(1-\left(\frac{\omega_{0}}{\omega_{s}}\right)^{2}\left(1-k_{m}^{2}\right)\right) \zeta}{-\left(\frac{\omega_{0}}{\omega_{s}}\right)^{4}\left(1+\frac{Q_{1}}{Q_{2}}\right)+\left(\frac{\omega_{0}}{\omega_{s}}\right)^{2}\left(1+\frac{Q_{1}}{Q_{2}} \zeta\right)}, \tag{6.87}
\end{equation*}
$$

where $\omega_{s}^{2}=1 / L_{2} C_{2}$, and $Q_{1}$ and $Q_{2}$ are, respectively, the Q -factors of the transformer's primary and secondary windings. It can be shown that $R_{i n}$ reaches its maximum when

$$
\begin{equation*}
\zeta_{R \max }=\frac{Q_{2}}{Q_{1}} \cdot\left(\frac{Q_{2}}{Q_{1}+Q_{2}} \cdot k_{m}^{2}+\frac{Q_{1}}{Q_{1}+Q_{2}}\right) . \tag{6.88}
\end{equation*}
$$

Note that the tank Q-factor is maximized at different $\zeta=Q_{2} / Q_{1}$ [24]. The maximum $R_{\text {in }}$ is obtained by inserting (6.88) into (6.87)

$$
\begin{equation*}
R_{\text {inmax }}=L_{1} \omega_{0} Q_{1} \cdot\left(1+k_{m}^{2} \cdot \frac{Q_{2}}{Q_{1}}\right) . \tag{6.89}
\end{equation*}
$$

Consequently, the transformer's coupling factor $k_{m}$ enhances $R_{i n}$ by a factor of $\sim\left(1+k_{m}^{2}\right)$ at $\zeta_{\text {Rmax }}$. For this reason, the switched-capacitor banks
are distributed between the transformer's primary and secondary to roughly satisfy (6.88). For $k_{m} \geq 0.5$, the voltage gain of the transformer-based tank may be estimated by

$$
\begin{equation*}
A_{0}=\frac{2 k_{m} n}{1-\zeta+\sqrt{1+\zeta^{2}+\zeta\left(4 k_{m}^{2}-2\right)}} \tag{6.90}
\end{equation*}
$$

As shown in Figure 6.15(c), $A_{0}$ increases with larger $\zeta$. Note that larger $R_{i n}$ and $A_{0}$ are desired to reduce $P_{D C}$ and $P_{b u f}$, respectively. To consider both scenarios, trans-impedance $R_{21}=R_{i n} \cdot A_{0}$ term is defined and depicted in Figure $6.15(\mathrm{~d})$. $R_{21}$ reaches its maximum at $\zeta=1$ for $Q_{1} \approx Q_{2}$, which is reasonable for monolithic transformers. We also define the maximum of $R_{21}$ as the transformer FoM $=\left(Q_{1} \| Q_{2}\right) \cdot\left(1+k_{m}\right)^{2} \cdot \sqrt{L_{1} L_{2}} \cdot \omega_{0}$. Consequently, the transformer dimensions and winding spacing are chosen to maximize this term.

### 6.7 Experimental Results

The oscillator was prototyped in TSMC 40 nm 1P7M CMOS. The chip micrograph is shown in Figure 6.16(a). $\mathrm{M}_{1,2}$ and $\mathrm{M}_{3,4}$ transistors are minimum-length low- $V_{\mathrm{t}}$ devices with a width of 32 and $256 \mu \mathrm{~m}$, respectively. The transformer's primary and secondary differential self-inductance is only 660 pH and 2 nH , respectively, with the coupling factor $k_{m}=0.76$. Both


Figure 6.16 (a) Chip micrograph; (b) measured oscillator phase noise and FoM at $3-\mathrm{MHz}$ offset frequency across the tuning range.
transformer's winding are realized with top ultra-thick metal ( $3.5 \mu \mathrm{~m}$ ). However, the transformer includes a floating M1-to-M6 shield to comply with the strict metal density rules ( $>10 \%-20 \%$ ) for manufacturability and also to alleviate the substrate loss. Note that the shield must be significantly thinner than the skin depth at the desired frequency to avoid any attenuation of the magnetic field. The skin depth of copper is $\sim 0.9 \mu \mathrm{~m}$ at 5 GHz . However, the thickness of M6 layer is $0.85 \mu \mathrm{~m}$. Hence, adding M6 dummy metal reduces the transformer's magnetic field, inductance, and Q-factor, and thus $R_{\text {in }}$ drops by $10 \%-20 \%$. The simulated Q-factor is 12 and 16 for the primary and secondary windings, respectively.

Figure 6.17 shows the measured PN at the highest and lowest frequencies $\left(\mathrm{f}_{\max }, \mathrm{f}_{\min }\right)$ with $V_{D D}$ of 0.5 V and $P_{D C}$ of 470 and $580 \mu \mathrm{~W}$, respectively. Thanks to the switching current-source technique, $1 / \mathrm{f}^{3} \mathrm{PN}$ corner of the oscillator is relatively low and varies between 250 and 420 kHz


Figure 6.17 Measured phase noise of this oscillator.

Table 6.3 Comparison table of low power oscillators

|  | This Work | $\begin{aligned} & {[40]} \\ & \text { JSSC'05 } \end{aligned}$ | $\begin{aligned} & {[23]} \\ & \text { JSSC' } 08 \end{aligned}$ | $\begin{aligned} & {[41]} \\ & \text { ESS-CIRC' } 14^{\dagger} \end{aligned}$ | $\begin{aligned} & {[26]} \\ & \text { ISSCC' } 14 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Technology | 40 nm | $0.18 \mu \mathrm{~m}$ | $0.13 \mu \mathrm{~m}$ | 28 nm | 40 nm |
| $\mathrm{V}_{\text {DD }}$ | 0.5 V | 0.5 V | 1 V | 0.5 V | 1 V |
| TR(\%) | 22.2 | 8.7 | 14 | N/A | 24.5 |
| $\mathrm{f}_{0}(\mathrm{GHz})$ | 4.8 | 3.8 | 4.9 | 2.35 | 2.44 |
| $\mathrm{PN}(\mathrm{dBc} / \mathrm{Hz})^{\ddagger}$ | -139 | -143 | -149.5 | -125.8 | -131.1 |
| $\mathrm{P}_{\mathrm{DC}}(\mathrm{mW})$ | 0.48 | 0.57 | 1.4 | 0.38 | 0.4 |
| FoM (dB) | 189.8 | 193 | 195.5 | 187.5 | 183 |
| $\mathrm{FoM}_{\mathrm{T}}(\mathrm{dB})^{*}$ | 196.7 | 191.7 | 198.5 | N/A | 190.8 |
| Freq pushing | $17 \mathrm{MHz} / \mathrm{V}$ | $273 \mathrm{MHz} / \mathrm{V}$ | N/A | N/A | N/A |
| Dummy fill | Yes | No | No | No | No |
| Area (mm ${ }^{2}$ ) | 0.14 | 0.23 | 0.11 | 0.2 | 0.15 |
| Oscillator topology | Switching current source | TRX feedback | Class-C | Class-D | Traditional |

${ }^{\dagger}$ Including LDO. LDO also performs a start-up role.
${ }^{\ddagger}$ At $\Delta f=10 \mathrm{MHz}$ normalized to $2.4-\mathrm{GHz}$ carrier.
${ }^{*} \mathrm{FOM}_{\mathrm{T}}=|\mathrm{PN}|+20 \log _{10}\left(\left(f_{0} / \Delta f\right)(\mathrm{TR} / 10)\right)-10 \log _{10}\left(P_{D C}(\mathrm{~mW})\right)$.
across the tuning range (TR). The oscillator has a $22.2 \% \mathrm{TR}$, from 4 to 5 GHz . Figure 6.16(b) displays plots of phase noise and FoM across the TR. The FoM reaches maximum 189.9 dBc at $\mathrm{f}_{\text {max }}$ and varies $\sim 1 \mathrm{~dB}$ across the TR.

Table 6.3 summarizes the oscillator performance and compares it with relevant state-of-the-art for $P_{D C}<2 \mathrm{~mW}$ and TR $>8 \%$. It is the only one with the all-layer dummy metal fills inside the LC tank for manufacturability. For the similar $P_{D C}(400-600 \mu \mathrm{~W})$, only the transformer-feedback VCO [40] shows better FoM but with a much larger area, lower TR, and extremely high frequency pushing. Class-C VCO [23] also shows better FoM but at a much higher $P_{D C}$. Furthermore, it needs additional complex biasing circuits (such as opamp) for proper operation, which can potentially limit its minimum $V_{D D}$ and thus $P_{D C}$.

It might be interesting to point out that switching current source oscillator is already adapted in a fractional-N ADPLL for BLE [42], in a fully integrated BLE transmitter [18], and a BLE transiver [43].

### 6.8 Conclusion

A switching current-source oscillator has been described and analyzed, providing deep insights into beneficial circuit operation. It combines advantages of low supply voltage operation of the conventional NMOS cross-coupled oscillator with high current efficiency of the complementary push-pull oscillator to reduce the oscillator supply voltage and dissipated power without sacrificing its start-up robustness or loading tank's Q-factor. The 28-nm CMOS prototype exhibits $189.5 \mathrm{dBc} / \mathrm{Hz}$ FoM, with $22 \%$ tuning range, dissipating 0.5 mW from 0.5 V power supply, while complying with the process technology manufacturing rules.

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[^0]:    ${ }^{1}$ Ring oscillators can also satisfy such a relaxed phase noise requirement. However, they consume much higher power than LC oscillators at $f_{0} \geq 1 \mathrm{GHz}$ [19].

[^1]:    $\dagger$ by considering $V_{O D}=0.5 V_{t}$ for the current source.
    $\ddagger$ at the minimum $V_{D D}$.

    * ideal value.

[^2]:    ${ }^{2}$ It should be noted that the "master/slave" view is mainly valid from a small-signal standpoint. Both are equally important when considering the large-signal switching operation.

[^3]:    ${ }^{3}$ The interested reader is directed to [35] for accurate closed-form equations of $L_{e q}, C_{e q}$, and $R_{i n}$.

