Feedback Linearization for Nonlinear Control of a Magnetic Levitation System

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Abstract.

In this research, a Feedback Linearization approach is devised for use in a Magnetic Levitation System (MLS). In order to levitate a ferromagnetic ball, a Non-Linear Control (NLC) with input and output feedback linearization algorithms of differential geometry was created. This method was used in conjunction with a controller for linear state feedback in the outer loop. During the process of putting together MLS, the real-time experimental data for the electromagnetic force were noticed. The amount of force exerted is directly related to the amount of current that is flowing as well as the location of the ferromagnetic ball that is going to be levitated by the electromagnetic force that is being applied. Unlike a standard PID controller, the output results highlight the usefulness of the newly built NLC.

Keywords. Electromagnetic, NLC, PID, Feedback Linearization

1. INTRODUCTION

This Magnetic levitation system (MLS) is an experimental setup that uses the principle of electromagnetism for levitates ferromagnetic ball. MLS concept minimizes friction by eliminating mechanical touch between moving parts and stationary parts [1]. MLS's many benefits can be attributed to the friction it eliminates minimum noise, the ability to work in a high-vacuum setting, and maximum accurate positioning system. MLS has three types of forces: the force that propulsion, the force that lifts, and the force that guides [2]. The propulsion force is responsible for moving portion ahead, the levitation force hang the moving part, and the guidance force keeps tracking it from derailing. If the nature of attractive force is present, then magnetic suspension works while the nature of force is repulsive then the principle of magnetic levitation occurs. MLS offers a wide range of uses, including the capacity to perform tasks in a very low-pressure setting [2]. Due to continuous need of levitation of ball the parameters also change with respect to time and mathematical modelling will consist of nonlinear in nature. Several attempts have been made to simulate and control the MLS [1-3]. Several initiatives to simulate and maintain control of the MLS have been made. [1-3]. The bulk of design techniques are linear in nature, magnetic levitation is a process that exhibits nonlinear behaviour which is characterised by a nonlinear differential equation [4-6]. Tracking performance in a linear model, on the other hand, degrades rapidly as the deviation from the nominal operating point grows. Nonlinear modelling, rather than linear modelling, is required to assure an extremely extended range of travel for a ferromagnetic ball while keeping effective tracking. As a result, for system modelling, Parameters are estimated in real time using experimental data.

The MLS is modelled in this paper using the electromagnetic concepts. combining input-output feedback control with nonlinearity as steady state. Fig. 1 Shows of the MLS law is devised based on this developed model. Additionally, real-time data is used to estimate the electromagnetic force, which depends on the input magnetising current and the location of the ferromagnetic ball that the input force would lift. The proposed controller's superiority over a typical PID controller is established at the conclusion.

2. Magnetic Levitation System Dynamic Modelling

The MLS under mathematical modelling is a magnetic field that can be adjusted by voltage, a ferromagnetic ball can be lifted. The actuator is a ferromagnetic core coil with the sensor detecting ball condition relative to the centre coil. The ferromagnetic ball, which only possesses one degree of freedom. Fig. 1 depict a schematic diagram of the system MLS parts as an electromagnet sensor, ferromagnetic ball and sensors along with a PC Connected interfacing card a signal conditioner and its accompanying cable. Total of two inputs are received by the system. Variations in power supply, coil temperature, and ball forces cause disturbances. To simulate the dynamic behaviour of a magnetic levitation system the electromagnet & mechanical subsystem can be studied. [3].

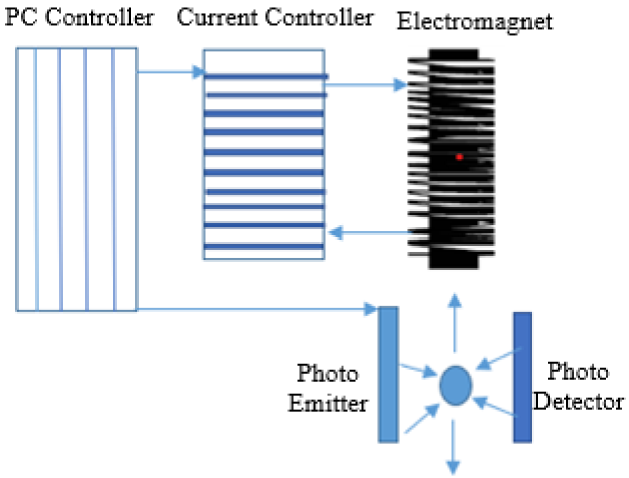

Fig. 1 Diagrammatic representation of the MLS

In magnetic levitation, the magnetic force is generated by the current running through the coil, this magnetic force is calculated by Kirchhoff's voltage law, and the magnetic field is computed by the principle of Biot-Savart law. The magnetic and gravitational fields are applied on the ferromagnetic ball. The free body model shown in Fig. 2 can be used to write the force equation using Newton's Laws of Motion.

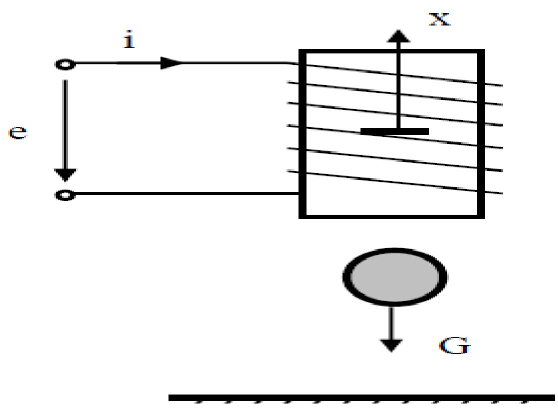

Fig. 2 MLS Free body diagram

In this system, there are three inputs. They are as follows:

1. Adjust the ball's vertical position using the set point.

2. Signal of reference input

3. Disturbances, such as fluctuation to the power supply, change in the coil's temperature, and external pressures applied on the ball A magnetic levitation system's dynamic behavior can be modelled by the study of electromagnetic and mechanical subsystems.

A ferromagnetic ball experiences both magnetic and gravity forces. The magnitude of an electromagnetic force is proportional to the amount of the current flowing through it and its distance of ball $F(x,i)$

$$F(x,i) - mg = ma \tag{1}$$

$$F(x,i) - mg = m\frac{d^2x}{dt^2} \tag{2}$$

$$m\frac{d^2x}{dt^2} = F(x,i) - mg \tag{3}$$

$$m\ddot{x} = F(x,i) - mg \tag{4}$$

$$\ddot{x} = \frac{F(x,i)}{m} - g \tag{5}$$

Now we must compute F(x, i) for the ball distance with respect to the Actuator. Given is the magnetic field produced by a brief section of wire dl carrying a current of I is given as

$$dB = \frac{\mu_o}{4\prod}\frac{idl \times r}{r^3} \tag{6}$$

Using current I and radius a to create a circular path then magnetic field is calculated as

$$B = \frac{\mu_o i}{2} \frac{a^2}{(a^2+d^2)^{3/2}} \quad (7)$$

$$r = \sqrt{a^2 + d^2} \quad (8)$$

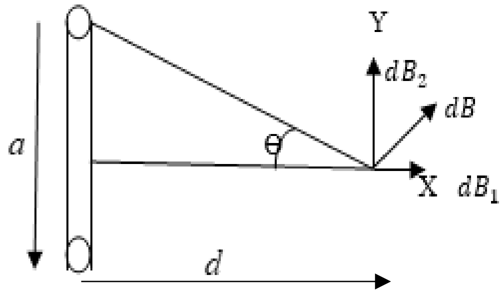

Fig. 3 shows the magnetic field of a circular contour

Because of field symmetry, the magnetic field in the y direction is 0, hence the magnetic field exists only in the x direction. N is the total number of turns, while n is the number of turns per unit length. The magnetic field of an electromagnet causes the ball to develop a magnetic dipole and get magnetized as a result. Thus, the magnetic force acting on the induced dipole and gravity are the two forces acting on the ball.

$$\sin\theta_1 = \frac{r}{\sqrt{(l+x)^2 + r^2}} \quad (9)$$

$$\sin\theta_2 = \frac{r}{\sqrt{r^2 + x^2}} \quad (10)$$

The magnetic field of an electromagnet induces Ball contains a magnetic dipole that magnetizes. The acceleration caused by gravity is responsible for the downward force that is operating on the ball. Additionally, the magnetic force that is acting on the induced dipole in the upward direction also contributes to this force. A solenoid (or an electromagnet) with N turns is denoted by ADEF having radius of r and a length of l. Total axial field generated by all turns We can deduce that

$$B = \int dB \quad (11)$$

$$B = \frac{\mu_o}{2} nI \frac{r^2}{(r^2 + l^2)^{3/2}} dx \quad (12)$$

$$B = \frac{\mu_O}{2} nI \left[\frac{X + l}{\sqrt{r^2 + (X+l)^2}} - \frac{X}{\sqrt{r^2 + X^2}} \right] \quad (13)$$

Because of the multiple turn's layers of the electromagnet coil, the inner radius r_1 and outer radius are r_2 . The magnetic field is

$$dB = \frac{\mu_O}{2} nI \left[\frac{X+l}{\sqrt{r^2+(X+l)^2}} - \frac{X}{\sqrt{r^2+X^2}} \right] ndr \tag{14}$$

$$dB = \frac{\mu_O}{2} n^2 I \left[\begin{array}{l} \dfrac{X+l}{\sqrt{r^2+(X+l)^2}} - \\ \dfrac{X}{\sqrt{r^2+X^2}} \end{array} \right] dr \tag{15}$$

We get by integrating eq. (12) r_1 from to r_2

$$B = \frac{\mu_O}{2} n^2 I \int_{r_1}^{r_2} \left(\frac{X+l}{\sqrt{r^2+(X+l)^2}} - \frac{X}{\sqrt{r^2+X^2}} \right) dr \tag{16}$$

$$B = \frac{\mu_O}{2} n^2 I \left[\begin{array}{l} (X+l) \displaystyle\int_{r_1}^{r_2} \frac{1}{\sqrt{r^2+(X+l)^2}} dr \\ - X \displaystyle\int_{r_1}^{r_2} \frac{1}{\sqrt{r^2+X^2}} dr \end{array} \right] \tag{17}$$

$$B = C_1 I G(X) \tag{18}$$

Where $C = \frac{\mu_o n^4}{8} s$

The Magnetic force on the ball is experienced as

$$f = \frac{B^2}{2\mu_o} s \tag{19}$$

S denotes the surface of the substance that is traversed by the magnetic flux.

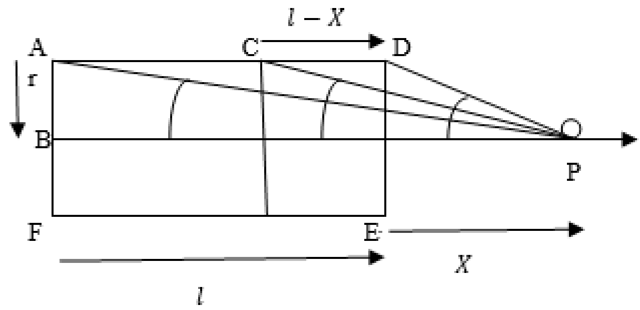

Fig. 4 Ball and coil combination demonstrating magnetic force

3. DESIGN OF NON-LINEAR CONTROLLER

3.1. Feedback Linearization

One of the most prevalent approaches in practical nonlinear control design is to turn nonlinear models into linear ones. The feedback linearization approach is divided into two parts: state and feedback. The first is input transformation, while the second is output transformation [5]. Using this strategy, we can algebraically turn a nonlinear MLS into a linear one without ignoring the nonlinear element. Feedback Linearization is not the same as Jacobian Linearization. Concept of Lie derivative use in Feedback Linearization technique, by the use of two vector fields f and g, where f (x) and g(x) are the system matrix and input matrix respectively.

$$\dot{X} = f(X) + g(X)u \tag{20}$$

$$y = h(X) \tag{21}$$

Where $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ i \end{bmatrix}$

Because the nonlinear dynamics equation is

$$i = k_1 . u \tag{22}$$

$$\begin{aligned} m\ddot{x} &= mg - k.\frac{i^2}{x^2} \\ \ddot{x} &= g - \frac{k}{m}.\frac{i^2}{x^2} \end{aligned} \tag{23}$$

$$\begin{aligned} x &= x_1 \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g - \frac{k}{m}.\left(\frac{i^2}{x^2}\right) \end{aligned} \tag{24}$$

$$\dot{x}_2 = g - \beta\left(\frac{u^2}{x_1{}^2}\right) \; where \;\; i = k_1 u \tag{25}$$

$$f(X) = \begin{bmatrix} x_2 \\ g \end{bmatrix} \;, \; g(X) = \begin{bmatrix} 0 \\ \frac{-\beta}{x_1{}^2} \end{bmatrix} \tag{26}$$

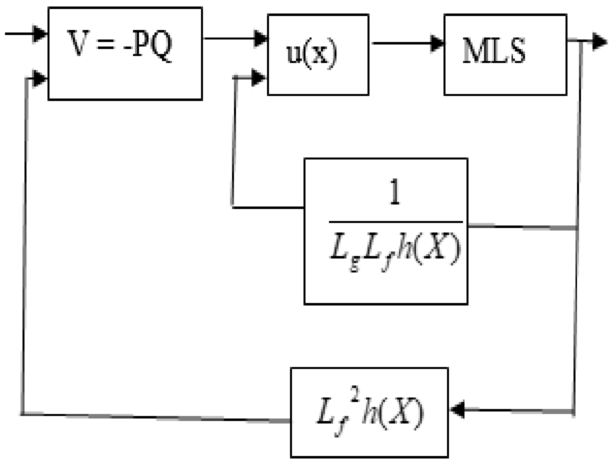

Fig. 5 Controller Design Block Diagram

The MLS nonlinear system can be linearized by in a region if two conditions are met by two vectors $f(x)$ and $g(x)$ and it would be linearizable in a region δ if two condition will be satisfied [6].$[g, ad_f g, ...ad_f{}^{n-1} g]$ should be controllable and other one $[g, ad_f g, ...ad_f{}^{n-2} g]$ satisfied for linear system. State vector for MLS is $x_3 =$ Speed $x_2 =$ Position & $x_1 =$ Current.

$$(x_1, x_2, x_3)^T = (I, y, \frac{dy}{dt}) \tag{27}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} I \\ y \\ \frac{dy}{dt} \end{bmatrix} \tag{28}$$

Lie derivative nonlinear dynamics equation [3]

$$h(x) = x_2$$
$$L_f h(x) = \frac{\partial h}{\partial x} f(x) = x_3 \tag{29}$$
$$L_f{}^2 h(x) = \frac{\partial h}{\partial x}\frac{\partial f}{\partial x} f(x)$$

$$L^2{}_f h(x) = g - \frac{1}{m}\left(\frac{x^2{}_1}{-1.07 + 14.58x_2 - 64.81x_2{}^2 + 119.10x_2{}^3}\right) \tag{30}$$

Space transformation $Q = \psi(x)$

$$Q = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \end{pmatrix} \tag{31}$$

$$\psi_1(x) = h(x), \psi_2(x) = L_f h(x), \psi_3(x) = L_f{}^2 h(x) \tag{32}$$

$$Q = \begin{pmatrix} x_2 \\ x_3 \\ g - \frac{1}{m}\left(\frac{x_1{}^2}{-1.07 + 14.58x_2 - 64.81x_2{}^2 + 119.10x_2{}^3}\right) \end{pmatrix} \tag{33}$$

For new states $Q_i = \psi_i(x)$

$$L^2{}_f h(x) = \left\{ g - \frac{1}{m}\left(\frac{x_1{}^2}{-1.07 + 14.58x_2 - 64.81x_2{}^2 + 119.10x_2{}^3}\right)\right\} \tag{34}$$

$$\dot{Q} = \begin{pmatrix} Q_2 \\ Q_3 \\ d(Q) + c(Q) \end{pmatrix} \tag{35}$$

$$\dot{Q} = CQ + DV_K \tag{36}$$

When differentiation output occurs up until we receive input u, a synthetic input v for nonlinear feedback u is being received [5].

$$y^r = L_f{}^r h + L_g (L_f{}^{r-1} h)U$$
$$V = L_f{}^r h + L_g (L_f{}^{r-1} h)U \tag{37}$$

Where r is the relative degree, the control variable for MLS (r=3) should be

$$u(X) = \frac{-L_f{}^3 h(X)}{L_g L_f{}^2 h(X)} + \frac{1}{L_g L_f{}^2 h(X)} v \tag{38}$$

The system is now linear and controllable, and it can be stabilised using the state feedback law. $v = -PQ$, Where $P(P_1, P_2, P_3)$ is pole placement technique.

4. RESULT AND DISCUSSION

4.1. Identification

If the electromagnetic force $F(x,i)\ \alpha \frac{i^2}{x^2}$ and the ball location from the coil act in polynomial form, then we can write

$$F = \frac{I^2}{b_0 + b_1 x + b_2 x^2 + b_3 x^3} \tag{39}$$

Here $b_0, b_1, b_2, and\, b_3$ is the polynomial coefficient, we experimented the data from the MLS Experimental setup.
Data are collected in such a way that the least value of the present particular levitation position is obtained. Using least square fitted data, the polynomial's parameter is obtained.

$$\begin{aligned} b_0 &= -1.07 \\ b_1 &= 14.58 \\ b_2 &= -64.81 \\ b_3 &= 119.10 \end{aligned} \tag{40}$$

Proportional Integral Derivative (PID) Controller is used to perform analysis on MLS. The most common type of PID Controller is

$$G_c(s) = K_p(1 + \frac{1}{\tau_i s})(1 + \tau_d s) \tag{41}$$

While the MLS linearized model is

$$G_p(s) = \frac{-0.0025}{S^2 + 0.0075} \tag{42}$$

From $G_p(s)$ Due to its inherently unstable character, the MLS mathematical model is linearized around the nominal operating point of $y = 5mm$.

4.2. Simulation

The proposed STC was numerically simulated in the MATLAB of SIMULINK® is used. The MLS in the Control and System Lab in BIT, Mesra, Ranchi, was subjected to the proposed STC. It was decided to use the goal trajectory as a step signal to validate the STC's trajectory tracking performance. Fig. 6 shows that MLS Identification technique for the applied force. Fig. 7 show the findings for both the desired & actual ball position. The ball position tracking error curves with a sinusoidal input were detected. calculates that PID has a tracking error of 0.66 mm, while NLC has a tracking error of 0.25 mm. The control input voltage with respect to desired PID and NLC are shown in Fig. 10.

TABLE 1: MLS PHYSICAL PARAMETERS

S.No	Physical Representation	Physical Parameter	Dimension
1	I	Input Current	(0 -3) Amp
2	V	Input Voltage	(0 -5) Volt
3	M	Ball Mass	0.021 Kg
4	X	Ball Start	5 mm
5	X	Ball's Destination	(5-25) mm
6	C	Magnetic Constant	0.0000824 Kg
7	g	Gravity Constant	9.8 m/Sec.Sec

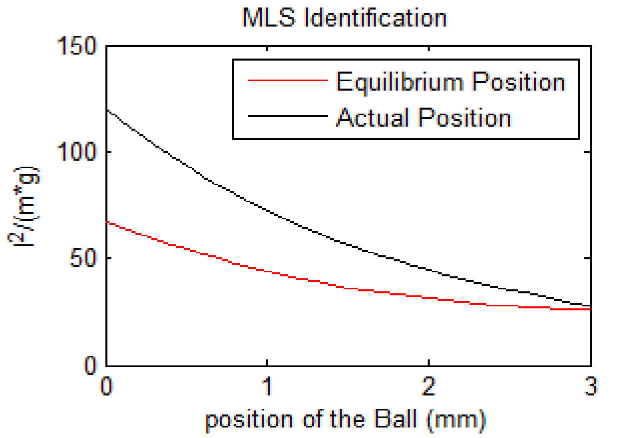

Fig. 6 Ball Position for Controller Design

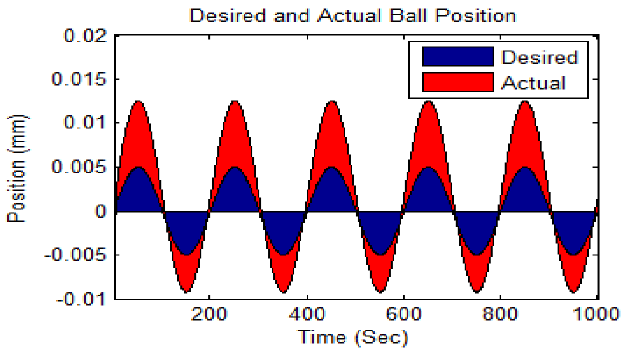

Fig. 7 Ball Position for Controller Design

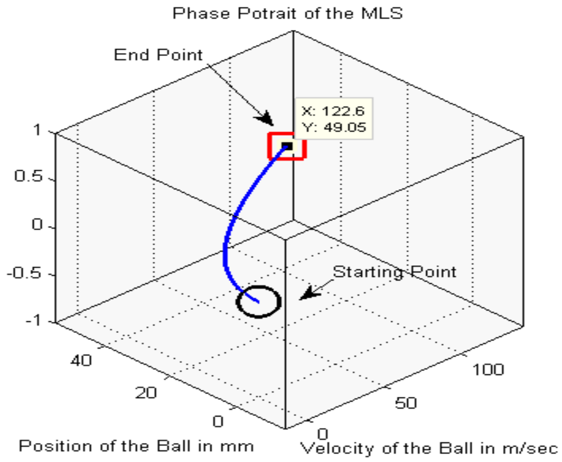

Fig. 8 Phase Potrait of MLS system without Controller

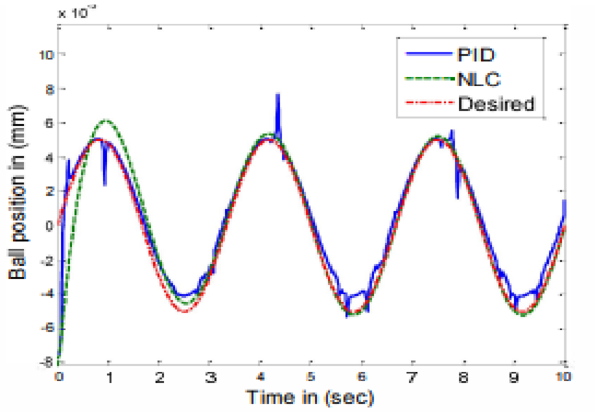

Fig. 9 Ball position due to intended sinusoidal input

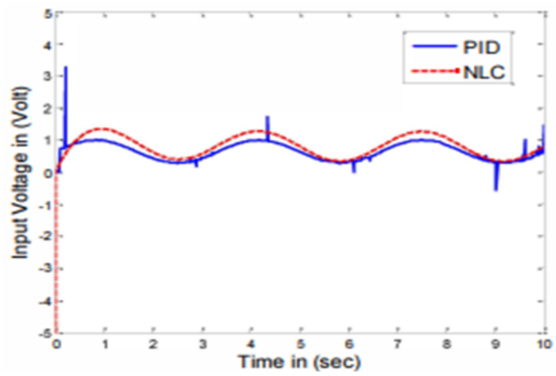

Fig. 10 Control input due to intended sinusoidal

Both the position and velocity of the ball are shown on the graph of the MLS state with respect to time that can be found in Fig. 6. The nature of both of these variables is nonlinear. The

MLS identification for the actual position and equilibrium position can be seen in Fig. 7. The actual position of the ball, as well as the position that should be, is shown in Fig. 8. The Phase Portrait of the MLS system is shown in Fig. 9, which does not include a Controller. As in Fig. 10, the result shows that there is minimum variation appears in between PID and NLC. The findings of the simulation demonstrate that the suggested NLC is capable of producing an adequate amount of adaptive torque despite the existence of sensor noise and nonlinearity in the system.

5. CONCLUSION

The concept developed for a Single input and single output MLS system & designed controller feedback linearization for lie derivatives to control the MLS in presence of nonlinearities. The developed NLC was applied to an MLS and produced the required outcome. The simulation findings show that the applied MLS provides adequate resilient compared to a fixed gain, torque under system nonlinearity and sensor noises traditional PID controller under comparable conditions. The work that came before could be improved in the future to construct reliable controllers that are nonlinear and are based on a nonlinear model of the MLS's dynamics.

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