Stabilization and Control of Quadcoptor

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Abstract.

The present work considers a sliding mode (SM) control strategy for control of quadcoptor and stabilization. First, a proportional derivative (PD) sliding surface is considered in order to achieve control of aircrafts. Variation in reference angles are allowed, and external disturbances are applied as well. The sliding mode control (SMC) with PD sliding surface achieves the reference tracking instantaneously. However, this control scheme cannot achieve tracking in presence of disturbances. Furthermore, an integral sliding surface is augmented with the proportional derivative one for disturbance rejection. Integral sliding surface is designed and implemented in simulation for quadrotor. The results are showcased which demonstrates that the PID SMC is successfully performing tracking even in presence of disturbances.

Key words: PID sliding surface, PD sliding surface, Sliding Mode control, Disturbance rejection, Quadcopter.

1. INTRODUCTION

In recent era, vehicle modernization is rapidly receiving interest of the researchers. Both unmanned ground and aerial vehicles are of great concern to scientists and engineers because of their field of application specifically in military surveillances, risk and rescue operations [1]-[6]. Unmanned aerial vehicles reach the places whwre UGV fails such hill top, flood affected areas, dense forest etc. Nowadays Quadcoptors are also replacing manned and fixed wing UAVs. Therefore stabilization and control of such systems plays a major role for fruitful operation of quadcoptor in specific object oriented applications. Literature review reveals that many works have already been reported in this domain. Proportional (P), Integral (I), Derivative (D) controllers, LQR control, Fuzzy based control, Sliding Mode (SMC) control are few approaches those have been applied for quadcoptors [7-11]. Out of these, SMC demonstrates a promising robust control approach which can achieve control even in presence of uncertainties. Keeping this in view, the objective of this work is set as development of a proportional derivative (PD) sliding surface for control and stabilization of UAVs. Variation in reference attitude angles are allowed, and some disturbances are applied as well. The SMC with PD sliding surface achieves satisfactorily the reference tracking. However, this control scheme fails to achieve disturbance rejection Hence, an integral sliding surface is introduced for disturbance rejection.

Arrangement of the present work is given next. Section II is on the presentation of the mathematical modeling of the quadrotor. Section III describes concepts of SMC with PD and PID sliding surfaces. Section IV showcases the simulation results. Section V includes Conclusions.

2. MATHEMATICAL MODEL OF QUADCOPTER

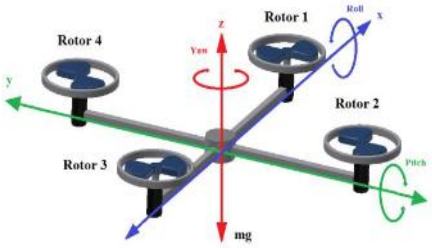

Fig. 1. Configuration of A Quadcoptor

The linear and angular positions and velocities are considered as the states of a quadrotor which altogether twelve in number where, x, y and z denote the linear positions along the x, y, and z directions. $\dot{x}$,$\dot{y}$ and $\dot{z}$ denote linear velocities in the x, y, and z directions. ϕ, θ, and ψ denote, respectively, the roll, pitch and yaw angles, and $\dot{\phi}, \dot{\theta}$ and $\dot{\psi}$denote the corresponding velocities. The configuration of the quadcopter under consideration is depicted in Fig. 1.

Dynamics characteristics of the quad-coptor is modeled by eqns. (1)-(6). $d_h(t), d_\phi(t), d_\theta(t), d_\psi(t)$ are the disturbances acting on quadrotor.

$$\ddot{x} = \frac{(\sin\psi \sin\phi + \cos\psi \sin\theta \cos\phi)U1 - A_x\dot{x}}{m} \qquad (1)$$

$$\ddot{y} = \frac{(\sin\psi \sin\phi - \cos\psi \sin\theta \cos\phi)U1 - A_y\dot{y}}{m} \qquad (2)$$

$$\ddot{z} = \frac{-mg + (\cos\phi \cos\theta)U1 - A_z\dot{z}}{m} + d_h(t) \qquad (3)$$

$$\ddot{\phi} = \frac{\dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) - J_r\dot{\theta}\Omega_r + l(U2)}{I_{xx}} + d_\phi(t) \qquad (4)$$

$$\ddot{\theta} = \frac{\dot{\phi}\dot{\psi}(I_{zz} - I_{xx}) + J_r\dot{\phi}\Omega_r + l(U3)}{I_{yy}} + d_\theta(t) \qquad (5)$$

$$\ddot{\psi} = \frac{\dot{\theta}\dot{\phi}(I_{xx} - I_{yy}) + (U4)}{I_{zz}} + d_\psi(t) \qquad (6)$$

Table 1.Quadcopter dynamic system parameters

System parameters	Description
$I_{xx}, I_{yy}, I_{zz}(kgm^2)$	Moments of inertia
m (kg)	Mass of the vehicle
l (m)	Arm length of Quad-copter frame
b (Ns^2)	Coefficient of Thrust
$J_r(kgm^2)$	Moment of inertia of the rotor

WhereU_i , i = 1, 2, 3, 4 are the control forces for the quad-copter, which are as:

$$U_1 = b\left(\Omega_1{}^2 + \Omega_2{}^2 + \Omega_3{}^2 + \Omega_4{}^2\right) \qquad (7)$$

$$U_2 = bl\left(-\Omega_2{}^2 + \Omega_4{}^2\right) \qquad (8)$$

$$U_3 = bl\left(-\Omega_1{}^2 + \Omega_3{}^2\right) \qquad (9)$$

$$U_4 = d\left(-{\Omega_1}^2 + {\Omega_2}^2 - {\Omega_3}^2 + {\Omega_4}^2\right) \tag{10}$$

$$\Omega_r = (-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) \tag{11}$$

$\Omega_r$denotes the resultant angular speed of the motors and gravitational acceleration $g = 9.81 m/s^2$.

3. SLIDING MODE CONTROL

Sliding Mode Control [3] involves sliding surface and equivalent control which is designed to enforce the error vector toward the sliding surface [6]. Based on the governing equations (1)-(6), there are four control equations described by (15)-(18) which are designed so as to maintain the reference set point even in presence of external disturbances. The signal U_1 is developed and designed for guaranteed reference tracking of altitude and the others control inputs U_2, U_3 and U_4 are designed for controlling the attitude angles.

A) PD SMC:

First, the sliding surface is taken as as follows:

$$s = \dot{e} + \lambda e \tag{12}$$

Where $e = z_d - z$ is the error between the reference and actual for the altitude. $\lambda > 0$ is a design parameter. The control force U(t), involves an equivalent part, $u_{eq}(t)$, which is continuous and a discontinuous part $u_D(t)$. That is:

$$U(t) = u_{eq}(t) + u_D(t) \tag{13}$$

Nonlinear $u_D(t)$ behaves as a switching component. Gain k_D is a design parameter which helps to reach the phase. The well-known problem of Chattering can be significantly attenuated for the control law which is represented as:

$$u_D(t) = k_D \frac{s(t)}{|s(t)|+\delta} \tag{14}$$

The parameter δ is to be tuned so as to achieve chattering reduction. The complete controller equations are [15] as follows.

$$U_1 = \left(g + \lambda(\dot{z}_d - \dot{z}) + \ddot{z}_d + \frac{k_D s}{|s|+\delta}\right) \frac{m}{\cos\theta \cos\phi} \tag{15}$$

$$U_2 = \left[\lambda\left(\dot{\phi}_d - \dot{\phi}\right) + \ddot{\phi}_d - \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) + \frac{J_r \dot{\theta} \Omega_r}{I_x} + \frac{K_D s}{|s|+\delta}\right] \frac{I_x}{l} \tag{16}$$

$$U_3 = \left[\lambda\left(\dot{\theta}_d - \dot{\theta}\right) + \ddot{\theta}_d - \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r \dot{\phi} \Omega_r}{I_y} + \frac{K_D s}{|s|+\delta}\right] \frac{I_x}{l} \tag{17}$$

$$U_4 = \left[\lambda\left(\dot{\psi}_d - \dot{\psi}\right) + \ddot{\psi}_d - \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{K_D s}{|s|+\delta}\right] I_z \tag{18}$$

B) PID Sliding Mode Control:

Now the sliding surface is given as:

$$s = \dot{e} + \lambda_1 e + \lambda_2 \int e\, dt \tag{19}$$

Where $\lambda_1, \lambda_2 (> 0)$ are tuning parameters. Applying the sliding condition, $\dot{s} = 0$, one can find out:

$$u_{eq} = (g + \lambda_1(\dot{z}_d - \dot{z}) + \lambda_2(z_d - z) + \ddot{z}_d) \frac{m}{\cos\phi \cos\theta} \tag{20}$$

Hence, one finally obtains

$$U_1 = (g + \lambda_1(\dot{z}_d - \dot{z}) + \lambda_2(z_d - z) + \ddot{z}_d + k_D \frac{s(t)}{|s(t)|+\delta}) \frac{m}{\cos\phi \cos\theta} \tag{21}$$

Assuming reference as step signal $\dot{z}_d = 0$, $\ddot{z}_d = 0$. $\ddot{z}$ is obtained as:

$$\ddot{z}+\lambda_1\dot{z}+\lambda_2 z=\lambda_2 z_d+k_D\frac{s}{|s|+\delta}+d \qquad (22a)$$

Which is for PD only is given by

$$\ddot{z}+\lambda\dot{z}=k_D\frac{s}{|s|+\delta}+d \qquad (22b)$$

λ_1 and λ_2 are responsible for the stability of response. A Lyapunov function is defined as $\boldsymbol{V}=\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{s}^{\mathbf{2}}>0$. $\dot{V}$ must be negative-definite which necessitates $k_D>0$. It follows from the dynamic of the quadrotor as described by (22a) and (22b) for PD and PID SMC, respectively, that a disturbance pulse gets attenuated exponentially with PID-SMC which remains present with constant amplitude for PD-SMC.

4. RESULTS

This portion demonstrates the performance of the PD SMC showcasing the response of the quadcoptor system under consideration corresponding to different reference points for altitude, attitude angles. Further, the response of the whole closed loop system is investigated in presence of disturbances. First, the PD SM controller is applied for the altitude and attitude control. The reference values are set as z_d = 1m, ϕ_d=5°, θ_d=5°, ψ_d=5°.

Table 2.Tuning parameters

Parameter	**Altitude and Attitude Angles**			
	z	$\boldsymbol{\phi}$	$\boldsymbol{\theta}$	$\boldsymbol{\psi}$
$\boldsymbol{\lambda}$	68	5	5	5
$\boldsymbol{K_D}$	100	100	100	100
$\boldsymbol{\delta}$	0.3	0.3	0.3	0.3

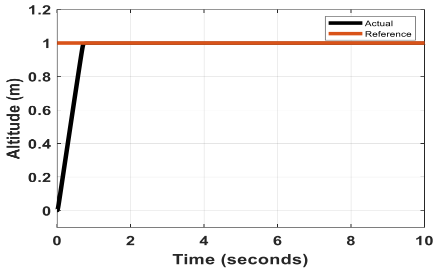

Fig.1 Altitude Response with PD SMC

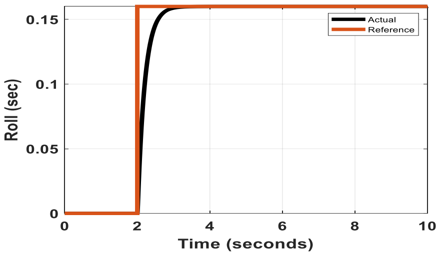

Fig.2 Roll response with PD SMC

A disturbance pulse train is applied at altitude of amplitude 30N and period 100 sec and pulse width 50%. Another disturbance pulse train is applied at roll of amplitude 2.5N and period 100 sec and pulse width 50%. So, PD sliding surface is responding to the disturbance but it is not rejecting the disturbance as shown in Figure 5 and 6 for altitude and roll, respectively. These figures show that the disturbance is present in the responses with constant magnitude. The performance may be improved by adding an integral part to the PD sliding Surface. The PID SMC is then implemented for the altitude and attitude angles. A disturbance is applied at 100s. The integral action rejects the disturbance and the responses are maintained at their reference values which are not possible with PD only. It is observed that the disturbance is attenuated in notable amount in case of PID SMC.

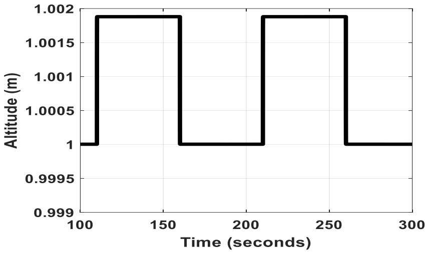

Fig. 6. Altitude Response with PD SMC in presence of disturbances

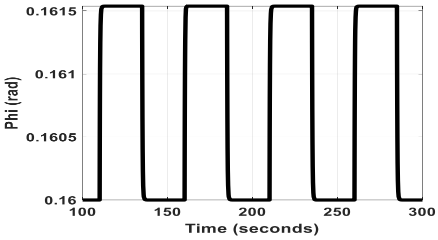

Fig. 7. Roll Response with PD SMC in presence of disturbances

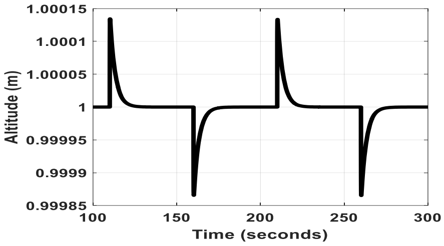

Fig. 8. Altitude Response with PID SMC in presence of disturbance

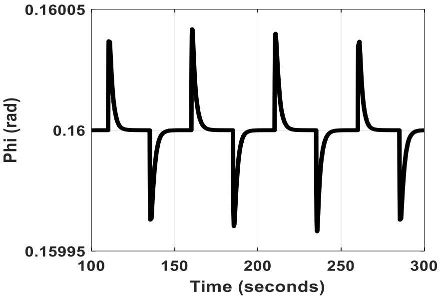

Fig. 9. Roll Response with PID SMC in presence of disturbance

5. CONCLUSION

The presented work develops a SMC based on PD sliding surface for full control of a quadrotor. The PD-SMC is tested analytically and through simulations. The simulation results show a successful tracking performance. However, this control scheme fails to achieve tracking when disturbances are applied. An integral sliding surface is augmented with the PD SMC to enable tracking of altitude and attitude commands even in presence of disturbances. The simulations verify the theoretical claim made towards disturbance rejection capability of PID-SMC.

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Biographies

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