OFDM AF FG Relaying as an Energy Efficient Solution for the Next Generation Mobile Cellular Systems

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Abstract

We analyzed the bit error rate (BER) and ergodic capacity performance improvement of dual-hop OFDM amplify and forward (AF) relay system with fixed gain (FG) at relay station (R), all attained through ordered subcarrier mapping (SCM) at R. In previous works we showed that in the region of small values of average signal-to-noise ratios (SNRs) on hops best-to-best SCM (BTB SCM) scheme should be applied, while in the region of medium and high values of average SNRs on hops, BER performance improvement is achieved by implementing best-to-worst SCM (BTW SCM) scheme. In this paper we analyzed a new solution, where in the assumed relay system with BTB SCM, one or more subcarriers having the lowest SNRs on each hop are omitted. Furthermore, we examined power balance of the system implementing SCM through comparison of BER performance expressed as a function of power transmitted per subcarrier, with BER performance of the ordinary OFDM AF FG relay system, as well as with the performance in case of direct transmission between source of information (S) and destination terminal (D). OFDM based relay systems are already included in IMT-Advanced standards, and all the obtained BER and ergodic capacity results have confirmed that OFDM AF FG relay system with appropriate SCM scheme implemented at
R station represents very interesting and energy efficient solution for the next generation mobile cellular systems.

**Keywords:** OFDM, amplify-and-forward, relay, fixed gain, ordered subcarrier mapping, BER, ergodic capacity, energy efficiency.

### 1 Introduction

The expansion of wireless communication systems over the last decade was enabled through implementation of appropriate technical solutions, which have succeeded to meet customer’s demands. In order to achieve the required high data rates and quality of service level, the next generation of cellular systems, IMT-Advanced systems, will incorporate, among other advanced technical solutions, OFDM (Orthogonal Frequency Division Multiplexing) based relay (R) stations, for the system capacity enhancement and coverage area extension, [1]. The accepted relay solution assumes dual-hop scenario, where the source terminal (S) cannot achieve direct communication with the destination terminal (D). Regenerative R stations, that decode the received signal and again re-encode it before forwarding toward destination (D) terminal, are already included in the accepted standards for IMT-Advanced systems, while non-regenerative R stations, that amplify and forward (AF) the received signal, are expected to become a part of the next IMT-Advanced specifications. R station implementing AF technique may amplify the received signal with the fixed gain (FG), or with the variable gain (VG), depending on its possibility to estimate the channel between the source of information (S) and R. Non-regenerative relay systems are less complex than regenerative systems, and they introduce shorter latency. Analyses presented in this paper consider the OFDM non-regenerative (AF) relay system applying FG at the R station.

Although the standards for both IMT-Advanced systems, LTE-Advanced and Wireless MAN-Advanced, have been accepted, there is still ongoing intensive research activity on the OFDM based relay systems, all with the goal to further improve their performance, energy efficiency or to optimize their functions. One of the methods for capacity enhancement and/or bit error rate (BER) improvement, which is at the same time energy more efficient, is implementation of ordered subcarrier mapping (SCM) at the R station, where the subcarriers from the S-R link (first hop) are mapped to appropriate subcarriers on the R-D link (second hop), all in accordance with their instantaneous signal-to-noise ratios (SNRs) [2–8]. The concept of sub-
carrier mapping (SCM) at the R station was introduced in [2], and shortly afterwards was discussed in [3] and [4]. In these papers it was proved that the capacity of dual-hop OFDM based relay system can be maximized if the subcarriers from the first hop are ordered according to their instantaneous SNRs, and then mapped to corresponding subcarriers on the second hop, which are also ordered in accordance with their instantaneous SNRs. This SCM scheme is denoted as Best-to-Best SCM (BTB SCM). However, when BER performance of the OFDM AF based relay systems is considered, it was shown that the BTB SCM scheme minimizes the BER results only in the region of small SNRs [5]. On the other side, for the medium and high SNR values, the SCM scheme denoted as Best-to-Worst SCM (BTW SCM) is proposed for implementation in order to minimize BER performance [5, 6]. BTW SCM scheme assumes that the subcarriers from the first hop, which are increasingly ordered according to their SNRs, are mapped to the decreasingly ordered subcarriers on the second hop. We derived closed-form BER performances of differentially phase shift keying (DPSK) and binary phase shift keying (BPSK) modulated OFDM AF FG relay systems with SCM in [6, 7], thus analytically proving the optimality of the previously described proposal. We have also examined analytically ergodic capacity performances of OFDM AF FG relay systems with SCM in [7]. However, some issues regarding energy efficiency, and possibility for implementation just one SCM scheme for both capacity enhancement and BER performance improvement, remained opened.

Thus, in order to examine the power balance of SCM solution, in this paper we compare BER performance of differentially phase shift keying (DPSK) modulated OFDM AF FG relay system implementing SCM, expressed as a function of power transmitted per subcarrier, with BER performance of ordinary OFDM AF FG relay system and with the case of direct OFDM transmission between S and D. Additionally, we analyzed if omitting one or more subcarriers having the lowest SNRs on both hops may sufficiently improve BER performance of OFDM AF FG relay system with BTB SCM, to attain or outperform, in all SNR regions, BER performance of the same system implementing BTW SCM. We proved in [8] that this kind of solution in OFDM AF VG relay system makes BTB SCM optimal SCM scheme for BER improvement for all SNR values on both hops. Also, in this paper we give capacity analyses of OFDM AF FG relay system with BTB SCM, through derivation of both upper and lower bounds of ergodic capacity, and present the level of capacity improvement in comparison with the same system without SCM. The paper is organized as follows. Section 2 describes
the analyzed system model. In Section 3, analytical derivation of BER and ergodic capacity performance is given. The obtained results are presented in Section 4. Finally, in Section 5 we present some concluding remarks.

2 System Model

We considered the same relaying system as the one presented in [7]. It assumes a dual-hop OFDM FG AF relay system, in which three communication terminals, all equipped with single antenna are involved in communication process. There is no direct link between S and D, and R operates in half-duplex mode. The basic block scheme of a relay terminal is presented in Figure 1. R first performs OFDM demodulation through FFT (Fast Fourier Transformation). Then, the subcarriers from the first hop are mapped to the appropriate subcarriers on the second hop in accordance to their instantaneous SNRs. It is assumed that the SCM block knows channel state information on both S-R and R-D links, The next processing step is getting back the signal into time domain through IFFT (Inverse Fast Fourier Transformation) block. The obtained OFDM signal is amplified by Furthermore, the gain G, and then transmitted toward destination terminal. In order to correctly demodulate the received signal D has to know subcarrier permutation function performed at R.

The post-FFT signal on the $i$-th subcarrier, received at the relay station, is given by:

$$Y_{R,i} = X_{1,i}H_{1,i} + N_{1,i}, \quad 1 \leq i \leq M,$$

where $M$ is the total number of subcarriers and $X_i$ is the data symbol sent by source on the $i$-th subcarrier. $N_{1,i}$ represents additive white Gaussian noise.
on the \( i \)-th subcarrier with variance \( \mathbb{E}(|N_{1,i}|^2) = N_{01} \), with \( \mathbb{E}(\cdot) \) denoting the expectation operator. Assuming that the SCM function \( \nu(i) \) performs mapping of the \( i \)-th subcarrier from the first hop to the \( k \)-th subcarrier on the second hop, the frequency domain signal at \( D \) can be written as

\[
Y_{D,k} = G H_{2,k} Y_{R,\nu(i)} + N_{2,k} = G H_{2,k} H_{1,i} X_i + G H_{2,k} N_{1,i} + N_{2,k}, \quad 1 \leq k \leq M \tag{2}
\]

where \( H_{2,k} \) denotes the \( k \)-th subcarrier transfer function on the second hop. \( N_{2,k} \) is the additive white Gaussian noise at the destination on the \( k \)-th subcarrier, with variance \( \mathbb{E}(|N_{2,k}|^2) = N_{02} \).

The fadings in the S-R and R-D channels are assumed to be independent and identically distributed (i.i.d.) among the subcarriers. Moreover, we assume Rayleigh fading in each subcarrier, so that the PDF and the CDF (Cumulative Distributive Function) of the SNR in each of the S-R subchannels is given by

\[
f_{SR}(x) = \lambda_{SR} \exp(-\lambda_{SR}x) \quad \text{and} \quad F_{SR}(x) = 1 - \exp(-\lambda_{SR}x),
\]

while the corresponding PDF and CDF of the SNR in each of the R-D subchannels are given by

\[
f_{RD}(x) = \lambda_{RD} \exp(-\lambda_{RD}x) \quad \text{and} \quad F_{RD}(x) = 1 - \exp(-\lambda_{RD}x),
\]

respectively. \( \lambda_{SR} = 1/\gamma_{SR} \) and \( \lambda_{RD} = 1/\gamma_{RD} \) denote the inverse of the average SNRs on the S-R and R-D links, respectively. Using (2) and the described gain, the end-to-end SNR on the \( k \)-th subcarrier can be presented as

\[
\gamma_k,\text{end} = \gamma_i,SR \gamma_k,\text{RD} \gamma_k,\text{RD} + \rho, \tag{3}
\]

where \( \rho \) is the coefficient that depends through the gain \( G \) as \( \rho = \epsilon_R / (G^2 N_{01}) \cdot \gamma_i,SR \) and \( \gamma_k,\text{RD} \) represent instantaneous SNRs on the \( i \)-th subcarrier of the S-R link and \( k \)-th subcarrier of the R-D link, respectively, while \( \epsilon_R \) is the symbol energy transmitted by \( R \).

### 3 Performance Analysis

In the following part we use the earlier obtained results on probability density function (PDF) and moment generating function (MGF) of SNR for the \( k \)-th subcarrier at \( D \) [6, 7], which are necessary tools for conduction of BER and ergodic capacity performance analysis.

#### 3.1 BER Performance Analysis

We have conducted BER performance analysis for the dual-hop DPSK modulated OFDM AF FG relay system with SCM, in the case of Rayleigh fading
channels on both hops in [6]. Using the ordered statistics, we first derived PDF of the increasingly ordered random variables having exponential distribution, and through the similar approach, PDF of SNR for decreasingly ordered exponentially distributed random variables. These PDF functions actually correspond to PDF of the SNRs of the ordered subcarriers from the first hop or from the second hop in the considered scenario with Rayleigh fading statistics. Using this, after some mathematical transformations, we derived PDF for the $k$-th subcarrier at D for the systems implementing BTB SCM and BTW SCM. However, for the case of DPSK modulation implemented, it is more convenient to use MGF based approach for BER performance analysis, as in that case BER for the $k$-th subcarrier at D is obtained through [9]:

$$P_{b,k} = 0.5 \cdot \mathcal{M}_{\gamma_{k,\text{end}}}(1),$$

with $\mathcal{M}_{\gamma_{k,\text{end}}}(\cdot)$ denoting MGF of SNR function. BER for the analyzed OFDM AF FG relay system with SCM is derived through averaging (4) over all $M$ subcarriers:

$$P_b = \frac{1}{M} \sum_{k=1}^{M} P_{b,k},$$

Thus, we derived MGF of SNR at the $k$-th subcarrier at D for the relay systems implementing both analyzed SCM schemes. In the scenario with BTB SCM scheme at the R station, MGF of SNR for the $k$-th subcarrier at D is obtained as [6]:

$$\mathcal{M}_{\gamma_{k,\text{end}}}(s) = \frac{1}{\bar{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{k-1} \frac{\alpha_j \alpha_i}{T_j(s)} \times \left[ \frac{1}{\beta_i} + \exp \left( \frac{\rho A_{j,i}}{T_j(s)} \right) E_1 \left( \frac{\rho A_{j,i}}{T_j(s)} \right) \left( \frac{\rho}{\bar{\gamma}_{RD}} - \frac{\rho A_{j,i}}{\beta_i T_j(s)} \right) \right].$$

where $E_1(\cdot)$ represents the exponential integral function defined in [10, (5.1.1)]. The coefficients $\alpha_i$ and $\beta_i$ are given through:

$$\alpha_i = (-1)^i M \binom{M-1}{k-1} \binom{k-1}{i} \quad \text{and} \quad \beta_i = i + M - k + 1$$

In (7), $(\cdot)$ denotes binomial coefficients. The coefficients $A_{j,i}$ and $T_j(s)$ are introduced in (6) for the easier representation of this relation, and they are equal to:

$$A_{j,i} = \beta_j \beta_i / \bar{\gamma}_{SR} \bar{\gamma}_{RD} \quad \text{and} \quad T_j(s) = s + \beta_j / \bar{\gamma}_{SR}. $$
Using the same approach, MGF of SNR for the $k$-th subcarrier at D in the case of BTW SCM scheme implemented at R is derived as [6]:

$$M_{\gamma_{\text{end}}}(s) = \frac{1}{\gamma_{\text{SR}}} \sum_{j=0}^{k-1} \sum_{i=0}^{M-k} \alpha_j \delta_i \left( T_j(s) \right)$$

$$\times \left[ \frac{1}{\varepsilon_i} + \exp \left( \frac{\rho B_{j,i}}{T_j(s)} \right) E_1 \left( \frac{\rho}{\gamma_{\text{RD}}} - \frac{\rho B_{j,i}}{\varepsilon_i T_j(s)} \right) \right]. \quad (9)$$

In (9) the coefficients $\delta_i$ and $\varepsilon_i$ are equal to

$$\delta_i = (-1)^i M \begin{pmatrix} M-1 \\ k-1 \end{pmatrix} \begin{pmatrix} M-k \\ i \end{pmatrix} \quad \text{and} \quad \varepsilon_i = i + k, \quad (10)$$

while $B_{j,i}$ can be written as

$$B_{j,i} = \beta_j \varepsilon_i / \gamma_{\text{SR}} \gamma_{\text{RD}}. \quad (11)$$

By substituting (6) or (9) in (4) and then in relation (5), BER performance of DPSK modulated OFDM AF FG relay system implementing BTB SCM or BTW SCM is obtained. As we already have mentioned, it is known that OFDM AF FG relay system implementing BTW SCM outperforms the system with BTB SCM in terms of achievable BER performance, in the regions of medium and high values of average SNRs on both hops. The same holds for the OFDM AF VG relay systems implementing SCM [9]. However, in [9] we have shown that in the OFDM AF VG relay system with BTB SCM, omitting just the worst subcarriers on both hops, i.e. the subcarriers with the lowest SNRs, makes the system with BTB SCM optimal solution for BER performance improvement in all the SNR regions. Thus, we wanted to examine if we do not use the subcarriers with the lowest SNRs on both hops in the OFDM AF FG relay system with BTB SCM may improve enough BER performance of this system to attain and prevail BER performance of OFDM AF FG system with BTW SCM, in all the SNR regions. Analytically, it means that in the relation (5) summation starts from $k = 2$ for the system with BTB SCM.

In order to examine if the additional signal processing in systems with SCM can be justified considering the issue of power consumption, the obtained BER results should be presented as a function of power transmitted per subcarrier in the whole system, $P_T$. We assumed equal power allocation among the S and R station, i.e. $P_S = P_R = P_T/2$ and among all the subcarriers. Now, the average SNRs on S-R and R-D links can be written as
\[ \tilde{\gamma}_{SR} = A_1 P_S \text{ and } \tilde{\gamma}_{RD} = A_2 P_R, \]

respectively, where \( A_1 \) and \( A_2 \) include parameters as the antenna gains, path loss, noise power and similar. For example, if using Friis propagation model, \( A_i \) \((i = 1, 2)\), can be written in the form

\[ A_i = \frac{G_{t,i} G_{r,i} \lambda^2}{(4\pi)^2 d_i^\alpha L N_0}, \]  

(12)

where \( G_{t,i} \) is the transmitter antenna gain on the \( i \)-th hop, \( G_{r,i} \) is the receiver antenna gain, \( \lambda \) is the wavelength, \( d_i \) is the distance between the transmitter and receiver on the \( i \)-th hop, \( L \) is the system loss factor, \( \alpha = 2 \) for free space and \( 3 < \alpha < 4 \) in urban environment, while \( N_0 \) is the noise variance at the \( i \)-th hop. Without loss of the generality, we took that the transmitter antenna gains at the S and R are equal, \( G_{t,1} = G_{t,2} \), and the receiver antenna gains at the R and D are also equal, \( G_{r,1} = G_{r,2} \), as well as that the noise variances at the R and D are the same, \( N_{01} = N_{02} \). Moreover, we assumed that in the case of relayed transmission, S, R and D are placed on a straight line, and that all the links are affected by the same shadowing environment. The average SNR at D in the case of direct transmission can be written as, where for this simplified propagation model, by taking \( \alpha = 3 \), \( A_{eq} \) is related to \( A_1 \) and \( A_2 \) through

\[ A_{eq} = \frac{A_2}{(1 + (A_2/A_1)^{1/3})^3}. \]  

(13)

For the case of direct OFDM transmission in Rayleigh fading environment, MGF of SNR for the \( k \)-th subcarrier has the form

\[ M_{\gamma_{k,eq}}(s) = \frac{1}{1 + s\tilde{\gamma}_{RD}} \]  

(14)

Substituting (14) into (4) and then in (5), BER of DPSK modulated OFDM system is obtained.

### 3.2 Ergodic Capacity Analysis

It is proven that BTB SCM scheme maximizes the achievable ergodic capacity of OFDM based relay systems [2–4]. Thus, in this part we will analyze only the ergodic capacity performance of the OFDM AF FG relay system with BTB SCM. For the ergodic capacity analysis it is necessary to know PDF of SNR for the \( k \)-th subcarrier at D. In the case of BTB SCM scheme
applied at R, this function has the form as given in [7]:

\[
\begin{align*}
    f_{\gamma_\text{end}}^{\text{BTB}}(x) &= 2 \bar{\gamma} \frac{1}{\bar{\gamma}_\text{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{k-1} \alpha_j \alpha_i \exp \left( -\beta_j \frac{x}{\bar{\gamma}_\text{SR}} \right) \\
    &\times \left[ \frac{\rho \beta_j x}{\beta_i \bar{\gamma}_\text{SR} \bar{\gamma}_\text{RD}} K_1 \left( 2 \sqrt{\rho \frac{\beta_j \beta_i x}{\bar{\gamma}_\text{SR} \bar{\gamma}_\text{RD}}} \right) + \frac{\rho}{\bar{\gamma}_\text{RD}} K_0 \left( 2 \sqrt{\rho \frac{\beta_j \beta_i x}{\bar{\gamma}_\text{SR} \bar{\gamma}_\text{RD}}} \right) \right].
\end{align*}
\]

(15)

where \( K_0(\cdot) \) and \( K_1(\cdot) \) are zero and first order modified Bessel functions of the second kind defined in [10, (9.6.21), (9.6.22)]. The presence of these functions in the expression for the PDF of SNR for the \( k \)-th subcarrier at D prevail the possibility for finding closed-form solution for the ergodic capacity of this system. However, taking into account the concave form of the logarithmic function, and the Jensen’s inequality, we found upper bound of the \( k \)-th subcarrier ergodic capacity through

\[
    C_k = 0.5 \cdot E(\log_2(1 + \gamma_{k,\text{end}})) \leq 0.5 \log_2(1 + E(\gamma_{k,\text{end}})).
\]

(16)

where \( E(\cdot) \) denotes the expectation operator, and factor 0.5 appears due to symbol transmission in two time slots (half-duplex relay). Closed-form expression is derived for \( E(\gamma_{k,\text{end}}) \) in [7] as

\[
    E(\gamma_{k,\text{end}}) = \frac{1}{M} \sum_{k=1}^{M} C_k
\]

(18)
In order to attain more complete insight in ergodic capacity performance of OFDM AF FG relay system with SCM, we also derived a tight lower bound of its achievable ergodic capacity. Using the expression of end-to-end SNR for the $k$-th subcarrier (3), ergodic capacity of the $k$-th subcarrier at D can be split in the following way [11]:

$$C_k = I_A - I_B = I_x - I_y - I_B.$$

$I_x$ is derived in closed-form as [11]:

$$I_x = \frac{4}{\ln(2)} \frac{\rho}{\gamma_{SR} \gamma_{RD}} \sum_{i=0}^{k-1} \alpha_i \exp \left( \frac{\beta_i}{\gamma_{SR}} \right) \frac{1}{\beta_i} \sum_{j=0}^{k-1} \alpha_j \sqrt{\frac{1}{\beta_j \beta_i} S_{-2,1} \left( 2 \sqrt{\frac{\rho \beta_j \beta_i}{\gamma_{SR} \gamma_{RD}}} \right)}.$$

(20)

where $S_{\mu,\nu}(\cdot)$ represents Lommel function defined in [12, (8.57)]. A closed-form solution for $I_B$ from (19) is also found in [11]:

$$I_B = \frac{-1}{2 \ln(2)} \sum_{i=0}^{k-1} \frac{\alpha_i}{\beta_i} \exp \left( \frac{\beta_i \rho}{\gamma_{RD}} \right) E_1 \left( \frac{\beta_i \rho}{\gamma_{RD}} \right).$$

(21)

The term $I_y$ in (19) could not be derived as a closed-form solution, but it can be upper bounded using again Jensen’s inequality, thus obtaining [11]:

$$I_y \leq \frac{1}{\gamma_{SR} \ln(2)} \sum_{i=0}^{k-1} \alpha_i \exp \left( \frac{\beta_i}{\gamma_{SR}} \right) \frac{1}{\beta_i} \sum_{j=0}^{k-1} \alpha_j \exp \left( \frac{\rho \beta_j}{\gamma_{RD} L(i)} \right) \Gamma \left( 0, \frac{\rho \beta_j}{\gamma_{RD} L(i)} \right),$$

(22)

where $\Gamma(\cdot, \cdot)$ denotes the upper incomplete Gamma function defined in [10, (8.350.2)]. $L(i)$ is equal to:

$$L(i) = \left( \frac{\gamma_{SR}}{\beta_i} \right)^2 - \left( \frac{\gamma_{SR}}{\beta_i} + \left( \frac{\gamma_{SR}}{\beta_i} \right)^2 \right)^2 \exp \left( -\frac{\beta_i}{\gamma_{SR}} \right).$$

(23)

The obtained upper bound for the $I_y$ actually presents the term which defines the lower bound for the ergodic capacity of the $k$-th subcarrier in the considered OFDM AF FG relay system with BTB SCP, and in final, through (18) it gives the lower bound of the ergodic capacity of the whole considered system.

## 4 Results

The subsequent presented analytical and simulation results assume perfectly synchronized dual-hop OFDM AF FG relay system implementing SCM. It
is assumed that the noise variances at R and D are equal, i.e. $N_{02} = N_{02}$, as well as the average symbol energies transmitted by S and by R, $\epsilon_S = \epsilon_R$. The so-called semi-blind scenario is considered, where R uses knowledge on channel state information (CSI) about the S-R link to calculate the gain $G$:

$$G^2 = \mathbb{E} \left[ \frac{\epsilon_R}{|H_{1,k}|^2 \epsilon_S + N_{01}} \right],$$

yielding

$$G^2 = \frac{\epsilon_R}{\epsilon_S \mathbb{E}[|H_{1,k}|^2]} e^{1/\gamma_{SR}} E_1 \left( \frac{1}{\gamma_{SR}} \right).$$

Simulation results are obtained through Monte Carlo simulations of the part of the OFDM system that belongs to frequency domain, what is possible and accurate approach as we have assumed perfect synchronization among the communication terminals. The subcarrier channel transfer functions on both hops are generated as independent Gaussian complex random variables with zero mean.

### 4.1 BER Performance

BER performances of DPSK modulated OFDM AF FG relay systems with SCM, as a function of total transmitted power per subcarrier $P_T$, are given in Figure 2. In order to get an insight in power balance of OFDM AF FG relay system with SCM, comparison of BER performances, with the OFDM AF FG relay system and with the direct transmission scenario are also provided. We have chosen parameters $A_1 = 2$ and $A_2 = 10$, which for downlink communication corresponds to the scenario where the distance between the R and D terminals is shorter than the one between the S and R terminals. For a direct transmission scenario $A_{eq}$ is calculated using (13).

From the given BER plots, the advantage of using relay transmission over the direct transmission is clear, as the power saving per subcarrier is about 2 dB for the BER values lower than $10^{-2}$. Comparing to basic OFDM AF FG relay system, additional 1dB of transmitted power can be saved for achieving the same BER performances if BTW SCM scheme is used, for all the BER values below $10^{-1}$. When BTB SCM scheme is concerned, it is shown that it achieves the best BER performances in the regime of very low transmitted power, i.e. in the cases where the average SNRs on both hops are low. In the assumed scenario, for $P_T$ values above 15 dB, the system with BTB SCM scheme has worse BER performances even than the system with direct transmission. Thus, in order to achieve the best BER performance, for
Figure 2 BER for DPSK modulated OFDM system and OFDM AF FG relay system with and without SCM, as a function of $P_T$.

The same transmitted energy, in the region of small SNRs, BTB SCM scheme should be implemented, while for the average and medium values of SNRs, BTW SCM scheme should be applied.

Figure 3 gives analytical and simulation results for DPSK modulated OFDM AF FG relay system implementing both BTB SCM and BTW SCM, in a scenario with equal average SNR values on both hops. For the sake of comparison, BER performance of the ordinary OFDM AF FG relay system is also presented. As it is obvious that in the relay system implementing BTB SCM, the subcarriers with the lowest SNRs on both hops, which are mapped at R station creating subcarrier pair $k = 1$, have the greatest contribution on increase of BER values, than we present the BER performances of relay system employing BTB SCM without “worst” subcarrier pair (w/o $k = 1$), for a different total of numbers of subcarriers in the system ($M = 16, 32, 64$).

The presented results confirm the accuracy of the undertaken analytical approach, as we have excellent matching between the analytical results and simulation results for all SNR values. As expected, in the region of low values of average SNR per hop, BTB SCM scheme achieves the best BER performance. It outperforms the system with BTW SCM scheme for the values
of average SNR per hop up to approximately 6.5 dB. For higher values of average SNR system with BTW SCM has the lowest BER, and its advantage in BER performances increases very fast as the values of average SNRs increase. Thus for example, for the BER value of $10^{-2}$, it already achieves SNR gain of 3 dB over the OFDM AF FG relay system and more than 7 dB SNR gain in comparison with the system implementing BTB SCM.

Let us now consider the system implementing BTB SCM, where the worst subcarrier pair is not used. From Figure 3 it can be seen that, besides for the very small values of average SNR per hop, only in the region of very high values of average SNR per hop (about 29 dB for $M = 32$ and 20 dB for $M = 16$), and for the cases of smaller number of subcarriers in OFDM system, this solution outperforms the system with BTW SCM in terms of BER. However, having in mind that in the real-case scenarios, such a big values of average SNRs on both hops will appear with very small probability, and that the OFDM systems with $M = 16$ and $M = 32$ subcarriers are not of the great practical importance, we wanted to examine if omitting several “worst” subcarrier pairs in the OFDM systems with greater number of subcarriers may bring enough BER performance improvement to have one SCM as an optimal solution for all SNR values of interest.

![Figure 3 BER for DPSK modulated OFDM AF FG relay system with and without SCM.](image-url)
Figure 4 presents BER performance of OFDM AF FG relay system with SCM having $M = 256$ and $M = 512$ subcarriers, where in the case of BTB SCM having $M = 256$ subcarriers $k = 6$ and $k = 8$ worst subcarrier pairs are not used, while in the system with $M = 512$ subcarriers, $k = 12$ and $k = 16$ worst subcarrier pairs are omitted. The given results confirm again that for the SNR values of interest, for the BER performance improvement in OFDM AF FG relay systems, BTB SCM should be implemented in the regions of small values of average SNRs on both hops, and BTW SCM should be implemented in the region of medium and high values of average SNRs on hops.

4.2 Ergodic Capacity

Figure 5 presents the obtained lower and upper bounds of the ergodic capacity for the considered system, as well as the ergodic capacity graph obtained through simulations. In order to identify the level of capacity improvement achieved through implementation of BTB SCM scheme, the simulation results for the OFDM AF FG relay system without (w/o) SCM are also given. The average ergodic capacity graphs are presented as a function of SNR.
The results presented in Figure 5 approve that the obtained analytical results present very tight upper and lower bounds, as they deviate very little from the simulation obtained results. This is more obvious through the data given in Table 1.

Table 1 presents values of the average ergodic capacity of OFDM AF FG relay system with BTB SCM, obtained through simulations and through analytical calculations of lower and upper bounds, as well as the ratios of analytically obtained results to simulation results expressed in percentages. For the sake of comparison, average ergodic capacity values of the OFDM AF FG relay system without SCM are also included, and the percentages of ergodic capacity enhancement attained through BTB SCM.

First of all, Table 1 shows that implementing BTB SCM can enhance ergodic capacity of OFDM AF FG relay system up to 30%, and the highest capacity enhancement is achieved in the region of small average SNR values on both hops. This is very important as it means the SCM technique may achieve the requested QoS in the case of very bad channel conditions on both hops, i.e. in the scenario where the regular OFDM AF FG relay system
may fail in fulfilling end-user’s demands. Besides this, from Table 1 we can see that the derived upper bound deviate from the simulation results up to 2.7%, while the derived lower bound differs from the simulation results up to 2.12%. Both these results confirm the accuracy of the undertaken analytical approaches, and the obtained ergodic capacity bounds may be effectively used for the assessment of the performance of OFDM AF FG relay system with SCM.

## 5 Conclusions

We analyzed the level of BER performance improvement and ergodic capacity enhancement achieved through implementation of ordered subcarrier mapping (SCM) at R station of OFDM AF FG relay system, which represents a very interesting solution for the upcoming specifications of IMT-Advanced mobile cellular systems. It has been shown analytically, and completely verified through simulations, that significant BER performance improvement may be achieved in the considered relay system if BTB SCM is implemented in the region of small values of average SNRs on both hops, and BTW SCM scheme in the region of medium and high values of average SNRs on both hops. Furthermore, we have shown that in OFDM AF FG relay system with BTB SCM, even when several subcarriers with the lowest SNRs from the both hops are not used, the BER performance in the region of medium and high values of average SNRs, which are of interest, cannot be improved enough to outperform the system implementing BTW SCM. Additionally, we presented energy efficiency of the considered SCM solution through presenting BER performance improvement achieved for the same power transmitted per subcarrier, in comparison with the ordinary OFDM AF FG relay system and with the direct case transmission.
When ergodic capacity of the system is considered, we analyzed only the solution assuming implementation of BTB SCM, which is the SCM scheme that is proven to maximize capacity for all the SNR values on both hops. We derived closed-form analytical expressions representing tight upper and lower bounds of the achievable ergodic capacity of the OFDM AF FG relay system with BTB SCM. These bounds can be efficiently used for the assessment of the ergodic capacity of the considered system, as they deviate from the simulation results less than 3% for all the SNR values. Ergodic capacity performance comparison of the analyzed relay system implementing BTB SCM, with the relay system without SCM has shown that ergodic capacity may be enhanced up to 30% in the region of small values of average SNRs on both hops. This is particularly important as it means significant capacity enhancement in the scenario with bad channel conditions on both hops.

All the presented BER and ergodic capacity performance have approved that dual-hop OFDM AF FG relay system with SCM at R station, represents very interesting energy efficient solution for the next generation mobile cellular systems.

References

Biographies

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