Analysis of Power Consumption in OFDM Systems

Irena Orović, Nikola Žarić, Srdjan Stanković, Igor Radusinović and Zoran Veljović

Faculty of Electrical Engineering, University of Montenegro, 20000 Podgorica, Montenegro; e-mail: zaric@ac.me

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Abstract

The energy efficiency of various OFDM systems, such as optical, mobile, Wireless and WiMaX systems, has been analyzed in this paper. High peak to average power ratio that may appear in modulation process is one of the main problems in OFDM systems. The influence of subcarriers number and modulation techniques to the peak-to-average power ratio in different OFDM based systems has been studied. The results of analysis are presented in numerous figures and tables. The main goal is to provide a comparative study that can be used for an optimal system selection with predefined power consumption requirements.

Keywords: OFDM, peak-to-average power ratio, transmission systems.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) provides transmission of multiple signals simultaneously over a single transmission path. Each signal is transmitted within its own frequency range (carrier), which is modulated by the data symbols. This technique provides high data rate even if relatively small frequency bandwidth is available. Also, OFDM based system has other favorable properties such as high spectral efficiency, robustness to

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channel fading and impulse interference [1, 2]. Therefore, OFDM has been used in numerous modern transmission systems, as for example, in Wireless IEEE 802.11a/g/n/e, digital audio broadcasting, digital video broadcasting (satellite, terrestrial, cable), fixed and mobile WiMaX, etc.

On the other hand, the OFDM systems are characterized by high peak-to-average power ratio (PAPR) [3–11]. A large PAPR appears as a consequence of the multicarrier OFDM signal nature. Namely, adding all carriers together can result in high maximum peak power, which can further increase with the number of carriers. For signals having large PAPR, the problem may appear during amplification at the transmitter. High signal values can be non-linearly amplified due to the small dynamic range of the amplifier, which will cause signal distortions. In order to avoid distortions, the mean signal power should be decreased, which then leads to a high power consumption and a low amplifier efficiency. Having in mind that the low power consumption is an important requirement in modern communication systems (e.g. in mobile systems), significant efforts have been made to develop techniques that are able to reduce PAPR [4–11].

In this paper we have analyzed the influence of subcarriers number to the PAPR for various OFDM based systems. Also, the influence of modulation techniques to the PAPR has been discussed. Further, we have considered different PAPR reduction techniques, as well as, their performances in different OFDM systems. A comparative analysis of these techniques is provided, which can be useful for optimal system selection.

2 OFDM Systems – Theoretical Background

OFDM based systems have been introduced to improve performances of previously used conventional transmission systems such as time division multiplex and frequency division multiplex based systems. High data transmission rates, high spectral efficiency, robustness, simple implementation are some of the characteristics that recommend OFDM as a standard for almost all modern communication systems

The block scheme of an OFDM system is shown in Figure 1. The input data sequence is usually processed with some digital modulator, such as BPSK, QPSK, 4QAM, 16QAM, 64QAM, etc. The choice of modulation technique significantly influences the data transmission rate. For example, if 16QAM is used, the transmission rate will be 8 times higher than if BPSK is used. However, the PAPR will be also significantly higher in the case of 16QAM, which will be discussed in the next section.

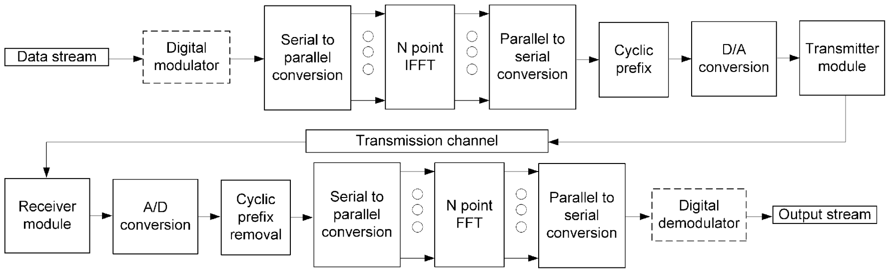

Figure 1 General block scheme of OFDM system.

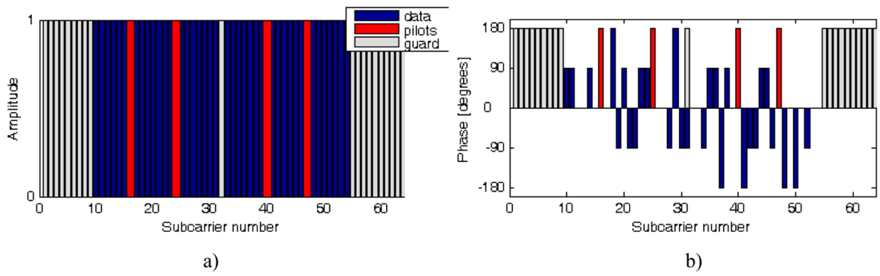

Figure 2 Frequency domain representation of one OFDM symbol: (a) amplitude of subcarriers, (b) phase of subcarriers.

The modulated sequence is converted into K parallel low bit rate data streams, where K represents the number of subcarriers used for data transmission. Each data stream is assigned to the appropriate subcarrier as follows:

$$s_k(t) = A_k e^{(2j\pi f_k t + \phi_k)}, \tag{1}$$

where A_k and φ_k are parameters of modulated symbol, while f_k represents the subcarrier frequency. The amplitude and phase representation of modulated subcarriers (in the case of QPSK modulation) for one OFDM symbol are shown in Figure 2. Note that, in this example, only $K = 40$ subcarriers are used to transmit data, while the remaining subcarriers are reserved for pilot symbols (4 subcarriers) and guard intervals (19 subcarriers). Also, having in mind properties of Fourier transform, the DC components (subcarrier at position 32) is not used. Pilot symbols are usually some known deterministic signals that are used for channel estimation and synchronization between transmitter and receiver. The guard intervals in OFDM systems are used to reduce the inter channel interference, and they are usually placed at the beginning and at the end of the spectrum.

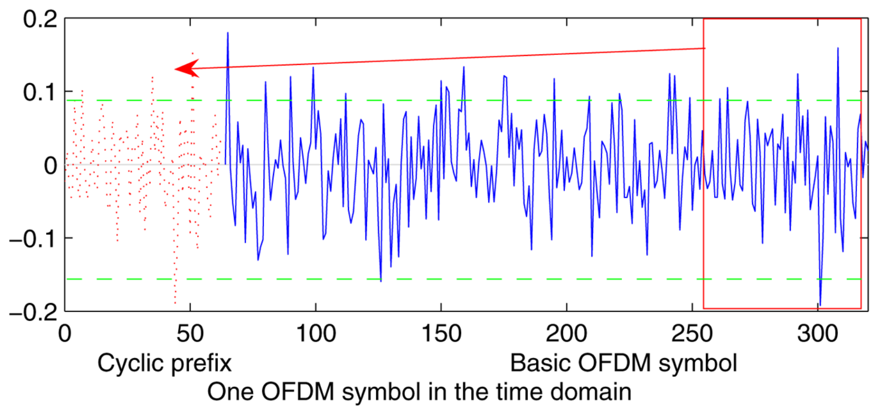

Figure 3 Time domain representation of an OFDM symbol, dotted line – cyclic prefix, solid line – basic OFDM symbol.

In order to obtain the OFDM symbol in the time domain, the inverse Fourier transform (IFFT block), is applied for each frequency bin:

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{(2\pi f_k t + \phi_k)}. \tag{2}$$

The output of the IFFT block is converted from serial to parallel stream, and the basic OFDM symbol of duration T_u is obtained (Figure 3). In order to avoid synchronization errors and to suppress inter symbol interference the cyclic prefix is added to the basic OFDM symbol. It is performed by copying last T_{cp} samples of the basic OFDM symbol to its beginning, as illustrated in Figure 3.

Finally, the D/A conversion is applied and the signal is amplified and transmitted.

OFDM demodulation requires the opposite procedure. After A/D conversion, the receiver discards the cyclic prefix and converts data from serial to parallel stream. The Fourier transform is then applied to recover the modulated symbols parameters. The output data stream is obtained after parallel to serial conversion and demodulation process.

3 Peak-to-Average Power Ratio Problem

A high peak-to-average power ratio is one of the problems in OFDM based transmission systems. It causes low energy efficiency of high-power amp-

Table 1 PAPR variations with respect to the number of subcarriers.

No. of carriers	64	128	256	512	1024	2048
Theoretical PAPR [dB]	18.06	21.07	24.08	27.09	30.1	33.11
PAPR for all carriers used [dB]	17.92	21	24.05	27.07	30.09	33.1
PAPR for 3/4 of carriers used [dB]	16.81	19.82	22.83	25.84	28.85	31.86
PAPR for 1/2 of carriers used [dB]	15.06	18.06	21.07	24.08	27.09	30.1
PAPR for 1/4 of carriers used [dB]	12.04	15.05	18.06	22.83	24.08	27.09

lifiers, D/A converters and other circuits used for transmission [1]. For the OFDM signal given by (2), the PAPR is defined as the ratio of maximal signal value and its average power:

$$\mathrm{PAPR}(x(t)) = \frac{\max(|x(t)|^2)}{E\{|x(t)|^2\}}, \tag{3}$$

where $E\{\cdot\}$ denotes the expected value. Due to the multicarrier nature, random subcarrier's phases and random subcarrier's amplitudes (in the case of QAM modulation) the maximal value of an OFDM signal can become significant.

The PAPR is very sensitive regarding number of carriers used for data transmission. Namely, by increasing the number of subcarriers used for data transmission, the maximal signal value increases and causes high PAPR [1]. The PAPR values for the cases of 64, 128, 256, 512, 1024 and 2048 available subcarriers and for different numbers of used subcarriers are reported in Figure 4 and Table 1 (QPSK modulation is considered). In the case of QPSK modulation, each subcarrier has the constant amplitude $|A_k| = 1$, and the maximal theoretical PAPR value in digital domain can be calculated as [1, 4]:

$$\mathrm{PAPR} = 10 \log 10(N). \tag{4}$$

Observe that PAPR increases by increasing the number of available and used subcarriers. Also, we can note that for the same number of available subcarriers PAPR decreases by reducing number of occupied subcarriers. However, it will also reduce the amount of data that can be transmitted within one OFDM symbol.

Beside the number of used subcarriers, the PAPR can be also influenced by the modulation scheme. The PAPR values for QPSK, 16QAM, 64QAM, and 256QAM modulation techniques are reported in Figure 5 and Table 2. One can observe that the PAPR is smallest in the case of QPSK, while it is largest for 256QAM.

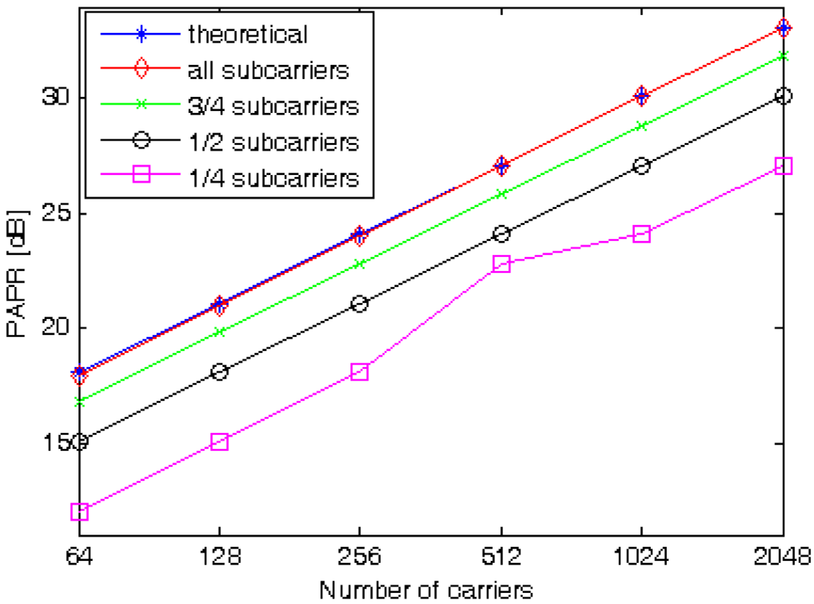

Figure 4 Influence of the number of subcarriers used for data transmission to the PAPR value.

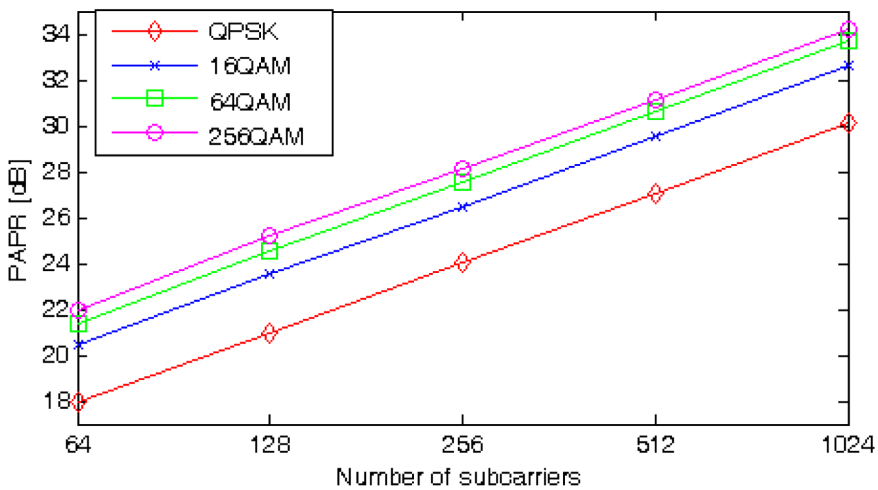

Figure 5 Influence of different modulation techniques to PAPR value.

Table 2 PAPR comparision for different modulation schemes.

No. of carriers	64	128	256	512	1024
PAPR for QPSK [dB]	17.92	21	24.05	27.07	30.09
PAPR for 16QAM [dB]	20.43	23.56	26.5	29.54	32.59
PAPR for 64QAM [dB]	21.38	24.56	27.57	30.66	33.7
PAPR for 256QAM [dB]	21.94	25.18	28.12	31.16	34.2

Table 3 Basic specifications of some OFDM based communication standards.

<table>
<tr><td>Application</td><td colspan="5">Number of available subcarriers</td><td>Modulation</td></tr>
<tr><td>WLAN</td><td colspan="5">64</td><td>BPSK, QPSK, 16QAM, or 64QAM</td></tr>
<tr><td rowspan="2">WiMaX</td><td>Fixed</td><td>Mobile Scalable</td><td>Mobile Scalable</td><td>Mobile Scalable</td><td>Mobile Scalable</td><td rowspan="2">BPSK, QPSK, 4,16,32,64, or 256QAM</td></tr>
<tr><td>256</td><td>128</td><td>512</td><td>1024</td><td>2048</td></tr>
<tr><td rowspan="2">DAB</td><td>Mode1</td><td colspan="2">Mode2</td><td>Mode3</td><td>Mode4</td><td rowspan="2">DQPSK</td></tr>
<tr><td>2048</td><td colspan="2">512</td><td>256</td><td>1024</td></tr>
<tr><td rowspan="2">DVB-T</td><td colspan="2">2K Mode</td><td colspan="3">8K Mode</td><td rowspan="2">QPSK, 16QAM or 64QAM</td></tr>
<tr><td colspan="2">2048</td><td colspan="3">8192</td></tr>
<tr><td>DVB-C</td><td>4096</td><td colspan="3">8192</td><td>32768</td><td>64-QAM up to 4096-QAM</td></tr>
</table>

In order to analyze some real cases the total number of subcarriers and the type of modulation techniques for different OFDM based systems are reported in Table 3. According to the previous analysis we have that, for example, in WLAN systems (Table 4) with $N = 64$ subcarriers and QPSK modulation PAPR is 18 dB, while in WiMaX and DAB systems with $N = 256$ subcarriers and QPSK PAPR is 24 dB holds. Since the analog OFDM signals are fed to the amplifiers, the equivalent PAPRs in analog domain [4, 11], for real systems with $N = 64$ and $N = 256$ (with QPSK) are 12.1 and 18.1 dB, respectively. Similarly, for WLAN system with $N = 64$ and 64QAM the equivalent PAPR is 17.16 dB is obtained [1].

It is important to note that even the value of 12.1 dB (obtained for $N = 64$ and QPSK) is considered as high PAPR, because it implies that the peak value is more than one order of magnitude stronger than the average signal value. In real cases the PAPR is usually somewhat lower than the theoretical one, but it is still high and thus the OFDM systems require the so called PAPR reduction techniques. Some of these techniques are analyzed in the sequel.

4 Analysis of PAPR Reduction Techniques

In the previous section, the performances of OFDM systems have been analyzed with respect to the PAPR value. In order to obtain more precise

Table 4 Performances of different PAPR reduction techniques.

	System	IFFT size	Modulation		Threshold λ [dB]		CCDF [dB]	BER
Clipping [6]	WiMaX	1024			No clipping	Standard Clipping	10^{-3}	10^{-1}
			BPSK		13.4	4.4		
			4QAM		14.7	6.3		
			16QAM		14.9	7.8		
			64QAM		15.1	9.6		
			256QAM		15.4	10		
Selective mapping [7]	Wireless IEEE802.11a	64	BPSK		12			no distortion
			QPSK		11			
			DQPSK		9.8			
Selective mapping [8]	Unknown	128	QPSK (RSC turbo code with K=4)	2 bits code	9.8		10^{-3}	no distortion
				4 bits code	8.2			
				5 bits code	7.7			
				6 bits code	7			
Constellation manipulations [4]	WiMaX	256	No reduction		11.7		10^{-3}	no distortion
			QPSK		8.3			
			16QAM		9.8			
			64 QAM		10.6			
Modulation adaptation and clipping algorithm [6]	WiMaX	1024	BPSK+4QAM+16QAM +64QAM+256QAM		9 dB		10^{-3}	10^{-5}
Time and frequency swapping [5]	Unknown	256	4FSK	Clipp. level 80%	5.4		10^{-2}	
				Clipp. level 90%	5			
				Clipp. level 95%	4.6			
Sequential algorithm [5]	Unknown	256	4FSK		6.3		10^{-2}	no distortion
Partial transmit sequence with original cross-entropy method [10]	Unknown	128	QPSK		5 iterations	6.25	10^{-3}	no distortion
					17 iterations	6.04		

analysis, the performances of OFDM system can be evaluated with respect to the probability that PAPR is above some predefined threshold λ (in dB). As a measure of performances, the complementary cumulative distribution function (CCDF) is used:

$$\Pr\{\text{PAPR} > \lambda\} = 1 - (1 - e^{-\lambda})^N. \tag{5}$$

Various techniques have been proposed in order to reduce PAPR in OFDM systems. Some of these techniques are: clipping and its modifications [5, 6]; partial transmit sequence [9, 10]; selective mapping [5, 7, 8]; constellations manipulations [4], etc. In the sequel we will consider some of these techniques and discuss their capabilities to reduce PAPR. The comparative results for different PAPR reduction techniques are summarized in Table 4. The characteristics of the applied PAPR reduction techniques are given in columns 2, 3, and 4. Column 5 contains the threshold values, while in the Column 6 the probabilities that PAPR exceeds specified threshold are reported.

Clipping is the simplest technique for PAPR reduction. It assumes that all amplitudes above predefined threshold λ are clipped to the threshold value. This value should be chosen to provide linear signal amplification and good power efficiency of high power amplifiers. For example, in WiMaX systems, the PAPR will be between 4.4 and 10 dB (depending on the modulation scheme) with the probability of 10^{-3}. However, the bit-error-rate (BER) is equal to 10^{-1} which is unacceptable in most of the applications. Thus, clipping based techniques may cause significant signal distortions and high BER.

An interesting modification of clipping technique is time and frequency swapping algorithm [5], where clipping is performed within a few iterative steps. In the first step, the random phases are assigned to each subcarrier and then transformed to time domain by using IFFT. The signal values are clipped in time domain and transformed back in the frequency domain. The procedure is repeated until the PAPR stops decreasing. It has been shown that, for $N = 256$ and 4FSK modulation, the probability that PAPR does not excide the range [4.6 dB, 5.4 dB] is 10^{-3} for clipping threshold between 80 and 95% of the maximal OFDM signal value. This system is characterized by high complexity and requires between 200 and 800 IFFT and FFT operations.

The algorithm based on modulation adaptation and clipping has been proposed in [6]. Based on predefined BER, the appropriate modulation is chosen for each sample. The clipping with high threshold value is applied to additionally reduce PAPR. The clipping is also used to control the switching between different modulation schemes. For this technique PAPR is 8 dB with probability 10^{-3}, while the BER is approximately 10^{-5}.

The sequential algorithm for PAPR reduction has been proposed in [5]. The different phases from the range $\{0, \pi\}$ are used to modify phases of used subcarriers. Modified signal is transformed in the time domain and PAPR is calculated. Then the phases are flipped and new PAPR is obtained. If the PAPR is not reduced, the original phases are kept. Probability that PAPR does not exceed value of 6.3 dB is approximately 10^{-2}. Although it provides worse performance compared to time and frequency swapping algorithm, the complexity of this system is much lower since it requires only 65 IFFT operations. Also, it does not introduce signal distortion, as it is the case with the time and frequency swapping or clipping techniques.

The partial transmission algorithm is another non-distortion technique [9, 10]. It divides the OFDM symbol into several shorter sequences that are separately optimized by using the cross-entropy method. The side information provided by the cross-entropy method should be sent to the receiver, which reduces the system capacity, i.e. the useful data rate.

Finally, the non-distortion solution based on selective mapping has been proposed in [5, 7, 8]. It uses additional bits to modify original sequence in a way to reduce PAPR. Namely, this algorithm iteratively adds different bits, while the lowest PAPR is obtained. The sequence that produces lowest PAPR is transmitted within OFDM symbol, and serves as side information at the receiver. By increasing number of bits used to modify original sequence, PAPR is reduced, but the channel utilization is decreased. For example, it has been shown in [8] that by using six bits to modify original sequence, PAPR can be reduced by 3 dB.

5 Conlusion

One of the main drawbacks of the OFDM based systems is high PAPR that causes high power consumption. Hence, these systems require some efficient PAPR reduction techniques. Generally, they can be classified into two categories: techniques that introduce signal distortion and non-distortion techniques. The techniques from the first group are usually much simpler and they can significantly reduce PAPR (e.g. PAPR is 4 dB in WiMAX systems) at the expense of lower signal quality. On the other hand, non-distortion techniques can provide limited PAPR reduction (in the best case the PARP can be reduced up to 6 dB). However, these techniques do not degrade the quality of original signal. Obviously, the compromise between PAPR reduction and signal quality should be made according to the specific application requirements. Finally, the PAPR reduction is still an open topic, since there is an increased demand for OFDM systems with improved energy efficiency, especially in mobile transmission systems.

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Biographies

Irena Orović (S'06) was born in Montenegro in 1983. She received the B.Sc., M.Sc., and Ph.D. degrees in electrical engineering from the University of Montenegro, Podgorica, Montenegro, in 2005, 2006, and 2010, respectively. From 2005 to 2010, she was a Teaching Assistant with the University of Montenegro. Since 2010, she has been an Associate Professor with the Faculty of Electrical Engineering, University of Montenegro. Her research interests include multimedia systems, digital watermarking, and time-frequency analysis.

Nikola Žarić was born in Montenegro in 1982. He received his B.Sc., M.Sc. and Ph.D. degrees in electrical engineering from the University of Montenegro, Montenegro, in 2005, 2006, and 2010, respectively. Since 2005, he has been employed as a Teaching Assistant at the Faculty of Electrical Engineering, University of Montenegro. In October 2010 he was promoted to Associate Professor at the Faculty of Electrical Engineering, University of Montenegro. His research interests are in digital watermarking and time-frequency analysis and related hardware realizations.

Srdjan Stanković was born in Montenegro in 1964. He received his B.S. (Hons.) degree in electrical engineering from the University of Montenegro, in 1988, his M.S. degree in electrical engineering from the University of Zagreb, Croatia, in 1991, and his Ph.D. degree in electrical engineering from the University of Montenegro in 1993. He is a Full Professor at the Faculty of Electrical Engineering, University of Montenegro. Since 2007, he has been the Dean of the Faculty of Electrical Engineering, University of Montenegro. His interests are in signal processing, multimedia systems, and digital electronics. He was the President of the Board of Directors in Montenegrin Broadcasting Company (2005–2006). In 1998 he spent a period of time with the Department of Informatics at the Aristotle University in Thessaloniki, supported by Greek IKY foundation. From 1999 to 2000, he was on leave at the Darmstadt University of Technology, with the Signal Theory Group, supported by the Alexander von Humboldt Foundation. In 2002, he spent three months at the Department of Computer Science, the University of Applied Sciences Bonn-Rhein-Sieg, as an Alexander von Humboldt Fellow. From 2004 to 2006, he stayed several times with the E^3I^2 Lab, ENSIETA, Brest. From 2007 to 2009 he visited (one month research stay) the Centre for Digital Signal Processing Research at King's College London, the Laboratory of Mathematical Methods of Image Processing, at Moscow State University Lomonosov, CAC at Villanova University PA, and the GIPSA Laboratory at INPG Grenoble. He has published several textbooks on electronics devices (in Montenegrin) and coauthored a monograph on time-frequency signal analysis (in English). He has published more than 100 papers in the areas of signal and image processing.

He is the Leading Guest Editor of the *EURASIP Journal on Advances in Signal Processing* special issue on "Time-frequency analysis and its applications to multimedia signals", as well as Guest Editor of the *Signal Processing* special issue on "Fourier related transforms".

From 2005 to 2009 Dr. Stanković served as an Associate Editor of the *IEEE Transactions on Image Processing*.

Igor Radusinovic, born in Montenegro in 1972, received his B.Sc. degree in electrical engineering from the University of Montenegro, Montenegro, in 1994. He received his M.Sc. and Ph.D. degrees in electrical engineering from the University of Belgrade, Serbia in 1997, and 2003, respectively. From 1994 to 2003, he was a Teaching Assistant with the University of Montenegro. From 2003 to 2008, he was an Assistant Professor with the

University of Montenegro. Since 2008, he has been an Associate Professor with the Faculty of Electrical Engineering, University of Montenegro. He has served as a reviewer for leading scientific journals in the different areas of telecommunications, as well as a member of Programme Committees of several conferences. His research interests are mainly in telecommunications network protocols and systems design. His current research topics are packet switch architectures, quality of service in wired/wireless networks, green communications and congestion control. In these areas he has published more than 100 referred publications in peer-review international journals and international proceedings.

Zoran Veljović was born in Serbia in 1968. He received his B.Sc. degree in electrical engineering from the University of Montenegro in 1992, his M.Sc. degree in electrical engineering from the University of Belgrade, Serbia, in 1997, and his Ph.D. degree in electrical engineering from the University of Montenegro in 2005. From 1992 to 2006, he was a Teaching Assistant with the University of Montenegro. Since 2006, he has been an Assistant Professor with the Faculty of Electrical Engineering, University of Montenegro. Since 2007 he has been Vice Dean for Academic Affairs. His research interests are mainly in digital communication systems, digital modulations, and mobile, satellite optical and maritime communications.