Minimum-Energy Broadcast Routing in Dynamic Wireless Networks*

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Abstract

One of the new challenges facing research in wireless networks is the design of algorithms and protocols that are energy aware. A good example is the *minimum-energy broadcast routing* problem for a static network in the plane, which attracted a great deal of attention these past years. The problem is NP-hard and its approximation ratio complexity is a solution proved to be within a factor 6 of the optimal, based on finding a Minimum Spanning Tree of the static planar network. In this paper, we use for the first time the evolving graph combinatorial model as a tool to prove an NP-Completeness result, namely that computing a Minimum Spanning Tree of a planar network in the presence of *mobility* is actually NP-Complete. This result implies that the above approximation solution cannot be used in dynamic wireless networks. On the positive side, we give a polynomial-time algorithm to build a rooted spanning tree of an on/off network, that minimizes the maximum energy used

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by any one node. Such tree could then be used in order to maximize the life-time of wireless communication networks.

Keywords: wireless networks, dynamic networks, routing, energy aware, evolving graphs, graph theoretical models, LEO satellite networks, fixed-schedule dynamic networks, sensor networks.

1 Introduction

Infrastructure-less mobile communication environments, such as mobile ad-hoc networks, sensor networks, and low earth orbiting (LEO) satellite systems, present a paradigm shift from back-boned networks, in that energy is usually a limited resource provided by batteries. In this setting, the generalized case of network routing using shortest paths or least cost methods are complicated by the need to save energy while ensuring communications. This naturally motivated studying energy aware algorithms and protocols for such ad-hoc networks (see [6, 13, 14] and references therein).

Power-saving communication problems and techniques in ad-hoc networks have received much attention recently. In addition to the above cited references, which belong to the networking community, one can find the same kind of concern in research done in theoretical aspects of communication networks, as witnessed by Clementi et al. [8] and Kirousis et al. [12].

Under the assumption that the nodes in an ad-hoc network can adjust their transmission power on demand, one particularly widely studied question is the following. Given a source in the network, find a power assignment for each node such that the total amount of energy required by the system to broadcast a packet is minimized. This problem is known as the *minimum-energy broadcast routing* (MEBR) problem [5, 8, 12, 15], which is NP-Hard ([7]). Its complexity approximation ratio was proven in [2] to be 6, based on the construction of a Minimum Spanning Tree (MST) on planar networks with no mobility, that were modeled as usual graphs.

The goal of the present paper is to show that in a setting where even a small amount of dynamics is allowed to the network, then computing a MST is itself already NP-Hard. To prove our results we only assume that nodes and edges in our dynamic network can go to sleep and wake-up, and this according to a known, fixed time schedule. Nodes positions may remain fixed during all the communications. This result evidently implies that computing a MST of a (planar) network with mobility is also NP-Hard.

As a positive note, we remind that, as pointed out in [6], when the nodes energy is finite, the energy may be the real hard constraint to be met, and the maximum life-time of the network should be the target of communication schemes. Under the same assumptions as above (fixed-schedule presence intervals, no mobility), we give a polynomial-time algorithm to build a rooted spanning tree of the network, that minimizes the maximum energy used by any one node. Such tree could then be used in order to maximize the life-time of wireless communication networks which respect to our assumption, like sensor networks.

In order to prove our results, we use the combinatorial model called *evolving graphs*, proposed in [10] as a formal abstraction for dynamic networks. Concisely, an evolving graph is an indexed sequence of subgraphs of a given graph, where the subgraph at a given index point corresponds to the network connectivity at the time interval indicated by the index number. The time domain is further incorporated into the model by restricting *journeys* (i.e., the equivalent of paths in usual graphs) to *never* move into edges which existed only in past subgraphs. Energy required by communications will then be modeled as edge weights, as usually.

We remark that this model may be as general as to allow for arbitrary changes between two consecutive time steps, with the possible creation and/or deletion of any number of vertices and edges. Furthermore, evolving graph edges can also be associated with traversal times. Algorithms were proposed for finding *foremost*, *shortest*, and *fastest* journeys in dynamic mobile networks modeled by evolving graphs [16] and then used to evaluate the performance of the best known protocols for ad-hoc mobile networks [11]. Evolving graphs may also be used to model other path problems, like those formulated under the MERIT approach [9], where the authors used the notion of competitive analysis [4] on a dynamic setting in order to assess the quality of protocols studied on snapshots describing the history of the network.

This paper is organized as follows. In the next section, basic definitions are given for various common graph theory terms in the context of evolving digraphs. In Section 2, we give an algorithm to compute rooted broadcast trees and show that this structure is minimal in evolving graphs. In Section 3, we prove that the minimum spanning tree problem is NP-hard, whereas in Section 4, we give a polynomial-time algorithm for min-max broadcast trees. Section 5 contains concluding remarks and scope for further research.

2 The Network Model

We use *evolving graphs* as a formal abstraction for dynamic networks, as follows.

Definition 1 (Evolving Graphs). *Let there be given a graph $G(V, E)$ and an ordered sequence of its subgraphs, $\mathcal{S}_G = G_1, G_2, \ldots, G_{\mathcal{T}}$ such that $\bigcup_{i=1}^{\mathcal{T}} G_i = G$. Let $\mathcal{S}_T = t_0, t_1, t_2, \ldots, t_{\mathcal{T}}$ be an ordered sequence of time instants. Then, the system $\mathcal{G} = (G, \mathcal{S}_G, \mathcal{S}_T)$ is called an evolving graph. We denote $|V| = N$ and $|E| = M$. We call G the underlying graph of $\mathcal{G}$.*

The duration of transmitting one packet over a link in the network is given as a function ζ representing the links' traversal times. Each G_i is the subgraph in place during $[t_{i-1}, t_i[$. The time domain $\mathbb{T}$ is further incorporated into the model in the definition of *journeys*, as follows. We call *route in* $\mathcal{G}$ a path $R = e_1, e_2, \ldots, e_k$ with $e_i \in E_{\mathcal{G}}$ in G. Let $R_\sigma = \sigma_1, \sigma_2, \ldots, \sigma_k$ with $\sigma_i \in \mathbb{T}$ be a time schedule indicating when each edge of the route R is to be traversed. We define a *journey* $\mathcal{J} = (R, R_\sigma)$ if and only if R_σ is in accordance with R, ζ, and $\mathcal{G}$, i.e., $\mathcal{J}$ allows for a traversal from u to v in $\mathcal{G}$. Note that journeys cannot go to the past. Corresponding to each edge e in $E_{\mathcal{G}}$ (respectively, node v in $V_{\mathcal{G}}$) we can define an *edge presence schedule* $P_E(e)$ (respectively, *node presence schedule* $P_V(v)$). We define the *activity of a vertex* v as the number of its activations/desactivations and the *activity of an edge* e in the same way. The *activity of an evolving graph* δ is defined as the maximum activity. And the *dynamics of an evolving graph* is defined as $(\delta - 1)/\mathcal{T}$. As a consequence, since usual graphs have $\delta = 1$, they have dynamics zero.

We will now introduce the notion of *sub-evolving graphs*. Informally, a sub-evolving graph is an evolving graph in which the presence intervals are contained in the presence intervals of the original evolving graph.

Definition 2 (Sub-evolving graph). *Let $\mathcal{G} = (G, \mathcal{S}_G, c\mathcal{S}_T)$ be an evolving graph. $\mathcal{G}' = (G', \mathcal{S}'_G, \mathcal{S}'_T)$ is called a sub-evolving graph of $\mathcal{G}$ if G' is a subgraph of G and if there is a monotonous function $f : \mathbb{N} \to \mathbb{N}$ such that for all i, G'_i is a subgraph of $G_{f(i)}$ and $[t_{i-1}, t_i[\subset [t_{f(i)-1}, t_{f(i)}[$.*

Definition 3 (Rooted evolving tree). *Given a vertex $r \in V$, an evolving tree in $\mathcal{G}$ with root r is a sub-evolving graph $\mathcal{G}_r \subset \mathcal{G}$, such that G_r is a tree, such that each edge has exactly one presence interval, and such that there is a journey from r to every vertex in the tree. We say that a rooted evolving tree is* spanning *if it contains all vertices in V.*

Observe that even if a sub-evolving graph allows less communication, its dynamics and space complexity may be greater than those of the original evolving graph. However, evolving trees have minimal space complexity.

2.1 Optimality of Evolving Trees

In [16] trees of foremost journeys were defined and computed. Their existence for every connected subgraph implies the following lemma:

Lemma 1 (Rooted evolving tree). *Given an evolving graph $\mathcal{G}$ and a vertex r, if there is a journey from r to every vertex in $\mathcal{G}$, then $\mathcal{G}$ contains a spanning evolving tree rooted in $\mathcal{G}$.*

This is particularly interesting because it means that rooted evolving trees are always part of a sub-evolving graph that allows communication from a single node. From this, we know that optimal structures for broadcasting are evolving rooted trees, provided that the cost function respects the inclusion property (the cost of a sub-evolving graph is smaller than cost of other sub-evolving graphs that contain it):

Theorem 1 (Optimality). *Given an evolving graph $\mathcal{G}$, a set X of vertices and a root r, given a cost function c on its sub-evolving graphs; if $\forall \mathcal{G}'' \subset \mathcal{G}' \subset \mathcal{G}$ we have $c(\mathcal{G}'') \leq c(\mathcal{G}')$ then the minimum structure with journeys from r to every vertex in X is an evolving tree.*

3 Minimum-Cost Spanning Trees

Given an energy consumption function $c : E \rightarrow \mathbb{R}_+$ on the edges of the network, the cost function of a broadcast structure we study in this section consists in adding the consumption (cost) of each edge in the structure.

This cost function on the structure respects the inclusion property, defined in Section 2.1, so according to Theorem 1 the minimum cost broadcast structure on an evolving graph will be a rooted evolving tree.

Unfortunately, although minimum-cost spanning trees can be found in polynomial-time in standard graphs, the problem of finding a minimum-cost rooted evolving tree is NP-hard, even if nodes are static in an Euclidean plane and the cost function on the edges is the square of their length. We show it by reducing the Steiner tree problem:

Theorem 2 (NP-hardness). *The problem of finding a single source minimum cost broadcast tree in an evolving graph is NP-hard, even if nodes are static* *in a Euclidean plane and the cost function on the edges is the square of their Euclidean length.*

Proof. We reduce the Steiner minimum cost tree problem in a planar graph with Euclidean distances, to our own problem. In the Steiner problem, we are given a planar graph $G = (V, E)$, and a set of vertices $X \subset V$ The problem consists in finding a tree in G containing all the vertices in X, and such that the sum of the costs of its edges is minimum. The cost of an edge is the square of its length in the plan.

Transformed problem: For each y in $Y = V \backslash X$ we create two new vertices y' and y''. y' and y'' are both positioned at distance ϵ from y. We call $Y' = \{y' such that y \in Y\}$ and $Y'' = \{y'' such that y \in Y\}$. We set $V' = V \bigcup Y' \bigcup Y''$. Let $E' = E \bigcup X \times Y' \bigcup Y' \times Y'' \bigcup Y'' \times Y$. $G' = (V', E')$ is the underlying graph of our evolving graph $\mathcal{G}$, defined as follows: every edge has a delay of 1. Every edge in E is available during the whole interval $[2, |V| + 2]$; every edge in $X \times Y'$ is available during $[0, 1]$, every edge in $Y' \times Y''$ is available during $[1, 2]$ and every edge in $Y'' \times Y$ is available during $[|V| + 1, |V| + 2]$. The cost function c' is already defined. Provided that X is non-empty, we arbitrarily single out a source $s \in X$. Now, the vertices in Y' can be reached only from s during $[0, 1]$. Vertices in Y'' may be reached during $[1, 2]$ for a small cost (less then $|V| \times \epsilon^2$). Vertices in Y may be reached during $[|V| + 1, |V| + 2]$ for an additional $|V| \times \epsilon^2$. Provided that X is non-empty, we arbitrarily single out a source $s \in X$. Our problem consists in finding a broadcast tree with source s, such that the sum of the costs of its edges minimum. Vertices in Y can be reached almost for free, that is for less than $2|V|\epsilon^2$ (where ϵ is as small as needed). □

4 Min-Max Rooted Evolving Tree

In this section, given an arbitrary cost function $c : E \to \mathbb{R}_+$, our goal is to minimize the edge with maximum cost in the broadcast tree. Energy-wise, it corresponds to maximizing the life span of the wireless network, where every component is independent. The problem of minimizing the maximum cost in a rooted evolving tree can be solved in polynomial time by the algorithm below.

Algorithm (minmax tree)
Complexity: $O(M^2 \log \delta + MN(\log n + \log \delta))$
Input: an evolving graph $\mathcal{G}$, a root node $r \in V$ and a cost function $c : E\mathbb{R}_+$

Output: an array *ead* of dates in the time domain $\mathbb{T}$, an array *father* of vertices which gives for every vertex except r its father in the evolving rooted tree

** the source has arrival date 0. The other vertices have arrival date ∞.*

** order all the edges of the evolving graph according to their cost.*

** while there is a vertex rich has arrival date ∞, do*

– *take the cheapest edge (x, y).*

– *Repeat*

  – *if it is possible to take the edge (x, y) after the arrival date of x, and if this makes for an earlier arrival date for y, then compute the earliest possible arrival date for y in this manner, update the arrival date for y, and state that x is the father of y*

  – *take the edge immediately more expensive*

– *Until a vertex has been updated*

In a worst-case scenario, our algorithm computes M times a new Dijkstra tree. This algorithm allows to store intermediate information on the network, so we can compute independently best paths towards any node. If we are never interested in independent paths, one can make a dichotomic search on the maximum cost, erase all edges above the cost and compute a tree. With this option, the worst case complexity drops to $O(\log M \times (M + N(\log N + \log \delta)))$.

5 Conclusion

In this paper we showed that computing a Minimum-Cost Rooted Spanning Tree on an Evolving Graph is NP-Hard, and that minimizing the maximum edge-cost on a Rooted Spanning Tree is polynomial. The former result implies that even a little dynamic behavior of the network precludes the use of the MST-based heuristic to solve the MEBR problem, as it was previously hinted in the literature. Fortunately, for fixed-schedule dynamic networks where energy is finite, like some sensor networks, the question of maximizing the network life-time can be solved in polynomial time according to the results shown in this paper.

With respect to perspectives, the theory of Evolving Graphs is in its infancy. In [1] the need for reproducibility of research outcomes through the development of reference models was expressed, as well as the fact that these new autonomous networks will be harnessed only through substantial innovation and paradigm shifts. The first results on Evolving Graphs show that old questions treated on usual graph models for static networks may be unsettled in a dynamic setting (scheduled transmissions, sleeping modes, mobility, etc.), requiring new insights. For instance, it was shown that, unlike usual graphs, finding connected components in evolving graphs is NP-Complete [3]. The way is wide open for building a new combinatorial and algorithmic toolkit for dynamic networks.

References

[1] Report of the National Science Foundation Workshop on Fundamental Research in Networking, 2003.
[2] C. Ambuehl. An optimal bound for the MST algorithm to compute energy efficient broadcast trees in wireless networks. In *Proceedings of 32nd ICALP*, pp. 1139–1150, 2005.
[3] S. Bhadra and A. Ferreira. Complexity of connected components in evolving graphs and the computation of multicast trees in dynamic networks. In S. Pierre, M. Barbeau, and E. Kranakis (eds.), *Proceedings of Adhoc-Now'03*, Montreal, October 2003. Lecture Notes in Computer Science, Vol. 2865, pp. 259–270, Springer Verlag, 2003.
[4] A. Borodin and R. El-Yaniv. *Online Computation and Competitive Analysis*. Cambridge University Press, 1998.
[5] W.T. Chen and N.F. Huang. The strongly connecting problem on multihop packet radio networks. *IEEE Transactions on Communications*, 37(3):203–295, October 1989.
[6] Imrich Chlamtac, Marco Conti, and Jennifer J.N. Liu. Mobile ad hoc networking: Imperatives and challenges. *Ad Hoc Networks*, 1:13–64, 2003.
[7] A. Clementi, P. Crescenzi, P. Penna, G. Rossi, and P. Vocca. On the complexity of computing minimum energy consumption broadcast subgraphs. In *Proceedings 18th Annual Symposium on Theoretical Aspects of Computer Science*. Lecture Notes in Computer Science, Vol. 2010, pp. 121–131, Springer-Verlag, 2001.
[8] A. Clementi, P. Penna, and R. Silvestri. On the power assignment problem in radio networks. In *Proceedings Electronic Colloquium on Computational Complexity*, 2000.
[9] A. Faragò and V.R. Syrotiuk. A unified framework for routing protocol. In *Proceedings ACM Mobicom 01*, pp. 53–60, 2001.
[10] A. Ferreira. Building a reference combinatorial model for manets. *IEEE Network*, 18(5):24–29, 2004.
[11] A. Ferreira, A. Goldman, and J. Monteiro. Performance evaluation of routing protocols for manets with known connectivity patterns using evolving graphs. *Wireless Networks*, 16(3):627–640, 2010.
[12] L.M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelk. Power consumption in packet radio networks. *Theoretical Computer Science*, 243:289–305, 2000.
[13] Vinod Namboodiri. *Algorithms & Protocols Towards Energy-Efficiency in Wireless Networks*. VDM Verlag, Saarbrücken, Germany, 2009.
[14] I. Stojmenovic (ed.). *Handbook of Wireless Networks and Mobile Computing*. John Wiley & Sons, 2002.

[15] J.E. Wieselthier, G.G. Nguyen, and A. Ephremides. On the construction of energy-efficient broadcast and multicast trees in wireless networks. In *Proceedings IEEE Infocom'2000*, 2000.
[16] B. Bui Xuan, A. Ferreira, and A. Jarry. Computing shortest, fastest and foremost journeys in wireless dynamic networks. In *Proceedings WiOpt'03*, 2003.

Biographies

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