Chapter 18
Finding the Limits of Beam Elements: Modeling Bolted Joints as an Effective Joint Region

Nicholas Pomianek, Casey Whitworth, Enrique Gutierrez-Wing, Trevor Jerome, and J. Gregory McDaniel

Abstract Interrupting a beam with a bolted joint introduces dynamic nonlinearity and time dependent variability arising from complex interfacial physics. High fidelity finite element simulation of large jointed structures is therefore expensive and has diminishing returns when considering only global behavior. We propose a simplified method to model bolted joints where a characteristic set of dynamic properties is determined for the jointed regions of a structure. Euler-Bernoulli beam elements are used to represent joints as continuous solid regions with tunable damping, stiffness, and density properties. These properties are updated by an optimization process based on experimental data using classical and recent frequency response similarity metrics. Assembly and reassembly effects are accounted for experimentally through variation of bolt tightening orders. The Brake-Reuß standardized beam is used as a test case for this work; results show robust agreement between the updated model and modal test data. This work develops a practical and effective framework for joint modeling that is readily applicable in any finite element software.

Keywords Joint Mechanics· Model Updating· Similarity Metrics· Frequency Response· Finite Elements

Introduction

It is easy to find structural design cases where an estimation of global response is required at the modeling stage but including nonlinearity is not strictly necessary. These are typically large structures which often contain many instances of the same joint and loading scenarios significantly below expected service levels. If the common joints in these designs are sufficiently standardized and prevalent, then prototypes should be available and easy to test *ex situ*. Methods that leverage the available information under these circumstances to produce a high fidelity model that is inexpensive enough for structural design model updating are of increasing interest to the joint modeling community.

Modeling nonlinear systems requires a choice between fidelity and computation time. The highest-fidelity techniques involve detailed scanning of the joint interface surfaces and modeling asperity interactions [1]. For modeling scenarios where joint surfaces are not available a priori, statistical descriptions of surface asperities are often employed alongside a combination of linear modal extraction and a quasi-static approach [2]. If the structure isn't expected to encounter significant macroslip, then a Iwan or Masing contact element formulation is typically used [3]. More recent techniques for this application include the definition of Contact-Zone elements [4], or the replacement of bolts with springs [5]. Commercial finite element packages offer joint elements that entirely replace joints but they require empirical stiffness data. Finally, sufficiently linear systems are adequately modeled using friction contact conditions such as the penalty method that are available in commercial finite element software. All these modeling techniques require validation and calibration with experimental data for best results. If these data are available and joints are expected to be in the linear regime, then a simplified and calibrated linear model that can predict the most significant dynamics of joints is a powerful tool for structural design.

Many joint modeling techniques have been developed to better predict the effects of friction across a wide range of loading scenarios but all require model updating and most require element types that are unavailable to finite element practitioners. Furthermore, the nonlinear approaches that many of these modeling techniques use prevent the use of frequency

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domain eigenvalue solvers. These limitations have inspired the recent work of Black et. al. [6], which addresses them by modeling joints as tied contact regions between solid elements. By updating the size of this contact region along with the stiffness and density of the surrounding elements, the global response of a jointed structure was predicted accurately. The present study also uses the idea of defining a 'virtual material' [6] but instead uses an approach tailored to situations where a structural prototype is unavailable.

This work seeks to develop an accessible and effective framework for joint modeling for use with standard FEA tools that does not require special contact elements or parameters. This work seeks to demonstrate that updating the damping, stiffness, and density characteristics of the elements within an effective joint region (EJR) can create a computationally efficient model that preserves the most important dynamic response features of a joint.

Experimental setup

The Brake-Reuß standardized test beam (BRB) [7] was used as a case study for this work because of the extensive modal testing data available in the existing literature. The prismatic geometry of the BRB also lends itself well as a best-case-scenario for the modeling approach developed in the following sections. Pictured in Figure 1, the test specimen was manufactured according to the machining specifications outlined in [8] but was made from ASTM A283 steel for material availability reasons. The procedure for assembling the test structure from [8] was followed, where both sides of the BRB are clamped to a flat surface, a shim is inserted in the interface parallel to the bolt axis, and an instrumented torque wrench is used to tighten the bolts to 20 Nm. This assembly procedure is followed during each re-assembly of the system throughout experimentation. The assembled system is suspended using monofilament fishing line to approximate free-free boundary conditions.

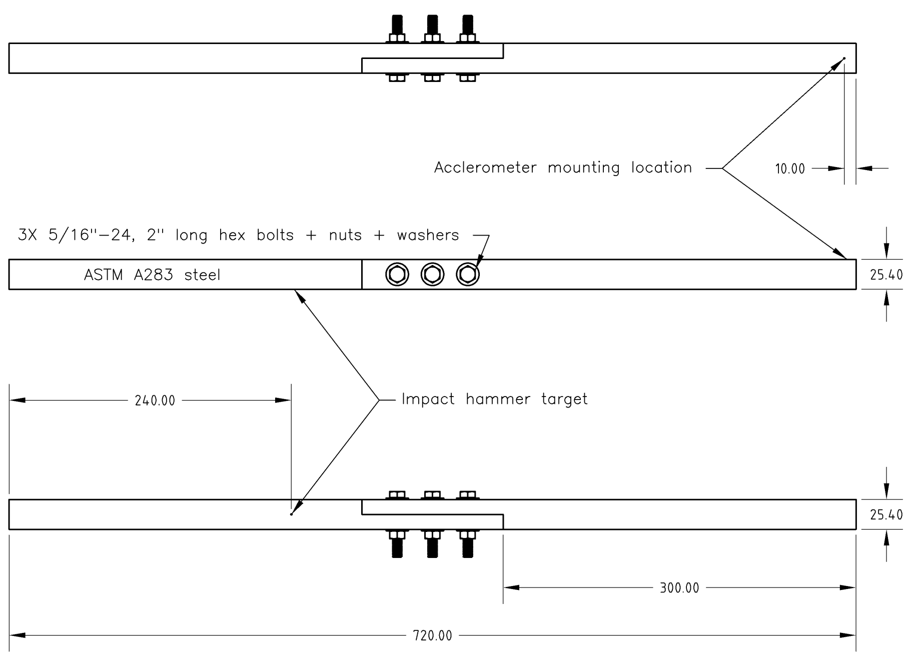

Fig. 1 Brake-Reuß beam test specimen, dimensions in mm.

Modal testing was performed using an aluminum-tipped Brüel & Kjær model 8206-002 impact hammer and a Brüel & Kjær model 4534-B accelerometer, the placement of the accelerometer and impact hammer target location are shown in Figure 1. Both the accelerometer and impact hammer signals are fed through PCB model 482A21 signal conditioners into a Digilent Analog Discovery 2 digital oscilloscope which records data to a computer running WaveForms software. The system is oriented such that the impact direction is perpendicular to the suspension lines so that they do not add stiffness to the beam. The experimental instrumentation setup is shown in detail in Figure 2.

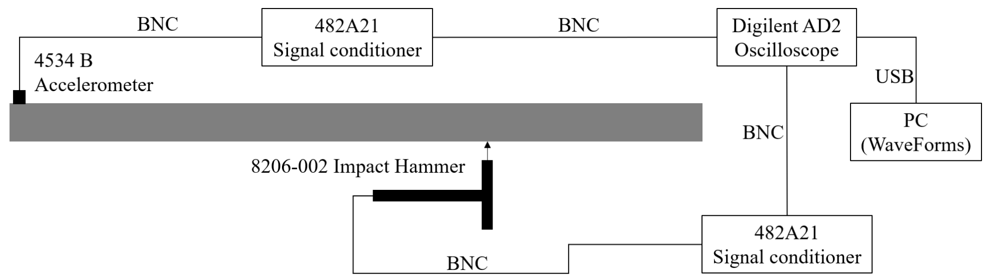

Fig. 2 Experimental instrumentation setup used for impact hammer modal testing.

Bolted joints, specifically the BRB, have been demonstrated to exhibit non-repeatability and sensitivity to bolt tightening order [8]. To account for this, six bolt tightening orders were tested, ten impacts were recorded for each order and the average frequency response function (FRF) was computed for each order in units of accelerance a as

$$A^{(n)}(f) = \tilde{a}^{(n)}(f)/\tilde{F}^{(n)}(f) \tag{1}$$

where $\tilde{a}$ is the frequency domain acceleration, $\tilde{F}$ is the frequency domain force, f is the frequency, and n is the hit number index. The average FRF is thus

$$A_{ave}(f) = \frac{1}{10}\sum_{n=1}^{10} A^{(n)}(f). \tag{2}$$

Finite element model updating

This work uses the Euler-Bernoulli beam element as a test case for the EJR method. Despite its lower performance compared to other finite element formulations for beams, the Euler-Bernoulli element is computationally inexpensive and widely accessible. The properties chosen for updating are the damping loss factor η, the Young's modulus E, and the density ρ. These properties were selected because joints have been observed to influence stiffness and damping, and the addition of bolts, nuts, washers and holes necessarily changes the density of an otherwise homogeneous region. Using this formulation, the dynamic response of an element at a given frequency ω as a function of these properties is

$$\vec{U}(\eta, E, \rho) = \left[(1+\eta i)\mathbf{K}(E) - \omega^2\mathbf{M}(\rho)\right]^{-1}\vec{F} \tag{3}$$

where $\vec{F}$ are the nodal applied forces, $\vec{U}$ are the nodal displacements, $\mathbf{K}$ is the stiffness matrix, and $\mathbf{M}$ is the mass matrix. In order to compare to modal test data, the response is converted to accelerance $\vec{A}$.

$$\vec{A} = (i\omega^2)\vec{U} \oslash \vec{F} \tag{4}$$

A unit force is applied at the instrumented hammer impact location by finding it's equivalent nodal location and substituting the corresponding element of $\vec{F}$ in equation (3). The frequency response function for comparison to modal data is calculated by finding the equivalent nodal location of the accelerometer and extracting the corresponding element from the accelerance vector in equation (4). All models evaluated in this work use free-free boundary conditions and neglect the added mass of the accelerometer. Mesh convergence studies were performed and an element length of 1 cm was chosen for model updating.

The EJR method separately defines properties for the joint region as η_j, E_j, and ρ_j, allowing them to differ from the known properties for the monolithic sections of the structure as shown in Figure 3. In the non-joint region, these material properties are assigned according to test specimen material data. EJR properties are assigned by finding the equivalent nodal locations of the joint boundaries and replacing η, E, and ρ with η_j, E_j, and ρ_j for the elements within these boundaries. The resulting model is that of a solid beam with properties η, E, and ρ varying as a function of beam length. The known material properties and bounds used for optimization of EJR properties are summarized in Table 1.

The finite element model used in this work was developed in MATLAB and benchmarked against Abaqus. Firstly, a set of frequencies is extracted from modal test data when the experimental frequency response function is calculated. Next, the finite element model is evaluated at each frequency in this set to construct a frequency response function that can be compared directly with the experimental modal test data. The degree of similarity between the experimental FRF and the

Table 1 Model properties and bounds set during optimization.

Property	Lower bound	Upper bound	Units
η	0.005	0.005	
η_j	0	0.03	
E	200	200	GPa
E_j	0	20000	GPa
ρ	7800	7800	$\mathrm{kg/m^3}$
ρ_j	7000	11000	$\mathrm{kg/m^3}$

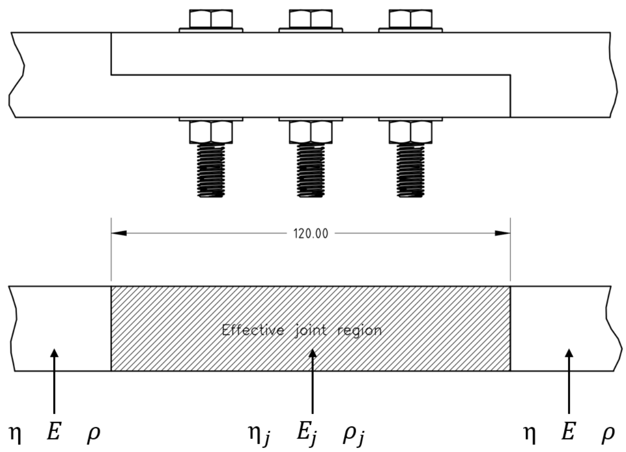

Fig. 3 The effective joint region is defined as any part of the structure that includes a frictional interface, dimension in mm.

finite element FRF is used as the cost function for a particle swarm optimization process. The EJR properties η_j, E_j, and ρ_j are the decision variables for this optimization process. Upon each iteration of the optimization, the EJR element properties are updated, the FEA model is evaluated, and the degree of similarity to the experimental dataset is calculated. The degree of similarity was calculated using a handful of different FRF similarity metrics from the literature.

The most straightforward metric tested is the root mean squared percent error (RMSPE) between the two FRFs being compared [9]. This is a simplistic approach that does not leverage the modal information contained in FRFs but is valuable as a point to compare against more sophisticated techniques. The R^2 value of a linear fit (ILR2) between the imaginary parts of each FRF serves as another metric focused on phase [9]. The FRF scaling factor (FRFSF) criterion is another alternative focused on the magnitude differences between FRFs.

$$\mathrm{FRFSF} = \frac{\sum_{f=f_1}^{f_2} |X(f)|}{\sum_{f=f_1}^{f_2} |Y(f)|} \tag{5}$$

In this equation, X is the reference FRF, Y is the comparison FRF, and f_1 and f_2 are the lower and upper bounds of the frequency range of interest. The frequency response assurance criterion (FRAC) is a well established inner-product approach that was used as a baseline to compare to more recent approaches [10].

$$\mathrm{FRAC} = \frac{\left|\sum_{f=f_1}^{f_2} X(f)Y^*(f)\right|^2}{\sum_{f=f_1}^{f_2} X(f)X^*(f) \ \sum_{f=f_1}^{f_2} Y(f)Y^*(f)} \tag{6}$$

Both FRAC and the modified frequency response assurance criterion (MFRAC), which scales FRAC by the ratio of the minimum to maximum powers of each FRF, are investigated. As an alternative to FRAC, the mean cross signature scale factor (CSF) was also applied due to its improved sensitivity to damping. [9].

$$\mathrm{CSF} = \left(\sum_{f=f_1}^{f^2} \frac{2\,|X^*(f)Y(f)|}{(X^*(f)X(f)) + (Y^*(f)Y(f))} \right) \frac{1}{N_f} \tag{7}$$

This equation includes the total number of frequencies of interest N_f. Finally the mean magnitude similarity function (MSF) and shape similarity function (SSF) criteria are the most recent and highest performing metrics used [11]. The similarity of magnitude and shape are calculated separately to increase the amount of information provided by each metric. For magnitude the metric is calculated as

$$|H(f)| = \left|\frac{Y(f)}{X(f)}\right| \tag{8}$$

$$M_\alpha(f) = 1 - \tanh\left(\frac{\ln 3}{2}\frac{|20\log_{10}|H(f)||}{\alpha}\right) \tag{9}$$

$$M \text{ index} = \frac{1}{f_2 - f_1}\int_{f_1}^{f_2} M_\alpha(f)df \tag{10}$$

where α is a scaling factor set to 20 in this work and M index is the average MSF value across the frequency range of interest. The phase similarity is calculated as

$$argH(f) = \phi_y(f) - \phi_x(f) \tag{11}$$

$$\tau(f) = \frac{1}{2\pi}\frac{d\, argH(f)}{df} \tag{12}$$

$$S_\beta(f) = 1 - \tanh\left(\frac{\ln 3}{2}\frac{|\tau(f)|}{\beta}\right) \tag{13}$$

$$S \text{ index} = \frac{1}{f_2 - f_1}\int_{f_1}^{f_2} S_\beta(f)df \tag{14}$$

where ϕ_x and ϕ_y are the phase of the FRFs X and Y and β is a scaling factor set to 1 for this work. The combined mean magnitude and shape similarity metric MSSF is

$$MS \text{ index} = w_m(M \text{ index}) + w_s(S \text{ index}) \tag{15}$$

where w_m and w_s are weighting factors such that $w_m + w_s = 1$. Only equal weighting is analyzed in this work.

The frequency window being used has been shown to influence model updating results when using single-number FRF similarity metrics [12]. In light of this concern, a windowed approach was developed where the frequency range of interest was subdivided into between one and ten sections which are each analyzed separately by the optimization process. Before optimization the experimental data is decimated and filtered using the Savitzky–Golay technique. The result of this numerical analysis setup is a set of EJR material properties that vary as a function of frequency which can be easily added to any commercial finite element software to model joints using beam elements.

Results

The aim of this work is to develop a computationally inexpensive joint modeling technique that yields acceptable results for applications where joints are not subject to high enough loads to induce highly nonlinear behavior. For comparison, a Euler-Bernoulli element model, a Timoshenko element model, and a solid element model were evaluated using known material properties. The properties applicable to the test specimen used are $\eta = 0.005$, $E = 200E9$ Pa, and $\rho = 7800$ kg/m^3, these properties are also assigned to the non-jointed region during EJR optimization. The Euler-Bernoulli and Timoshenko models are that of a monolithic beam with geometry equivalent to the BRB. The solid element model was developed in Abaqus following standard practices for joint modeling outlined in [13]. All geometry of the beams and bolts are modeled explicitly, the surface contact between beams and between washers is modeled explicitly, loading is applied internal to the bolts, and friction is modeled using a penalty formulation. As seen in the upper plot in Figure 4, agreement of all nominal models is poor when compared to experimental data and this comes at a computational cost of 7.83 minutes for the solid element model, 10 seconds for the Timoshenko model, and 0.055 seconds for the Euler-Bernoulli model. The EJR method, pictured in the lower plot of Figure 4, shows notably better agreement with the significant features of the experimental FRF. Because the optimized EJR model uses only Euler-Bernoulli elements, it also takes 0.055 seconds to run. The EJR optimization process that updated the nominal Euler-Bernoulli model takes 13.52 seconds.

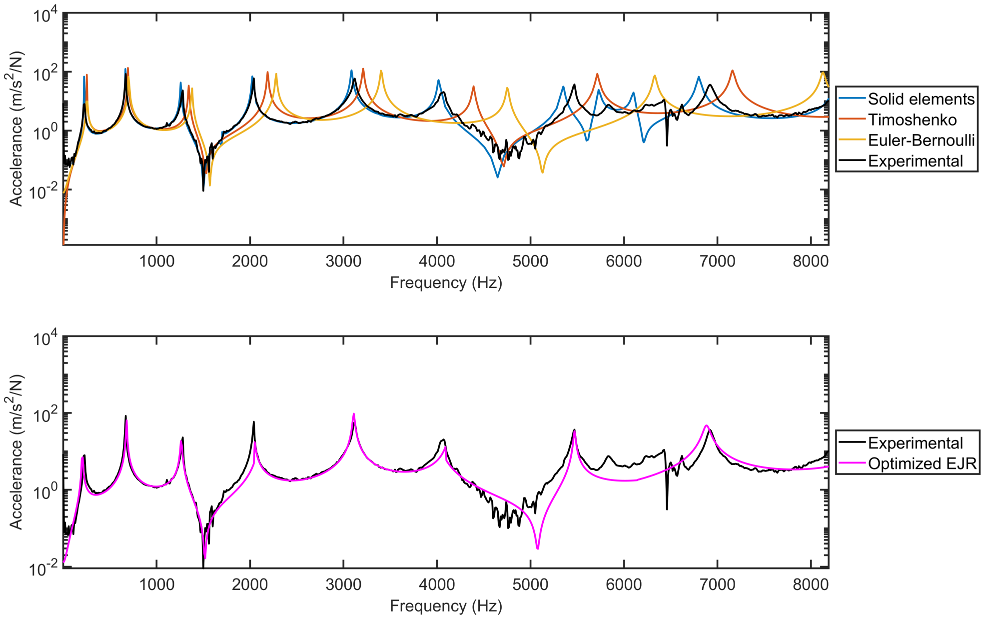

Fig. 4 Comparison of nominal FE models with EJR method optimization results. Solid element simulation run in Abaqus using explicitly modeled joint geometry, contact interactions, and bolt forces. MSSF similarity metric, bolt tightening order 3-2-1.

The effect of different FRF similarity metrics was also considered. With the exception of the RMSPE and FRFSF metrics, all considered similarity metrics performed well. Figure 5 shows the optimized EJR models generated using the eight similarity metrics analyzed in this work. The optimal number of frequency window divisions depended on the similarity metric but for the ILR2, FRAC, MFRAC, CSF, MSF, and MSSF optimized models the highest performing number of windows was four. The highest performing similarity metric overall was found to be MSSF.

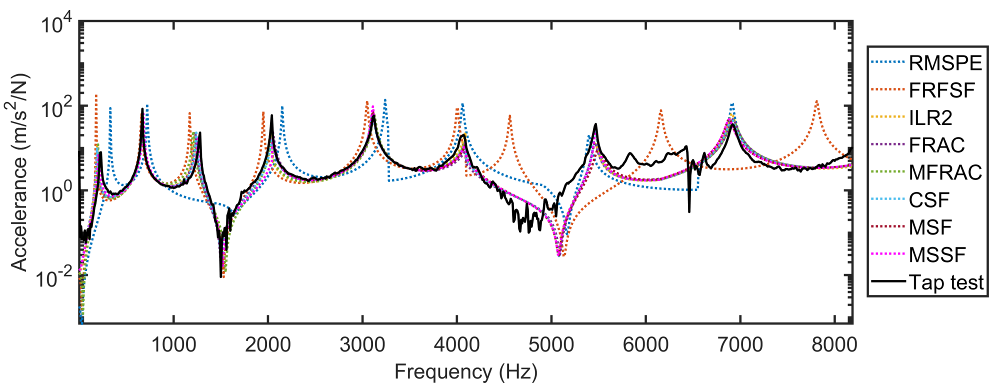

Fig. 5 Performance comparison of FRF similarity metrics, bolt tightening order 3-2-1.

A repeatability study was performed to test the sensitivity of the EJR method to system variability due to assembly conditions. Experimental data was taken on the same BRB assembled with six bolt tightening orders, as shown in the upper plot of Figure 6. An EJR optimized model was then generated for each data set, showing minor sensitivity to joint repeatability. The lower plot of Figure 6 shows the EJR model results corresponding to each bolt tightening order.

Because four windows were used to update the models shown in the previous figures, the EJR properties vary as a function of frequency. Figure 7 shows the variation in properties in the different frequency windows used for optimization. The upper and lower bounds of each variable available to the particle swarm optimization algorithm are shown by the vertical axis limits of each plot. While many of these results are non-physical, the resulting FRF is in good agreement with experimental data. The MSSF similarity metric between the EJR models and experimental data in Figure 6 ranges from 0.924 to 0.967 (its maximum value is 1). One of the inherent limitations of beam elements is their lower performance at

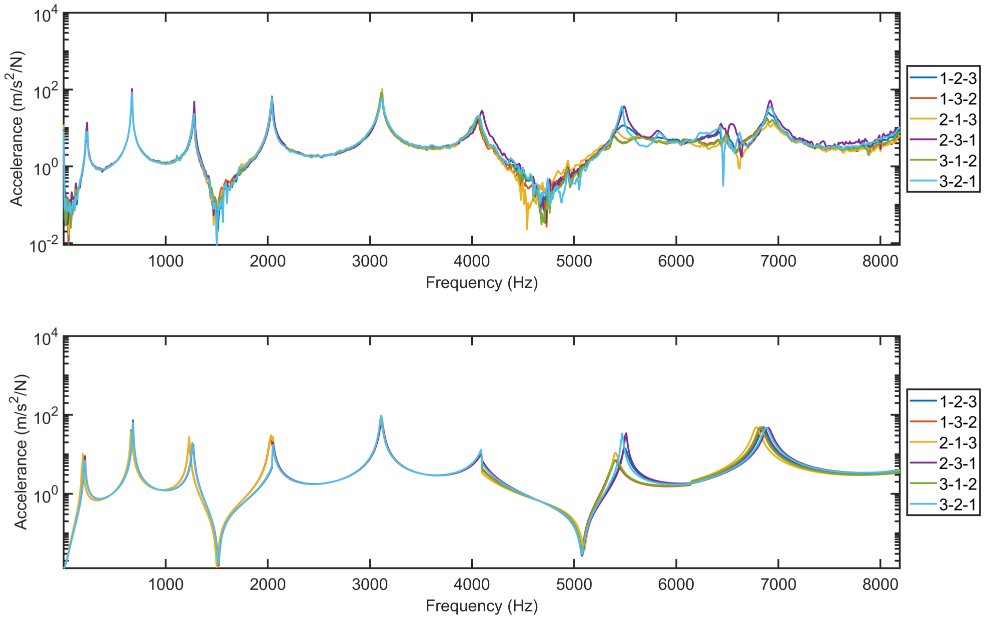

Fig. 6 Comparison of experimental data and optimization results for six bolt tightening orders, MSSF similarity metric.

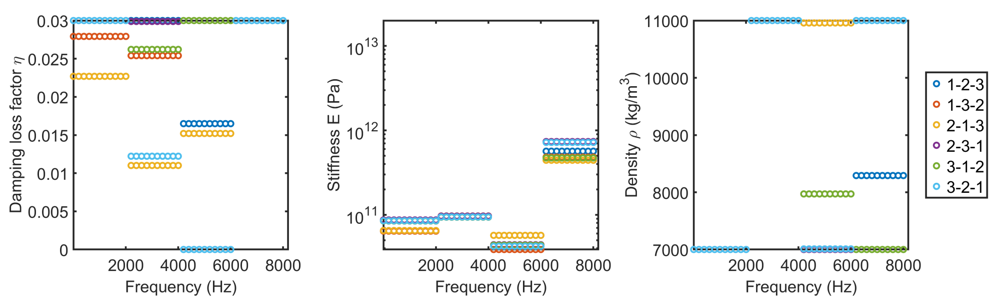

Fig. 7 Comparison of EJR properties across six bolt tightening orders, frequency range divided into four windows, MSSF similarity metric.

higher frequencies, the optimization process was required to compensate for this which likely contributed to the prediction of non-physical properties.

Conclusion

A method for modeling joints using inexpensive element formulations was developed and demonstrated to have good performance in the case of the BRB benchmark structure. A 99.988% reduction in computation time was achieved when compared to standard practice solid element joint modeling with better agreement to experimental data. The EJR method was shown to have low sensitivity to joint non-repeatability and similarity metric cost function formulation for optimization. The performance of various FRF similarity metrics for model updating in joints was also explored. In comparison to previous model updating techniques that parameterize joints, the EJR method can be applied in any commercial finite element modeling package which gives it the potential to be a powerful and accessible tool for modeling the dynamic response of jointed structures. Further research into the methods developed by this work must be done to validate EJR models against a structure containing many joints. Improvements in property physicality can likely be made by further dividing the EJR elements into bolt regions and non-bolt regions. Additionally, generalization of the EJR method to more than just Euler-Bernoulli beam elements is a necessary step in creating a complete method of analysis for real-world structures.

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