Chapter 20
Topology Optimization of Isolated Response Curves in 3D Geometrically-nonlinear Beam

Enora Denimal Goy, Yichang Shen, Samuel Fruchard, Adrien Mélot, and Ludovic Renson

Abstract Topology optimisation is a powerful tool for designing efficient and light structures. However, classical topology optimisation methods (SIMP, LSF), which are gradient-based, are not adapted to deal with nonlinear vibrations in the context of geometrical nonlinearities as the simulation of such systems is computationally expensive, and the strong nonlinear behaviour makes the objective function non-convex with many local minima. The present work investigates the potential of using global optimisation methods to topology optimise those structures. To provide more robust nonlinear features in the optimisation, the bifurcations are directly tracked and optimised. The strategy is applied to a 3D finite element model of a beam.

Keywords Bifurcation · Topology optimisation · Global optimisation · Geometric nonlinearities · Reduced-order Model

Introduction

A major transformation in design methods for mechanical structures (transport, aeronautics, space, etc) is taking place in order to continue to make them lighter and increase their performances while benefiting from the arrival of new technologies such as 3D printing. Optimising the dynamics of these structures is becoming extremely complex due to the intrinsic presence of non-linearities leading to dynamic phenomena rarely observed in industrial mechanical systems. For example, in the presence of non-linearities, the dynamics of structures can exhibit bifurcations.

Previous studies have focused on the structural and topology optimisation of mechanical structures to limit the level of amplitudes or avoid bifurcation behaviour in the context of localised nonlinearities [1–3]. The present work has for objective to expand these works to the context of geometric nonlinearities. Such nonlinearities imply a significant increase in the computational cost, making the use of reduced-order models (ROM) as well as efficient optimisation techniques crucial. In [4], the authors employed gradient-based optimisation for nonlinear resonances mitigation in the context of geometrical nonlinearities, yet the approach is limited to structures composed of beams and can only do shape optimisation.

In this work, a framework is proposed to perform topology optimisation of geometrically nonlinear structures to control the appearance of bifurcations for 3D structures. For each new geometries, a ROM is constructed to solve the nonlinear problem and bifurcations are tracked. To reduce the computational cost of the optimisation, a global optimisation algorithm coupling the CMA-ES to the PSO is proposed. The topology is parametrised with the Moving Morphable Component [5] framework and unconnected geometries are removed in the optimisation loop to reach higher efficiency.

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Methodology

Geometric Nonlinearity & Model Reduction

In the design of thin structures, geometric nonlinearity plays an important role because of the large deflections that they exhibit and the resulting complex nonlinear phenomena. Finite element (FE) simulations are frequently employed to predict the nonlinear response. A large number of elements is usuallu needed to obtain a converged discretization. As a consequence, full-order nonlinear analysis of complex structures featuring important nonlinearity is computationally expensive, setting a clear limitation in terms of analysis and parametric design. To address this challenge, one can transform the full-order model by reduction methods, leading to a Reduced-order model (ROM) with less computational complexity. An ideal ROM should accurately capture the nonlinear behavior and characteristics of the full-order model with minimal approximation error while remaining computationally efficient.

Invariant-manifold-based techniques for model reduction of nonlinear mechanical systems have been explored in recent years [6]. These methods ensure the ROM's trajectories are consistent with those of the full-order system due to the invariance property, and thus provide mathematically rigorous model reduction for nonlinear systems [7]. In this work, the Direct Normal Form (DNF) approach [8] was applied to obtain the ROM of the FE models. The approach allows direct computation of the nonlinear mapping enabling it to pass from the physical space (dofs of the FE mesh) to lower-dimensional invariant manifolds. The nonlinear mapping between the physical and reduced coordinates derived by the DNF methods reads:

$$\mathbf{q} = \sum_{i}^{n} \phi_i X_i + \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{\mathbf{a}}_{ij} X_i X_j + \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{\mathbf{b}}_{ij} \dot{X}_i \dot{X}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{\mathbf{c}}_{ij} X_i \dot{X}_j, \tag{1}$$

where n is the number of master modes used to build the ROM, vector $\mathbf{q}$ is generalized displacements at the nodes, and X_i together with its velocity $\dot{X}_i$ are the coordinates used to span the i-th invariant manifold, ϕ_i is the eigenvector and $\bar{\mathbf{a}}_{ij}$, $\bar{\mathbf{b}}_{ij}$ and $\bar{\mathbf{c}}_{ij}$ are second-order tensors. The reduced dynamics is then given by the normal form of the problem, up to the second-order, by taking into account the Rayleigh damping $\mathbf{C} = \zeta_M \mathbf{M} + \zeta_K \mathbf{K}$ with an assumption of small damping ratios on the master modes. It reads, $\forall\, p = 1, ...m$:

$$\ddot{X}_p + (\zeta_M + \zeta_K \omega_p^2)\dot{X}_p + \omega_p^2 X_p + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} (A_{ijk}^p X_i X_j X_k + B_{ijk}^p X_i \dot{X}_j \dot{X}_k + C_{ijk}^p X_i X_j \dot{X}_k) = F_p, \tag{2}$$

where the coefficients $A_{ijk}^p, B_{ijk}^p, C_{ijk}^p$ arise from the cancellation of non-resonant quadratic terms. The full expressions of the tensors and coefficients in Eqs. (1, 2), which are not given here for the sake of conciseness, can be found in [8]. Figure 1 shows an example of reduced-order modeling of a clamped beam discretized with 3D elements and the corresponding backbone curves computed by using ROM.

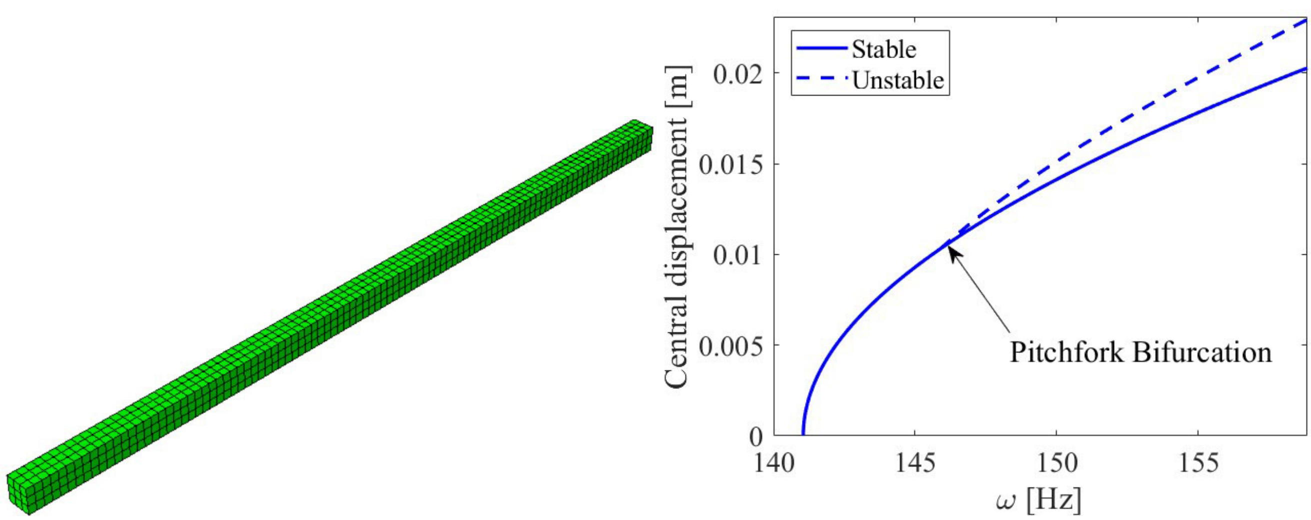

Fig. 1 Left: The clamped beam discretized with 3D elements. Right: Backbone curves of the first mode computed by the ROM

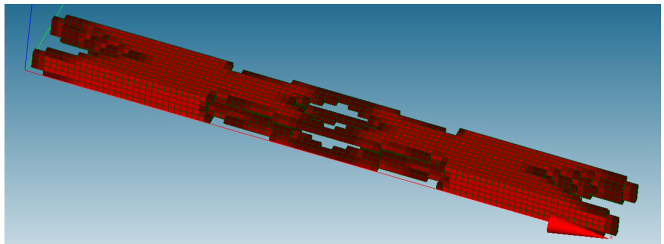

Fig. 2 Example of topology with the 3D MMC framework

Bifurcation detection and optimisation

A computational framework based on the harmonic balance method is employed to compute the dynamic response. Hill's stability analysis is performed to detect and locate the potential bifurcations. Similarly to [2], the objective function for the optimisation is based on a bifurcation measure, able to handle multiple bifurcations and to locate them at chosen frequency and/or amplitude levels. It is written as follows:

$$f(T, P(\theta)) = |T - P|\Psi(\theta) + \frac{1}{|T|} \sum_{\tau \in T} \prod_{\pi\theta \in P} \left| \frac{\pi(\theta) - \tau}{\tau} \right|^{1/|P|} \tag{3}$$

with T the set of target bifurcation, $P(\theta)$ the set of bifurcations predicted for the model parameters θ. The notation $|T|$ represents the number of elements in the set T.

Geometry Parameterisation

Traditional topology optimisation algorithms, such as SIMP or LSM, require the analytic definition of the gradient to ensure reasonable computational time. As this information is unavailable in our case, and as the problem is nonlinear, non-convex, with many local minima, the choice is made to go for global optimisation algorithms. As they are parametric, the Moving Morphable Component (MMC) framework is employed here to ensure the parametrisation of the beam. Coupled to the global optimisation algorithm, it was proven to be efficient in dealing with topology optimisation for nonlinear vibration mitigation [1]. A 3D individual component is characterised by 9 parameters, which define explicitly a Level-Set Function (LSF) [5]. The global geometry is defined as the union of the unitary LSF. By moving, rotating, and shrinking each component, complex topologies can be described. The LSF is then projected on the initial mesh to define the density field. An example of topology obtained with the MMC framework is given in Figure 2.

Global Optimisation

The optimisation problem is non-convex with many local minima. For this reason, a global optimisation algorithm is considered. The strategy proposed here couples two algorithms, namely the CMA-ES [9] and the PSO [10]. A preliminary study has been done on the case of linear vibrations, and it appears that the CMA-ES is very robust and efficient to reach approximately the global minima, but then requires many model calls to reach the precise solution. On the other hand, the PSO algorithm is very efficient when initialised near the optimal point. The solution proposed here couples the two algorithms.

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