Chapter 22
Multi-level Stochastic Model Identification of Complex Aeroelastic Systems Considering Aerodynamic Nonlinearities

Michael McGurk and Jie Yuan

Abstract This study investigates the nonlinearities in aeroelastic systems that lead to Limit Cycle Oscillations (LCO) below the linear flutter threshold. We employ an adaptive multi-level data-driven model updating framework within a Bayesian inference scheme, achieving up to a 20% improvement in accuracy over conventional methods. However, previous models struggled to fully capture experimental data, particularly at high angles of attack, due to reliance on a quasi-steady linear aerodynamic model. To enhance model fidelity, we incorporate the nonlinear Beddoes-Leishman (B-L) model, which effectively accounts for dynamic stall and vortex shedding. The resulting probabilistic bifurcation diagram successfully encompasses all experimental data within 95% confidence bands, demonstrating significant improvement in accuracy compared to the quasi-steady model. This study confirms the multi-level framework's effectiveness in accurately representing experimental behaviour through improved mathematical modelling.

Keywords Bayesian model updating · Beddoes-Leishman · LCO · Aeroelasticity

Introduction

Nonlinearities in aeroelastic systems can result in subcritical post-flutter responses, leading to self-sustaining oscillations, known as Limit Cycle Oscillations (LCO), at velocities below the linear flutter threshold. These behaviours have been experimentally observed in high-aspect-ratio flexible wings, and recent studies have identified subcritical LCO formation in highly flexible wings during low-speed flutter experiments. Accurately validating mathematical models to capture these nonlinear behaviours is, therefore, crucial. However, generating the bifurcation diagrams required to describe LCO behaviour is often computationally expensive.

In previous work, an adaptive multi-level data-driven model updating framework was demonstrated, in which data-driven models were developed and employed within a Bayesian inference scheme to obtain probabilistic estimates of nonlinear parameters and LCO behaviour [1]. This process was applied to a simple aerofoil test case with a structural nonlinearity in the pitch degree of freedom and a linear quasi-steady (Q-S) aerodynamic model. The proposed methodology provided up to a 20% improvement in accuracy compared to conventional deterministic methods, while significantly increasing computational efficiency in both the updating and uncertainty quantification processes when applied to a nonlinear aeroelastic test case with experimental data.

Despite these improvements, none of the probabilistic responses fully captured the experimental data, indicating potential noise or a mismatch between the mathematical model and experimental results, especially at high angles of attack. It is hypothesised that this discrepancy arises from the use of a quasi-steady linear aerodynamic model, which fails to account for nonlinear aerodynamic effects at high angles of attack. To address this, the nonlinear Beddoes-Leishman (B-L) model, which accounts for dynamic stall and vortex shedding, has been employed [2]. The aim is to improve the agreement between the probabilistic behaviour estimates and the experimental data. The B-L model has proven successful in low-airspeed analyses and has shown the capability to capture high-amplitude nonlinear oscillations, while remaining relatively computationally

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efficient compared to alternative models [3]. In this study, the vortex shedding phase is of particular interest, as it can induce additional lift [4]. In theory, this could lead to higher amplitude LCO at greater pitch angles and improve the fit of the data to the bifurcation curves from the previous study. The new model also introduces more degrees of freedom, allowing this study to further demonstrate the improved computational efficiency of the multi-level updating framework. Additionally, the inclusion of aerodynamic nonlinearities enables the consideration of more nonlinear parameters in the model updating process.

Methodology

The Beddoes-Leishman model, as outlined in [2], was applied to the two-degree-of-freedom structural model used in the previous study [1]. This new model introduces additional complexity, expanding the system from a 6-degree-of-freedom problem to a 16-degree-of-freedom system, by adding twelve aerodynamic states (denoted x_1– x_{12}) alongside the four structural states. As in the previous study, a structural nonlinearity is introduced in the pitch degree of freedom (α), defined by the two nonlinear parameters $K_{\alpha 2}$ and $K_{\alpha 3}$

$$f_{nl} = K_{\alpha 2}\alpha^2 + K_{\alpha 3}\alpha^3$$

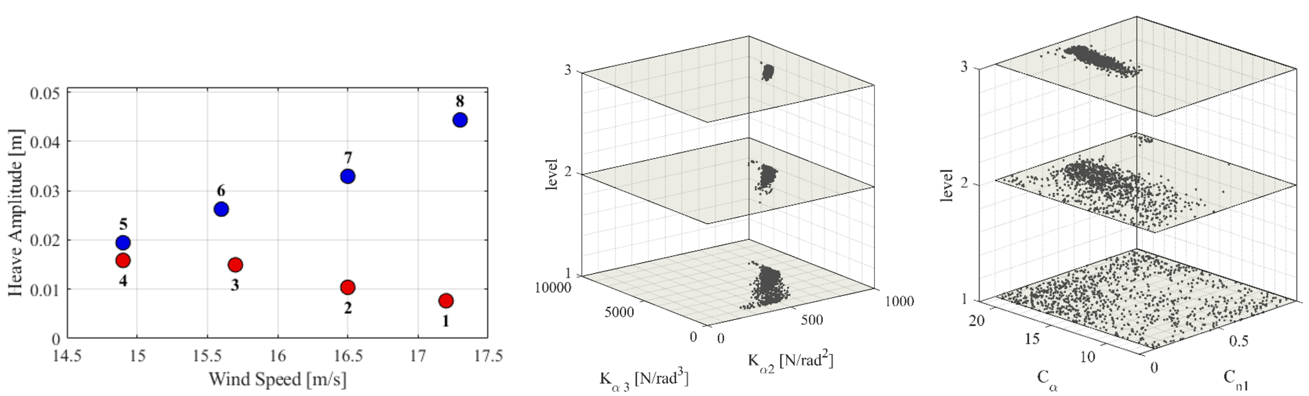
These were the only parameters estimated by the multilevel data-driven Bayesian framework in the previous case. However, with the introduction of a nonlinear aerodynamic model, we can now identify key nonlinear aerodynamic terms that influence vortex-induced lift and estimate these terms as well. The primary aerodynamic state influencing the vortex shedding phase in this study is x_{12}, which represents the normal force coefficient due to vortex shedding. This acts as an additional lift force during the vortex shedding phase and is evaluated with:

$$\dot{x}_{12} = \left(\frac{2V}{c}\right)\frac{-x_{12} + \dot{C}_v}{T_v}$$

Where V is the airspeed, c is the wing chord, and T_v is a time constant. $\dot{C}_v$ is a derivative dependent on the constant C_α, meaning the magnitude of the normal force due to lift is influenced by C_α. Another term that significantly affects the impact of the vortex-induced lift force is C_{n1}. This term essentially defines the points in heave and pitch where vortex shedding occurs. In summary, four key terms have been identified as having a significant influence on the shape of the bifurcation diagrams: the structural terms $K_{\alpha 2}$ and $K_{\alpha 3}$, and the aerodynamic terms C_α and C_{n1}.

Results

The multi-level data-driven Bayesian framework was applied to the new model with the aim of estimating $K_{\alpha 2}$, $K_{\alpha 3}$, C_α, and C_{n1} over three levels of model updating based on the experimental points shown in Figure 1a [5]. The resulting scatter



(a) (b) (c)

Fig. 1 a) Control based continuation experimental data adapted from [5], b) Structural parameter scatter plots, c) Aerodynamic parameters scatter plots

plots for each level are shown in Figure 1. The mean estimates for $K_{\alpha 2}$ and $K_{\alpha 3}$ were 722.35 N/rad^2 and 4762.8 N/rad^3, respectively, representing an increase of up to 23% compared to the results obtained using the quasi-steady aerodynamic model. The mean estimates for C_α and C_{n1} were 17.7 and 0.474, respectively. This reflects a 30% increase in C_α from the value reported in ref [1] and a 12% decrease in C_{n1}.

The resulting probabilistic bifurcation diagram is presented in Figure 2a, showing that all experimental points are captured within the 95% confidence bands. When compared to the probabilistic results from the quasi-steady study in Figure 2b, a significant improvement is evident, as 37.5% of the data in the previous model falls outside the confidence region. A comparison of the mean estimations from each model is provided in Figure 3. From Figure 3a, it is clear that the results from the B-L model offer a better fit to the experimental data, particularly in the upper portion of the diagram, as anticipated. A direct comparison of accuracy, shown in Figure 3b, demonstrates that the B-L model improves accuracy at every data point. Both models capture the system's behaviour well before the turning point; however, beyond this point, there is a noticeable decline in accuracy for both models, although the BL model performs significantly better.

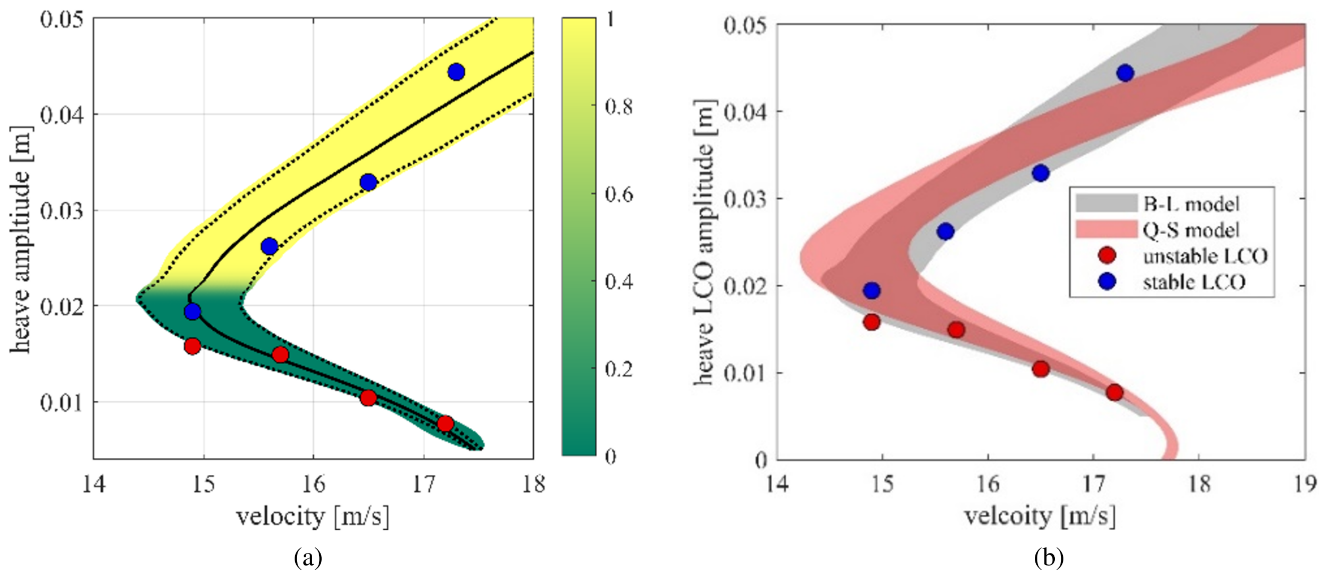


Fig. 2 a) Probabilistic bifurcation diagram, b) Probabilistic bifurcation diagram comparison

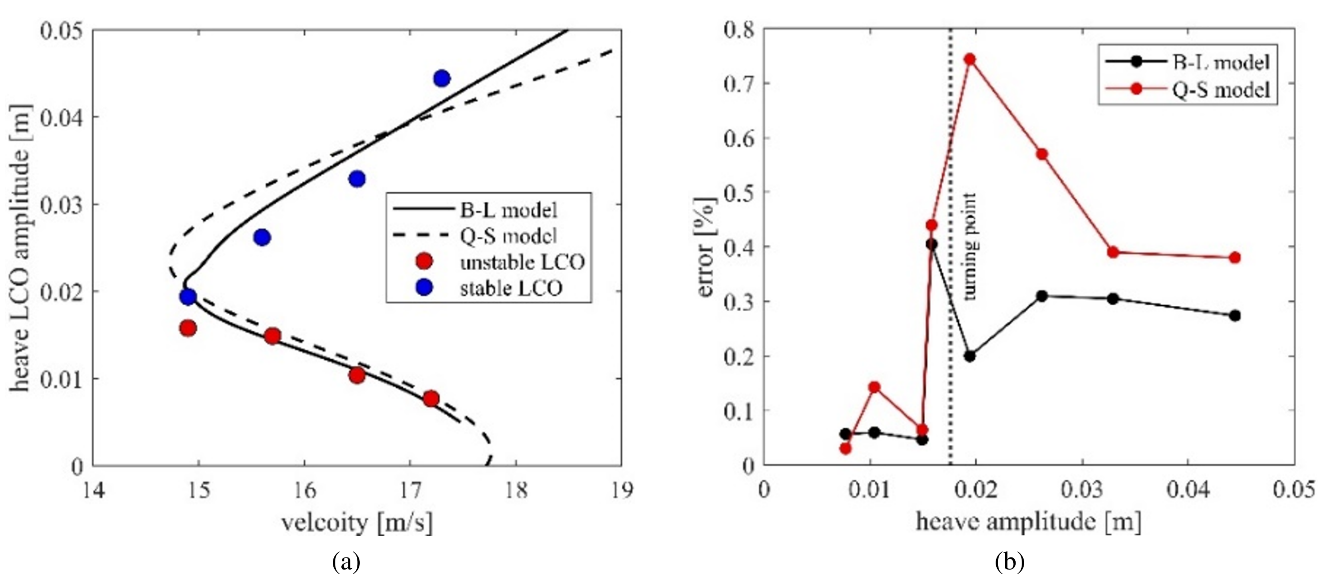


Fig. 3 a) Mean bifurcation diagrams, b) Mean bifurcation diagrams comparison

Conclusion

The findings of this study demonstrate that when applying the multi-level framework to the B-L model, the behaviour observed in the experimental data is captured with greater accuracy. The probabilistic bifurcation diagram successfully encompassed all the data using the B-L model, and the mean plot showed improved accuracy at every point when compared to the Q-S model. Further investigation is required to determine whether the aerodynamic parameters can be altered to such an extent and whether this reflects the true aerodynamics of the system. However, what this study does confirm is that if the mathematical model can represent the experimental behaviour, the multi-level framework will function effectively.

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