Chapter 23
Resonance Tracking of Nonlinear Structures by Amplitude Controlled Sine-Dwell Testing

Gleb Kleyman, Hans-Jürgen Borutta, and Sebastian Schwarzendahl

Abstract Sine-dwell testing is a common methodology in high cycle fatigue (HCF) testing. During a sine-dwell test the structure is excited with a single sine tone exactly at a resonance frequency until structural failure or sufficient time/cycles passed. When the resonance frequency shifts due to accumulated damage or temperature changes the controller tracks the resonance frequency by adjusting the drive frequency to maintain a defined phase relation between two channels.

Nonlinear systems are known to have modal parameters varying with the excitation amplitude. The evolution of the resonance frequency over the excitation level is often referred to as the backbone-curve of a system. Backbone-curves can provide useful insights into the physical mechanisms involved in the nonlinearity.

For many nonlinear structures, the backbone-curve can be experimentally determined using the sine-dwell method combined with amplitude control. In this approach, the resonance frequency is tracked as the excitation level is increased step by step. This iterative phase control technique is an extension of traditional sine-dwell testing and is now available in the latest m+p VibControl software. It allows test engineers to better understand the nonlinear behavior of structures using familiar technology.

This paper discusses the application of this method to a cantilever beam excited within an amplitude range where it exhibits geometric nonlinear behavior. When excited using conventional amplitude-controlled sweeps, the system experiences a jump phenomenon, with both resonance frequency and damping varying. It is demonstrated that these amplitude-dependent modal parameters can be characterized through nonlinearity plots derived from data obtained during sine-dwell testing. These results can be directly compared to classical methods, offering a reduction in testing time.

Keywords Resonance Tracking · Sine Dwell · Backbone-Curve · Phase Resonance · Nonlinear Modal Analysis

Introduction

Nonlinearities are a common occurrence in vibration and modal testing. It is likely that all practical engineering structures exhibit some degree of nonlinearity. Typically, this nonlinearity results in modal parameters that change with load levels. These modes require careful consideration, as the resonance may shift to critical frequencies when the excitation level changes.

Various factors, such as structural joints, boundary conditions, and material properties, can be identified as sources of nonlinearity [1]. However, geometric nonlinearity is often overlooked. Mechanical systems are classified as geometrically nonlinear when the nonlinear effects arise solely from the system's kinematics. In practical applications, slender structures like beams tend to demonstrate geometrically nonlinear behavior when subjected to sufficiently large deformation amplitudes, even though the strains and stresses remain within the linear elastic range. It has been demonstrated that, disregarding modal interaction, plane beam systems can be represented as Duffing oscillators [2]:

$$m\ddot{q} + d\dot{q} + kq - \alpha q^3 = f, \tag{1}$$

Here, q represents the (modal) displacement, while m, d, and k denote the structural parameters. The term f corresponds to the excitation force, and α represents the cubic stiffness term. In this context, it is important to differentiate between the two primary types of geometrically nonlinear effects: (1) midplane stretching, which results from the coupling of transverse

Gleb Kleyman
m+p international Mess- und Rechnertechnik GmbH, Thurnithistraße 2, 30519 Hannover, Germany
e-mail: gleb.kleyman@mpihome.com; hans.borutta@mpihome.com; sebastian.schwarzendahl@mpihome.com

Ludovic Renson et al. (eds.), *Nonlinear Structures & Systems, Vol. 1*, Conference Proceedings
of the Society for Experimental Mechanics Series, https://doi.org/10.13052/97887-438-0146-7_23

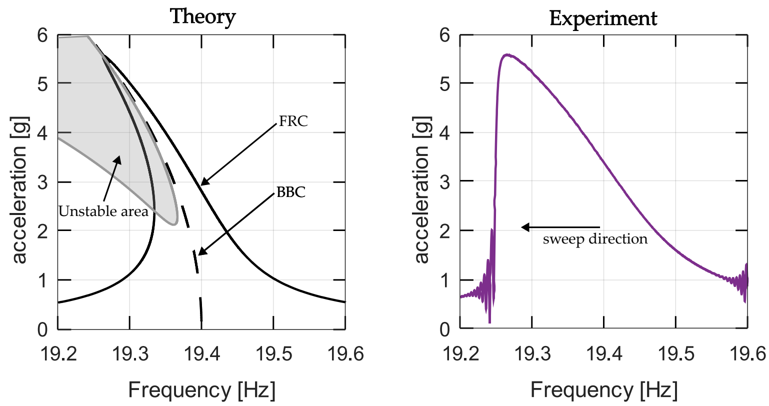

Fig. 1 Theoretical frequency responce curve vs. swept sine measurement of a cantilever beam

vibrations with longitudinal strain, and (2) nonlinear elasticity, arising from large cross-sectional rotations. The latter effect tends to exhibit a softening behavior, which here is reflected as a negative nonlinear stiffness term in the Duffing model. In the left subplot of Fig. 1, the frequency response curve (FRC) and backbone-curve (BBC) of a softening Duffing oscillator are shown, estimated using the Harmonic Balance method [3]. The FRC is depicted as a solid black line, while the BBC is represented by a dashed line. The unstable region is shaded in gray. For comparison, the right subplot presents the measured response of a base-excited cantilever beam, where the base acceleration was maintained at a constant excitation level during a swept sine test.

It can be observed that as the measured response approaches the upper bifurcation point, where the system typically loses stability, the curve abruptly jumps from the upper to the lower stable branch. A similar phenomenon occurs in the opposite direction at the lower turning point of the FRC (not shown in the figure). In both cases, the middle branch of unstable solutions remains undetected. Traditional experimental modal analysis methods are insufficient to fully characterize such systems due to the non-uniqueness of the frequency response. As a result, new approaches are required. Common alternatives include using the backbone-curve or plotting the resonance frequency and damping as functions of the excitation amplitude (nonlinearity plots).

With growing attention to the identification of nonlinear systems over the past two decades [4, 5], the automated extraction of backbone-curves has become increasingly desirable. In recent years, advanced control methods have been introduced [6, 7]. These approaches are primarily based on two key principles: (a) stabilizing the system when it nears an unstable oscillatory state, and (b) tracking the backbone-curve as the excitation level increases. It can be observed that the BBC approaches the unstable region at higher oscillation levels. Consequently, specialized nonlinear identification methods, such as the previously mentioned PLL and CBC, incorporate time-domain feedback control to maintain stable oscillations at higher levels. However, as noted in [8, 9], this classical control strategy is not always required, as the driving point position and shaker properties can influence the instability region. When this is the case, a simpler frequency-domain approach, like the one presented here, is sufficient for tracking the resonance frequency. In this method, a defined phase relationship between two channels is maintained by iteratively adjusting the excitation frequency.

Experimental Setup and control strategy

A photograph and schematic representation of the experimental setup are shown in Fig. 2. The specimen used was a cantilever beam with dimensions of 190x4x0.8 cm, base-excited in the horizontal direction. Both the acceleration of the base and the beam were measured. The control system consisted of m+p VibRunner hardware and m+p VibControl 2.17 software. The third bending mode of the beam was investigated, with a linear natural frequency of approximately 19.4 Hz. The beam is made from a single piece of steel, with the only bolted connection being at the fixture to the shaker's slip table, minimizing frictional contacts. Hence, the observed nonlinear behavior is likely due to the large deflection, as discussed in the previous section.

For phase control, a relative acceleration $\ddot{q}$ between the beam and the base was introduced. As explained in [10], a 90° phase shift between $\ddot{q}$ and $\ddot{q}_{\mathrm{base}}$ was selected as the control target. The relative phase is calculated from the fundamental harmonics of these signals, and if it deviates from the reference phase, the excitation frequency is adjusted until the target is achieved. The amplitude controller functions similarly. However, the reference value for the amplitude controller can be set in a dwell table, allowing different excitation levels to be sequentially targeted during testing. This control strategy does not

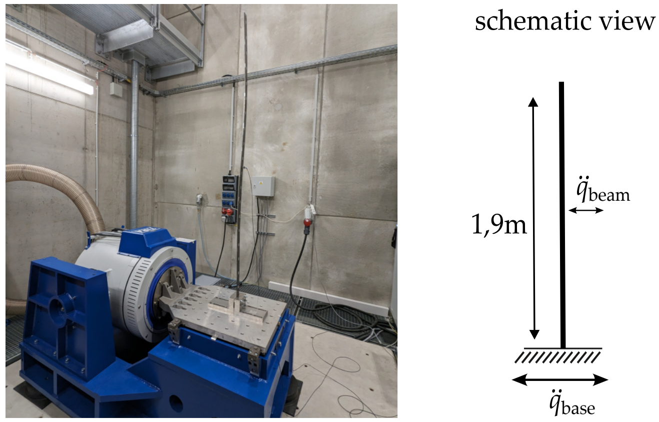

Fig. 2 Experimental setup of a base excited cantilever beam

inherently stabilize the system, as it operates in the frequency domain and cannot prevent the system from jumping between stable branches if a disturbance occurs. However, by keeping the amplitude increments small between different levels, it is possible to trace the backbone curve deep into the overhanging range, as the following results will demonstrate. Larger amplitudes than those shown in Fig. 4 were not targeted due to concerns about the structural integrity of the system.

Experimental Results

Two types of measurements were conducted: (1) resonance tracking using the sine-dwell testing mode and (2) classical amplitude-controlled swept sine excitation at various amplitude levels. The three subplots in Fig. 3 display the time records of the two acceleration signals, along with the corresponding phase and frequency values over the entire duration of the sine-dwell test. An initial frequency of $f_0 = 19.3$ Hz was set. Starting from this frequency, the control algorithm initiates a frequency sweep at the defined sweep rate to locate the targeted phase. Once the desired phase value is reached, the controller locks onto it and maintains tracking for the remainder of the test duration, which was set to 120 seconds in this instance. The relevant area is shaded in gray.

When the time period elapses, the next amplitude level is approached, with each level maintained for 90 seconds. This duration could be extended to minimize variations in the BBC measurement caused by transients; however, the current experiment was designed to minimize the overall testing time. After each increment, the resonance frequency is detuned, as indicated by the individual peaks in the phase curve. The controller then adjusts the excitation frequency to restore the phase resonance condition.

The left subplot of Fig. 4 displays the backbone curve (BBC), estimated from the results shown in Fig. 3, alongside the frequency response curves obtained from base acceleration-controlled sweeps. To calculate the BBC, measurement points were selected from the phase curve that fall within an interval of 88° to 92°. Each data point of the BBC represents an average over the corresponding interval at a specific amplitude level. The sweeps were conducted in a descending frequency

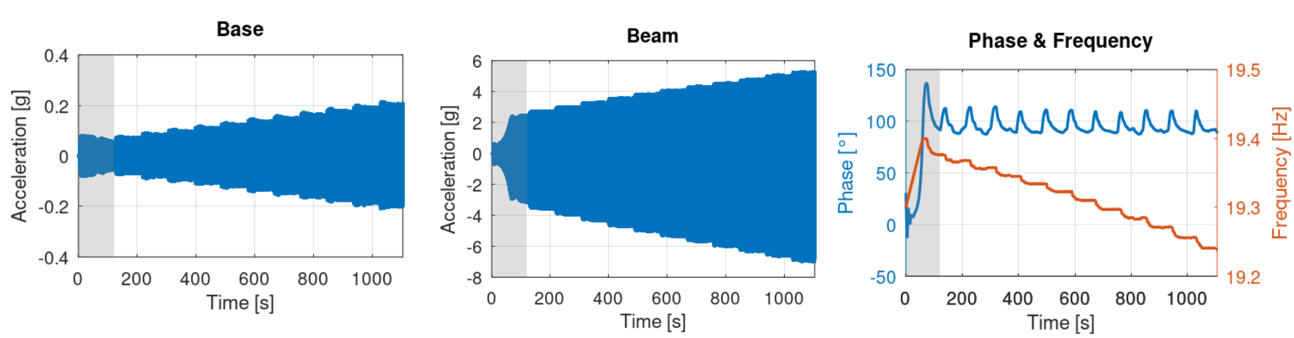

Fig. 3 Time domain data of a sine dwell test for resonance tracking

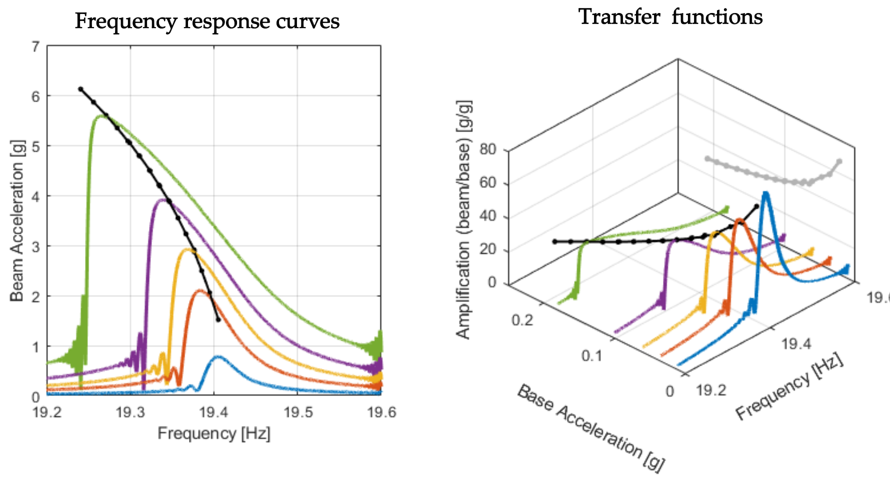

Fig. 4 Comparison of results from amplitude controlled swept-sine excitation and resonance tracking (backbone-curve)

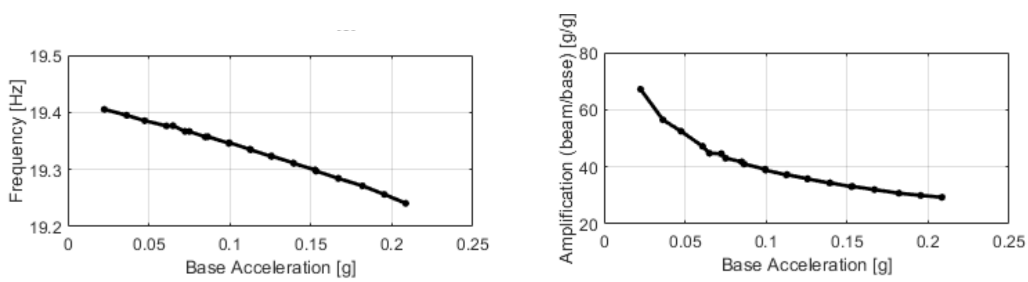

Fig. 5 Nonlinearity plots

direction at five different amplitude levels. It is evident that the BBC accurately captures the resonance peaks of the frequency response curves. Additionally, the resolution of the BBC is significantly finer due to the larger number of measurement points compared to the number of sweep levels. Notably, the measurement time for obtaining the BBC through sine-dwell testing is only half that of sequential sweep measurements.

The right subplot of Fig. 4 presents the transfer functions, obtained by dividing the output signals by their corresponding input values. In engineering applications, the transfer function is often considered the most important characteristic of a dynamic structure, as it defines the maximum response amplitude for a given excitation. Linear systems exhibit proportional behavior, which means their transfer functions remain constant for different excitation levels. Therefore, in a three-dimensional representation similar to Fig. 4, where different functions are plotted at the corresponding excitation levels, the locus of resonance peaks in the amplitude subspace (shown as a gray curve) would appear as a horizontal line. However, this is not the case for the nonlinear system in this example. Instead, amplification decreases with increasing excitation levels within the examined range. Typically, a decreasing amplification factor in nonlinear systems is associated with an increasing damping ratio. A potential explanation for the observed nonlinear damping was provided by [11] in a similar experimental setup. Large amplitude vibrations can produce quadratic damping due to aerodynamic drag, which is negligible at smaller amplitudes. Similar findings were also reported by [12]. Unfortunately, classical methods for estimating damping, such as the half-power method, cannot be used to validate this theory due to the asymmetrical shape of the frequency response curves. However, a novel approach for estimating nonlinear damping from phase-resonant base excitation tests was proposed by [10]. Due to the extensive experimental effort involved, further investigations into the damping characteristics were not conducted in this study but will be considered in future work.

If amplification can serve as an alternative measure for the modal damping of the system, nonlinearity plots can be generated from the collected measurement data, as illustrated in Fig. 5. In these plots, the resonance frequency and amplification (the maximum value of the transfer function) are displayed against the excitation amplitude. This approach offers valuable insights into the nonlinear structural properties of the mode under investigation. In this particular example, a nonlinear damping behavior emerged that was not considered in the mechanical model.

Conclusion

This study successfully demonstrates the application of the sine-dwell testing method combined with amplitude control to effectively investigate the nonlinear behavior of a cantilever beam. By tracking the resonance frequency as the excitation level is increased, the backbone-curve was measured, offering critical insights into the modal parameters that vary with excitation amplitude. The results indicate that traditional modal analysis techniques may be insufficient for fully characterizing nonlinear systems due to their complex response characteristics. The findings also reveal a notable nonlinear damping behavior that was not captured in the existing mechanical model, emphasizing the necessity for further refinement in future research. Overall, the research provides valuable methods and insights for engineers and researchers who want to better identify and model the complexity of nonlinear dynamic systems in practical applications.

References

1. Worden, K. and Tomlinson, G. *Nonlinearity in Structural Dynamics - Detection, Identification and Modelling*. IOP Publishing (2001).
2. Thomsen, J. *Vibration and Stability*. Springer-Verlag Berlin Heidelberg (2003).
3. Hagedorn, P. *Nichtlineare Schwingungen*. Akademische Verlagsgesellschaft (1978).
4. Kerschen, G., Worden, K., Vakakis, A.F., and Golinval, J.C. "Past, present and future of nonlinear system identification in structural dynamics". *Mechanical Systems and Signal Processing*, 20(3):505–592 (2006).
5. Noël, J. and Kerschen, G. "Nonlinear system identification in structural dynamics: 10 more years of progress". *Mechanical Systems and Signal Processing*, 83:2–35 (2017).
6. Hippold, P., Scheel, M., Renson, L., and Krack, M. "Robust and fast backbone tracking via phase-locked loops" (2024).
7. Tatzko, S., Kleyman, G., and Wallaschek, J. "Continuation methods for lab experiments of nonlinear vibrations". *GAMM-Mitteilungen*, 46(2):e202300009 (2023).
8. Stanbridge, A., Ewins, D., Sanliturk, K., and Ferreira, J. "Experimental investigation of dry friction damping and cubic stiffness nonlinearity". *Proceedings of the ASME Design Engineering Technical Conference*, 6:2141–2148 (2001).
9. Robbins, E., Kuether, R., Pacini, B., and Moreu, F. "Stabilizing a strongly nonlinear structure through shaker dynamics in fixed frequency voltage control tests". *Mechanical Systems and Signal Processing*, 190:110118 (2023).
10. Müller, F., Woiwode, L., Gross, J., Scheel, M., and Krack, M. "Nonlinear damping quantification from phase-resonant tests under base excitation". *Mechanical Systems and Signal Processing*, 177:109170 (2022).
11. Colin, M., Thomas, O., Grondel, S., and Cattan, E. "Very large amplitude vibrations of flexible structures: Experimental identification and validation of a quadratic drag damping model". *Journal of Fluids and Structures*, 97:103056 (2020).
12. Debeurre, M., Benacchio, S., Grolet, A., Grenat, C., Giraud-Audine, C., and Thomas, O. "Phase resonance testing of highly flexible structures: Measurement of conservative nonlinear modes and nonlinear damping identification". *Mechanical Systems and Signal Processing*, 215:111423 (2024).