Chapter 4
Fatigue Damage Potential in a Nonlinear Dynamical System with Isolated Resonances

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Abstract Cyclical loading can cause fatigue damage in a structure, even at relatively low stress levels. If the structure is subject to nonlinear behaviors, then it may also experience complex phenomena such as super- or sub-harmonic resonances, chaotic motion, and isolated resonances, also known as isolas. Isolas in particular may be especially dangerous for a structure because they are dependent on initial conditions or perturbations to the structure, and they can drive the structure at amplitude outputs of an order of magnitude or higher than the expected amplitude levels. Although fatigue damage is a widely researched field, there is currently little work available on quantifying the effects of fatigue damage in a system that is experiencing isola motion. The goal of this work is to evaluate the fatigue damage potential of a harmonically driven, nonlinear dynamical system that is known to have isolas. The damage that can be accumulated while on an isola is compared to the damage accumulated at other points along the main frequency response branch of the system. Rainflow cycle counting is used to compute the fatigue damage potential, given a representative material with a range of S-N curve coefficients.

Keywords Fatigue, cyclical loading, nonlinear dynamics, isolated resonance, rainflow

Introduction

Nonlinearities give rise to a wide variety of strange phenomena in dynamical systems. Of particular interest in this work is the isolated resonance, which is a type of nonlinear behavior that forms closed solution curves separate from the main solution curve in a system's frequency response. Other names for this phenomenon include detached resonance, isolated response, detached response, or simply isola. Isolas have a tendency to occur at relatively high amplitudes and over narrow frequency ranges, and they can form near to or far from the system's primary resonances, depending on the system parameters. Isolas have been observed and studied since at least 1938, with Rauscher [1] analyzing a dynamical system whose frequency response contained solution curves that originated from "the delicate relation between damping and response in the present problem". Abramson [2] similarly observed isolas in a system with softening nonlinearity. As the excitation force was varied, two solution curves moved closer and eventually merged at a single point before detaching again into a classical softening resonance and a separate set of curves that "corresponds to motions of large amplitude and low frequency". Research on isolas picked up significantly in the 2010s, following the increased popularity of numerical techniques such as harmonic balance to simulate nonlinear systems. The inclusion of numerical continuation methods allows isolas to efficiently be traced out in the frequency space, and even with respect to multiple parameters. Mélot et al. [3] and Habib et al. [4] provide nice reviews on the historical development of isolas.

In many structural dynamics applications, isolas are considered dangerous for two main reasons: their susceptibility to perturbations, and their often relatively large amplitudes. As isolas are detached from the main frequency response curve, they are capable of forming far from resonances where the amplitude is otherwise expected to be small. A small bump, shock, or other perturbation sending a structure into unforeseen resonance is, understandably, quite an undesirable situation that places life and property at unacceptable risk. However, there has been seemingly little work to actually quantify the danger that an isola can pose. In this work, we evaluate the risk of isolas in a harmonically excited nonlinear dynamical system [5] using the Palmgren-Miner fatigue damage metric [6]. Our objective is to compare the damage accumulated due to motion on an isola to the damage accumulated at other co-existing solutions. The results of this work could help inform to

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what extent an isola motion can eat up the fatigue life of a structure, and how to better qualify a structure designed to operate in an environment that may be at risk of experiencing nonlinear behaviors such as isolas.

Methodology and System Model

Three simple and well-known methods are combined for this fatigue analysis: the log-linear (i.e., Basquin) model of the stress-cycles curve $\mathrm{N} = \mathrm{cS}^{-\beta}$, Palmgren-Miner linear damage $\mathrm{D} = \sum_{\mathrm{i}=1}^{\mathrm{k}} \frac{\mathrm{n_i}}{\mathrm{N_i}}$, and rainflow cycle counting [7]. An experimental apparatus of the dynamical system, along with parameter values, can be found in [3] and is modeled as follows:

$$\ddot{x} + 2\omega_n \zeta \dot{x} + \omega_n^2 x + \frac{\alpha}{m} x^3 + \frac{F_c(x)}{m} = \frac{p}{m} cos\left(\omega t\right) \quad , \tag{1a}$$

$$F_c = \begin{cases} K_c\left(x + j_1\right), & x < -j_1 \\ 0, & -j_1 \leq x \leq j_2 \\ K_c\left(x - j_2\right), & x > j_2 \end{cases} \tag{1b}$$

$$\sigma\left(t\right) = \frac{12Eb}{L^2} x\left(t\right) \tag{2}$$

Euler-Bernoulli beam theory, for a fixed-fixed beam with centered load, is assumed in order to approximate the maximum stress in the system in Equation (2) [8]. Figure 1 shows the nonlinear forced response curves for the system model, including superharmonic, subharmonic, and isola solutions.

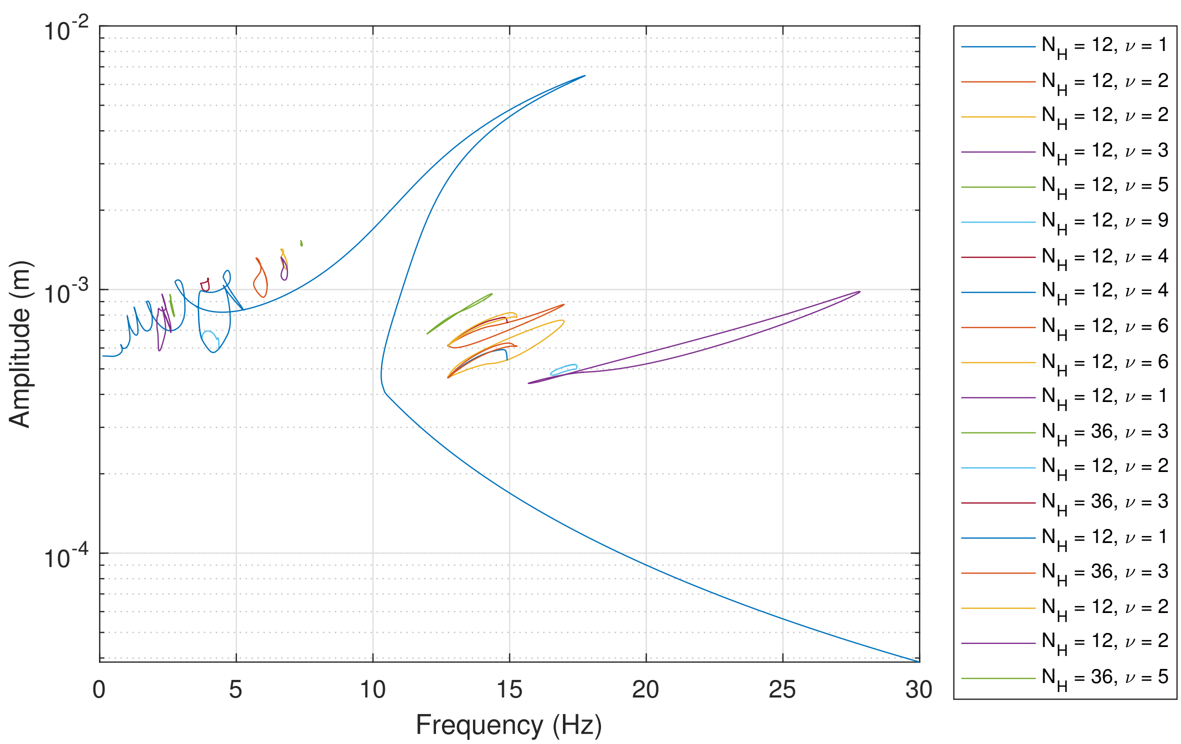

Fig. 1 Nonlinear forced response curves for the dynamical system described by Eq. (1). N_H is a solver convergence parameter, and ν is the subharmonic order of each solution curve

Fatigue Damage Analysis

Figure 2 shows the calculated fatigue damage for each of the solution curves in Figure 1. A fatigue damage coefficient of $\beta = 4.22$ was used in the following calculations. Complete analysis details and parameter values can be found in the final conference presentation. Notice that while the amplitudes span from 0.4 mm to 6 mm, the damage values span a much larger range from $3 * 10^{-13}$ to $4 * 10^{-4}$. Because the damage metric scales linearly with time duration, we can perform relative comparisons. At 20 Hz, for instance, the subharmonic isola of order 3 (in purple) has an amplitude approximately 6 times larger than the off-resonance amplitude that one would traditionally expect to encounter in this frequency region, but the fatigue damage for the same time duration is almost 2,400 times higher on the isola—a difference greater than three orders of magnitude. This difference means a 20-year life of operation at the off-resonance motion is reduced to a mere 3 days and 2 hours on the isola. For a material with fatigue coefficient $\beta = 6.67$, that lifetime is reduced to just 45 minutes.

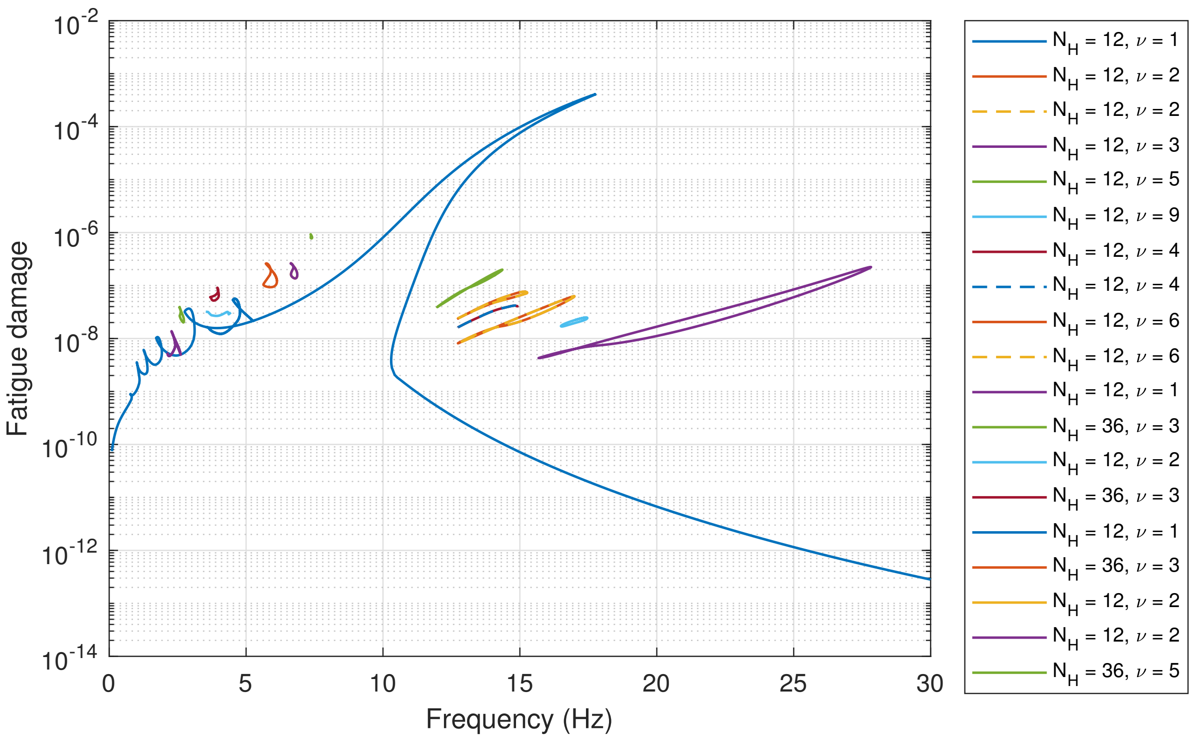

Fig. 2 Fatigue damage for the solution curves in Fig. 1 for a fatigue damage coefficient of $\beta = 4.22$. N_H is a solver convergence parameter, and ν is the subharmonic order of each solution curve

Conclusions

In this work, we assessed the risk of encountering an isola in a nonlinear dynamical system. Palmgren-Miner fatigue accumulation and rainflow stress cycle counting were applied to a structure treated as a loaded Euler-Bernoulli beam with a Basquin model of fatigue life. It was shown that the fatigue damage due to an isola motion can easily be orders of magnitude higher than an off-resonance motion, significantly eating into the fatigue life of a structure. The most straightforward defense against this type of damage is to treat an isola like a regular resonance: design the operating characteristics of the structure around the frequency ranges of any isolas; or design the structure to simply not have isolas in the operating frequency range. There are several avenues for future work on this topic.

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