Chapter 5
Robust Optimization of Surface Topology for Jointed Connections

Aalokeparno Dhar, Justin H. Porter, and Matthew R. W. Brake

Abstract Bolted connections are ubiquitous in mechanical and aerospace engineering. However, these connections introduce significant uncertainties in vibration responses. Given sufficient information about a jointed connection, a recent physics-based rough contact modeling has provided significant improvements towards predicting nonlinear trends in frequency and damping. For jointed connections, increasing vibration amplitudes result in a decrease in the natural frequency and an increase in damping as the system transitions from sticking to slipping. While the physics-based model showed improvements towards predicting these trends, it required knowledge of the as-built mesoscale topology of the interface. Specifically, an approximately 100 micron gap in the center of the nominally flat interface resulted in an approximately 7% shift in the natural frequency compared to simulations with a flat interface. This uncertainty is due to manufacturing tolerances and cannot easily be eliminated for mass production of engineering structures. To address this challenge, the present work seeks to understand how the contact interface within a joint can be designed to reduce uncertainty due to uncontrolled machining deviations. Thus, the physics-based rough contact model that represents the best available predictions for jointed connections is employed to simulate the Brake-Reuß Beam. These simulations consider the effects of different interface designs while also analyzing the sensitivity of the responses to mesoscale topology variations. The extended periodic motion concept (EPMC) is utilized to obtain nonlinear modal properties. Sensitivities are calculated directly based on the solution points and verified with finite difference calculations. These sensitivities provide groundwork for optimizing designs to reduce uncertainty due to manufacturing deviations. Overall, this work provides insights into the effects of machining variations on jointed connections and how to improve designs to limit such effects.

Keywords Friction · Jointed Connection · Mesoscale Topology

Introduction

Bolted connections in a structure can introduce significant nonlinearities, which makes it challenging to analyze and predict the structure's behavior. In terms of the system's dynamic characteristics, the nonlinearity due to the bolted connections affects both the free vibration response's resonant frequencies and dissipative behavior. The damping factor and modal frequency can vary with the types of bolted interfaces, as seen across a number of experiments [1–7]. Lap joints can exhibit a 1-2% decrease in resonant frequency as the excitation amplitude is increased; however, over the same range the damping capacity can increase by an order of magnitude [1]. Flange joints, by contrast, can exhibit a much smaller increase, or even no increase, in damping as compared to lap joints [2, 3]. Dovetail joints can even exhibit a decrease in damping ratio as excitation amplitude increases (for some modes, and for other modes the damping ratio increases) [6, 7]. Across each of these types of joints, though, it is clear that the meso-scale topography and relative alignment of the joint surfaces can lead to much greater variability in the response of the structure than is due to preload or vibration amplitude [3, 7, 8].

The system characteristics of a jointed structure can be derived from free vibration responses, as demonstrated in [9–12]. In looking at different types of jointed connections, the participating modes significantly influence the system's characteristic damping frequency, while joint parameters, such as bolt torque, have been shown to affect modal interaction behavior [3, 13]. It is also found that changing the bolt torque can cause a different pressure distribution around the bolted joint [7, 14]. Upon dynamic excitation, the pressure distribution can exhibit highly nonlinear behavior. Variations in pressure distribution and the resulting system response can also stem from non-conformal interfaces, often caused by manufacturing uncertainties.

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After applying prestress through bolt torque, gaps between connected surfaces can exist at macro, meso, or micro levels. Chen et al. [15] analyzed the effect of non-flat interfaces in a Brake-Reuß Beam (BRB) and found that end-gap configurations reduce the natural frequency at low amplitudes, while central gaps increase it. To mitigate such uncertainties, deliberate macro-level variations can be introduced at joints, such as adding padding around bolt holes. This approach creates a high-pressure field at predictable locations [1]. Although this design enhances system determinism, concentrated local stresses may not be a desirable for a design. On the micro-scale, contact between two surfaces is governed by variations in asperity heights, which primarily drive frictional contact. Pressure distribution tends to be more localized in interfaces with higher surface roughness than those with smoother surfaces.

Surface errors due to manufacturing can also occur at the meso-scale. Once the bolt torque is applied, the mesoscale profile at the connecting surfaces can create an interference contact at some locations and a gap in some other locations. However, the nonlinear response of various gap topologies can also be something the designer can use to increase the joint interface's robustness. In this paper, we hypothesize that one can design a topology which makes the damping parameter robust to the modal amplitude. That is, it will be beneficial to find a surface profile that is less sensitive to the manufacturing perturbation than a flat, conformal interface. Designing such an interface, however, requires a predictive modeling methodology of structure dynamics with bolted joints with various mesoscale gap topologies.

To analyze the effect of mesoscale variability due to manufacturing error and to design a robust surface topology, it is essential to have a detailed model of the system that includes enough information related to the joint interface. Numerous studies [16] have attempted to understand contact mechanics, how to model the joints and friction and study its dynamic behavior. Among different models, physics-based modelling [17] is applicable to a wide range of joints as long as underlying surface properties are captured well, and is more suitable for the current problem. This model takes care of the micro, meso, and macro contacts at a hierarchical dependency from smaller to a larger scale. The microslip modelling [8] is preferred over the traditional stick-slip models because the former is found to be more accurate while compared with experimental behaviors. The rough contact modeling framework makes use of the statistical representation of the micro-scale asperity heights obtained from the experimental measurements.

In this work, the Brake-Reuß Beam to study the effect of mesoscale gap topology under modal vibration. Modelling provides a brief description of the methodology and the contact model used. The Extended Periodic Motion Concept [18] is used to solve this model using the response amplitude as a continuation parameter. Different gap profiles have been fed to the model to check their implications for the frequency and damping response. The sensitivity of these topologies over perturbations has been studied to gain an understanding of the variability of the response. The paper concludes with thoughts on the more quantitative approaches to obtain a robust manufacturable surface profile.

Modeling

The Brake-Reuss Beam (BRB) is a an assembly of two half beams connected with a three bolted lap joint. The finite element modelling of BRB is initially done in Abaqus (Figure 1a), and model reduction is applied to the interface degrees of freedom plus 50 fixed interface modes using the procedure described in [8, 19]. The model is then further reduced using relative coordinates to a model with 588 Zero Thickness Elements (ZTEs) and 14 fixed interface modes [8]. For a faster model, the mesh is further reduced to 232 ZTEs using the procedure in [20] (and shown in Figure 1b). These ZTEs are modeled to capture the interfacial kinematics and asperity heights obtained from the statistical distribution and the mesoscale gaps or interferences. Friction inside each of the ZTEs is modeled with a physics-based rough contact model [17] that has been reimplemented in Python using automatic differentiation [21]. The normal contact between the asperities at each element is described using the ellipsoidal Hertzian contact law and principles of strain hardening, depending on the elastic and plastic regimes. The tangential contact is modeled using the tangent asperity model [17]. Experimental data as well as the plasticity models have been used to obtain a friction coefficient. A friction coefficient value of 0.55 is obtained for this system. A preload of 12.249kN is applied to the bolts. The latter implementation is adapted for the present models with simulation routines and examples available at [22]. The schematic of the overall procedure is shown in Figure 2.

The Extended Periodic Motion Concept (EPMC) [18] is the primary solution method adopted for this work, which is suitable for lightly damped systems. This method helps obtain a periodic response of a free dissipative system by including an artificial negative damping to the system. This way, the method is useful for obtaining the natural frequency as well as the mass proportionate damping factor, which are the two additional variables along with the harmonic coefficients of each DOFs. A nonlinear prestress analysis and eigenvalue analysis around the preload condition provide the initial guess for the frequency and displacement coefficients for each DOF under calculation. the EPMC calculation requires the Fourier coefficients of the nonlinear force terms associated with the model, which is obtained using the Alternating Frequency-Time (AFT) procedure.

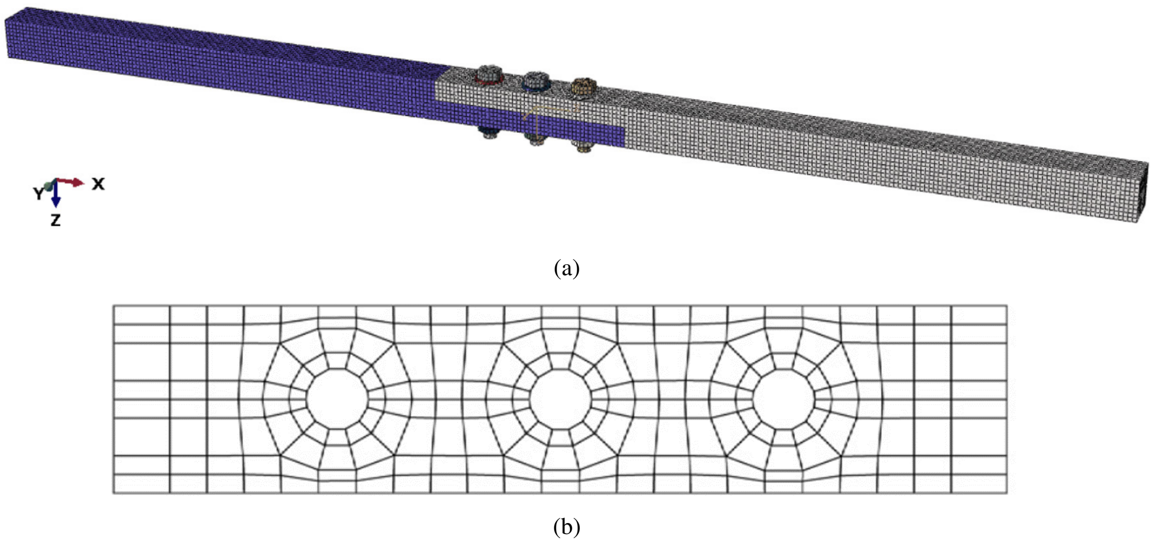


Fig. 1 Original mesh of BRB model in ABAQUS (a) and Reduced interface mesh of ZTEs (b)

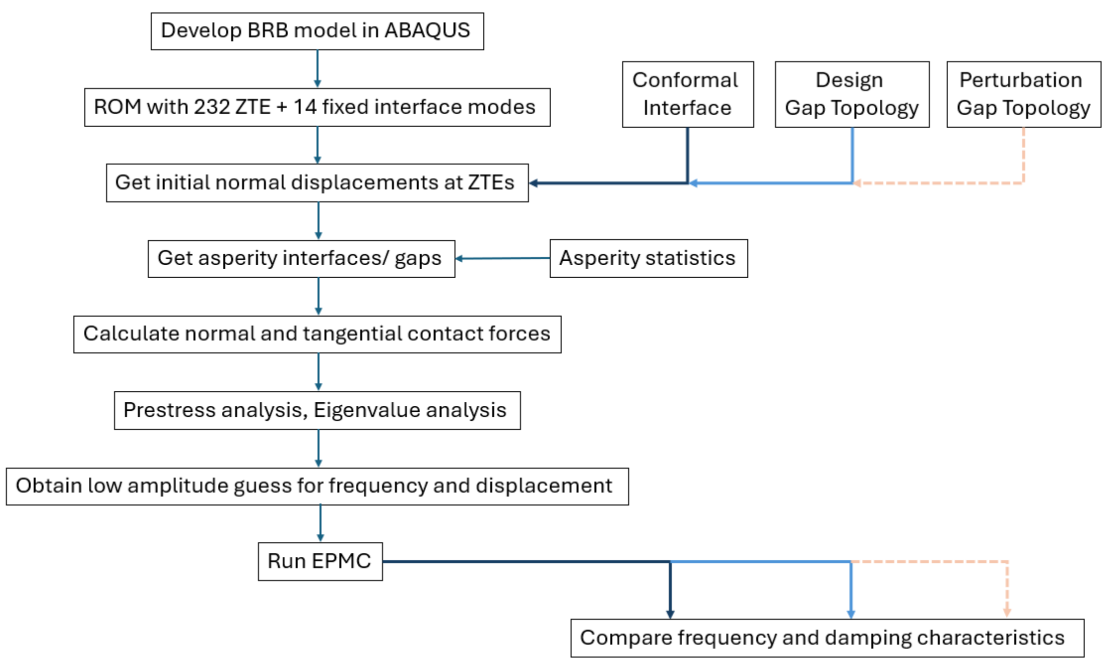


Fig. 2 Flowchart of the modelling approach.

The mesoscale topologies are applied within the ZTEs to offset the initial distances prior to contact of each ZTE. This approach allows for a single Abaqus model of a nominally flat interface to be used for all perturbations. It is expected that this approach is only valid for perturbations up to a certain amplitude; in the subsequent parameter studies, it is demonstrated that the perturbations considered in the present work are within the limit of validity for this modeling approach before a new Abaqus model of a larger perturbed surface is necessitated. This approximation can be validated by a detailed ABAQUS modelling of the perturbes surfaces in future work, where one can estimate the maximum height of the topology allowable by the ZTE.

The possible types of topologies are taken form the rectangular Legendre polynomials [23] and rectangular Zernike polynomials [24]. The Legendre polynomials, traditionally defined over a continuous interval, have been adapted to represent two-dimensional surface topologies over a rectangular domain. They are orthogonal polynomials that provide a systematic way to describe surface variations, offering a basis for approximating complex shapes. The zeroth order polynomial resembles flat topology, whereas the first-order polynomials represent y-tilt and y-tilt. The second-order Legendre polynomials introduce quadratic surface features with terms like x^2, xy, and y^2. These quadratic terms are obtained by combining one-dimensional Legendre polynomials in X and Y directions (in the original Legendre polynomials there are constant terms associated with x^2 and y^2 that are ignored for this application as the polynomials need to be scaled according to the domain boundaries and gap height). These terms describe more complex curvatures, such as parabolic features in either axis and coupled variations in both directions. Zernike polynomials, another class of orthogonal polynomials, are particularly well-suited for describing wavefront deformations and are often applied in optical systems for their radial symmetry. Though these polynomials are defined over a unit circle, there are ways to extend them for the rectangular domain. The transformation from a circular domain to a rectangular domain does not alter the form of the polynomials up to the second-order. While the zeroth and first-order polynomials are similar to the Legendre polynomials, the second-order polynomials include the shapes $x^2 + y^2$, xy, and $x^2 - y^2$. These rectangular polynomials are illustrated in Figure 3.

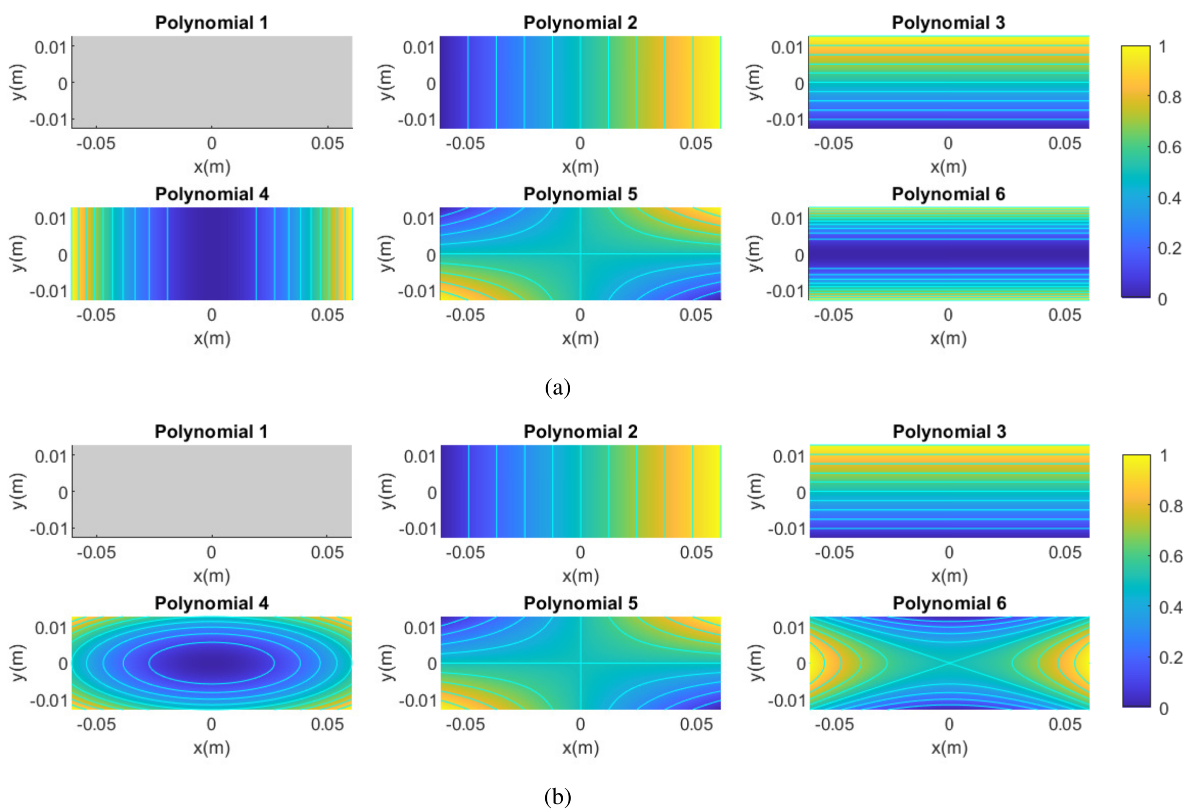


Fig. 3 First six shapes of rectangular Legendre (a) and Zernike (b) polynomials. The values are scaled with respect to the BRB interface dimension and normalized.

Over these designs, perturbation topologies have been added to determine the sensitivity of these above surface profiles. From the literature on manufacturing defects [25–27], various topologies have been observed. Based on that, five simple one-dimensional perturbations have been chosen for the preliminary analysis perturbation. The models are as follows: Perturbation 1: full sine wave with a maximum height of 10µm, Perturbation 2: full sine wave with a maximum height of 50µm, Perturbation 3: reverse of Perturbation 2, Perturbation 4:Perturbation 2 with the wavelength 2/3rd of the length of an interface, and Perturbation 5: a 50µm step at 2 cm away from the center of the interface. Perturbation 2 is a scaled version of Perturbation 1, and a comparison between these two perturbations can be useful to assess how much the scaling of the perturbation affects the response. The perturbations are shown in Figure 4.

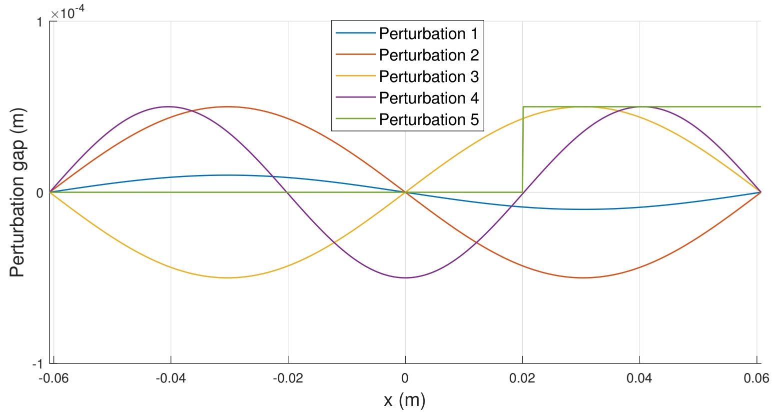


Fig. 4 The perturbation gaps used for the BRB interface

Results

The amplitude-dependent frequency and damping of the BRB is characterized with different mesoscale topologies by using the EPMC method. The initial goal is to understand the effect of different mesoscale topologies on the nonlinear dynamics of the system and to assess how the nonlinear dynamics change with the addition of perturbations. To investigate this, each topology effect is compared with respect to a flat, conformal interface. Then, a perturbation is added to each topology to study the effect of it. But how the conformal interface itself behaves under perturbation is studied first, and the results are shown in Figure 5. The 'flat' response is the baseline topology that contains no mesoscale curvature but a statistically described set of asperities (that are held constant across all simulations), and its frequency and damping characteristics are obtained using both 232 ZTE and 588 ZTE elements to assess model convergence. The 232 ZTE and 588 ZTE interface models yield identical responses, indicating that subsequent simulations can be conducted using just 232 ZTE. Also, it is found that the prestress and linearized eigenvalue analysis provides a useful initial guess for low-amplitude responses in the context of solving the EPMC problem. With the five different perturbations, a significant change in both frequency and

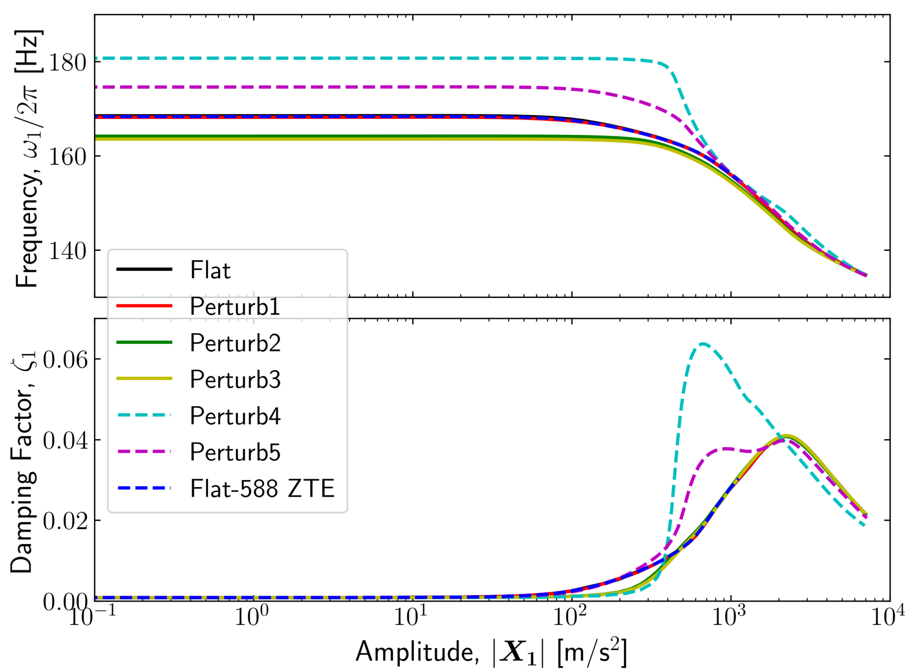


Fig. 5 EPMC results for the BRB with conformal interface and five different perturbations shown in Figure 4

damping is observed. These design perturbations chosen for this initial study are at the extreme of realistic perturbations, which indicates that this is a valid modeling approach for considering smaller perturbations as the design trends can be validated against experimental results [15].

In order to rigorously explore the design space of the meso-scale topology, the x-tilt, y-tilt and xy-tilt (addition of both) topology is taken into account. The maximum gap in x-tilt and y-tilt design is taken as 100µm. Later, a superposition of both these two makes the xy-yilt configuration. However, the frequency and damping characteristic curves are found to be precisely similar to the flat topology, as shown in Figure 6. Even with the perturbation (Perturbaton 2 in Figure 4) the the response is similar to the conformal topology. The deviation from the flat topology in the prestress frequency is in the fourth decimal in the case of x-tilt, and it is in the second decimal in the case of y-tilt. The relative position of the contacting surfaces after preloading is similar for both the flat and tilt cases, which resulted in similar characteristic curves. That is, upon assembly, the initial gap is able to be closed completely in the case of a planar tilt of the interface, as expected.

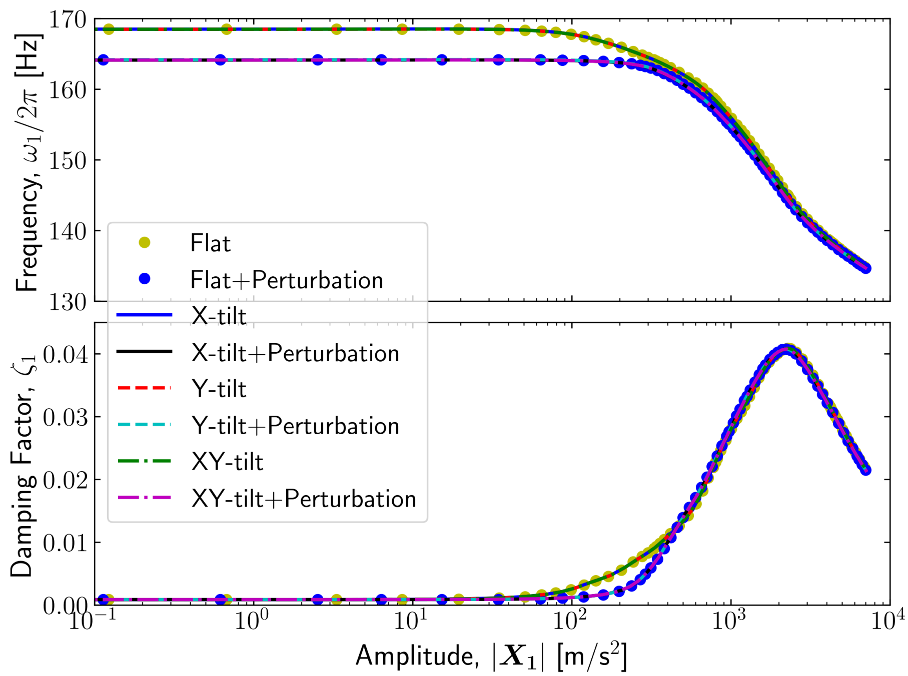


Fig. 6 Comparing linear and flat topology, with and without mesoscale perturbation (Perturbation 2)

Next, the quadratic topologies (x^2, y^2, xy, and $x^2 - y^2$) are investigated. Three different heights are considered for the profiles: 10µm, 100µm and 1000µm, and similar gaps are considered for the reverse profile. In case of the x^2 and y^2 profiles, the positive and negative/ reverse profile resembles edge gap and central gap configuration, respectively. Figure 7 shows the effect of the quadratic topologies compared to the flat interface. One clear trend observed from the x^2 topology is that the edge gap causes a reduction in the frequency and damping factor, whereas the central gap increases it. This is also confirmed by the experimental observations [15]. Among different quadratic topologies, the xy topology has shown minimal change in its characteristic curve compared to the flat topology. These quadratic topologies are further analyzed with perturbations, shown in Figure 8. While studying perturbations, the maximum gap of the design topology is considered to be 200µm.

Perturbation topologies 2 and 3 are symmetrically opposite to each other, and they have shown similar responses for all the topologies, which is as expected. Amongst various perturbations, perturbation 4 is the most sensitive one. This indicates that the shorter wavelength can cause more nonlinear interface profiles once preloaded, which increases the uncertainty in the system. After a detailed analysis of various profiles, xy showed the most robust behavior against perturbations. The sensitivity to perturbation in the case of the xy topology is lesser for both the frequency and damping. Thus, topologies with higher-order polynomials show promise for being able to make the interface robust to manufacturing variability.

If one compares the variation in frequency and damping, the relative error in damping is much larger than the frequency. So, making the topology less sensitive to the damping characteristics is of more importance. From Figure 8, it can be found that the first three perturbations do not make huge change in the value of the damping factor. However, the perturbations 4 (sine with shortened wavelength) and 5 (step) show significant changes in the damping value. The step perturbation increased the maximum damping factor over the response domain, and the increase was more than double in the case of flat

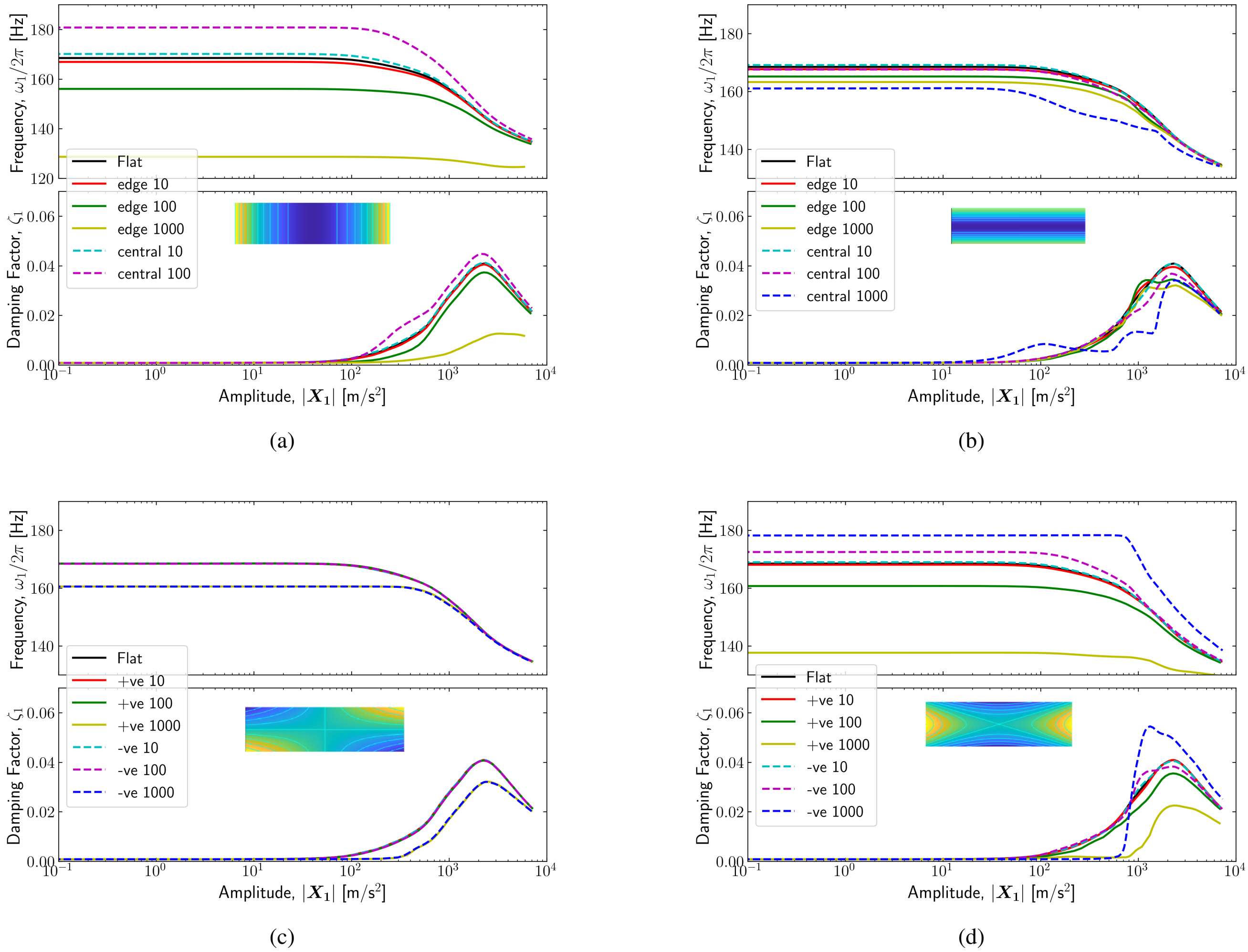


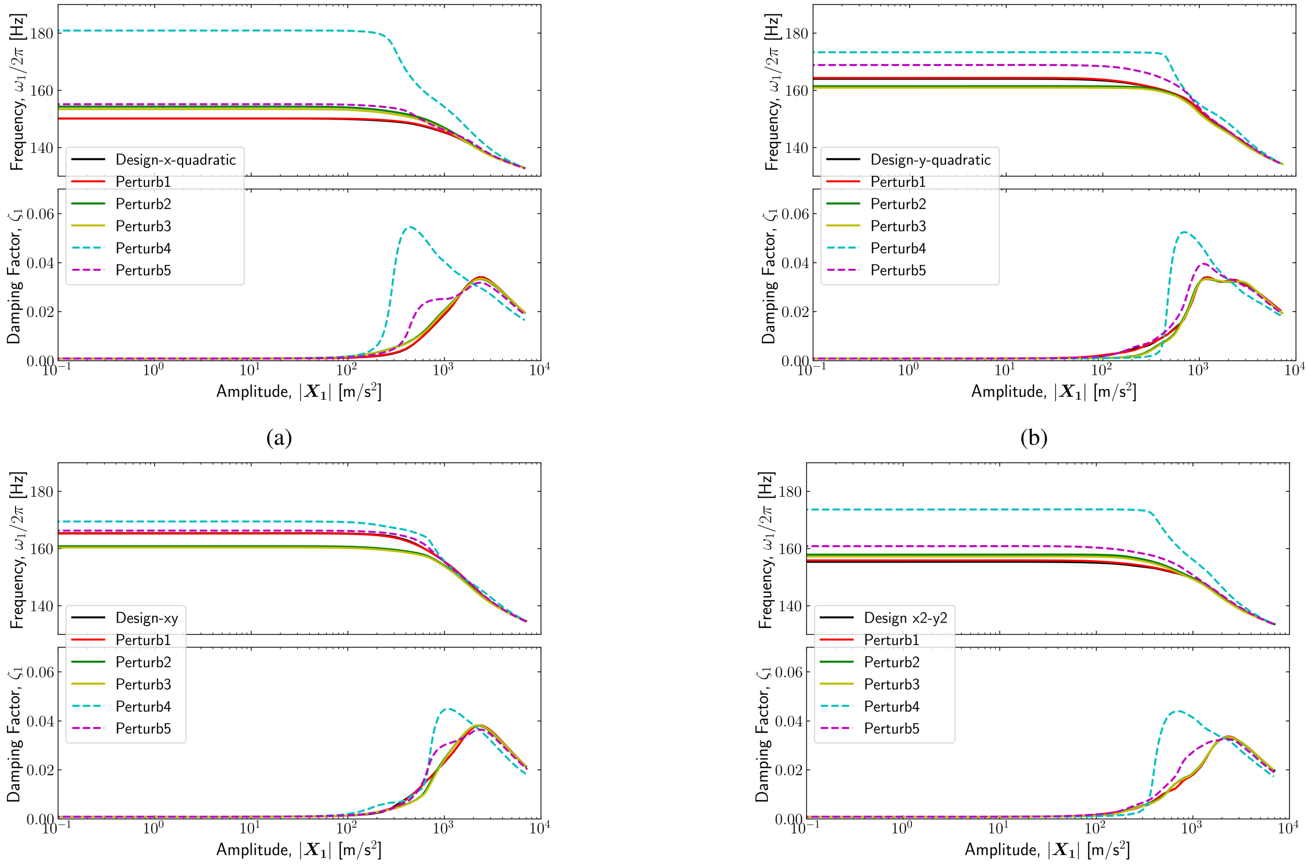
Fig. 7 EPMC results for BRB with quadratic topologies: (a) x^2, (b) y^2, (c) xy (d) $x^2 - y^2$. Each topology compared with flat and amplitude varied as 10µm, 100µm and 1000µm, for both positive and negative direction

and x-quadratic profiles. For the xy-topology, the growth is less, though significant. The increase in damping factor due to perturbation 4 is 55.8% for flat design (Figure 5), where it got reduced to 17.7% for xy design. A more detailed investigation with the surface profile might be able to mitigate the effect of these strong perturbations.

Conclusion and Future Work

This work represents a foundational step towards robust optimization of bolted connections. Specifically, it focuses on analyzing various interface topologies to identify an optimal design with minimal sensitivity to manufacturing errors. The BRB is employed as an example problem; it is found that by modifying the interface to have xy curvature, the design can be made robust to manufacturing variability. The EPMC method has proven effective for analyzing vibration characteristics at higher amplitude levels. Additionally, the use of ZTEs at the interface offers a highly effective modeling approach, allowing for detailed interfacial contact mechanics to be captured without the need for re-meshing with each surface topology change.

Future work will aim to experimentally validate these findings and explore the use of more sophisticated optimization algorithms. A key requirement will be the development of metrics to quantify the variability in behavior for each interface topology. One potential approach is to introduce a sensitivity metric to measure the variability of the damping factor ζ such as $||(\zeta_{\text{Design + Perturb}} - \zeta_{\text{Design}})/\zeta_{\text{Flat}}||_2$. Other than this response metric, another system parameter metric can also be defined as the ratio of gap variance in the perturbed system to that of the nominal design. Additionally, the design of such surfaces must



(a) (b) (c) (d)

Fig. 8 Perturbation on various topologies: (a) x^2, (b) y^2, (c) xy, (d) $x^2 - y^2$

be informed by the capabilities of both conventional and non-conventional manufacturing processes [28, 29], as fabricating complex surface profiles will pose significant challenges.

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References

[1] M. R. W. Brake, C. W. Schwingshackl, and P. Reuß. "Observations of variability and repeatability in jointed structures". In: *Mechanical Systems and Signal Processing* 129 (2019), pp. 282–307.

[2] T. W. Jerome, M. R. Shepherd, and S. A. Habric. "Acoustic excitation of a flanged joint". In: *36th International Modal Analysis Conference (IMAC XXXVI)*. Orlando, FL, 2018.
[3] T. Jerome, M. Shepherd, and S. Hambric. "Variability in measured resonance frequencies and loss factors of a bolted panel structure". In: *INTER-NOISE and NOISE-CON Congress and Conference Proceedings*. Auckland, New Zealand, 2023.
[4] B. J. Deaner, M. S. Allen, M. J. Starr, D. J. Segalman, and H. Sumali. "Application of Viscous and Iwan Modal Damping Models to Experimental Measurements From Bolted Structures". In: *Journal of Vibration and Acoustics* 137.2 (2015), p. 021012. ISSN: 1048-9002. URL: https://doi.org/10.1115/1.4029074.
[5] S. I. Zare Estakhraji, M. Wall, J. Capito, and M. S. Allen. "A thorough comparison between measurements and predictions of the amplitude dependent natural frequencies and damping of a bolted structure". In: *Journal of Sound and Vibration* 544 (2023), p. 117397. ISSN: 0022-460X. URL: https://www.sciencedirect.com/science/article/pii/S0022460X22005806.
[6] H. Dastani, D. Botto, and M. Glorioso. "Experimental and numerical investigation of contact parameters in a dovetail type of blade root joints". In: *Applied Sciences* 11 (2021), p. 12008.
[7] J. Yuan, L. Salles, D. Nowell, and C.Schwingshackl. "Influence of mesoscale friction interface geometry on the nonlinear dynamic response of large assembled structures". In: *Mechanical Systems and Signal Processing* 187 (2023), p. 109952.
[8] N. N. Balaji, W. Chen, and M. R. W. Brake. "Traction-based multi-scale nonlinear dynamic modeling of bolted joints: Formulation, application, and trends in micro-scale interface evolution". In: *Mechanical Systems and Signal Processing* 139 (2020), p. 106615. ISSN: 0888-3270.
[9] M. W. Sracic, M. S. Allen, and H. Sumali. "Identifying the modal properties of nonlinear structures using measured free response time histories from a scanning laser Doppler vibrometer". In: *30th International Modal Analysis Conference (IMAC XXX)*. Jacksonville, FL, 2012.
[10] J. P. Noël and G. Kerschen. "Nonlinear system identification in structural dynamics: 10 more years of progress". In: *Mechanical Systems and Signal Processing* 83 (2017), pp. 2–35.
[11] M. Scheel, G. Kleyman, A. Tatar, M. R. W. Brake, S. Peter, J.-P. Noel, M. S. Allen, and M. Krack. "System identification of jointed structures: nonlinear modal testing vs. state-space model identification". In: *36rd International Modal Analysis Conference (IMAC XXXVI)*. Orlando, FL, 2018.
[12] M. Jin, W. Chen, M. R. W. Brake, and H. Song. "Identification of instantaneous frequency and damping from transient decay data". In: *Journal of Vibration and Acoustics* (2020), pp. 1–36.
[13] W. Chen, D. Jana, A. Singh, M. Jin, M. Cenedese, G. Kosova, M. R. W. Brake, C. W. Schwingshackl, S. Nagarajaiah, K. J. Moore, and J.-P. Noël. "Measurement and identification of the nonlinear dynamics of a jointed structure using full-field data, Part I: Measurement of nonlinear dynamics". In: *Mechanical Systems and Signal Processing* 166 (2022), p. 108401. ISSN: 0888-3270.
[14] T. Dreher, M. R. W. Brake, B. Seeger, and M. Krack. "In situ, real-time measurements of contact pressure internal to jointed interfaces during dynamic excitation of an assembled structure". In: *Mechanical Systems and Signal Processing* 160 (2021), p. 107859. ISSN: 0888-3270.
[15] W. Chen, M. Jin, I. G. Lawal, M. R. W. Brake, and H. Song. "Measurement of slip and separation in jointed structures with non-flat interfaces". In: *Mechanical Systems and Signal Processing* 134 (2019), p. 106325.
[16] M. R. W. Brake, M. S. Allen, D. D. Quinn, D. Nowell, and D. R. Roettgen. "Testing and Modeling of Friction and Slip in Mechanical Interfaces: State of the Art and Perspectives for the Next Decade". In: *Journal of Structural Dynamics* (To be published).
[17] J. H. Porter and M. R. W. Brake. "Towards a predictive, physics-based friction model for the dynamics of jointed structures". In: *Mechanical Systems and Signal Processing* 192 (2023), p. 110210. ISSN: 0888-3270.
[18] M. Krack. "Nonlinear modal analysis of nonconservative systems: Extension of the periodic motion concept". In: *Computers & Structures* 154 (2015), pp. 59–71. ISSN: 0045-7949.
[19] N. N. Balaji. *An ABAQUS-MATLAB tutorial for jointed systems*. 2024. URL: https://nidish96.github.io/Abaqus4Joints/.
[20] N. N. Balaji, T. Dreher, M. Krack, and M. R. W. Brake. "Reduced order modeling for the dynamics of jointed structures through hyper-reduced interface representation". In: *Mechanical Systems and Signal Processing* 149 (2021), p. 107249. ISSN: 0888-3270.
[21] J. H. Porter and M. R. W. Brake. "Efficient model reduction and prediction of superharmonic resonances in frictional and hysteretic systems". In: *Mechanical Systems and Signal Processing* arXiv:2405.15918 (Under Review). arXiv: 2405.15918.
[22] J. H. Porter and M. R. W. Brake. *TMDSimPy: Tribomechadynamics Simulations for Python*. URL: https://github.com/tmd-lab/tmdsimpy.
[23] Y. Li, J. Guo, and R. Liu. "Exploring wavefront detection in imaging systems with rectangular apertures using phase diversity". In: *Sensors* 24.4 (2024). ISSN: 1424-8220. URL: https://www.mdpi.com/1424-8220/24/4/1191.
[24] K. Niu and C. Tian. "Zernike polynomials and their applications". In: *Journal of Optics* 24.12 (2022), p. 123001. URL: https://dx.doi.org/10.1088/2040-8986/ac9e08.
[25] C. Zhao, J. Lv, S. Du, and Y. Deng. "Geometric error modeling and monitoring of the 3d surface by gaussian correlation model". In: *2019 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*. 2019, pp. 1022–1025.
[26] Y. Chen, H. Tang, Q. Tang, A. Zhang, D. Chen, and K. Li. "Machining error decomposition and compensation of complicated surfaces by emd method". In: *Measurement* 116 (2018), pp. 341–349. ISSN: 0263-2241. URL: https://www.sciencedirect.com/science/article/pii/S0263224117307364.
[27] J. Wu, L. Qiao, and Z. Huang. "Deviation modeling of manufactured surfaces from a perspective of manufacturing errors". In: *International Journal of Advanced Manufacturing Technology* 98 (2018), 1321–1337. URL: https://doi.org/10.1007/s00170-018-2305-2.
[28] C. Zuo, X. Zhou, Q. Liu, R. Wang, J. Lin, P. Xu, and X. Zhang. "Analytical Topography Simulation of Micro/Nano Textures Generated on Freeform Surfaces in Double-Frequency Elliptical Vibration Cutting Based Diamond Turning". In: *Journal of Manufacturing Science and Engineering* 140.10 (2018), p. 101010. ISSN: 1087-1357. URL: https://doi.org/10.1115/1.4040616.
[29] D. Zhao and W. Guo. "Shape and Performance Controlled Advanced Design for Additive Manufacturing: A Review of Slicing and Path Planning". In: *Journal of Manufacturing Science and Engineering* 142.1 (2019), p. 010801. ISSN: 1087-1357. URL: https://doi.org/10.1115/1.4045055.