Chapter 6
Nonlinear Vibration Control on Complex Structures via Model Predictive Control Exploiting Koopman Model

Yichang Shen and Ludovic Renson

Abstract Active vibration control of thin structures is often necessary to enhance or ensure performance, and it can also be used to characterize their dynamic behavior during experimental testing through techniques like control-based continuation. In this work, Model Predictive Control (MPC) is applied to control the nonlinear vibration of a thick (i.e. linear) cantilever beam with artificial nonlinearity and track periodic signals. The Koopman operator is used to linearise the dynamics of the system, thereby obtaining an effective real-time MPC implementation. Virtual experimental results show that using quadratic programming MPC with the Koopman model leads to efficient and accurate control performance.

Keywords Vibration control · Model predictive control · Koopman operator · Nonlinear vibration

Introduction

The constant drive to improve the performance of mechanical structures is leading to new designs that are increasingly lighter, more flexible, and inherently nonlinear. Active vibration control of thin structures is sometimes necessary to enhance or guarantee their performance. Active control can also be used to characterize the dynamic behavior of nonlinear structures during experimental tests using techniques like control-based continuation (CBC) [1]. Model Predictive Control (MPC) is an advanced control technique widely employed in industrial settings. MPC allows for the use of existing models to control multiple-input, and multiple-output systems and handle input-output interactions, making it appealing for complex systems. Moreover, MPC uses constraints to limit response amplitudes and reduce the risk of failure during large amplitude vibrations. Thus, it serves as a potential advanced feedback controller that may extend CBC to more complex, real-life applications.

To implement MPC effectively, it is important to develop an accurate yet computationally efficient model of the system's dynamics. The author's previous study [2] validates that, although exploiting nonlinear reduced-order models (NLROM) in (N)MPC algorithms leads to good control performance and low-dimensional representations of the system's dynamics, nonlinear models remain computationally expensive to solve numerically. To take advantage of powerful convex programming solvers, such as quadratic programming (QP), in this work, the Koopman operator has been utilized to convexify the originally nonconvex problem by generating a linear model from experimental data. Preliminary virtual experiments indicate that MPC, exploiting the Koopman model, is able to control nonlinear structural vibrations for tracking periodic signals.

Experimental Set-up of Benchmark Structure

The benchmark structure is a straight, thick cantilever beam with arbitrary artificial nonlinearity generated by an electrodynamic shaker. Key components of the setup include a real-time controller (dSPACE MicroLab Box), an electrodynamic shaker, a stinger, an impedance head attached to the beam for measuring the force and acceleration at the excitation point, and a laser sensor for measuring the beam's displacement. The assembly details and schematic view of the experimental setup are shown in Figure 1. A similar benchmark structure was previously investigated in [3]. The nonlinear system is investigated as a single-input, single-output system. The nonlinearity in the system was artificially implemented using the real-time

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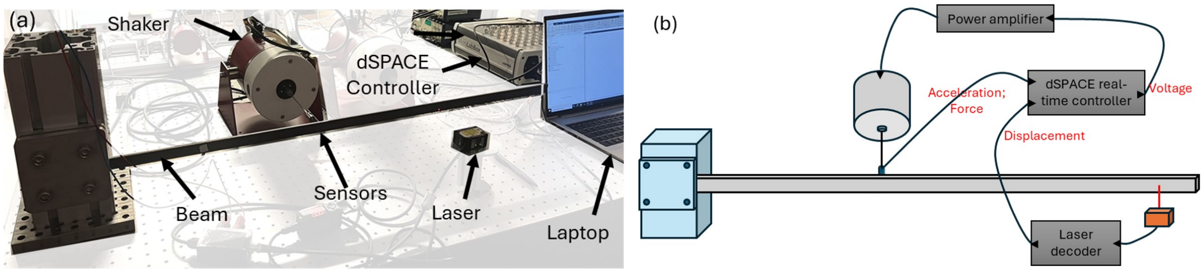

Fig. 1 Control performance of using the Carleman linearised model to generate model predictions. (a): Experimental set-up. (b): Schematic view of the experiments.

feedback control based on the measurement displacement. The force applied by the shaker to the structure was composed of the control output $f_{ex}(t)$ and virtual nonlinearity $f_{nl}(t)$. In this work, $f_{nl}(t) = K_c y^3$, where K_c is the nonlinear restoring force coefficient, and y is the beam's tip displacement measured by the laser.

Koopman Linear reconstruction and Control objective

The Koopman operator is an effective tool for providing a global linear representation of a nonlinear system [4]. This linearization allows for the application of well-established control techniques for linear systems. However, its infinite-dimensional nature presents considerable challenges for practical use. To address this issue, the method employed in this work is the EDMD method [5], which enables the approximation of the Koopman operator's eigenvalues, eigenfunctions, and modes from data. This data-driven approach allows us to obtain a linear reconstruction of the nonlinear system's dynamics from experimental data in the form of

$$\dot{\mathbf{z}}^{[\mathbf{k}]} = \bar{\mathbf{A}}\mathbf{z}^{[\mathbf{k}]} + \bar{\mathbf{B}}u. \tag{1}$$

The class of functions used for lifting $\mathbf{z}^{[\mathbf{k}]}$ used here includes polynomials up to third-order and first-order Fourier modes. In this model predictive control application, the objective is for the nonlinear structure vibration to follow a periodic reference signal X_{ref} composed of multiple sine functions. The cost function minimized by the MPC algorithm at time step k is expressed by

$$J_k = \sum_{i=0}^{n_p-1} \left((x_{k+i} - X_{ref}(t_k))^T \mathbf{Q}(x_{k+i} - X_{ref}(t_k)) + u_{k+i}^T \mathbf{R} u_{k+i} \right) + x_{k+n_p}^T \mathbf{P} x_{k+n_p}, \tag{2}$$

where x_{k+i} are the future system states predicted using the system model (Koopman) for a number of time steps, n_p, ahead of the current time t_k. The input u and the beam tip displacement are constrained to remain in a desired range to avoid damaging the experimental set-up.

Virtual Experimental results

Virtual tests were conducted in SIMULINK to validate the overall control design methodology. The virtual experiment model was tuned to match the physical setup, as shown in the SIMULINK model in Figure 2(a). The cantilever beam was represented as a single degree of freedom (1-DOF) system derived using MATLAB's n4sid.m system identification function based on experimental data obtained using a multiple sine excitation signal. The Koopman model employed in the MPC is measured by experimental data with the same input signal but with the virtual cubic nonlinearity included. The shaker is system-identified to the input voltage and output force transfer function. Fig. 2 (b) illustrates the performance of the model predictive control, demonstrating that the displacement simulated by the beam model closely aligns with the target signal. Figure. 2 (c) and (d) depict the artificially induced nonlinear component of the system dynamics and the optimal control output computed by MPC in real-time, respectively.

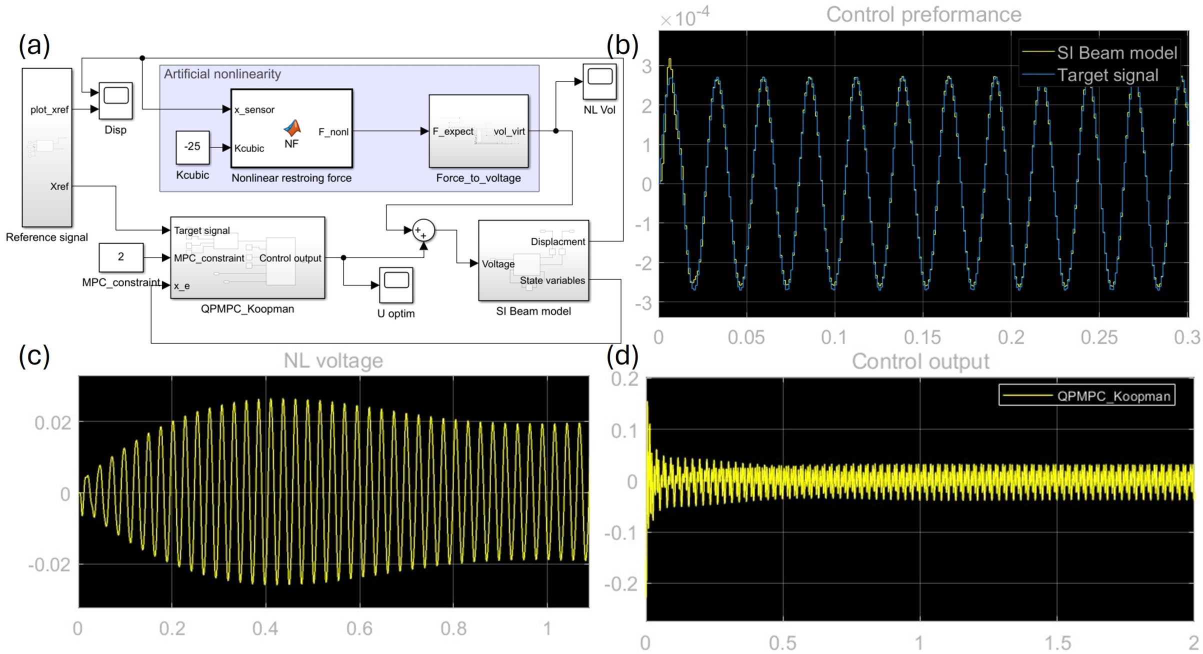

Fig. 2 Virtual experimental set-up and results. (a): SIMULINK model. (b): MPC Control performance. (c) Virtual nonlinear component. (d) Control output.

Conclusion

In this work, a cantilever beam with artificial nonlinearity is investigated, and model predictive control (MPC) exploiting the Koopman operator is applied to control the nonlinear structure vibrations to track a desired, multi-harmonic periodic signal. Results from virtual experiments demonstrate that using Koopman linearized models within MPC enables efficient and accurate control performance. Full-scale experimental testing is currently being prepared and will be conducted in the next phase of the research.

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