Chapter 7
Stochastic Dynamics and Surrogate Modeling for Nonlinear Aerospace Nozzle Systems with Partial Observations

Evangéline Capiez-Lernout, Olivier Ezvan, and Christian Soize

Abstract This research addresses the computational updating of a statistical surrogate model using a partial target dataset. A parameterized nonlinear stochastic computational model of a three-dimensional engine nozzle undergoing large displacements under a stochastic pressure jet is considered with a small, incomplete target dataset of normal displacements at the nozzle exit. The probabilistic PLoM algorithm is used and adapted to the target constraint in order to construct a surrogate computational model, enabling updates that allow for the response to better match the target response despite data limitations.

Keywords uncertainty quantification · probabilistic learning · nonlinear stochastic dynamics · nozzle · surrogate model

Introduction

Numerous works have highlighted the significance of nozzles in various engineering aspects, including flow analysis, jet noise source characterization and acoustics, material properties, design optimization, and the impact of uncertainty on nozzle computational models. More precisely, the vibrational behavior of nozzle structures is a critical concern in the aerospace engineering field. This research focuses on a computational framework for analyzing the updating process in the nonlinear stochastic dynamics of a nozzle, accounting for model uncertainties, partial observability, and limited data. A parameterized high-fidelity stochastic nonlinear uncertain computational model of the nozzle is meticulously designed, optimizing of the numerical parameters to balance accuracy and efficiency of the numerical solvers [1], yielding a small training dataset to be available. Then, the Probabilistic Learning on Manifolds (PLoM) algorithm adapted to an existing incomplete target data constraint [2, 3] and which relies on a purely probabilistic approach, is used in order to construct an updated surrogate computational model [4].

Background

The inherent complexity of this nozzle computational application emerges from multiple sources: (1) the high number of degrees-of-freedom present in the nozzle computational model whose mesh is represented in Figure 1, and which is a consequence of its structural complexity driven by its small thickness; (2) the geometric nonlinear effects in its dynamic behavior, induced by displacements of the structure that are comparable to the nozzle thickness; (3) the presence of model uncertainties in the representation of homogenized materials that arise from an uncontrolled parameter represented by random vector $\mathbf{U}$, (4) the influence of an external load represented by a stationary stochastic process. This latter one is modeled by a time-dependent random pressure at the nozzle wall $P(z, \theta, t; \mathbf{w})$, using cylindrical coordinates and dependent on the control parameter $\mathbf{w}$. The load intensity is assumed to be sufficiently high for triggering nonlinear displacements responses $\mathbf{Y}(t; \mathbf{w})$, (5) the under-observability and the limited data availability, complicating the resolution of the statistical inverse problem required for the updating, and (6) the high-computational cost associated with the construction of a single realization of the stationary random displacements responses, as this requires solving a set of nonlinear stochastic coupled differential equations in the time domain, using iterative algorithms combined to multiple time steps. The observations of interest $\mathbb{O}(\nu; \mathbf{w})$ are located at the nozzle exit and are then expressed as displacement amplitudes in the frequency domain. Such parameterized

Evangéline Capiez-Lernout · Olivier Ezvan · Christian Soize
MSME UMR 8208 CNRS, Université Gustave Eiffel, 77454 Marne-La-Vallée, France
e-mail: evangeline.capiez-lernout@univ-eiffel.fr; olivier.ezvan@univ-eiffel.fr; christian.soize@univ-eiffel.fr

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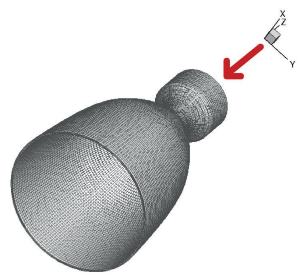
Fig. 1 Finite element mesh of the nozzle.



computational stochastic model is meticulously designed, allowing for computing accurate statistics of output observations but for which only a small training dataset is available.

The updating of the nonlinear uncertain stochastic computational model is carried out with respect to a given target dataset that is constituted of an incomplete data, corresponding for instance, to the case of a limited number of sensors located at the nozzle exit wall. In order to construct the statistical surrogate model of quantities of interest with respect to control parameter $\mathbf{w}$, using the PLoM algorithm under constraints, vector $\mathbf{w}$ is modeled by a vector-valued random variable $\mathbf{W}$. The statistical surrogate model is not represented by a computational model but by a learned probability measure. The quantities of interest are then derived from conditional statistics of the quantities of interest for given value of the control parameter, using the learned dataset for the updated stochastic system. To aid in the comprehension of this paper, the figure 2 presents a simplified flowchart of the used methodology.

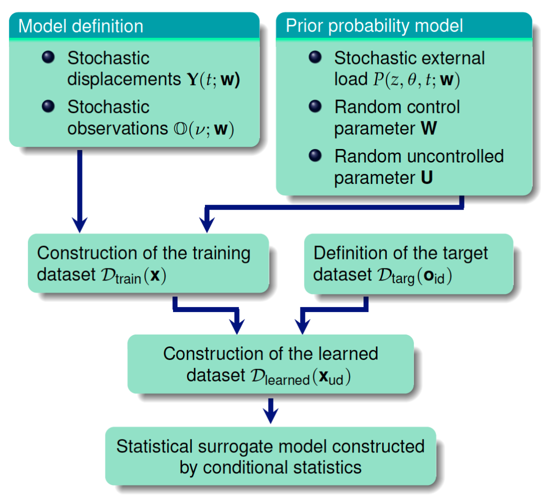


Fig. 2 Flowchart of the methodology.

Analysis of the Results

Figure 3 present the computational results corresponding to the responses as a function of the frequency for a given observation point. Each figure includes: (a) the mean response of the training dataset, both nonlinear and its linear counterpart (left upper figure), (b) a comparison between the mean response of the training dataset and the target (right upper figure), (c) the mean response of the learning dataset and the target (left lower figure), and (d) the mean response of the training dataset, the target, and the conditional confidence region given $\mathbf{w}_0$ at a 95% probability level. In (a), nonlinear effects are highlighted by shifts in resonance peaks and reduced amplitudes due to energy spreading across the excitation band. In (b), it is seen that the mean response of the training set does not match with the target, showing shifts in the second and third resonance peaks. In (c), it is shown that the constrained learning process captures the mean statistics of the target, even if limited observation data is available. Finally, in (d), three features are emphasized: (1) the mean target response is centered within the confidence region, (2) the confidence envelope peaks align well with the target's, and (3) the confidence region remains relatively narrow, reflecting the influence of uncontrolled parameters and excitation variability. A similar analysis is observed across various observation points, whether they correspond to identification points (used during the updating process) or validation points (not used), which validates the proposed methodology.

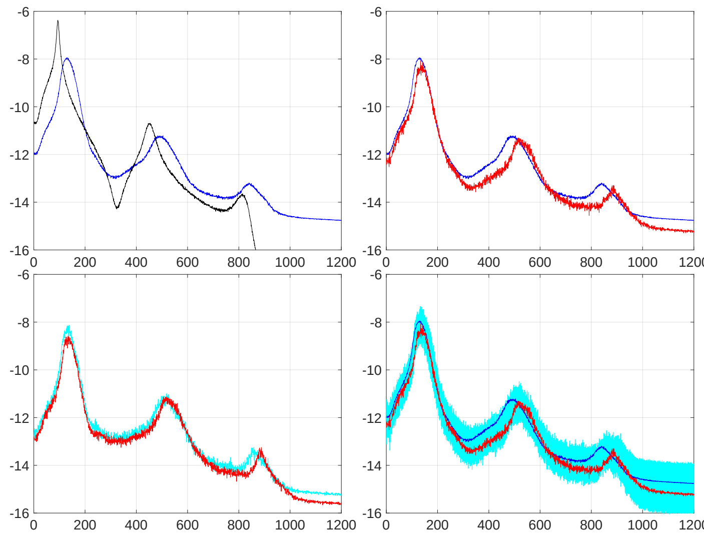


Fig. 3 Analysis of the responses as a function of the frequency at a given observation point. Left upper figure (a) : mean responses of the linearized and non linear system issued from the training set. Right upper figure (b) : mean nonlinear responses issued from both training and target data sets. Left lower figure (c) : mean nonlinear responses issued from both target and learning data sets. Right lower figure (d) : confidence region of the nonlinear response issued from the learning dataset and the corresponding mean nonlinear responses issued from both target and training data sets.

Conclusion

We have proposed a methodology for updating a high-dimensional nonlinear computational model of a nozzle structure under the constraints of limited and partially observable data. The model's complexity and computational cost are driven by uncertain parameters, geometric nonlinearities, and stochastic external forces. Using the PLoM approach, the updated surrogate model is used for estimating conditional statistics of the observations and shows its capability of capturing the characteristics of the target dataset, even for unobserved variables.

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