Chapter 9
Decoupling of Nonlinear Substructures using Quasi-linear Frequency Response Functions

Furkan K. Çelik, Taylan Karaağaçlı, M. Bülent Özer, and H. Nevzat Özgüven

Abstract Although decoupling methods based on measured frequency response functions (FRFs) have been extensively studied for linear systems, the development of decoupling methods for nonlinear systems is still in its infancy. A recently developed nonlinear system identification technique, namely the Response-Controlled Stepped-Sine Test (RCT) method, showed that nonlinear systems, in general, exhibit quasi-linear behavior independent of the location, type, and severity of nonlinearity provided that the displacement amplitude of a specific coordinate (generally excitation point) is kept constant in a single input sine testing with closed-loop control. This finding opened the way for adapting linear modal analysis and substructuring techniques to nonlinear systems. This study extends the previously developed FRF Decoupling Method for Nonlinear Systems (FDM-NS), which is limited to a localized nonlinearity, to the case of multiple nonlinearities distributed all over the coupled structure. After obtaining the quasi-linear FRFs of the coupled nonlinear structure and the known nonlinear substructure using RCT, linear decoupling formulations are applied. These formulations are used to decouple the structure at single or multiple connection points to predict the quasi-linear FRFs of the unknown nonlinear substructure. Then, response-level dependent nonlinear modal model parameters are identified by applying linear modal analysis methods to these quasi-linear FRFs. Finally, the nonlinear modal model can be used to synthesize the constant force FRFs of the unknown nonlinear substructure. The extended approach is verified using case studies, and its accuracy and limitations are discussed.

Keywords Nonlinear Structural Dynamics · Nonlinear Decoupling · Nonlinear Substructuring · Substructuring · Response-Controlled Testing

Introduction

Decoupling techniques aim to determine the dynamic characteristics of unknown substructures or joints within larger assemblies without additional experimental or computational effort. Extensive research on substructuring [1–3] has demonstrated that decoupling can be applied across frequency and modal domains. Studies have elaborated on the theoretical foundations of substructuring and have demonstrated its application through various linear case studies, e.g. [3–6].

Recent studies also applied coupling/decoupling techniques to nonlinear systems. For example, [7–9] implemented linear structural modification techniques to nonlinear systems using response-controlled testing. In [10], FDM-NS was proposed to decouple nonlinear systems with a localized nonlinearity in the frequency domain. Another method conducted coupling of nonlinear numerical systems using optimization [11]. One of the early works showed the applicability of modal coupling on numerical systems with nonlinear connection elements [12] .

With the increasing interest and need for nonlinear mechanical system identification, this area has improved rapidly in the last decade. New generation modal identification methods such as Control-Based Continuation (CBC) [13] , Phase-Locked Loop (PLL) [14] , and Response-Controlled stepped-sine Testing (RCT) [15] have been proposed and validated in

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various theoretical and experimental cases. Among these techniques, the application of RCT on nonlinear structures results in quasi-linear FRFs, which opens the way for using linear substructuring methods with nonlinear systems.

The theory of the method discussed in this paper is quite straightforward [16] and built on quasi-linear FRFs, which can be obtained by employing RCT. The method makes it possible to apply the decoupling techniques for linear structures to nonlinear structures exhibiting nonlinearities in any or all substructures. In that respect, it extends FDM-NS [10] to nonlinear systems regardless of the location, type, or severity of the nonlinear elements. The application of the method on a real structure with weak and local nonlinearity [17] gave promising results. However, the applications and limitations of the method are not studied extensively. This paper aims to provide an investigation on the applicability, accuracy and limitations of the method. Firstly, the underlying theory of RCT and linear decoupling will be presented, and then examples of decoupling from a single and multiple connection degree of freedoms (DoFs) will be presented to reach conclusions about the accuracy and limitations of the method.

Theory

The method suggested in [7,10] was restricted to cases where the nonlinearity is localized at the excitation DoF, or the nonlinearity is between two DoFs where the relative displacement of these DoFs is kept constant with closed loop control. This restriction can be bypassed by the application of RCT, which is used to identify various other real system nonlinearities, such as a piezo-actuator shown in Fig. 1 [18] , a control fin actuation mechanism with high friction [19] and a bolted lap-jointed structure [20] .

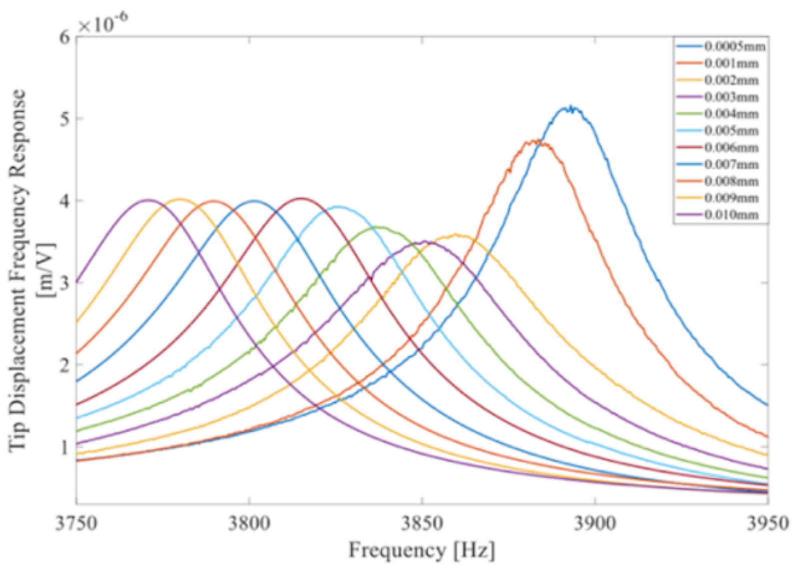

Fig. 1 Quasi-linear constant-response FRFs of Nonlinear Piezoelectric Actuators measured by RCT

The equation of motion of a nonlinear system with structural damping can be written in the frequency domain as follows,

$$-\omega^2 \boldsymbol{MX} + i\boldsymbol{HX} + \boldsymbol{KX} + \boldsymbol{\Delta}\left(\boldsymbol{X}\right)\boldsymbol{X} = \boldsymbol{F} \tag{1}$$

Herein, $\boldsymbol{M}$, $\boldsymbol{K}$, and $\boldsymbol{H}$ are the underlying linear system's mass, stiffness, and structural damping matrices, respectively. $\boldsymbol{\Delta}$ denotes the response level dependent nonlinearity matrix [21] . $\boldsymbol{X}$ is the response amplitude vector, $\boldsymbol{F}$ is the harmonic excitation amplitude vector, and ω is the excitation frequency.

Eq. (1) assumes that the system is under harmonic excitation and that the effect of sub- and super-harmonics is negligible. By keeping any DoF of the system under a constant harmonic displacement amplitude, the displacement amplitudes of all DoFs remain nearly constant around the resonance region due to nonlinear normal mode theory [22] . This approach ensures that the nonlinearity matrix [21] behaves as an equivalent stiffness or damping matrix, making the system quasi-linear. Subsequently, classical modal identification techniques like peak-picking are applied to the resulting response-level-dependent quasi-linear FRFs, yielding a response-level-dependent modal model of the system. The constant-force frequency responses of the nonlinear system can then be synthesized by solving the following equation with the Newton-Raphson method and the arc-length continuation algorithm.

$$X_j\left(q_r\right)=\frac{\overline{A}_{jkr}\left(q_r\right)F_k}{\overline{\omega_r^2}\left(q_r\right)-\omega^2+i2\overline{\xi}\left(q_r\right)\omega\overline{\omega_r}\left(q_r\right)} \tag{2}$$

Herein, X_j denotes the response amplitude of the j^{th} DoF, F_k denotes the harmonic forcing amplitude at k^{th} DoF, $\overline{A}_{jkr}$, $\overline{\omega_r}$, $\overline{\xi}$ and q_r denote the modal constant, natural frequency, modal damping ratio, and modal amplitude of the r^{th} mode, respectively. A more detailed explanation of the theoretical background and derivations can be found in [15] .

The quasi-linear FRFs can be used to apply structural modifications to a nonlinear system or couple it with another structure [7–9]. They can also be used to decouple a nonlinear structure from a single or multiple connection points [16] . Linear dual decoupling method can be used to decouple the structure. Dual decoupling assumes force equilibrium from the outset. Linear FRFs can be directly used in these equations, but nonlinear systems must also satisfy compatibility conditions because they contain response-level dependent elements. In a previous work [11] , it is shown that compatibility conditions can be satisfied iteratively. However, since RCT enforces a constant displacement level at the control DoF, the compatibility condition is not required to be satisfied iteratively. Any available linear dual decoupling formulation can be used alongside RCT to decouple the quasi-linear FRFs. In this study, the formulation given in Equation (3) is used. Its derivation can be found in [3] .

$$\boldsymbol{H}^B=\begin{bmatrix}\boldsymbol{H}^C & 0\\ 0 & -\boldsymbol{H}^A\end{bmatrix}-\begin{bmatrix}\boldsymbol{H}^C & 0\\ 0 & -\boldsymbol{H}^A\end{bmatrix}\begin{bmatrix}\left(\boldsymbol{B}^C\right)^T\\ \left(\boldsymbol{B}^A\right)^T\end{bmatrix}\left(\begin{bmatrix}\boldsymbol{B}^C & \boldsymbol{B}^A\end{bmatrix}\begin{bmatrix}\boldsymbol{H}^C & 0\\ 0 & -\boldsymbol{H}^A\end{bmatrix}\begin{bmatrix}\left(\boldsymbol{B}^C\right)^T\\ \left(\boldsymbol{B}^A\right)^T\end{bmatrix}\right)^{-1}$$
$$\begin{bmatrix}\boldsymbol{B}^C & \boldsymbol{B}^A\end{bmatrix}\begin{bmatrix}\boldsymbol{H}^C & 0\\ 0 & -\boldsymbol{H}^A\end{bmatrix} \tag{3}$$

where $\mathbf{H^A}$, $\mathbf{H^B}$, and $\mathbf{H^C}$ indicate the FRFs of subsystems A, B, and the coupled system C, respectively. $\mathbf{B^A}$ and $\mathbf{B^C}$ correspond to the Boolean matrices of subsystems A and the coupled system C, respectively.

Systems Coupled From A Single Point

The method is initially demonstrated by using the discrete system shown in Fig. 2. The mass and stiffness values used in this case study are $0.1\ kg$ and $1x10^4\ N/m$, respectively. The structure has 1% structural damping, and the cubic stiffness value is $k_{cubic}=1x10^7 N/m^3$.The mass at the connection point is divided into two equal parts.

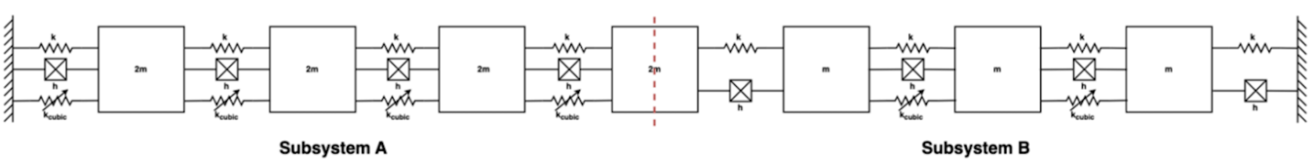

Fig. 2 Lumped coupled system with distributed cubic stiffness nonlinearities

Simulated RCT experiments are performed on both the coupled system C and subsystem A while maintaining the connection DoF (shown as a red dashed line in Fig. 2) at constant displacement amplitude levels and applying force to the same DoF, to decouple subsystem B from subsystem A. These simulations result in quasi-linear FRFs for the coupled system C and subsystem A. The quasi-linear FRFs at different displacement amplitude levels are then used with the dual decoupling formulation given in Eq. (3) to obtain the quasi-linear FRFs of subsystem B. The modal identification using these quasi-linear FRFs provides the modal parameters for each response level. A spline-fitted, response-level dependent modal model is then used in Eq. (2) to synthesize constant-force FRFs of the system, as shown in Fig. 3.

The synthesized constant force FRFs of Subsystem B are validated by comparing them with the ones directly obtained from constant-force simulations of Subsystem B, as shown in Fig. 3. The subscripts in the FRF correspond to the decoupling DoF in Fig. 2, which is named as the first DoF of Subsystem B. As can be seen from Fig. 3, the match between the synthesized and directly simulated FRFs is excellent. In this scenario, the linear modes of systems C and A are at 95.71 rad/s and 87.25 rad/s, respectively, and are relatively near the identified mode of subsystem B. In a real engineering system,

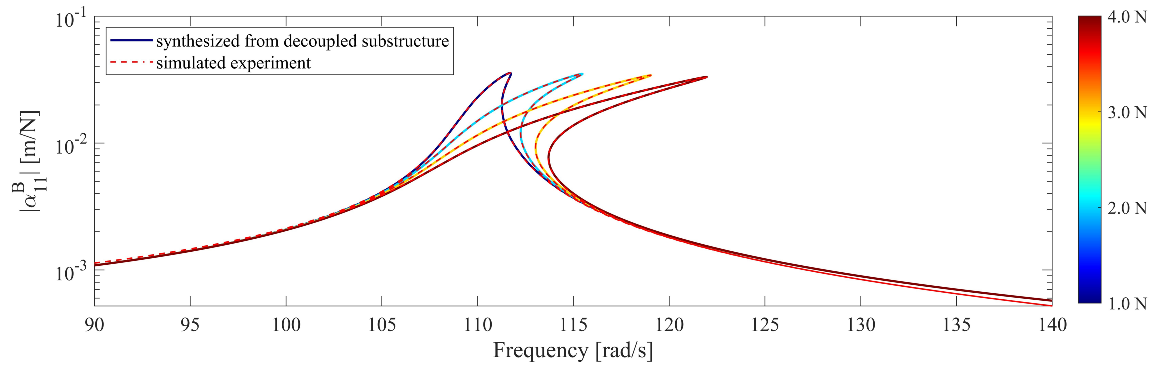

Fig. 3 Constant force direct point FRFs of Subsystem B

as the resonance frequencies of the subsystems become further away from each other, issues such as increased error as well as noisy appearance of the decoupled quasi-linear FRFs may occur due to noise becoming more dominant at off-resonance frequencies. Another practical challenge will be achieving the required force levels to reach higher values of constant displacement amplitudes at frequencies where the receptance values are significantly lower than at resonance.

Systems Connected From Multiple Points

Lumped parameter system

The method is also tested when two substructures are connected from multiple connection points. To see the effect of the strength of nonlinearity on the inaccuracies, the nonlinearities are attached in two configurations to the system, as shown in Fig. 4. In the first configuration, there is only the green nonlinearity (between mass 9-12). The second configuration has both the green (between mass 9-12) and blue (between mass 13-14) nonlinearities. The parameters of the system are given in Table 1. The structure has 1% structural damping, and the cubic stiffness is taken as $k_{cubic} = 1 \times 10^7 N/m^3$. DoFs 6, 7, and 8 are the connection DoFs that connect subsystem B to subsystem A.

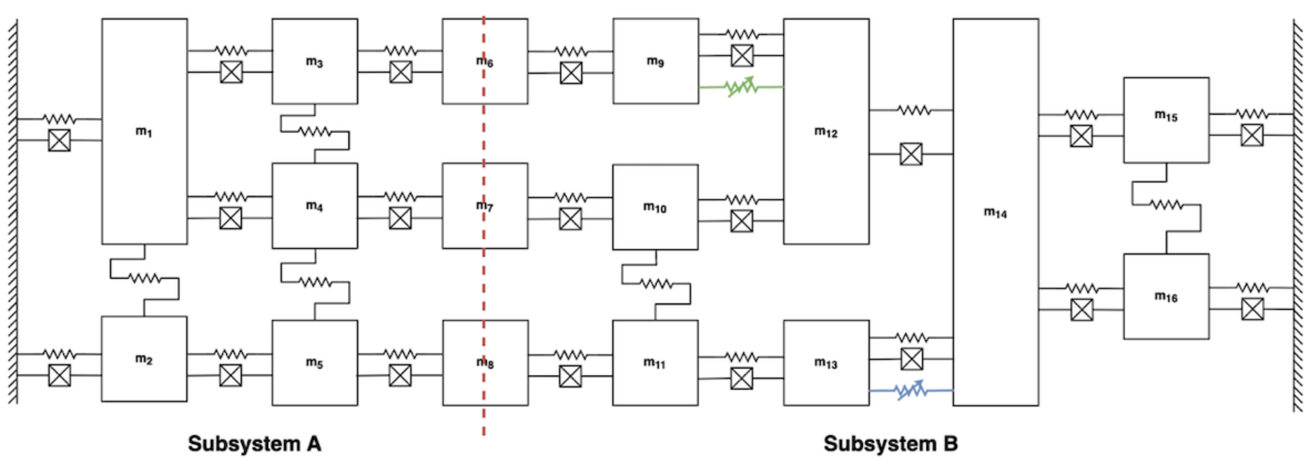

Fig. 4 Coupled nonlinear system with three connections (DoFs)

The displacement of the 6^{th} DoF is kept constant while external excitation forces are applied in three separate sets of simulations at the 6^{th}, 7^{th}, and 8^{th} DoFs. These simulations are designed to capture the quasi-linear FRFs of both subsystem A and the coupled system including H^{A}_{77}, H^{C}_{77}, H^{A}_{88} and H^{C}_{88}. These FRF matrices are then used in Eq. 3 to determine the FRFs of subsystem B.

Table 1 Properties of the lumped coupled system with multiple connections.

Index	1-5	6-8	9-16
Mass [kg]	1	1.5	1
Stiffness [N/m]	10000	10000	10000

In the case of multiple connection points, errors in the FRF estimates of Subsystem B arise because the system is connected through an nxn matrix where n is the number of connection DoFs (in this example, n = 3). The constant force FRF for the 2^{nd} configuration is shown in Fig. 5. Zoom-box around the resonance peaks in Fig. 5 aims to show the amplitude errors more clearly. The comparison of two versions of the system for all force levels is given in Table 2.

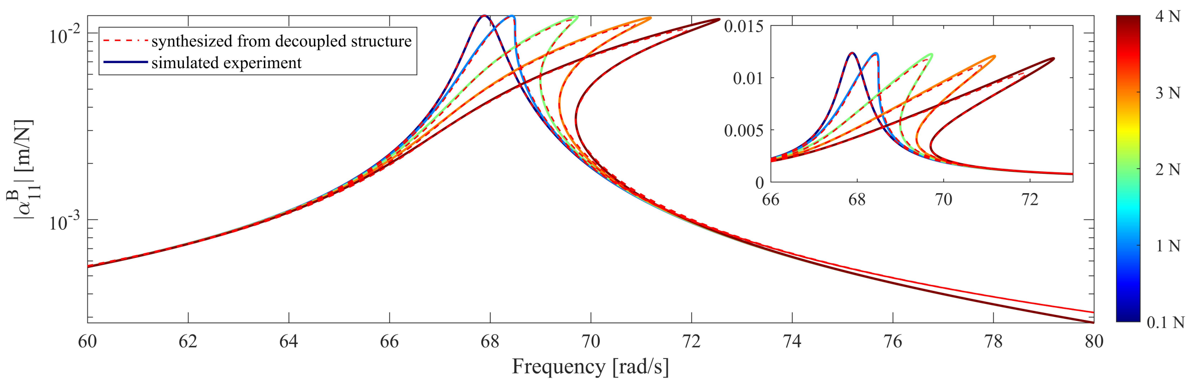

Fig. 5 Constant force FRF comparison for configuration 2 with two nonlinearities

Table 2 Natural frequency and peak level errors of the two configurations of synthesized nonlinear constant force FRFs.

-	Force [N]	0.1	1	2	3	4
Configuration 1	Error in Natural Frequency [%]	0.001	0.033	0.127	0.282	0.415
	Error in Peak [%]	0.019	1.170	3.441	5.296	6.624
Configuration 2	Error in Natural Frequency [%]	0.001	0.027	0.150	0.443	0.880
	Error in Peak [%]	0.020	1.342	4.318	7.642	11.728

The linear modes of system C and A are at 71.31 rad/s and 73.66 rad/s, respectively, and are near the identified mode of subsystem B. Table 2 shows that the errors increase with higher force levels. Also, when the number of nonlinearities in the system increase, the errors also increase.

Continuous system

The effectiveness of the method is also tested on the continuous beam shown in Fig. 6, coupled from a single connection point, but two DoFs (translational and rotational). Since rotational forcing cannot be applied experimentally, and measuring rotational displacement is also very difficult, D'Ambrogio and Fregolent [6] suggested modifying the dual decoupling procedure for continuous systems. One of these methods is the mixed interface decoupling. This method uses some of the DoFs from the connection interface and some of the DoFs from the internal nodes of the known substructure.

To decouple substructure B from the coupled system C, RCT is applied to the translational DoFs of the 5^{th} and 9^{th} nodes for displacement levels 0.01 mm to 0.4 mm. However, in both simulated experiments, the displacement of the translational DoF of the 9^{th} node is kept constant while changing the force location. The decoupled quasi-linear FRFs are used to obtain the modal model of the nonlinear cantilever beam. The obtained modal model is used to synthesize the constant force FRFs in Fig. 7.

As the internal DoF is also used to decouple the nonlinear beam, the errors given in Table 3 are relatively low compared to the system in Fig. 4, despite the higher degree of nonlinearity. The low error shows that using a mixed interface decoupling with RCT can effectively decouple nonlinear systems, whether continuous or lumped.

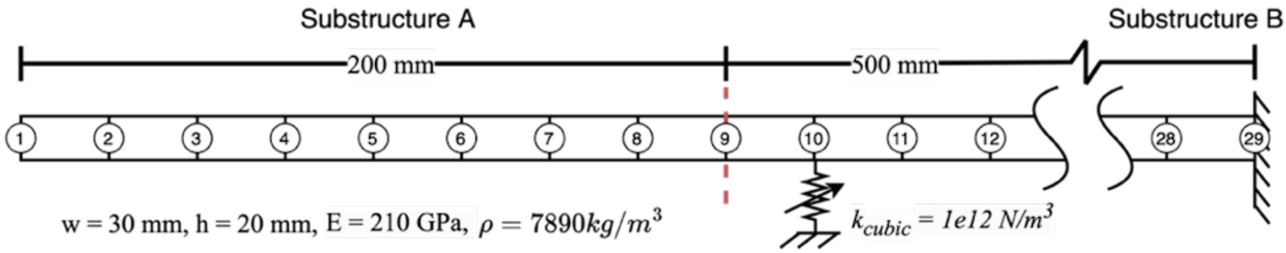

Fig. 6 Nonlinear beam subjected to decoupling

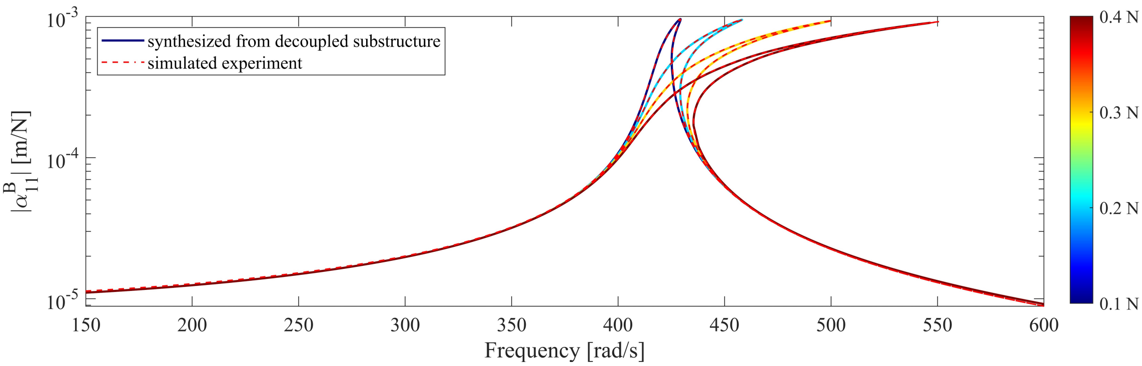

Fig. 7 Constant force FRF comparison of decoupled nonlinear beam

Table 3 Natural frequency and peak level errors of the synthesized nonlinear constant force FRF of the decoupled nonlinear beam.

Force [N]	0.1	0.2	0.3	0.4
Error in Natural Frequency [%]	0.003	0.020	0.279	0.789
Error in Peak [%]	0.132	0.529	1.130	1.906

Conclusion

This study examines the applicability, accuracy, and limitations of the method proposed in [16] to extend FDM-NS [10] , originally developed for systems with localized nonlinearities, to address multiple nonlinearities in one or both substructures of a coupled system. The method's accuracy in estimating a decoupled subsystem's constant force and constant amplitude frequency response functions is demonstrated through numerical simulation case studies. The results are generally highly successful, even for higher levels of nonlinearity. While the error in frequency response estimation increases with the degree of nonlinearity in multiple DoF decoupling, incorporating internal DoFs in the process significantly reduces these errors. A key limitation of the method arises from the relationship between the resonance frequency regions of the substructures and the coupled system. When the resonance frequency of the unknown substructure falls in off-resonance regions of the coupled system, noise may complicate the estimation of the unknown substructure. Additionally, achieving the required force levels in frequency regions where the receptance is lower than near-resonance values may present practical challenges. Nevertheless, the case studies, including those not reported here, show promising results, suggesting that the method holds significant potential for practical applications.

References

1. D'Ambrogio, W., and Fregolent, A., Direct decoupling of substructures using Primal and dual formulation. Conference Proceedings of the Society for Experimental Mechanics Series, 47–76., 2011.

2. de Klerk, D., Rixen, D. J., Voormeeren,S., General Framework for Dynamic Substructuring: History, Review and Classification of Techniques., AIAA Journal 46.5., 2008.
3. W. D'Ambrogio, A. Fregolent, The Role of Interface DOFs in Decoupling of Substructures Based on the Dual Domain Decomposition, Mechanical Systems and Signal Processing, 24 (7) 2035-2048., 2010.
4. van der Seijs, M. V., Pasma, E. A., van den Bosch, D. D., and Wernsen, M. W. M., A benchmark structure of validation of experimental substructuring, transfer path analysis and source characterisation techniques., 35th IMAC, 2017.
5. Mahmoudi, A. E., Rixen, D. J., and Meyer, C. H., Comparison of different approaches to include connection elements into frequency-based substructuring., Experimental Techniques, 44(4), 425–433., 2020.
6. D'Ambrogio, W., and Fregolent, A., Are Rotational DoFs Essential in Substructure Decoupling?., Proceedings of the 32nd IMAC, A Conference and Exposition on Structural Dynamics., 2014.
7. Arslan, Ö. and Özgüven, H. N., Modal Identification of Nonlinear Structures and the Use of Modal Model in Structural Dynamic Analysis, Proceedings of the 26th International Modal Analysis Conference, The Society of Experimental Mechanics, Orlando, Florida, 2008.
8. Ekinci, E. C., Özer, M. B., Özgüven, H. N., A Novel Approach for Local Structural Modification of Nonlinear Structures., Proceedings of the 41st International Modal Analysis Conference, The Society of Experimental Mechanics, Austin, Texas. Springer, 2023.
9. Ekinci, E. C., Karaağaçlı, T., Çelik, F. K., Özer, M. B., and Özgüven, H. N., Extension and Experimental Verification of an Efficient Re-Analysis Method for Modified Nonlinear Structures., Proceedings of the 42nd International Modal Analysis Conference, The Society of Experimental Mechanics, Orlando, Florida. Springer, 2024.
10. Kalaycıoğlu, T., Özgüven, H. N., FRF decoupling of nonlinear systems, Mechanical Systems and Signal Processing, 102, 230–244. ,2018.
11. Tepe, Ç., Ciğeroğlu, E., Structural Coupling of Two-Nonlinear Structures, Proceedings of the 33rd IMAC, A Conference and Exposition on Structural Dynamics, 2015.
12. Cömert, M.D., Özgüven, H.N., A Method for Forced Harmonic Response of Substructures Coupled by Nonlinear Elements, IMAC 13, 1995.
13. Renson, L., Gonzalez-Buelga, A., Barton, D. A. W., and Neild, S. A., Robust identification of backbone curves using control-based continuation., Journal of Sound and Vibration, 367, 145–158., 2016.
14. Peter, S., Scheel, M., Krack, M., and Leine, R. I., Synthesis of nonlinear frequency responses with experimentally extracted nonlinear modes., Mechanical Systems and Signal Processing, 101, 498–515., 2018.
15. Karaağaçlı, T., Özgüven, H. N., Experimental modal analysis of nonlinear systems by using response-controlled stepped-sine testing, Mechanical Systems and Signal Processing, 146, 107023, 2021.
16. Özgüven, H. N., Advanced Topics in Structural Dynamics: Seminar 3: Nonlinear Structural Modification, Coupling and Decoupling, Class and Seminar Notes, Technical University of Munich, Fall 2022.
17. Di Manno, M., Linear and nonlinear mechanical joint identification through substructure decoupling, Doctoral dissertation, Sapienza University of Rome, 2023.
18. Koyuncu, A., Karaağaçlı, T., Şahin, M., Özgüven, H.N., Experimental modal analysis of nonlinear amplified piezoelectric actuators by using response-controlled stepped-sine testing, Exp. Mech., 62, pp. 1579–1594, 2022.
19. Karaağaçlı, T., Özgüven, H.N., Experimental modal analysis of structures with high nonlinear damping by using response-controlled stepped-sine testing, Proceedings of the 41^{st} International Modal Analysis Conference (IMAC), Austin, TX, USA, 2023.
20. Gürbüz, M.F., Karaağaçlı, T., Özer, M.B., Özgüven, H.N., Bypassing the repeatability issue in nonlinear experimental modal analysis of jointed structures by using the RCT-HFS framework, Proceedings of the 41^{st} International Modal Analysis Conference (IMAC), Austin, TX, USA, 2023.
21. Tanrıkulu, Ö., Kuran, B., Özgüven, H.N., Imregün, M., Forced harmonic response analysis of nonlinear structures using describing functions, AIAA J., 31 (7), pp. 1313–1320, 1993.
22. Szemplinska-Stupnicka, W., The modified single mode method in the investigations of the resonant vibrations of nonlinear systems, J. Sound Vib., 63(4), pp. 475-489, 1979.