Chapter 2

Response Modification of Structures with Supplemental Rotational Inertia

Nicos Makris and Gholamreza Moghimi

Abstract Tall, multistory, buildings are becoming increasingly popular in large cities as a result of growing urbanization trends. As cities continue to grow, many of them along the coasts of continents which are prone to natural hazards, the performance of tall, flexible buildings when subjected to natural hazards is a pressing issue with engineering relevance. The performance of structures when subjected to dynamic loads can be enhanced with various response modification strategies which have been traditionally achieved with added stiffness, flexibility, damping and strength. Together with the elastic spring that produces a force proportional to the relative displacement of its end-nodes and the viscous dashpot that produces a force proportional to the relative velocity of its end-nodes; the inerter produces a force proportional to the relative acceleration of its end-nodes and emerges as the third elementary mechanical element (in addition to the spring and dashpot) capable for modifying structural response. Accordingly, in this lecture we examine the seismic performance of multistory and seismic isolated structures when equipped with inerters upon deriving selective frequency- and time-response functions of the inerter and other simple inertoelastic and inertoviscous network in association with the mathematical operations involved.

Keywords Inerter · Supplemental rotational inertia · Response modification · Seismic protection · Nonlinear analysis

Introduction

Response modification of civil structures has been traditionally achieved with added stiffness, flexibility damping and strength ([1–12] and references reported therein). Nevertheless, together with the elastic spring that produces a force proportional to the relative displacement of its end-nodes and the viscous dashpot that produces a force proportional to the relative velocity of its end nodes, the inerter produces a force proportional to the relative acceleration of its end-nodes and emerges as the third elementary mechanical element (in addition to the spring and dashpot) for the synthesis of mechanical networks [13–16]. While a driving spinning top as the one shown in Figure 1, which is a physical realization of the inerter, has been familiar to several generations; inerters were apparently first proposed for the response modification of buildings in the mid-1980s by Kawamata [17] and subsequently by Ishimaru [18] and Arakaki et al. [19, 20] in the mid to late 1990s in Japan.

Following the pioneering work in Japan and the subsequent theoretical and experimental studies by Smith and coworkers [13, 21–24] who established, within the context of linear networks, the analogy of the inerter to the electric capacitor; a growing number of publications have proposed the use of inerters for the response modification of buildings by installing them at all floor levels [25–28]; at selected levels [29–33] or at the ground level, either within the context of protecting a first soft-story [34–36], or within the context of enhancing a seismic isolation system [37–41].

Nicos Makris · Gholamreza Moghimi
Department of Civil and Environmental Engineering, Southern Methodist University, Dallas, TX 75205
e-mail: nmakris@smu.edu; nmakris@smu.edu

Matthew Whelan et al. (eds.), *Dynamics of Civil Structures, Vol. 2*, Conference Proceedings
of the Society for Experimental Mechanics Series, https://doi.org/10.13052/97887-438-0147-4_2

Fig. 1 A physical realization of the inerter in which the force output is proportional only to the relative acceleration between nodes 1 and 2.

Background

Supplemental rotational inertia can be achieved either with a rack-and-pinion- flywheel arrangement, a ball-screw assembly as shown in Figure 1 [19–21, 26, 28, 42–44] or fluid inerters [45–48]. The mechanical system shown in Figure 1 is a mechanical analogue of the electrical capacitor in a force-current/velocity-voltage analogy between mechanical and electrical networks. This missing analogy was first recognized by Smith [13] who coined the term inerter for any mechanical arrangement where the output force is proportional only to the relative acceleration between its end-nodes. The constant of proportionality of the inerter is coined the inertance $= M_R$ [13] and has units of mass [M]. Accordingly, with reference to Figure 1 if $F_1,\ u_1$ and $F_2,\ u_2$ are the forces and displacements at the end-nodes of the inerter with inertance, M_R, its constitutive relation is [15, 35, 37]

$$\begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} = \begin{bmatrix} M_R & -M_R \\ -M_R & M_R \end{bmatrix} \begin{Bmatrix} \ddot{u}_1(t) \\ \ddot{u}_2(t) \end{Bmatrix} \tag{1}$$

The basic frequency and time-response functions of the mechanical element defined by equation (1) have been derived by Makris [14, 15] and are covered in the lecture together with those of simple inertoelastic and inertoviscous elements. The unique characteristic of the inerter is that it has an appreciable inertia mass as opposed to a small gravitational mass.

The potential unfavorable situation when the rotational inertia stored in the inerter may drive the structure was first recognized by Makris and Kampas [34] who introduced the implementation of two parallel inerters together with the use of a clutch (pair of clutching inerters), so that the rotating flywheels only resist the motion of the structure without inducing any deformations. The benefits of using a pair of counter-rotating clutching inerters were subsequently examined on a 2DOF elastic structure [35]. About the same time De Domenico andRicciardi [39] examined the response of a multi-degree-of-freedom seismic isolated elastic structure where the isolation system was enhanced with a tuned mass damper inerter. In that study the nonlinear behavior of the isolator is accounted for, and subsequently, the study proceeds with a stochastic linearization technique.

A seminal contribution among the aforementioned studies is the work of Furuhashi and Ishimaru [25] who showed that when inerters are installed without being interrupted, starting from the first level, then the inertance of the inerters can be adjusted to eliminate the participation of higher modes. At the same time, the unique ability of the inerters to eliminate the contribution of higher modes vanishes when they are placed at a higher level without having inerters at all the floors down below to the first level. Following Furuhashi and Ishimaru's [25] seminal contribution, Takewaki et al. [28] explained that for inerters to suppress the induced ground acceleration they need to be installed without being interrupted, starting from the ground level; otherwise, the inerters which are installed above the "interrupted" level can no-longer suppress the ground induced acceleration. In spite of Takewaki et al. [28] important finding-that solitary inerters when placed at higher levels do not suppress the ground induced acceleration, several subsequent publications investigated the response of structures equipped with a solitary inerter at a floor level other than the first level [29–33] within the context of proposing an enhanced tuned mass damper. Soon after Takewaki et al. [28] publication, Lazar et al. [31] examined in detail the role of a tuned inerter damper (TID)—which is essentially an inerter supported on a compliant support (a spring-dashpot parallel connection), as an alternative tuned mass damper (TMD) and they correctly concluded that the TID is more effective when placed at the ground level, reaffirming the findings of Takewaki et al. [28].

In view of the appreciable number of publications that examine the response of tall buildings when equipped at a higher level with a solitary mechanical network that involves inerters [32, 33, 49], in association that this concept has enjoyed full-scale implementations [29, 30], this lecture reviews, the dynamics of structures equipped with inerters and how inerters modify elastic and inelastic structural response.

Analysis

The Distinguishable Characteristics of the Inerter

a) Suppression of the induced ground acceleration

Figure 2 depicts a single-degree-of-freedom undamped structure with stiffness k and mass m. A stiff chevron frame supports a flywheel with radius R and mass m_W that can rotate about an axis O. In the interest of simplicity, we consider the case of a non-compliant chevron frame whose deformation is negligible to the translational displacement, $u(t)$, of the SDOF structure. The case of a compliant chevron frame has been examined in depth in past publications [34–36]. Concentric to the flywheel, there is an attached pinion with radius ρ engaged to a linear rack connected to the bottom of the vibrating mass m of the SDOF. With this arrangement when the mass m undergoes a positive displacement, $u(t)$, the flywheel is subjected to a clockwise rotation, $\theta(t)$. Given that there is no slipping between the rack and the pinion, $u(t) = \rho\theta(t)$, the internal force $F_I(t)$ that develops along the rack-pinion interface satisfies dynamic moment equilibrium

$$F_I(t)\rho = I_W\ddot{\theta}(t) \tag{2}$$

where $I_W(t) = (1/2)m_W R^2 =$ moment of inertia of the flywheel about point O. With reference to Figure 2, for a positive displacement, $u(t)$, to the right, the internal force, $F_I(t)$ given by equation (2) opposes the motion of the mass, m (to the left). Accordingly, dynamic equilibrium of the vibrating mass when subjected to a ground acceleration, $\ddot{u}_g$, gives [34]

$$\ddot{u}(t) + \frac{\omega_0}{1+\frac{1}{2}\frac{m_W}{m}\frac{R^2}{\rho^2}}u(t) = -\frac{1}{1+\frac{1}{2}\frac{m_W}{m}\frac{R^2}{\rho^2}}\ddot{u}_g(t) \tag{3}$$

where $\omega_0 = \sqrt{k/m}$ =natural frequency of the structure when the pinion-flywheel is disengaged and m_W is the mass of the flywheel.

Equation (3) indicates that the engagement of the flywheel lengthens the vibration period of the structure [50]; yet, most importantly, it suppresses the level of ground shaking given that the denominator in the right-hand side is always larger than

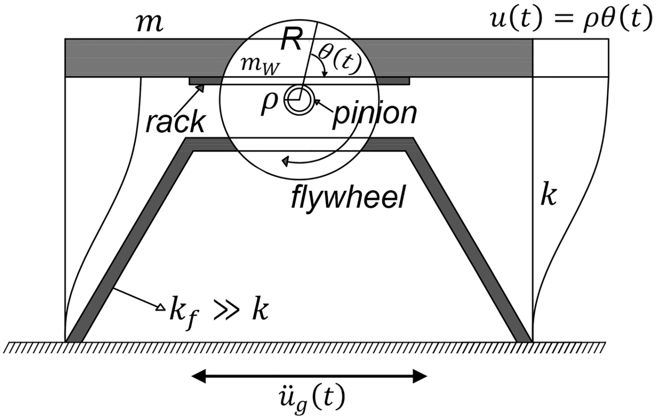

Fig. 2 Single-degree-of-freedom structure with mass m and stiffness k with supplemental rotational inertia from a flywheel with radius R supported on a chevron frame with stiffness k_f that is much larger than k.

unity [34]. The quantity $M_R = (1/2)M_W(R^2/\rho^2)$ is defined as the inertance of the inerter with units of mass $[M]$, whereas the ratio $\sigma = M_R/m$ appearing in the denominators of equation (3) is the inertance ratio.

b) Elimination of the participation of a higher mode

In a study with remarkable insight, Furuhashi and Ishimaru [25] showed that when the inertance of an inerter placed at the ground level is judiciously selected, the supplemental rotational inertia eliminates the contribution of a higher mode. In a recent study that investigated the implementation of inerters in seismic isolated structures, Makris and Moghimi [41] showed that while a small amount of supplemental rotational inertia is needed to eliminate the participation of the second mode of the 2DOF linear isolated structure; the effect of this elimination is marginal on the structure response, since the participation of the second mode is invariably small even when isolation systems without inerters are used. The nonlinear response analysis of the same 2DOF isolated structure is examined by adopting a bilinear behavior for the isolation system in association with a formulation that accounts for the compliance of the support of the inerter. Our study shows that supplemental rotational inertia aggravates superstructure displacements and accelerations at larger isolation periods ($T_b > 2.5sec$). In view of these findings in association with the small gains in reducing displacements above isolators, the use of inerters in isolation systems is not recommended.

Conclusion

In this lecture we first present the dynamics of single-story and multistory structures with supplemental rotational inertia and how inerters modify their elastic or inelastic response. Because of the participation of the inerter, some basic time-response functions of the examined mechanical networks exhibit a causal oscillatory response and this behavior complements the decaying-exponential response due to the participation of the dashpot. It is shown that the inerter emerges as an attractive response-modification element given that in some cases it "absorbs" the impulsive response of the solitary spring or dashpot present in the mechanical network. The basic response-functions derived for the inerters in association with the mathematical operations outlined, extends the well-established theory of linear viscoelasticity to the inertoelastic and inertoviscoelastic behavior (combination of inerters, dashpots and springs) and introduces the subject of inertoviscoelasticity.

Supplemental rotational inertia controls effectively the displacements of the first story of a 2DOF elastic structure along a wide range of the response spectrum. The proposed seismic protection strategy can accommodate large relative displacements without suffering from the issue of viscous heating ([51–53]) and potential leaking that challenges the implementation of fluid dampers under prolonged cyclic loading. When the chevron frame that supports the rotational inertia system is stiff, the use of two parallel rotational inertia systems offers improved results for the response of the 2DOF structure. However, as the compliance of the chevron frame that supports the inerters increases, the use of a single rotational inertia system offers more favorable response other than increasing the forces transferred to the chevron frame.

While a small amount of supplemental rotational inertia is needed to eliminate the participation of the second mode of the 2DOF linear isolated structure; the effect of this elimination is marginal on the structure response, since the participation of the second mode is invariably small even when isolation systems without inerters are used ([5, 54]). The nonlinear response analysis of the same 2DOF isolated structure is examined by adopting a bilinear behavior for the isolation system in association with a formulation that accounts for the compliance of the support of the inerter. Our study shows that supplemental rotational inertia aggravates superstructure displacements and accelerations at larger isolation periods ($T_b > 2.5sec$). In view of these findings in association with the small gains in reducing displacements above isolators, the use of inerters in isolation systems is not recommended.

The response analysis of a SDOF elastoplastic and bilinear structure reveals that when the yielding structure is equipped with supplemental rotational inertia (inerters), the equal-displacement rule is valid starting from lower values of the preyielding period given that the presence of inerters lengthens the apparent preyielding period. Furthermore, inerters suppress effectively the inelastic displacements of SDOF yielding structures; while the resulting base shears are systematically lower than when large values of supplemental damping ($\xi_d = 25\%$) are used. The forces transferred at the mounting of the inerters are appreciably lower than the corresponding forces originating from an elastic structure. Consequently, the implementation of inerters emerges as an attractive response modification strategy for elastoplastic and bilinear SDOF structures with larger preyielding periods. The use of a pair of clutching inerters does not offer any additional benefits compared to the case where a single inerter is used. Pair of clutching inerters are found to be attractive when suppressing the response of elastic structures.

Inerters are also effective in suppressing the inelastic response of 2DOF yielding structures with pre-yielding periods up to $T_1 = 1.5$sec without aggravating the inelastic response of the superstructure. The effectiveness of inerters to suppress the inelastic response of the 2DOF yielding structure outperforms the effectiveness of large values of supplemental damping

($\xi_d = 25\%$) when the support frame of the response modification device is compliant. In view of these findings the use of inerters emerges as an attractive response modification strategy and is recommended for yielding structures. For larger pre-yielding periods (say $T_1 > 2.0sec$), the effectiveness of inerters to suppress the inelastic response of 2DOF yielding structures reduces; and for very flexible first stories; as in the case of isolated structures, the use of inerters at the first level (isolation system) is not recommended.

References

1. Kelly, J.M., Skinner, R., and Heine, A. "Mechanisms of energy absorption in special devices for use in earthquake resistant structures". *Bulletin of the New Zealand Society for Earthquake Engineering*, 5(3):63–88 (1972) (doi: https://doi.org/10.5459/BNZSEE.5.3.63-88).
2. Skinner, R.I., Kelly, J.M., and Heine, A. "Hysteretic dampers for earthquake-resistant structures". *Earthquake Engineering and Structural Dynamics*, 3(3):287–296 (1974) (doi: https://doi.org/10.1002/eqe.4290030307).
3. Clough, R. and Penzien, J. *Dynamics of structures*. Mc Graw-Hill, New York, U.S.A. (1975)
4. Skinner, R.I., Robinson, W., and GH, M. *An Introduction to Seismic Isolation*. John Wiley and Sons, New York, NY, U.S.A. (1993)
5. Kelly, J.M. *Earthquake-resistant design with rubber*, volume 7. Springer, London, U.K. (1997)
6. Soong, T. and Dargush, G. *Passive Energy Dissipation Systems in Structural Engineering*. John Wiley and Sons, New York, NY, U.S.A. (1997)
7. Constantinou, M., Soong, T., and Dargush, G. *Passive energy dissipation systems for structural design and retrofit*. Multidisciplinary Center for Earthquake Engineering Research Buffalo, NY, U.S.A. (1998)
8. Makris, N. and Chang, S. "Effect of viscous, viscoplastic and friction damping on the response of seismic isolated structures". *Earthquake Engineering and Structural Dynamics*, 29(1):85–107 (2000) (doi: https://doi.org/10.1002/(SICI)1096-9845(200001)29:1⟨85::AID-EQE902⟩3.0.CO;2-N).
9. Whittaker, A.S., Bertero, V.V., Thompson, C.L., and Alonso, L.J. "Seismic testing of steel plate energy dissipation devices". *Earthquake Spectra*, 7(4):563–604 (1991) (doi: https://doi.org/10.1193/1.1585644).
10. Black, C., Makris, N., and Aiken, I. *Component testing and modeling of buckling restrained "unbonded" braces*. Proc., Conference on Behaviour of Steel Structures in Seismic Areas, Routledge, Netherlands. (2003)
11. Symans, M., Charney, F., Whittaker, A., Constantinou, M., Kircher, C., Johnson, M., and McNamara, R. "Energy dissipation systems for seismic applications: current practice and recent developments". *Journal of Structural Engineering, ASCE*, 134(1):3–21 (2008) (doi: https://doi.org/10.1061/(ASCE)0733-9445(2008)134:1(3)).
12. Sarlis, A.A., Pasala, D.T.R., Constantinou, M., Reinhorn, A., Nagarajaiah, S., and Taylor, D. "Negative stiffness device for seismic protection of structures". *Journal of Structural Engineering, ASCE*, 139(7):1124–1133 (2013) (doi: https://doi.org/10.1061/(ASCE)ST.1943-541X.0000616).
13. Smith, M.C. "Synthesis of mechanical networks: the inerter". *IEEE Transactions on Automatic Control*, 47(10):1648–1662 (2002) (doi: https://doi.org/10.1109/TAC.2002.803532).
14. Makris, N. "Basic response functions of simple inertoelastic and inertoviscous models". *Journal of Engineering Mechanics, ASCE*, 143(11):04017123 (2017) (doi: https://doi.org/10.1061/(ASCE)EM.1943-7889.0001348).
15. Makris, N. "Time-response functions of mechanical networks with inerters and causality". *Meccanica*, 53(9):2237–2255 (2018) (doi: https://doi.org/10.1007/s11012-018-0822-6).
16. Makris, N. and Efthymiou, E. "Time-response functions of fractional derivative rheological models". *Rheologica Acta*, 59(12):849–873 (2020) (doi: https://doi.org/10.1007/s00397-020-01241-5).
17. Kawamata, S. "Control of structural vibration by inertia pump damper: part i theoretical model and response to harmonic excitation". In *Proc., Summaries of Technical Papers of Annual Meeting Architectural Institute of Japan, 1986*. National Institute of Informatics (CiNii), Japan (1986)
18. Ishimaru, S. "Outline of response control of structures against earthquakes, design mechanism and control dynamics of building structure". *Architectural Institute of Japan*, pages 199–202 (1994)
19. Arakaki, T., Kuroda, H., Arima, F., Inoue, Y., and Baba, K. "Development of seismic devices applied to ball screw: Part 1 basic performance test of rd-series". *AIJ Journal of Technology and Design*, 5(8):239–244 (1999) (doi: https://doi.org/10.3130/aijt.5.239_1).
20. Arakaki, T., Kuroda, H., Arima, F., Inoue, Y., and Baba, K. "Development of seismic devices applied to ball screw: Part 2 performance test and evaluation of rd-series". *AIJ Journal of Technology and Design*, 5(9):265–270 (1999) (doi: https://doi.org/10.3130/aijt.5.265).
21. Papageorgiou, C. and Smith, M.C. "Laboratory experimental testing of inerters". In *Proc., 44th IEEE Conference on Decision and Control*, pages 3351–3356, Seville, Spain (2005) IEEE.
22. Papageorgiou, C., Houghton, N.E., and Smith, M.C. "Experimental testing and analysis of inerter devices". *Journal of Dynamic Systems, Measurement, and Control*, 131(1) (2009) (doi: https://doi.org/10.1115/1.3023120).
23. Chen, M.Z., Papageorgiou, C., Scheibe, F., Wang, F.C., and Smith, M.C. "The missing mechanical circuit element". *IEEE Circuits and Systems Magazine*, 9(1):10–26 (2009) (doi: https://doi.org/10.1109/MCAS.2008.931738).
24. Kuznetsov, A., Mammadov, M., Sultan, I., and Hajilarov, E. "Optimization of improved suspension system with inerter device of the quarter-car model in vibration analysis". *Archive of Applied Mechanics*, 81:1427–1437 (2011) (doi: https://doi.org/10.1007/s00419-010-0492-x).
25. Furuhashi, T. and Ishimaru, S. "Mode control seismic design with dynamic mass". In *Proc., 14thWorld Conference on Earthquake Engineering*, Beijing, China (2008)
26. Ikago, K., Saito, K., and Inoue, N. "Seismic control of single-degree-of-freedom structure using tuned viscous mass damper". *Earthquake Engineering and Structural Dynamics*, 41(3):453–474 (2012)

27. Ikago, K., Sugimura, Y., Saito, K., and Inoue, N. “Modal response characteristics of a multiple-degree-of-freedom structure incorporated with tuned viscous mass dampers”. *Journal of Asian Architecture and Building Engineering*, 11(2):375–382 (2012) (doi: https://doi.org/10.3130/jaabe.11.375).
28. Takewaki, I., Murakami, S., Yoshitomi, S., and Tsuji, M. “Fundamental mechanism of earthquake response reduction in building structures with inertial dampers”. *Structural Control and Health Monitoring*, 19(6):590–608 (2012) (doi: https://doi.org/10.1002/stc.457).
29. Sugimura, Y., Goto, W., Tanizawa, H., Nagasaku, T., Saito, K., Ninomiya, T., and Saito, K. “Response control effect of hi-rised steel building structure using tuned viscous mass dampers”. *AIJ Journal of Technology and Design*, 18(39) (2012) (doi: https://doi.org/10.3130/aijt.18.441).
30. Ogino, M. and Sumiyama, T. “Structural design of a high-rise building using tuned viscous mass dampers installed across three consecutive storeys”. In *Proc., 12th International Conference on Computational Structures Technology*, volume 225, Naples, Italy (2014)
31. Lazar, I., Neild, S., and Wagg, D. “Using an inerter-based device for structural vibration suppression”. *Earthquake Engineering and Structural Dynamics*, 43(8):1129–1147 (2014) (doi: https://doi.org/10.1002/eqe.2390).
32. Marian, L. and Giaralis, A. “Optimal design of a novel tuned mass-damper–inerter (tmdi) passive vibration control configuration for stochastically support-excited structural systems”. *Probabilistic Engineering Mechanics*, 38:156–164 (2014) (doi: https://doi.org/10.1016/j.probengmech.2014.03.007).
33. Taflanidis, A., Giaralis, A., and Patsialis, D. “Multi-objective optimal design of inerter-based vibration absorbers for earthquake protection of multi-storey building structures”. *Journal of the Franklin Institute*, 356(14):7754–7784 (2019) (doi: https://doi.org/10.1016/j.jfranklin.2019.02.022).
34. Makris, N. and Kampas, G. “Seismic protection of structures with supplemental rotational inertia”. *Journal of Engineering Mechanics, ASCE*, 142(11):04016089 (2016) (doi: https://doi.org/10.1061/(ASCE)EM.1943-7889.0001152).
35. Makris, N. and Moghimi, G. “Displacements and forces in structures with inerters when subjected to earthquakes”. *Journal of Structural Engineering, ASCE*, 145(2):04018260 (2019) (doi: https://doi.org/10.1061/(ASCE)ST.1943-541X.0002267).
36. Moghimi, G. and Makris, N. “Seismic response of yielding structures equipped with inerters”. *Soil Dynamics and Earthquake Engineering*, 141:106474 (2021) (doi: https://doi.org/10.1016/j.soildyn.2020.106474).
37. Saitoh, M. “Simple model of frequency-dependent impedance functions in soil-structure interaction using frequency-independent elements”. *Journal of Engineering Mechanics, ASCE*, 133(10):1101–1114 (2007) (doi: https://doi.org/10.1061/(ASCE)0733-9399(2007)133:10(1101)).
38. Saitoh, M. “On the performance of gyro-mass devices for displacement mitigation in base isolation systems”. *Structural Control and Health Monitoring*, 19(2):246–259 (2012) (doi: https://doi.org/10.1002/stc.419).
39. De Domenico, D. and Ricciardi, G. “Optimal design and seismic performance of tuned mass damper inerter (tmdi) for structures with nonlinear base isolation systems”. *Earthquake Engineering and Structural Dynamics*, 47(12):2539–2560 (2018) (doi: https://doi.org/10.1002/eqe.3098).
40. Ye, K., Shu, S., Hu, L., and Zhu, H. “Analytical solution of seismic response of base-isolated structure with supplemental inerter”. *Earthquake Engineering and Structural Dynamics*, 48(9):1083–1090 (2019) (doi: https://doi.org/10.1002/eqe.3165).
41. Makris, N. and Moghimi, G. “Response of seismic isolated structures with supplemental rotational inertia”. *Earthquake Engineering and Structural Dynamics*, 51(12):2956–2974 (2022) (doi: https://doi.org/10.1002/eqe.3709).
42. Hwang, J.S., Kim, J., and Kim, Y.M. “Rotational inertia dampers with toggle bracing for vibration control of a building structure”. *Engineering Structures*, 29(6):1201–1208 (2007) (doi: https://doi.org/10.1016/j.engstruct.2006.08.005).
43. Watanabe, Y., Ikago, K., Inoue, N., Kida, H., Nakaminami, S., Tanaka, H., Sugimura, Y., and Saito, K. “Full-scale dynamic tests and analytical verification of a force-restricted tuned viscous mass damper”. In *Proc., 15th World Conference on Earthquake Engineering.*, Lisbon, Portugal (2012)
44. Ishii, M., Kazama, H., Miyazaki, K., and Murakami, K. “Application of tuned viscous mass damper to super-high-rise buildings”. In *Proc., 6th World Conference of the International Association for Structural Control and Monitoring (6WCSCM), Barcelona, Spain* (2014)
45. Swift, S., Smith, M.C., Glover, A., Papageorgiou, C., Gartner, B., and Houghton, N.E. “Design and modelling of a fluid inerter”. *International Journal of Control*, 86(11):2035–2051 (2013) (doi: https://doi.org/10.1080/00207179.2013.842263).
46. Wang, F.C., Hong, M.F., and Lin, T.C. “Designing and testing a hydraulic inerter”. In *Proc., Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, volume 225, pages 66–72. SAGE Publications Sage, London, U.K. (2011)
47. Liu, X., Jiang, J.Z., Titurus, B., and Harrison, A. “Model identification methodology for fluid-based inerters”. *Mechanical Systems and Signal Processing*, 106:479–494 (2018) (doi: https://doi.org/10.1016/j.ymssp.2018.01.018).
48. De Domenico, D., Deastra, P., Ricciardi, G., Sims, N.D., and Wagg, D.J. “Novel fluid inerter based tuned mass dampers for optimised structural control of base-isolated buildings”. *Journal of the Franklin Institute*, 356(14):7626–7649 (2019) (doi: https://doi.org/10.1016/j.jfranklin.2018.11.012).
49. Giaralis, A. and Taflanidis, A. “Optimal tuned mass-damper-inerter (tmdi) design for seismically excited mdof structures with model uncertainties based on reliability criteria”. *Structural Control and Health Monitoring*, 25(2):e2082 (2018) (doi: https://doi.org/10.1002/stc.2082).
50. Chen, M.Z., Hu, Y., Huang, L., and Chen, G. “Influence of inerter on natural frequencies of vibration systems”. *Journal of Sound and Vibration*, 333(7):1874–1887 (2014) (doi: https://doi.org/10.1016/j.jsv.2013.11.025).
51. Makris, N. “Viscous heating of fluid dampers. i: Small-amplitude motions”. *Journal of Engineering Mechanics, ASCE*, 124(11):1210–1216 (1998) (doi: https://doi.org/10.1061/(ASCE)0733-9399(1998)124:11(1210)).
52. Makris, N., Roussos, Y., Whittaker, A.S., and Kelly, J.M. “Viscous heating of fluid dampers. ii: Large-amplitude motions”. *Journal of Engineering Mechanics, ASCE*, 124(11):1217–1223 (1998) (doi: https://doi.org/10.1061/(ASCE)0733-9399(1998)124:11(1217)).
53. Black, C.J. and Makris, N. “Viscous heating of fluid dampers under small and large amplitude motions: experimental studies and parametric modeling”. *Journal of Engineering Mechanics, ASCE*, 133(5):566–577 (2007) (doi: https://doi.org/10.1061/(ASCE)0733-9399(2007)133:5(566)).
54. Kelly, J.M. “Base isolation: linear theory and design”. *Earthquake Spectra*, 6(2):223–244 (1990) (doi: https://doi.org/10.1193/1.1585566).