Chapter 8
Initial Validation of a Novel Output-to-Output Frequency Response Function Mapping Method for Impact Localization in Dispersive Media

Andrew T. Gothard and Steven R. Anton

Abstract In this study, a novel impact localization mapping algorithm is proposed to solve the source localization problem in solid media, namely the output-to-output frequency response function (OO-FRF) mapping method. Previous work has shown that in media where mechanical waves travel with a constant velocity the time difference of arrival (TDOA) method can be used to solve the source localization problem. In solid media, the TDOA method is an insufficient solution due to mechanical waves being distorted by dispersion, wave reflections, and frequency-dependent damping. The ΔT mapping method was developed as a data driven Modified-TDOA (M-TDOA) method, which is able to account for path dependent velocities in complex structures by creating TDOA mapping functions across the surface of a structure; however, the ΔT mapping method does not account for the dispersive effects when different frequency ranges are excited by impacts. In order to improve upon the previous work, this work proposes the OO-FRF mapping method. The OO-FRF mapping method is a frequency response function (FRF) localization method and does not rely on TDOA values but instead uses the relative frequency response between pairs of sensors to localize impacts. By using OO-FRFs, this method naturally captures frequency dependent relationships across the structure without needing to know the input force on the system. The OO-FRF method is validated using a numerical study with a 1D finite element beam and compared against the ΔT mapping method. The results show that the localization error of the ΔT mapping method and OO-FRF mapping method are $0.0957\ m$ and $0.0029\ m$, respectively. Based on these results, the OO-FRF mapping method shows clear improvement over the ΔT mapping method, and the results demonstrate the potential for these methods to be used to localize impacts in complex dispersive structures.

Keywords Impact localization · Output-to-output FRF · Mapping function · Dispersive media

Introduction

The general source localization problem can be defined as the determination of the location of an event using the characteristics of the signals emitted by that event. The source localization problem has been widely studied for wireless communication systems where a mobile station emits a signal, such as radio waves, and is located using access points [1, 2]. Source localization in solid media is also an important area of research that has been studied for a variety of structural health monitoring (SHM) and damage localization applications in composite and metallic structures, such as planes, naval vessels, civil infrastructure, and others [3–6]. Two of the major subsets of the source localization problem in solid media are impact localization and acoustic emission (AE) source localization. The AE emission source localization problem is typically focused on the localization of cracks or other damage within a structure. Impact localization in solid media has been largely studied to identify the location of potentially damaging impacts; however, more recently with the development of vibration based smart building technology, impact localization has also been used to monitor human building interactions (HBIs) for occupant tracking, gait analysis, and fall detection to improve occupant safety and comfort [7–9]. Typically, the source localization problem is solved using the time difference of arrival method (TDOA), which estimates the source location based on the speed of the wave and the difference in the arrival times of the wave at multiple sensors. The TDOA method works well for

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Matthew Whelan et al. (eds.), *Dynamics of Civil Structures, Vol. 2*, Conference Proceedings
of the Society for Experimental Mechanics Series, https://doi.org/10.13052/97887-438-0147-4_8

wireless communication systems due to the fact that the emitted signals travel at a constant speed and receive little interference from the environment. However, for source localization in solid media, the emitted mechanical waves are affected by dispersion, frequency dependent damping, and reflections from the boundaries of the structure. Due to these complexities, the velocity of the mechanical waves is not constant throughout the media, which can cause large localization errors when using the TDOA method even for simple structures.

Previous work has been done to improve source localization in solid media using modified TDOA (M-TDOA) methods. While many types of algorithms exist, the majority of the M-TDOA methods fall into the following major categories: velocity modeling methods [10–12], dispersion compensation methods [13, 14], machine learning methods [15, 16], and ΔT mapping methods [17–19]. Velocity modeling methods typically relax the assumption of a single propagation velocity by fitting a velocity model to a set of training data, which is then used by a TDOA algorithm to localize an impact. Another type of velocity modeling method uses a TDOA optimization scheme to predict the most likely velocity and impact location from a set of possible velocities. By relaxing the constant velocity assumption, the velocity modeling methods reduce localization errors due to dispersion. The dispersion compensation methods typically use a continuous wavelet transform (CWT) to perform a time-frequency analysis on the signals received at each sensor to remove the effects of dispersion. Since wavelets act as bandpass filters with a dominant center frequency, the CWT can be used to extract particular center frequencies from a signal. The center frequencies are not perfectly isolated due to the tradeoff between temporal and frequency resolutions inherent to the CWT; however, the effect of dispersion on the mechanical waves is reduced. A TDOA algorithm can then be used to localize an impact based on the arrival times of the isolated center frequency. While the velocity modeling and CWT dispersion compensation methods reduce error due to dispersion and are an improvement over regular TDOA methods for simple plate like structures, these methods do not account for the effects of path dependent velocities that are often present in complex structures. Other groups have proposed improved TDOA methods, which use machine learning models. The machine learning methods often use neural networks (NNs) or convolutional neural networks (CNNs) and a set of training impacts in order to learn complex patterns in the data and then predict the location of test impacts. While machine learning methods have been successful in localizing impacts in complex structures, they have the disadvantage of being black box models, which means it is difficult to determine if the model is learning physically relevant features that will be applicable to new impacts. Additionally, machine learning algorithms often require large amounts of training data that sufficiently covers the type of impacts that could be seen in the structure, which is difficult to define. Finally, ΔT mapping methods are another type of M-TDOA method that have been developed to handle impact localization in complex structures. ΔT mapping methods use a data driven approach where a grid of calibration impacts is taken across the surface of a structure and TDOA values are calculated for each grid location and for each sensor pair. TDOA maps (called ΔT maps) are generated by interpolating between impact locations. Impacts are localized by minimizing the difference between the TDOA value of the test impact and the ΔT maps. By using mapping functions for a structure, the ΔT mapping methods compensate for path dependent velocities for a particular type of impact and demonstrate the usefulness of mapping functions for localization in complex structures. While the ΔT mapping method allows for calibration across complex structures, it makes the assumption that the ΔT maps are independent of the type of impact on a structure. In reality, different impacts can excite various frequency ranges in a structure and the dispersive effects of the wave propagation can cause variabilities and inaccuracies in the ΔT maps leading to localization errors.

In this work, the output-to-output frequency response function (OO-FRF) mapping method is proposed as a dispersion compensated mapping method. The OO-FRF mapping method is a modified version of the spatially sparse FRF impact localization method developed by Alajlouni et al., which uses input-to-output calibration FRFs to localize impacts [20]. The OO-FRF mapping method does not rely on TDOA calculations but instead calibrates a surface using OO-FRF maps. The calibration OO-FRFs take the ratio between the frequency domain representations of the outputs of two sensors at a particular location. By mapping the ratio between frequency domain representations across a structure, the proposed method naturally accounts for frequency dependent relationships and path dependent velocities in the structure. Another benefit of the OO-FRFs is that the input force is not required to generate the calibration OO-FRF maps. Since the input force is not needed, any object that excites the desired frequency range in the structure can be used for calibration. This may be useful for calibrating vibration based smart building systems for HBIs using natural impacts like footsteps. The OO-FRF mapping method was inspired by the work done by Davis et al. who showed that OO-FRFs could be used to localize impacts, but the proposed method improves upon the work done by this group by incorporating interpolation and a minimization scheme that allows for impact prediction between calibration locations [21]. The OO-FRF mapping method is validated through a numerical study of a 1D homogenous beam using a finite element model (FEM) and compared to the ΔT mapping method.

Methods

Output-to-Output FRF Mapping Method:
The OO-FRF mapping method is a data driven method developed to localize impacts in complex mechanical structures using a set of dynamic sensors and a calibration dataset. The proposed algorithm has two main stages: a calibration stage and a localization stage.

In the calibration stage, impacts are recorded across a structure and OO-FRFs are formed for each impact location. The OO-FRFs are formed by taking the ratio between the frequency domain representations of the outputs of pairs of sensors. As an example, for a pair of sensors s_1 and s_2 at calibration impact location k, the OO-FRF is formed using the following expression,

$$\overrightarrow{R}_{s_1 s_2,k}(f) = \frac{\overrightarrow{H}_{s1,k}\ (f)}{\overrightarrow{H}_{s2,k}(f)} = \frac{\overrightarrow{A}_{s1,k}(f)}{\overrightarrow{F}_k(f)} \frac{\overrightarrow{F}_k(f)}{\overrightarrow{A}_{s_2,k}(f)} = \frac{\overrightarrow{A}_{s_1,k}(f)}{\overrightarrow{A}_{s_2,k}(f)} \tag{1}$$

where $\overrightarrow{R}_{s_1 s_2,k}(f)$ is the OO-FRF between sensors s_i and s_j, respectively, $\overrightarrow{H}_{s1,k}\ (f)$ and $\overrightarrow{H}_{s2,k}(f)$ are the accelerance FRFs between s_1 and s_2 and point k, respectively, $\overrightarrow{A}_{s1,k}(f)$ and $\overrightarrow{A}_{s_2,k}(f)$ are the FFT of the acceleration response at s_1 and s_2, respectively, and $\overrightarrow{F}_k(f)$ is the FFT of the input force at k. Note that the $\overrightarrow{F}_k(f)$ term cancels so that $\overrightarrow{R}_{s_1 s_2,k}(f)$ is dependent only on $\overrightarrow{A}_{s1,k}(f)$ and $\overrightarrow{A}_{s_2,k}(f)$. As previously mentioned, the benefit of using OO-FRFs is that you do not need to know the input force to perform calibrations and can perform the calibration with any object or even a person walking. It should be noted that because input-to-output FRFs are not used the force of the impact cannot be reconstructed. Once the calibration OO-FRFs are formed, cubic spline interpolation is used to increase the spatial resolution between calibration locations.

Next, in the localization stage, test impacts are localized by finding the location that minimizes the difference between the calibration OO-FRFs and the ratio between the frequency domain representation of the acceleration response between pairs of accelerometers. For a set of N sensors $S = \{s_1, \ldots, s_N\}$ and a set of M interpolated calibration points $K = \{1, \ldots, M\}$ the minimization is described using the following equation,

$$k_{impact} = \min_{k \in K} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} a_{s_i s_j,k} \left\| \overrightarrow{R}_{s_i s_j,k} - \overrightarrow{A}_{s_i} \oslash \overrightarrow{A}_{s_j} \right\| \right) \tag{2}$$

where k_{impact} is the predicted impact location, $\| \cdot \|$ represents the L_2 norm, $a_{s_i s_j,k} = \frac{1}{\left\| \overrightarrow{R}_{ij,k} \right\|}$ is a normalization factor, $\overrightarrow{R}_{s_i s_j,k}$ is the calibration OO-FRF between sensors s_i and s_j, respectively, at location k, $\overrightarrow{A}_{s_i}$ and $\overrightarrow{A}_{s_j}$ are the FFT of the sensor outputs of s_i and s_j, respectively, and $\oslash$ is the Hadamard division operator which represents element wise division. Sensor pairs are not repeated for reciprocal relationships since this would not add information to the localization problem. It should be noted that the minimization scheme should only include the range of frequencies excited in the structure during the calibration stage.

1D FE Beam Model:
While the OO-FRF mapping method presented is designed for complex structures, it is first necessary to validate the method on a simple structure. Therefore, in this work, the OO-FRF mapping method is validated on a 1D homogenous finite element (FE) aluminum 3003-H14 beam model with fixed-fixed boundary conditions. The results are also compared to the ΔT mapping method. The parameters of the beam model can be found in Table 1. Aluminum 3003-H14 is used as the material due

Table 1 1D FE beam model parameters.

Parameter	Parameter Value
Material	Aluminum 3003-H14
Modulus	68.9 GPa
Poisson's Ratio	0.33
Density	2730 kg/m^3
Length	0.4572 m
Width	0.0254 m
Thickness	0.00635 m

to the availability of the material for future experimental investigation. The FE model is developed using the commercially available Ansys Mechanical software package and is validated against analytical natural frequencies and mode shapes from the literature [22]. The model uses quadratic Timoshenko beam elements of size $0.00635\ m$. It should be noted that manual mesh refinement is performed to determine that the results converged at this mesh size. Proportional damping is assumed in the model and is estimated by using realistic mass coefficient (α) and stiffness coefficient (β) values from the literature for a lightly damped aluminum structure [8]. The α and β damping values are selected as 1 and $7 * 10^{-8}$, respectively. The sampling rate for the model is selected to be $25\ kHz$ to provide enough temporal resolution to accurately capture the difference in arrival times for the ΔT mapping method.

Localization Algorithm Evaluation:
Once the model is developed, sensor locations, calibration impact locations, and test impact locations are selected, which can be seen in Figure 1. Two sensor locations are selected $0.0254\ m$ from each end of the beam. The locations are chosen to be at each end of the beam due to the fact that for 1D TDOA localization only impacts between two sensors can be distinguished if a constant propagation velocity is assumed, which is required for the ΔT mapping method. Next, 10 evenly spaced calibration points are placed along the beam between the two sensors. The test locations are chosen to be at 0.2L, 0.3L, 0.4L, and 0.5L, where L is the length of the beam. It should be noted that 0.1L is excluded because this is outside of the calibration locations. Impacts at each calibration and test location are simulated using a narrow Gaussian force distribution, which has been used in previous literature to represent impulses [23, 8]. The Gaussian distribution can be described using Equation (3),

$$f_g\left(t\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \tag{3}$$

where $f_g\left(t\right)$ is the Gaussian impulse, σ is the standard deviation parameter of the distribution, and μ is the mean parameter of the distribution. σ and μ control the width and center of the distribution, respectively. In order to demonstrate that the ΔT mapping method is affected by dispersion from different frequency ranges being excited and that the OO-FRF mapping method is able to handle impacts that excite different frequency ranges, two types of impacts are used at each calibration and test location. For Impact Type 1 and Impact Type 2, the σ parameter is selected as 0.001 and 0.003, respectively, and μ is selected to place the impulse in the center of the impulse window. These impact functions are selected because Impact Type 1 excites a much broader frequency range than Impact Type 2, which can be seen from the FFT of the impact types shown in Figure 2. Average calibration maps are formed for both the ΔT and OO-FRF mapping methods by taking the average of the calibration maps for each impact type across the structure. Then the average calibration maps are interpolated to increase spatial resolution. It should be noted that the OO-FRF mapping method does not require averaging across multiple impact types since it accounts for frequency dependent relationships; however, the ΔT mapping method requires averaging across multiple impact types since the ΔT calibration maps are dependent on the type of impact. Therefore, in order to fairly compare the ΔT and OO-FRF mapping methods, average calibration maps are used for localization in both algorithms. Each localization method is then used to predict the location of the four test impacts for each impact type and localization errors are calculated by finding the difference between the predicted locations and the actual test locations. Average localization errors are computed for the ΔT and OO-FRF methods by taking the average of the localization errors for both types of test impacts for each method. It should be noted that the interpolation resolution of the cubic spline interpolation for the OO-FRF mapping method is selected to be $\Delta x = 0.004\ \ m$. Additionally, the frequencies used for the localization calculation are limited to $200\ Hz$ and below since this is where the majority of the frequency content exists for the beam. ΔT maps are also generated using the same calibration dataset and interpolation resolution. Arrival times are calculated using the AIC as performed in previous literature [18].

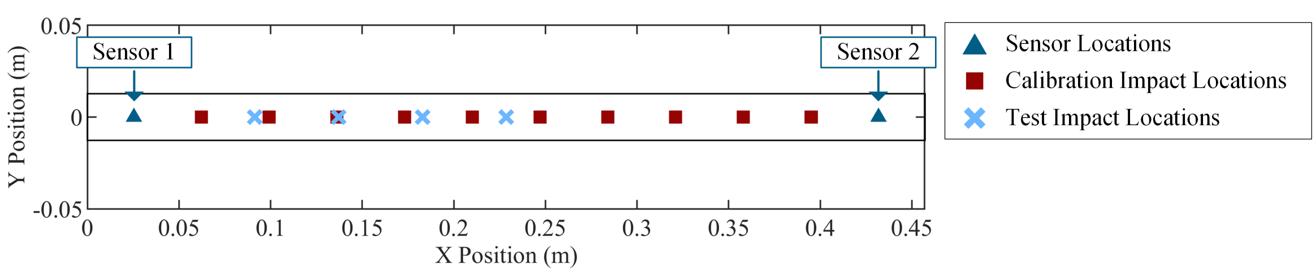

Fig. 1

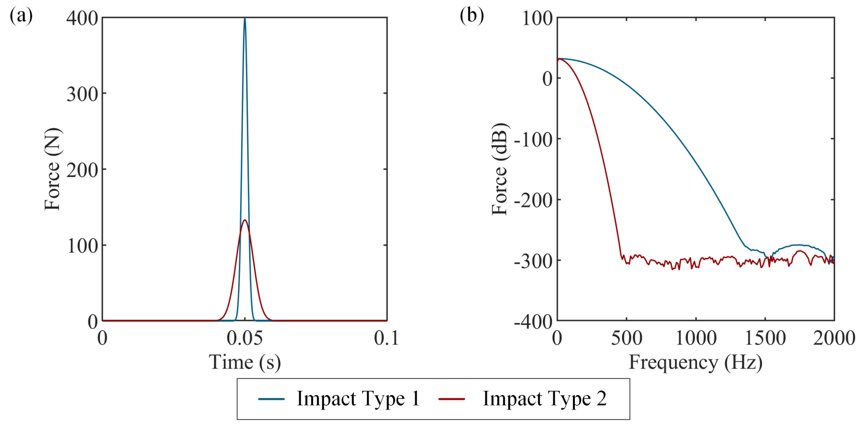

Fig. 2

Results

The localization accuracy of the ΔT and OO-FRF mapping methods are calculated and compared. The average localization error for the ΔT and OO-FRF mapping methods are $0.0957\ m$ and $0.0029\ m$, respectively. Based on these results the OO-FRF mapping method shows significant improvement to the localization error of the ΔT mapping method. This demonstrates that, even when using averaged calibration maps, the ΔT mapping method struggles to localize different impact types; however, the OO-FRF mapping method is able to compensate for the dispersive effects of different impact types and localize the different impact types accurately.

Conclusions

In this work, the output-to-output frequency response function (OO-FRF) mapping method is proposed as a novel source localization algorithm in dispersive solid media. The OO-FRF mapping method naturally compensates for dispersive effects that occur when different frequency ranges are excited in a structure by using OO-FRF maps to calibrate a structure. The OO-FRF maps use the ratio between the frequency domain representations of pairs of sensors to form calibration maps across a structure. The OO-FRFs do not require TDOA calculations and naturally capture frequency dependent relationships of the structure. Additionally, the OO-FRF mapping method does not require a known input force to calibrate the system. The proposed method is validated and compared to the ΔT mapping method using a simulated 1D FE beam model. The results show that the proposed mapping method has a better localization accuracy than the ΔT mapping method. Future work will include experimental validation of the OO-FRF mapping method on both a simple beam structure and structures with increased complexity.

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