Chapter 12
Uncertainty Quantification in Railway Bridge Dynamics via on-board Accelerations Collected from Traversing Trains

Charikleia Stoura, Szymon Greś, and Eleni Chatzi

Abstract Structural Health Monitoring (SHM) offers a pragmatic approach to inform on bridge condition and performance given the exposure of these structures to increasing loads, adverse environments, and ageing. Traditional SHM methods, which rely on sensors installed directly on bridge structures, face limitations such as high costs and insufficient infrastructure coverage. To address these issues, mobile sensing approaches have emerged in recent years. These methods utilize sensors mounted on vehicles traversing bridge elements, collecting data as they pass. In railway systems, this can involve dedicated diagnostic rail vehicles equipped with sensors or even regular in-service trains fitted with low-cost accelerometers. The data gathered by these vehicles enable the use of indirect monitoring approaches for identifying the dynamic properties of bridges, which can then be used to detect potential damage or deterioration. One significant challenge with indirect monitoring is ensuring that the identified modal properties can be attributed to the monitored structure, as opposed to the vehicle and other interaction phenomena, especially when no prior information about the expected properties is available. This study introduces an on-board monitoring approach that estimates the modal properties of railway bridges using acceleration data collected from passing trains, while simultaneously measuring the uncertainty of the identified modal parameters. The acceleration data is processed through a model-based methodology employing Bayesian inference to estimate the states and input (contact force) of the vehicle. Subsequently, a subspace identification approach, combined with a Delta method-based uncertainty quantification methodology, is used to estimate the modal frequencies of the traversed bridge and the associated uncertainty. The proposed methodology is demonstrated on a simple vehicle-bridge system using simulated acceleration data. Long-term monitoring of these frequencies can serve as an indicator of damage, facilitating timely maintenance and ensuring the safety and reliability of rail transportation.

Keywords Structural health monitoring · On-board monitoring · Subspace identification · Delta method · Uncertainty quantification

Introduction

Bridges constitute a fundamental component of railway infrastructure, especially in the case of high-speed rail (HSR) systems. In HSR networks, bridges may constitute more than 50% of an HSR line's total length [1]. As rail networks expand, and with many bridges in older systems nearing the end of their operational lifespan, ensuring the structural integrity of bridges is critical for avoiding accidents or disruptions. Structural Health Monitoring (SHM) plays a key role in this respect, allowing for early detection of damage and hence facilitating maintenance planning [2]. Traditional SHM methods, such as deployment of fixed sensors on the structure or conducting visual inspections, are effective but costly and labor-intensive, especially for extensive rail networks [3]. To address this challenge, mobile sensing approaches, also known as indirect SHM, have been developed [4]. Such approaches rely on the vehicle-bridge interaction (VBI) phenomenon, i.e., on the coupled dynamics of vehicles and bridges when a vehicle traverses a bridge [5]. In these methods, vibrations recorded from vehicles

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crossing the bridge can provide valuable information on the bridge's structural state. This paper focuses on improving the long-term monitoring of railway bridges using advanced SHM methods.

Background

One key problem in indirect SHM of bridges is extracting the modal properties of the bridge, e.g., eigenfrequencies, from vehicle data in an accurate and reliable manner. Main challenges to this end are the interference of the recording vehicle frequencies and the roughness profile of the pavement (for highway bridges) or the tracks (for rail bridges). Subspace Identification (SID) algorithms have been widely used to identify frequencies, damping ratios, and mode shapes of bridges by modeling the system as a linear, time-invariant state-space representation and, at the same time, liberating from the presence of the vehicle frequencies in the identification task [6, 7, 8]. However, on-board monitoring relies on a recording vehicle moving on the sustaining bridge, which renders the state-space system of the VBI system time-variant. One approach to convert such a system into a time-invariant one is to assume slow variation of the dynamics of the vehicle with respect to the dynamics of the bridge by limiting the traversing speed of the vehicle [6]. Based on this assumption, Jin et al. [6] estimated frequencies of highway bridges based on an output-only short-time stochastic subspace identification (SSI) technique. Nevertheless, such an approach can not be applied to railway bridges, especially in the case of monitoring based on in-service trains, where moving speeds are typically much higher. To overcome this limitation, Jin et al. [8] approximated the first mode of the bridge as a linear or sinusoidal function, thus estimating the location of the moving vehicle on the bridge, and applied an input-output SID algorithm.

A common element of these SID approaches is that they assume known states of the vehicle at all degrees of freedom (DOFs) as well as known contact force. In practice, measuring such portions is difficult, if not impossible. Therefore, we proposed a methodology to estimate the modal properties of bridges based solely on the availability of measured accelerations from a recording railroad vehicle [9]. In doing so, a Bayesian inference approach was applied, namely a Dual Kalman Filter (DKF) [10], to first estimate the states of the vehicle and the exerted contact force [11], and then an input-output Multivariable Output Error State Space (MOESP) SID algorithm is used to estimate the frequencies of the bridge [8].

However, a significant issue with SID approaches is the uncertainty in the identified modal parameters, particularly when distinguishing actual bridge frequencies from those caused by noise or roughness. To address this, uncertainty quantification becomes essential. Greś et al. [12] introduced a first-order Delta method that propagates uncertainty in the data covariance onto the estimated modal parameters, allowing for a statistical evaluation of system parameters and providing confidence in the identified frequencies, damping ratios, and mode shapes.

This study presents an approach to estimate bridge modal properties from acceleration data of passing trains with a concurrent evaluation of the uncertainty in the identified modal parameters. Similar to our previous studies, we first estimate the states and input to the vehicle based on a Bayesian inference framework [11]. Then, we set up a subspace identification formulation for the bridge that eliminates the effect of rail roughness and assumes a known mode shape for the bridge in order to render the problem time-invariant [9]. Lastly, different from existing approaches, the subspace identification scheme concurrently performs an uncertainty quantification, ensuring that the identified bridge frequencies are not only accurately inferred, but also belong to the examined system [12].

Analysis

The methodology is developed on a generic three-dimensional railroad vehicle-bridge system fig. 1 that assumes a known vehicle model. The dynamics of the vehicle are captured using a second-order equation of motion (EOM):

$$\mathbf{m}^{\mathrm{v}}\ddot{\mathbf{u}}^{\mathrm{v}}(t) + \mathbf{c}^{\mathrm{v}}\dot{\mathbf{u}}^{\mathrm{v}}(t) + \mathbf{k}^{\mathrm{v}}\mathbf{u}^{\mathrm{v}}(t) = \mathbf{W}^{\mathrm{v}}\boldsymbol{\lambda}(t) \tag{1}$$

where $\mathbf{u}^{\mathrm{v}}(t)$ is the response vector of the vehicle, and $\mathbf{m}^{\mathrm{v}}$, and $\mathbf{c}^{\mathrm{v}}$ and $\mathbf{k}^{\mathrm{v}}$ represent the damping and stiffness matrices, respectively. $\boldsymbol{\lambda}(t)$ is the contact force vector between the vehicle and the underlying bridge system, and $\mathbf{W}^{\mathrm{v}}$ is the contact direction matrix connecting the DOFs of the vehicle to the contact force elements.

In turn, the bridge dynamics are described via an EOM of the form:

$$\mathbf{m}^{\mathrm{b}}\ddot{\mathbf{u}}^{\mathrm{b}}(t) + \mathbf{c}^{\mathrm{b}}\dot{\mathbf{u}}^{\mathrm{b}}(t) + \mathbf{k}^{\mathrm{b}}\mathbf{u}^{\mathrm{b}}(t) = -\mathbf{W}^{\mathrm{b}}(x)\boldsymbol{\lambda}(t) + \mathbf{F}^{\mathrm{b}} \tag{2}$$

where $\mathbf{u}^{\mathrm{b}}(t)$ is the response vector of the bridge, and $\mathbf{m}^{\mathrm{b}}$, $\mathbf{c}^{\mathrm{b}}$, and $\mathbf{k}^{\mathrm{b}}$ are, respectively, the mass, damping, and stiffness matrices. $\mathbf{W}^{\mathrm{b}}(x)$ is the contact direction matrix of the bridge, which changes with time t. Finally, the expression of the

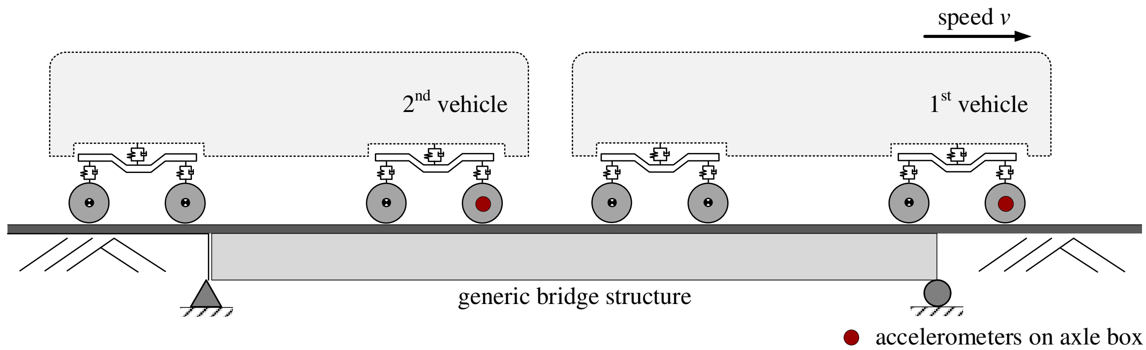

Fig. 1 Two railroad vehicles traversing a generic bridge with accelerometers mounted on the axle boxes of their first wheelsets (red circles).

contact force follows an elastic contact model as:

$$\boldsymbol{\lambda} = k\left[\left(\mathbf{W}^{\mathrm{b}}\left(x\right)\right)^{\mathrm{T}}\mathbf{u}^{\mathrm{b}}\left(t\right) - \left(\mathbf{W}^{\mathrm{v}}\right)^{\mathrm{T}}\mathbf{u}^{\mathrm{v}}\left(t\right) + \mathbf{r}\left(x\right)\right] \tag{3}$$

where k is a contact stiffness parameter that multiplies the relative displacement between the wheels of the vehicle and the sustaining bridge and $\mathbf{r}\left(x\right)$ is the vector containing the roughness at each contact point, assuming that the vehicle traverses the bridge running on tracks with a rough surface.

After modeling the dynamics of the coupled system, the first step is to estimate the unknown contact force between the railroad vehicle and the bridge. Based on the assumption that the railroad vehicle dynamics are known while accelerations are measured at the axle boxes of the vehicle, the states (displacement, velocity, and acceleration) of the system and the contact forces acting on it can be estimated. To this end, this study employs a DKF [10] for simultaneous state and input estimation. In this case, the vehicle is written in a state-space form as:

$$\dot{\mathbf{x}}^{\mathrm{v}}(t) = \mathbf{A}^{\mathrm{v}}\mathbf{x}^{\mathrm{v}}(t) + \mathbf{B}^{\mathrm{v}}\boldsymbol{\lambda}(t) \tag{4}$$

where $\mathbf{x}^{\mathrm{v}}(t)$ is the state vector of the vehicle containing displacements and velocities, and $\mathbf{A}^{\mathrm{v}}$ and $\mathbf{B}^{\mathrm{v}}$ are the state and input matrices containing the known vehicle properties. Accordingly, the output equation, which links the measured accelerations $\mathbf{z}(t)$ to the vehicle's states and contact forces, is:

$$\mathbf{z}(t) = \mathbf{C}^{\mathrm{v}}\mathbf{x}^{\mathrm{v}}(t) + \mathbf{D}^{\mathrm{v}}\boldsymbol{\lambda}(t) \tag{5}$$

where $\mathbf{C}^{\mathrm{v}}$ and $\mathbf{D}^{\mathrm{v}}$ are the output and feedforward matrices, respectively, also estimated via the known vehicle properties. Finally, the system of equations is discretized, and a random walk model is assumed for the evolution of the unknown contact force.

Upon estimation of the railroad vehicle states and inputs, the bridge modal properties are inferred following a SID approach. To this end, the bridge EOM is written in a state-space form:

$$\dot{\mathbf{x}}^{\mathrm{b}}(t) = \mathbf{A}^{\mathrm{b}}\mathbf{x}^{\mathrm{b}}(t) + \mathbf{B}^{\mathrm{b}}\mathbf{W}^{\mathrm{b}}(x)\boldsymbol{\lambda}(t) \tag{6}$$

where $\mathbf{x}^{\mathrm{b}}(t)$ is the state vector of the bridge, and $\mathbf{A}^{\mathrm{b}}$ and $\mathbf{B}^{\mathrm{b}}$ are the state and input matrices. In eq. (6), $\boldsymbol{\lambda}(t)$ is known as it was estimated from Eqs. eqs. (4) and (5) via the DKF. To apply a SID algorithm on eq. (6), an output equation is also necessary, which is formulated based on the EOM of the vehicle eq. (1). However, to bring the vehicle EOM into a proper output equation that also includes the bridge response, the expression of the contact force (eq. (3)) is substituted into eq. (3). In this case, the bridge response $\mathbf{u}^{\mathrm{b}}(t)$ and roughness profile $\mathbf{r}\left(x\right)$ appear in the output equation. The roughness profile cannot be directly identified and is, thus, eliminated from the output equation by assuming measurements by two consecutive vehicles crossing the same roughness profile, as demonstrated in fig. 1 and considering the residual response of the two vehicles. The residual response is:

$$\Delta\mathbf{s}\left(t\right) = \mathbf{W}^{\mathrm{v}}k\left(\mathbf{W}^{\mathrm{b}}\left(x\right)\right)^{\mathrm{T}}\left(\mathbf{u}_2^{\mathrm{b}}\left(t\right) - \mathbf{u}_1^{\mathrm{b}}\left(t\right)\right) \tag{7}$$

where $\Delta\mathbf{s}\left(t\right)$ contains the states of the first and second vehicles that were estimated via the DKF. Note that in eq. (7) no roughness profile appears. Eventually, the state-space EOM of the bridge (eq. (6)) is also written as the difference of the

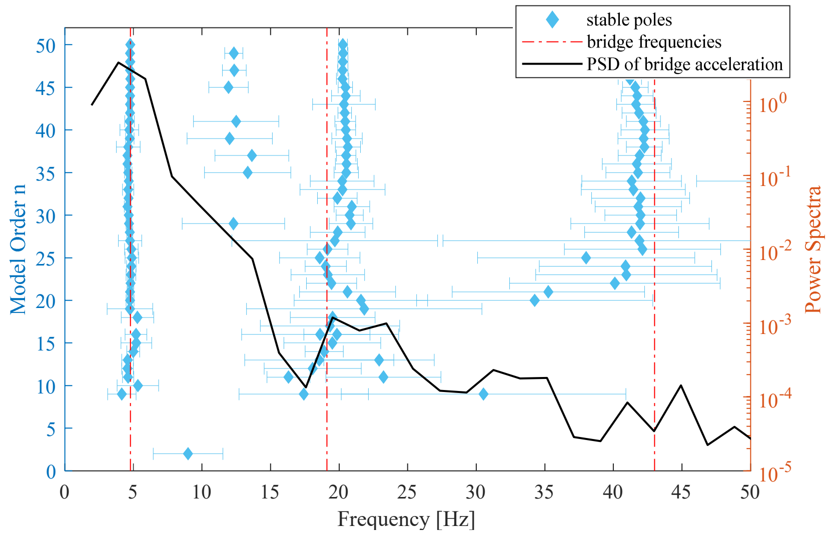

Fig. 2 Example of stabilization diagram from the residual response of two single-degree-of-freedom vehicles traversing a simply supported bridge with a simultaneous estimation of the uncertainty of the estimated bridge frequencies.

bridge response when the two vehicles cross the bridge as:

$$\dot{\mathbf{x}}_2^{\mathrm{b}}(t) - \dot{\mathbf{x}}_1^{\mathrm{b}}(t) = \mathbf{A}^{\mathrm{b}} \left(\mathbf{x}_2^{\mathrm{b}}(t) - \mathbf{x}_1^{\mathrm{b}}(t)\right) + \mathbf{B}^{\mathrm{b}}\mathbf{W}^{\mathrm{b}}(x) \left(\boldsymbol{\lambda}_2(t) - \boldsymbol{\lambda}_1(t)\right) \tag{8}$$

and the output equation (eq. (7)) in state-space assumes the form:

$$\Delta\mathbf{s}\,(t) = \mathbf{H}(x) \left(\mathbf{x}_2^{\mathrm{b}}(t) - \mathbf{x}_1^{\mathrm{b}}(t)\right) \tag{9}$$

where $\mathbf{H}(x)$ includes quantities associated with $\mathbf{W}^{\mathrm{b}}(x)$ which is here considered to be known, assuming a sinusoidal shape mode for the bridge.

The solution of eqs. (8) and (9) follows an input-output covariance-driven SID algorithm that uses an orthogonal projection of the future output Hankel matrix onto the future input Hankel matrix, which is then right-multiplied with the transpose of the part output data Hankel matrix. The Hankel matrix is then decomposed using Singular Value Decomposition (SVD) to extract the systems's modal parameters. To estimate the uncertaintty of the estimated modal parameters, this study adopts the Delta method, as demonstrated by Greś et al. [12]. Specifically, it propagates the covariance of the data onto the modal parameter estimates and, hence, estimates the covariance of the estimated modal parameters. Eventually, the uncertainty of the modal properties of the system, e.g., modal frequencies, is estimated for the different orders assumed for the subspace identification, as demonstrated in fig. 2.

Conclusion

This study proposes a methodology to perform on-board monitoring of rail bridges based on acceleration data of passing railroad vehicles with a simultaneous estimation of the uncertainty of the identified modal parameters. By employing a DKF for state-parameter estimation we only require measured accelerations from the recording vehicle, overcoming the limiting assumption of known vehicle displacements, velocities, and contact forces for the performance of the identification task, which is a common assumption in previous studies. Subsequently, by incorporating an uncertainly quantification scheme within the modal identification task we enhance the reliability of subspace identification for modal parameter estimation. This methodology offers a cost- and time-efficient solution for indirect monitoring of rail infrastructure and paves the way for scalable online monitoring of rail assets.

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