Chapter 16

A Surrogate Modelling Approach Based on Anti-Resonance Properties for FRF Prediction of Uncertain Dynamical Systems

Enora Denimal Goy

Abstract Quantifying uncertainties of dynamical responses is crucial for the design of robust mechanical structures. Computing the Frequency Response Function (FRF) is a classical tool in this context. So far, basic approaches to propagate uncertainties have led to poor prediction accuracy due to convergence issues around the peaks. Some previous works proposed numerical strategies to deal with this limitations for small systems and in specific cases. The present work proposed a new approach based on resonance and antiresonance properties to split the FRF in different sections, a surrogate can then be constructed on each section leading to better performances than classical strategies. The methodology is illustrated on a 2 dof academic system.

Keywords Frequency Response Function · Uncertainty propagation · Antiresonance · Gaussian process

Introduction

Mechanical structures are subjected to numerous uncertainties impacting their dynamic response. To reach reliable prediction levels, those uncertainties must be considered when predicting the dynamical response. These uncertain parameters are usually described with a probability law and the dynamic response becomes stochastic. However, characterising this stochastic response can represent a substantial numerical cost, especially when the number of uncertainties is high or when one wants to predict the structure full frequency response function (FRF). To reduce this cost, a well-known strategy consists of constructing a surrogate model of the FRF. Straight approaches based on constructing one surrogate for the full FRF directly proved to experience convergence issues at the resonance peaks [1]. Some recent strategies proposed to cut the FRF intervals based on numerical properties to construct one surrogate per interval [2]. However, those approaches fail to deal with complex mechanical structures with more complex dynamical behaviour.

The current work proposes to exploit the resonance and antiresonance properties of mechanical structures to cut the FRF in different intervals and construct a surrogate model of the FRF on each interval. Gaussian Processes are used for the construction of the surrogate models. By splitting the FRF using physical properties, convergence issues at resonance peaks are avoid and realistic mechanical structures can be considered. The method is illustrated on an academic example. As a result, no convergence issues are observed and the approach gives promising results for future application on 3D finite element models.

Proposed Methodology

General objective

A linear dynamical system is described by the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{1}$$

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where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, the displacement vector, the dot denotes the derivative over time and $\mathbf{F}$ the excitation vector of the form $\mathbf{F}_e \cos \omega t$. For such system, it is classical to study the Frequency Response Function (FRF), i.e. the response of the system when ω varies on an interval. In uncertainty quantification, many studies have been devoted to the prediction and the amplitude response. The objective of the work is to propose a new and efficient strategy for this. The general workflow is explained first, and the different steps are then detailed.

General workflow

In the following, the uncertain parameters are denoted $\mathbf{x}$, ω_{AR} denotes an antiresonance pulsation and ω_R a resonance pulsation. The surrogate of a quantity of interest z is referred with a: $\hat{z}$.

- Step 1: for each dof i of the system, a surrogate model of each resonance and antiresonance is constructed, i.e. the different $\hat{\omega}_{AR}^{(i)}$ and $\hat{\omega}_R^{(i)}$ are constructed. The creation of the training set is done based on the solving of a modified eigenvalue analysis. For each dof, it defines zones between consecutive resonance and/or antiresonance.
- Step 2: a training set of the FRF is generated. It is composed of N input points $((\omega_k, \mathbf{x}_k))_{k\in[1,N]}$ and their evaluation $\mathbf{y}_k$. For each dof, the training set points are grouped in the correct zone defined in Step 1.
- Step 3: for each zone and each dof, a surrogate model of the FRF is constructed.

Antiresonance computation

The antiresonance at dof i can be obtained by resolving a modified eigenvalue problem where the i*th* column of the sitffness matrix is replaced by the force vector, and where the i*th* column of the mass matrix is replaced by a vector of 0 [3]. This strategy is employed in Step 1 to generate the training set of the antiresonance surrogates. For the resonance, a classical eigenvalue analysis

Gaussian process regression

In the present study, Gaussian processes (GP) are employed for the creation of the surrogates. It is assumed that the data are generated by a process $Y_i = f(X_i) + \epsilon_i$ with $i \sim \mathcal{N}(0, \sigma^2)$. The the current context, no noise is considered as the data comes from numerical simulations. From a training set of N points (X_i, Y_i), a GP is constructed based on the covariance function κ (which depends on hyperparameters θ) and the mean function m. It defines the matrix $K_{ij} = \kappa(\theta; X_i, X_j)$ and the vector $\mu = (f(X_1), \ldots, X_N)$. The hyperparameters θ are optimised to minimise the likelihood. Then, the prediction at a new points X_* is $\mu_* = K_*^T K^{-1} f$ with K_* the covariance matrix between X_* and the training set. The GPs are constructed with the GPflow python package [4].

Application on a 2-D model

Model presentation

The model studied is a 2dof system represented in Figure 1. The parameters are $m = 1$ kg, $k = 15000$ N/kg and $c = 1$ Ns/m. In the considered case, only the stiffness is uncertain and can vary up to 10%. The frequency range of interest for the FRF prediction is $[60, 220]$ Hz.

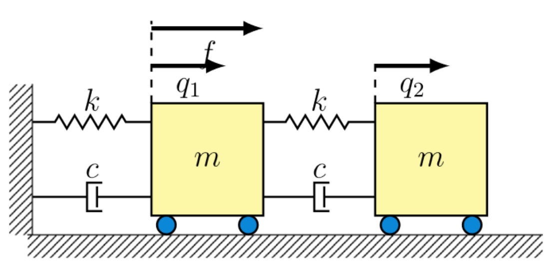
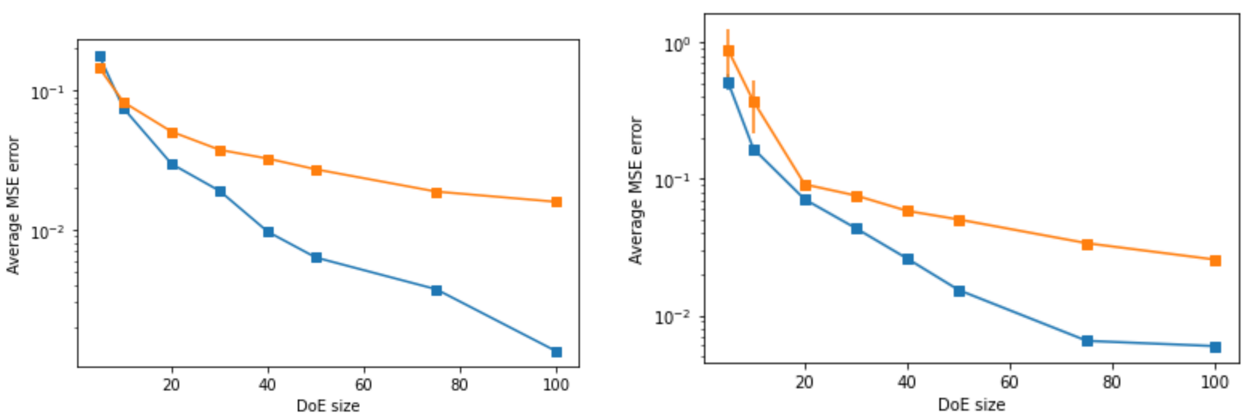


Fig. 1 Model under study

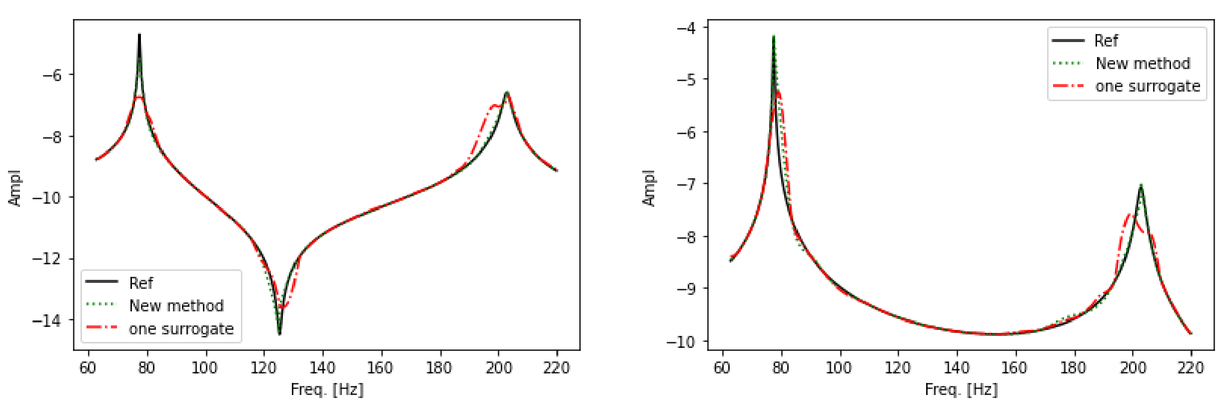
Results

The strategy proposed previously is compared to a strategy where all points from the training set are used to create a unique surrogate of the FRF. In both cases, the data are centred before constructing the surrogate. A Matérn 5/2 kernel is taken for GPs related to the proposed strategy, and a Matérn 1/2 kernel is taken for the "unique surrogate" strategy. Concerning the present approach, for the first dof, there is one antiresonance between the two resonances, and so there are 4 zones (and 4 surrogates for the amplitude). For the second dof, there is no antiresonance over the frequency range of interest, so there are only 3 zones (and 3 surrogates for the amplitude). The quality of the surrogate is assessed by computing the Mean Squared Error (MSE) over a validation set of 100 points per zone (400 points for dof1 and 300 points for dof2). The error is computed for training set of different size, generated by LHS. For each size, 10 different training sets are generated and the average error over the 10 training sets is computed. The evolution of the error is displayed in Figure 2. The blue line corresponds to the error with the proposed strategy, whereas the orange line corresponds to the error with a "unique surrogate" strategy. The error level is one order of magnitude lower with the proposed strategy compared to the unique surrogate strategy. It shows clearly the interest in cutting the FRF into different parts between resonance and antiresonance. As an illustration, one predicted FRF is displayed in Figure 3, where the black line is the reference, the red line the results from the unique surrogate strategy and the green line from the proposed strategy. With the proposed strategy, the peaks of the FRF are well predicted, whereas oscillations are observed around the peaks with the unique surrogate strategies, leading to poor prediction quality.



(a) Dof 1 (b) Dof 2

Fig. 2 Average error on the amplitude prediction for the two dofs - proposed algorithm (orange) and classic method (blue)



(a) Dof 1 (b) Dof 2

Fig. 3 Prediction of one FRF - reference (black), proposed approach (green) and classic method (red)

Conclusion

The present study has for objective to propagate uncertainty for FRF prediction of linear mechanical system. A new strategy is proposed by cutting the training set in subsets based on physical properties, namely resonance and antiresonance, and creating a surrogate for each of this subset. The computation of the antiresonance is done with a modified eigenvalue analysis. The strategy is compared to the case where a unique surrogate model is constructed. This is applied on an academic 2dof system and results proved the efficiency of the approach as lower error level and quicker convergence are observed. The results are promising for future extension to 3D structures.

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