Chapter 3
Likelihood-Free Bayesian Inference for Localized Corrosion in Miter Gates

Guofeng Qian, Jice Zeng, Zhen Hu, and Michael D. Todd

Abstract Multi-scale corrosion simulation models have the capability of predicting the evolution of corrosion damage on large civil structures like miter gates under different environmental and operational conditions. However, updating such models using Bayesian methods is challenging because they often involve heterogeneous uncertainty sources and/or complex model architectures, making the likelihood function analytically intractable. This paper proposes a likelihood-free probabilistic model updating method for a complex multi-scale corrosion model based on a conditional invertible neural network (cINN). The multi-scale physics-based model consists of a local phase-field corrosion model, a global structural analysis model, and a statistical distribution model. Due to the lack of observational data, the cINN is first trained with simulation data. Once trained, the model can update multiple key parameters using image observations. Specifically, pit dimensions are identified from these observations using a convolutional neural network (CNN). Features extracted from the CNN are then used to infer the posterior distribution with the pre-trained cINN, eliminating the need to evaluate the likelihood function. The inference results show improved accuracy and significantly lower computational costs compared to the existing likelihood-free model updating methods. The proposed method is demonstrated on a miter gate example to showcase its effectiveness.

Keywords Likelihood-Free Bayesian Inference · Condictional Invertible Neural Network· Structural Health Monitoring

Introduction

Localized corrosion is one of the most dangerous forms of deterioration in large metal civil structures, not only because it can initiate stress corrosion cracking (SCC) [1], but also due to its difficulty to be detected. Vision-based damage detection and diagnostics [2, 3] have been extensively studied in the structural health monitoring (SHM) literature. This approach has the advantage of efficiently detecting local damage at a low cost, making it well-suited for localized corrosion detection.

Traditional Bayesian inference requires the evaluation of an explicit likelihood function to extract information from available observations. However, in physics-based simulations, such likelihood functions are often intractable due to the complexity of coupled partial differential equations [4]. In these cases, likelihood-free or simulation-based Bayesian inference approaches are necessary. However, these methods typically require large numbers of simulations, making the inference process computationally expensive. Flow-based amortized Bayesian inference [5, 6] offers a more efficient alternative, providing fast inference compared to other simulation-based approaches. The goal of this paper is to perform amortized Bayesian model updating for complex multi-scale corrosion simulation. In what follows, we first introduce the forward simulation model of the localized corrosion damage. Then, we further introduce the inference process with the conditional invertible neural network. The inference results are compared with a classical approximate Bayesian computation method followed by the conclusion.

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Forward Model for Multiscale Corrosion Simulation

The overall framework of this study is shown in Figure 1. There are three parts in the forward multi-scale corrosion simulation model. The first part is the macroscale finite element (FE) model of the Greenup miter gate structure (as shown in the upper left of Fig. 1). This model was previously validated with strain gauge measurements [7]. This high-fidelity FE model consists of about 65,000 elements. Most of these elements are 3D shell elements since the most common component in the gate is the thin steel plate. The diagonals and anchor bars are simulated with beam-column elements. The 4-node, reduced integration linear quadrilateral elements are primarily used with 3-node full-integration linear triangular elements for complex geometry. This model is responsible for simulating the strain response $\mathbf{s}(\mathbf{d})$ of the gate at any spatial location $\mathbf{d}$ for any given operating load, hydrostatic pressure, and gap length due to the loss of contact between the quoin block and the wall [8]. The local strain response $\mathbf{s}$ provides the strain load to the local corrosion evolution model, as illustrated in the upper right of Fig. 1. This localized corrosion evolution model is constructed using the phase-field method with four partial differential equations together, i.e., Butler-Volmer equation for kinetics, diffusion equation, Poisson's equation, and mechanical equilibrium [9]. The model is able to simulation the corrosion under mechanical strain $\mathbf{s}(\mathbf{d})$ and environmental parameter $\theta(\mathbf{d})$ which describes the influence from the local environment at $\mathbf{d}$ on the corrosion reaction.

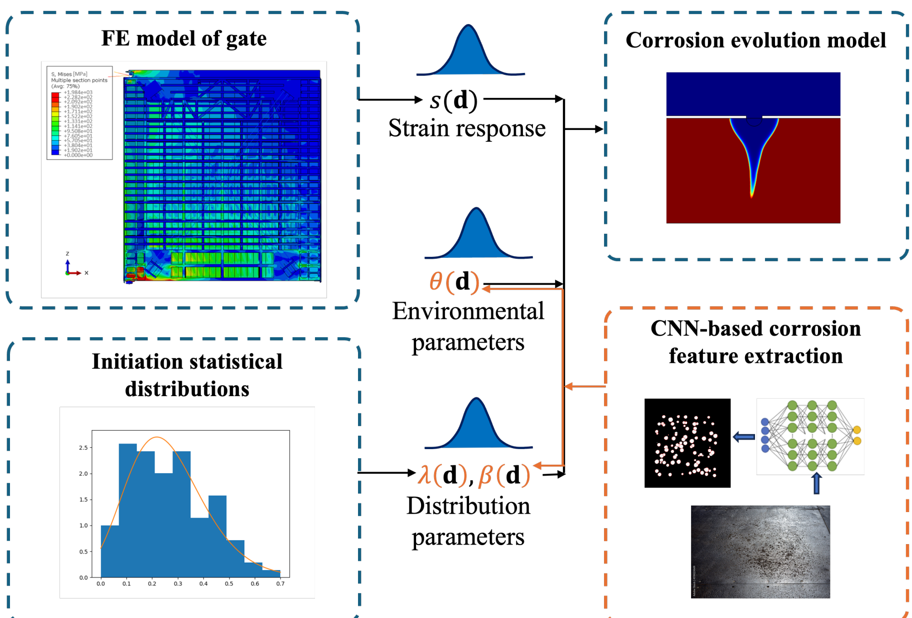

Fig. 1 Overall framework for multi-scale corrosion simulation

Corrosion initiation must be simulated to inform the pit evolution process. The initiation process is influenced by different factors including temperature, chemical composition, material, and potential [10] at local position $\mathbf{d}$. However, detailed mechanism of corrosion initiation is still under active research [11], making it difficult to simulate it using physics-based simulation models. Therefore, we simulate the initiation process statistically as shown in the lower left in Fig. 1. A Weibull distribution is employed for the modeling of the corrosion initiation time. This distribution is governed by two parameters, namely $\lambda(\mathbf{d}), \beta(\mathbf{d})$. Combined with the environmental parameter $\theta(\mathbf{d})$, we simulate localized corrosion growth using a high-fidelity corrosion model within a multiscale approach applied to the large structure.

Likelihood-Free Bayesian Inference with Invertible Neural Network

There are three parts in the forward model as mentioned above. The strain response $\mathbf{s}$ is measurable with strain gauges. Even it is not practical to measure every location on the gate, the gate model can be validated and updated with a few strain measurements at the global model level. However, some of the parameter values are difficult to measure. The environmental parameter θ is almost impossible to measure locally as it is influenced by the temperature, water composition, material, and other factors. The distribution parameters λ, β are not measurable directly either. Therefore, we implemented likelihood-free approach to infer these immeasurable parameters, shown in orange color in Fig. 1, including λ, β and θ.

We implemented amortized Bayesian inference with conditional invertible neural network (cINN) based on normalizing flow. Due to its invertibility property, we can leverage the relationship from model parameters to observable results from simulations. By inverting this relation, we can perform the inference from observable results back to model parameters in an amortized way [5]. The flow of amortized inference is shown in Fig. 2.

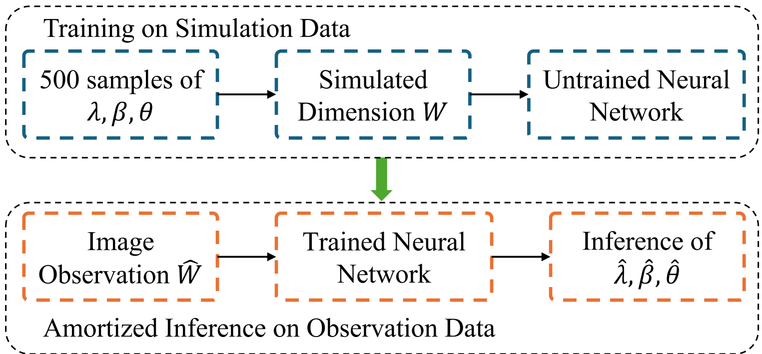

Fig. 2 Amortized inference approach

The pit opening width distribution, W, is selected as the observation from synthetic images since actual gate data were not available. A CNN-based feature extraction technique has been developed [12].

As shown in Figure 2 above, the proposed cINN is trained on the parameters λ, β, θ and the observation W. After training with five hundred samples of input parameters and simulation results, cINN is able to perform the amortized Bayesian inference for $\hat{\lambda}, \hat{\beta}, \hat{\theta}$ given one real observation $\hat{W}$.

Comparison of inference results

The classic approximate Bayesian computation (ABC) is also performed as a reference with the same number of simulations used to train cINN. The simulation results from prior samples are compared with the observation to determine whether the sample is accepted or rejected. The inference time for each observation will take about 4 hours. While the cINN learns the relationship between different parameters and according to results. Although the training stage takes 5 hours, the inference can be completed within seconds for one observation. This allows fast inference for large amounts of observation which is not possible with traditional likelihood-free inference methods like ABC.

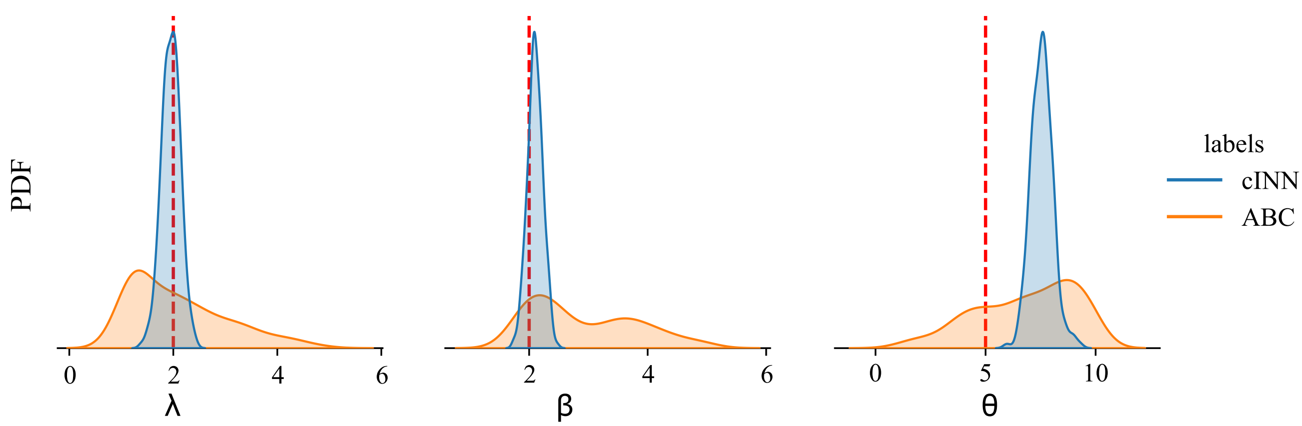

Fig. 3 Posterior comparison between cINN and ABC

The comparison of inference results is shown in Fig. 3. It is obvious that the cINN approach has more accurate and confident posterior for λ and β than ABC approach. However, for environmental parameter θ, both approaches has some bias for larger θ in the inference results. This bias probably comes from the observation bias as small pits might not be detected. As a result of detecting pits later, simulation-based approaches will infer a faster growth speed to match the big pit width [13]. In conclusion, cINN-basd inference method has more accurate inference results with less variance. It reduces the inference time significantly compared to other approaches. It might be more sensitive to observation bias as well.

Conclusions

The application of amortized Bayesian inference with cINN for model updating for physics-based multiscale corrosion simulations offers several notable advantages, which are summarized as below

- Efficiency: The model can be inferred and updated within seconds, making it highly suitable for time-sensitive applications.
- Accuracy: cINN consistently outperforms other likelihood-free methods, such as ABC, by significantly reducing the variance in inference.
- Potential Limitation: While cINN provides reliable results, it may sometimes exhibit overconfidence in its inferences, particularly in cases where observation bias is present.

Overall, cINN proves to be an effective tool for complex corrosion modeling that supports an SHM process for miter gates. With further refinement, pit observation bias will be considered more carefully.

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