



Chapter 6

Uncertainty Quantification of Fatigue Damage from Random Vibration and Sine-sweep

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Abstract A damage-based approach is often used for different types of fatigue failure to quantify the relative severity of vibration specifications. Efficient numerical codes based on structural response allow rapid evaluation of the effect of both input and model uncertainties on ‘damage’. Damage index is computed using a fatigue damage model (S-N curve) based on stress cycles. Because these approaches are approximate, and are the only means to provide engineered guidance, quantification of uncertainties is essential to their use.

Comparing the severity of various vibration test specifications is complicated since the Auto-spectral Density (ASD) can be higher or lower at different frequencies. Test durations are also different. An added complexity is that sine-sweep test specifications (from directly coupled engine vibration) can be prescribed to be concurrent with random vibration ASDs (structure borne vibration). A system or component may be subject to vibration during transport and handling vs. during launch or flight. It then becomes important to quantify the consequences of various test specifications to prioritize testing, or to ensure tests done to previous specifications can be used to ascertain safety when requirements and usage has changed later in the development phase.

The paper presents the technical basis for comparing fatigue damage from various types of vibration test specifications. Codes developed and associated theoretical approximations will be discussed, along with associated uncertainty quantifications.

Keywords Fatigue damage · Vibration · Specification · Modal

Background

Amplification at resonance is $Q = 1/(2\xi)$, where ξ is the viscous damping factor. The half-power (3db) bandwidth (Δf) captures the frequency band around the resonant frequency f_n within which most of the energy associated with resonance is concentrated: $\Delta f = 2\xi f_n = f_n/Q$. The basic premise of equivalencing sine-tone into random-vibe and vice-versa is to look at each frequency, consider the energy within the half power (3db) bandwidth (Δf), and evaluate the equivalent imparted damage. Fackler [1] provides detailed discussion on the technical aspects relevant to such equivalencing and shows that the peak amplitude G_s of the sine-tone is proportional to $\sqrt{(\pi Q G_{xx} f_n / 2)}$, where G_{xx} is the Auto-spectral Density (ASD) of the equivalent random-vibe. Details of the equivalencing process accounting for the different duration for which the sine-sweep dwells in each octave band have been provided by Maji [2]. It uses R_f to account for the Raleigh distribution of peaks in a narrow-band random to get

$$G_s = R_f \sqrt{(\pi G_{xx} f_n / (2Q))} \quad (1)$$

Sandia National Laboratories is a multimission laboratory managed and operated by National Technology & Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International Inc., for the U.S. Department of Energy's National Nuclear Security Administration under contract DE-NA0003525.

This paper describes objective technical results and analysis. Any subjective views or opinions that might be expressed in the paper do not necessarily represent the views of the U.S. Department of Energy or the United States Government.

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In the research reported here, a sequence of numerical simulations with progressively greater complexity were undertaken to quantify the uncertainty associated with each approximation:

- Using a Single Narrow-band Random Input.
- Influence of Adjacent Bands on Conservatism.
- Convergence of damage with increasing duration.
- Effect of sampling frequency for pure sinusoid.
- Full-Frequency range, one bandwidth at a time ($R_f = 1.0$, unconservative).
- Full-Frequency range one bandwidth at a time ($R_f = 1.4$, conservative).
- Full-Frequency range with one bandwidth at a time (most accurate, band-specific R_f).
- All bandwidths simultaneously, with realistic simulated transfer functions.

Numerical Simulations

Table 1 shows the various simulations with increasing complexity that are reported in this document.

Table 1 Numerical Simulations with Increasing Complexity

Simulation Type	Simulation #	Input Frequency Bands	Response Modes	Analysis Type	Objective of Simulation
Numerical Simulations with Increasing Complexity	Simulation 1	950-1050Hz	1000Hz	single band	Narrow-band accuracy
	Simulation 2	950-1050Hz, & broader	1000Hz & 1100Hz	single band	Effect of adjacent bands and off-center resonance
	Simulation 3	950-1050Hz	1000Hz	single band	Statistical Variation & Convergence
	Simulation 4	1000-4000Hz	center of each band	1 band at a time	Accuracy with $R_f = 1.0$
	Simulation 5	1000-4000Hz	center of each band	1 band at a time	Accuracy with $R_f = 1.4$
	Simulation 6	1000-4000Hz	center of each band	1 band at a time	Accuracy with band-specific R_f
	Simulation 7	1000-4000Hz	Transfer Function	concurrent	Realistic input, response and full range simulation

Simulation #1: Using a Single Narrow-band Random Input

The objective of this simulation was to examine and validate that the response due to narrow-band input (acceleration, relative acceleration, and displacement) all have a Rayleigh distribution, which forms the fundamental basis for the factor R_f . In this validation process, a narrow-band random-vibration (950–1050 Hz with ASD = 0.01 g^2/Hz) was convolved with the transfer function of a 1 kHz single degree of freedom (SDOF) system. Figure 1 shows the steps by which the damage due to a narrow-band ASD can be determined for a 1 kHz SDOF response. Damage ($\propto \Delta\sigma^b$) is computed based on Miner's rule and the number of fatigue cycles obtained using MATLABTM 'rainflow'. Figure 2 shows the random-vibe input convolved with the transfer function at 1 kHz and the resulting response (approximate amplification of 10 using damping $\xi = 0.05$). Displacement was obtained by numerical integration of the acceleration followed by high-pass filtering to eliminate the low-frequency (< 10 Hz) drift. Figure 3 shows that the input and response accelerations and response displacement all have Rayleigh distributions which relates to the parameter R_f in the conversion process [2]. The same is true for relative

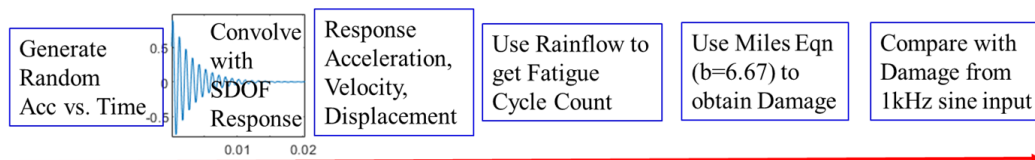


Fig. 1 Schematic of simulation for a single narrow-band input.

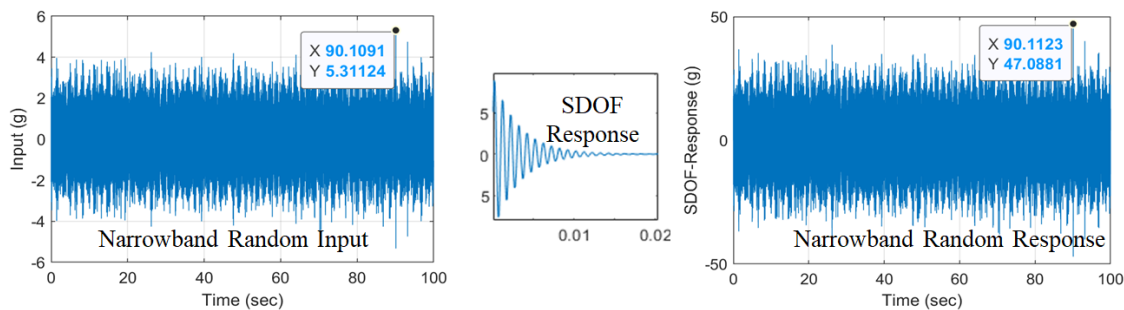


Fig. 2 Narrow-band random input and response via convolution.

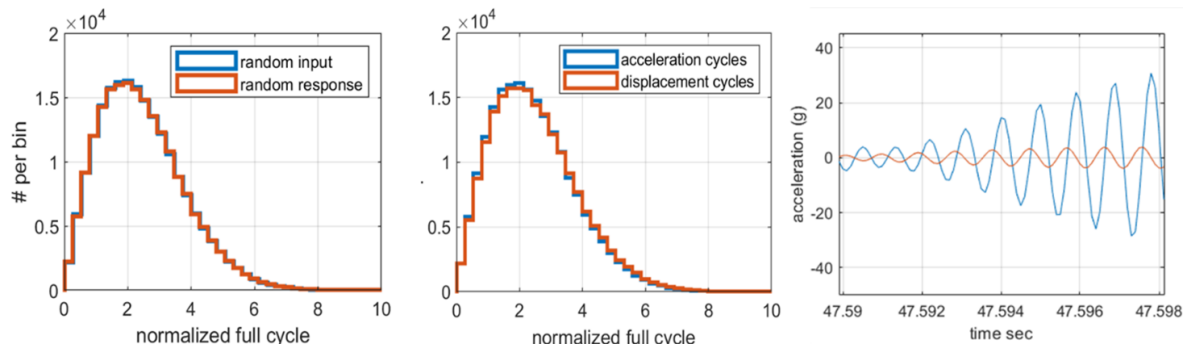


Fig. 3 Normalized distribution of peaks in narrow-band input and response.

acceleration (response minus input) since the input and response acceleration have a phase offset of π (See Figure 3, right), so the peak acceleration remains the same as that of the response acceleration.

Simulation #2 Influence of Adjacent Bands on Conservatism

The objectives of this simulation were to validate the equivalencing equation for a single narrow-band and to investigate the effect of inputs from adjacent bands. The process of equivalencing is based on converting the sine-tone in each octave-spaced band (3dB bandwidth) into an equivalent ASD. This is achieved by setting equal the damage due to the response of a SDOF structure due to these two inputs (sine-tone centered in that band and the equivalent random-vibe ASD). Dividing the overall frequency range of interest into octave-bands is necessary to account for the fact that the underlying random-vibration ASD and the sinusoid are varying with frequency and time of test.

It is also necessary to recognize that a SDOF structure also responds to inputs outside of the narrow-band. It will be shown in the results below that a SDOF that has a natural frequency at the edge of a frequency bin will likely show greater conservatism from the equivalent ASD. These two issues were therefore explored with four sets of simulations. Figure 4 helps understand the adjacent bands and the SDOF resonances.

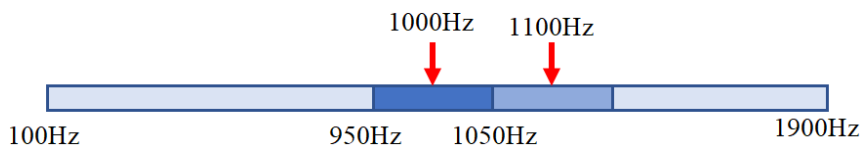


Fig. 4 Adjacent bands and SDOF resonant frequencies considered in simulation (arrows).

A 1kHz sinusoid of peak amplitude 1.772g and a narrow-band ASD ($0.01g^2/Hz$) between 950Hz and 1050Hz were compared first. A sampling rate of 10000Hz was used and the length of simulation was 100 seconds. These two inputs are equivalent per [Eq.1] and should therefore lead to the same damage to a SDOF with natural frequency $f_n = 1000Hz$. This is shown to be true in rows 1 and 2 in Table 2; the damages from the two inputs are essentially the same (difference by 0.03dB only).

Next, the natural frequency of the SDOF was changed to 1100Hz (outside of the narrow-band and centered approximately on the next adjacent narrow-band). Note that since the SDOF is outside of the input band-width, response is very low, and

the consequent damages are lower by -5.2dB and -6.9dB respectively (rows 3 and 4 in Table 2). It is worth noting that the SDOF at 1100Hz responds more to the sine-equivalent ASD than the sine-tone input at 1kHz and the consequent damage is 3.7 times higher ($1.7e^{15}$ vs. $4.6e^{14}$). Even though these values are much lower than the damage due to the SDOF centered at 1kHz ($9.3e^{16}$), due to the nonlinear nature of damage (exponent $b = 6.67$), small differences in response amplitude result in significant differences in damage. As a simple example, if one is to think of the responses of the 1000Hz and 1100Hz SDOF as a and b , where $a \gg b$, then the damage due to both of them together $\propto (a+b)^{6.67}$ can be equal to $(1+0.1)^{6.67} = 1.89$ or $(1+0.05)^{6.67} = 1.38$, or 0.5dB lower even though $b \ll a$.

Therefore, damage due to concurrent inputs at 1000Hz and 1100Hz to a SDOF at 950Hz were examined and the result shown in rows 5 and 6 of Table 2 show that the resulting total damage is now higher (than row 1 for the SDOF at 1000Hz only) for both the sine-equivalent ASD and the sinusoid input (2.52dB and 1.82dB, respectively). More importantly, damage from the sine-equivalent ASD is 0.7dB higher than that from the sinusoid.

The last simulation extended the bandwidth of the ASD over a wider band (100-1900Hz), while keeping the ASD magnitude the same ($0.01g^2/Hz$). Note that this results in the damage increasing by $10\times$ to $9.3e^{17}$. This is expected according to theory, since the response of a SDOF to a wide-band ASD is $\sqrt{2}\times$ higher than that due to a narrow-band ASD, consequently the damage is 3dB higher. In real tests, inputs will exist in all of the various narrow-bands, and these will affect the response at each resonant frequency of the structure of interest. The resulting increase in damage is disproportionately higher for the equivalent random-vibe input vs. the sinusoid input as per the results in the previous paragraphs.

Table 2 Comparison of Damage from Wide vs. Narrow Band and Off-centered SDOF Frequency.

Type of Input	Frequency (Hz)	ASD g^2/Hz	Sinetone-Peak (g)	Inputs (kHz)	Damage	Difference (dB)
Narrow-Band ASD, centered SDOF	950-1050	0.01		1.0	$9.30E+16$	0
Sinusoid only, centered SDOF	$1.00E+03$		1.77	1.0	$9.50E+16$	0.03
Narrow-Band ASD, centered SDOF	950-1050	0.01		1.1	$1.70E+15$	-5.2
Sinusoid only, centered SDOF	$1.00E+03$		1.77	1.1	$4.60E+14$	-6.9
Narrow-Band ASD, centered SDOF	950-1050	0.01		1.0&1.1	$6.42E+17$	2.52
Sinusoid only, 2 SDOFs	$1.00E+03$		1.77	1.0&1.1	$3.76E+17$	1.82
Wide-Band ASD, centered SDOF	100-1900	0.01		1.1	$9.30E+17$	3.0

Simulation #3 Convergence of Damage with Increasing Duration

In order to examine the convergence of ‘damage’ from simulations of different duration, the previous simulation was repeated 10 times each for durations of 10sec, 100sec and 1000sec to examine the statistical variation. The damage values from the 10 and 1000 second duration were multiplied and divided by 10 respectively to make them comparable to those of the 100sec simulations. Figure 5 shows the progressive convergence with increased length of simulation. Comparing the highest and lowest value for each duration in Figure 5 shows that the variation for 10sec, 100sec and 1000sec simulations were 0.75dB, 0.18dB and 0.06dB, respectively. This result can provide guidance on the length of simulation necessary to achieve a certain

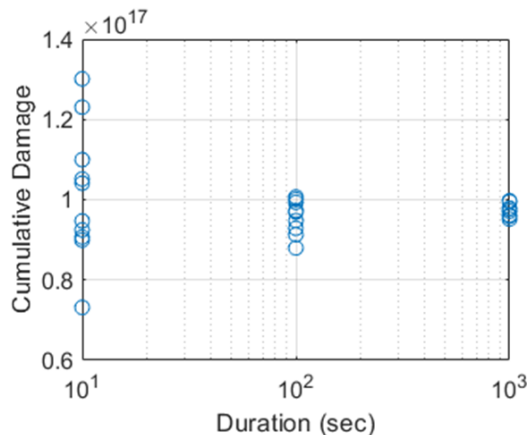


Fig. 5 Convergence of damage with increasing duration.

level of accuracy; simulation longer than 100sec would not be worth the added time and associated cost if an accuracy of 0.18dB is acceptable.

A 100sec simulation of the equivalent sine-tone (peak = 1.772g) provided a damage of $9.4e16$. It is important to note that even at a sampling rate of 10000Hz ($10 \times f_n$) the peak of the sine-tone is clipped (Figure 6); peak is 1.69 vs. actual value of 1.77. The clipping would vary depending on the phase of the sine-tone and cause a small decrease in damage $\propto (amplitude)^b$.

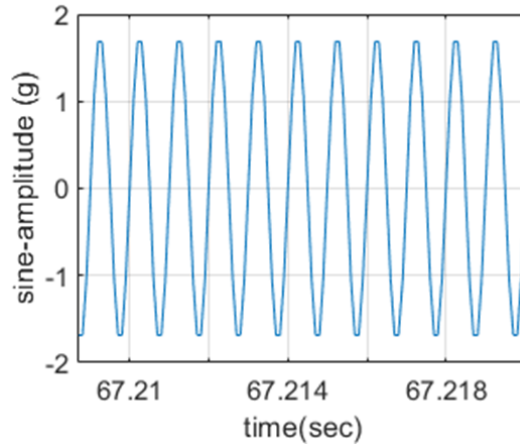


Fig. 6 Effect of sampling frequency for pure sinusoid.

Simulation #4 Full-Frequency Range One Bandwidth at a Time ($R_f = 1.0$, conservative)

The next set of simulations involved using inputs over the entire frequency range of interest (1-4kHz), but for only one bandwidth at a time with the SDOF’s natural frequency f_n at the center of the respective bands. Table 3 shows the input sine-sweep and the random-vibe ASD test specifications. The Sine-tone specification processed one octave band at a time to examine the accuracy of the process for each octave band. Note that in this simulation, the duration of dwell in each band and actual modal response of the structure were not considered, it simply examined the validity of [Eq.1] in providing a conservative estimate of equivalent ASD for the sine-tone in each band before a full-up simulation (simultaneous with all frequency bands) would be done.

Table 3 Inputs for Broadband Simulation.

Freq (Hz)	ASD (g^2/Hz)	Freq (Hz)	Sinusoid Peak (g)
1000	0.0025	1000	1.5
4000	0.0018	1600	2
		2100	1
		2500	1
		3200	1.9
		4000	2.2

Table 4 shows the octave bands and columns along with the octave-band input sine-tone and the random-vibe ASD test specifications. For each octave band the equivalent ASD for the sine-tone in that octave band was calculated using [Eq.1]. The conservative limit value of $R_f = 1.0$ was first used followed by more accurate values of R_f in subsequent simulations. A damage coefficient $b = 6.67$ was used throughout since it is more commonly used and more sensitive than $b = 4.0$. The simulation involved the following steps:

Acceleration vs. time data was generated for each octave band using a sampling rate of $10 \times$ the band’s mid-frequency. For each band 10^6 instances of time were used. So for the sixth band with a mid-frequency of 1710Hz, the duration of simulation was $10^6 / (10 \times 1710) = 58.5$ seconds. This was done for each of the sine-tone peak, original ASDs, equivalent ASDs (shown in the 3rd, 4th and 5th columns in Table 4, respectively) and for the sum of ASDs (sum of the 4th and 5th columns).

These acceleration vs. time data were combined as follows:

- Signal A: Acceleration response synthesized from the sine tone was added to the acceleration response synthesized from the original ASD (total_sine_acc).

- Signal B: Acceleration response synthesized from the sine-tone equivalent ASD was added to the acceleration response synthesized from the original ASD (total_acc).
- Signal C: Acceleration response synthesized from the sum of the sine-tone equivalent ASD and the original ASD (totalASD_acc).

Each of these three accelerations vs. time data were convolved with the SDOF impulse response centered at each octave band frequency (sample rate = $10 \times$ mid-band frequency). Damage cycles from each of the three resulting accelerations (using MATLABTM 'rainflow') and the corresponding total damage was calculated using the damage coefficient $b = 6.67$.

Results in Table 4 shows that the Damage-C is very similar to Damage-B, which is expected since the only difference is summing the ASDs followed by generation of acceleration vs. time or vice versa. Comparing Damage B vs. Damage A, it can be seen that the conservative R_f provided a Damage-B that exceeded Damage-A for each frequency band by 0.9-2.1dB (last column). No damage value is provided for the first row because there was no previous mid-frequency to generate sine-tone-equivalent ASD. Previous numerical simulations showed that the accurate R_f needs to account for the sine/original ASD rms ratio. The relevant sine/original ASD rms ratios for each octave band are also presented in Table 4, ranging from 1.03 to 2.34. Note that R_f can be as high as 1.4 (vs. conservative limit of 1.0 used in this simulation), which would result in a 3dB lower estimate of the equivalent ASD. Therefore a 0.9-2.1dB overestimation is to be expected unless a more appropriate R_f is used, which was therefore examined in future simulations.

Table 4 Results of Damage Simulation for Each Octave Band (Using $R_f = 1.0$)

Mid-band Frequency (Hz)	Duration of Dwell (min)	Sine-tone Peak (g)	Original ASD (g^2/Hz)	Sine-tone Equivalent ASD	Damage-A (sinetone + Original ASD)	Damage-B (Equivalent ASD + original ASD)	Damage-C (Equivalent ASD + Original ASD)	Sine/ASD rms ratio	dB ratio of DamageB vs. DamageA
1050	1.03E+00	1.54E+00	2.47E-03	1.44E-02					
1158	1.13E+00	1.63E+00	2.41E-03	1.46E-02	8.30E+15	4.16E+16	3.85E+16	2.18	2.10
1276	1.25E+00	1.73E+00	2.36E-03	1.49E-02	1.19E+16	6.09E+16	5.64E+16	2.23	2.12
1407	1.37E+00	1.84E+00	2.31E-03	1.53E-02	1.74E+16	8.73E+16	9.11E+16	2.28	2.10
1551	1.51E+00	1.96E+00	2.25E-03	1.58E-02	2.41E+16	8.25E+16	9.04E+16	2.34	1.61
1710	1.67E+00	1.78E+00	2.20E-03	1.18E-02	1.69E+16	3.73E+16	3.92E+16	2.05	1.03
1886	1.84E+00	1.43E+00	2.15E-03	6.89E-03	7.18E+15	9.73E+15	1.09E+16	1.59	0.40
2079	2.03E+00	1.04E+00	2.10E-03	3.33E-03	2.71E+15	5.45E+15	5.81E+15	1.11	0.91
2292	2.24E+00	1.00E+00	2.05E-03	2.78E-03	2.86E+15	5.68E+15	5.72E+15	1.03	0.89
2527	2.47E+00	1.03E+00	2.01E-03	2.70E-03	3.99E+15	1.18E+16	1.17E+16	1.03	1.41
2786	2.72E+00	1.37E+00	1.96E-03	4.27E-03	9.84E+15	4.20E+16	3.96E+16	1.31	1.89
3072	3.00E+00	1.73E+00	1.92E-03	6.24E-03	2.62E+16	1.10E+17	1.08E+17	1.60	1.86
3386	3.31E+00	1.97E+00	1.87E-03	7.30E-03	4.71E+16	1.82E+17	2.06E+17	1.75	1.76

Simulation #5 Full-Frequency Range One Bandwidth at a Time ($R_f = 1.4$, non-conservative)

The previous simulation was then repeated using $R_f = 1.4$ (upper non-conservative limit) and the results are shown in Table 5. Note that the higher R_f value (1.4 instead of 1.0) has lowered the sine-tone-equivalent ASDs by a factor of 2 (3dB) as expected. It has also resulted in a corresponding decrease in Damage-B and Damage-C relative to the same columns in Table 4. It is also possible to compare the variation between two simulations (using the same sine-tone + original ASD data) by comparing the column for Damage-A between Tables 4 and 5. Comparing Damage-B vs. Damage-A, it can now be seen that the non-conservative R_f provided a Damage-B that is consistently lower than Damage-A for each frequency band by up to -1.35dB (last column). These two simulations prove that the factor R_f appropriately captures the lower and upper limit of damage within a particular frequency band for a SDOF response centered in that frequency band. Further improvement of accuracy is explored next.

Simulation #6 Full-Frequency Range with One Bandwidth at a Time (Most Accurate, Band-specific R_f)

Previous simulations [1] had shown how R_f varies based on the ratio of the sine-tone peak vs. grms of the original ASD. From this analysis the accurate values of R_f could be tabulated (Table 6) for values of the fatigue coefficient b . These more accurate R_f values can then be used for more accurate determination of equivalent ASD from each sine-tone. Using the look-up Table 6 below and the rms ratio of the sine-tone and random-vibe specification in each octave band, the appropriate R_f for each band was determined and used in the equivalencing equation.

Table 5 Results of Damage Simulation for Each Octave Band (Using $R_f = 1.4$)

Mid-band Frequency (Hz)	Sine-tone Peak (g)	Original ASD (g^2/Hz)	Sine-tone Equivalent ASD (g^2/Hz) $b = 6.67$	Damage-A (sinetone + Original ASD)	Damage-B (Equivalent ASD + ASD)	Damage-C (Equivalent ASD - Original ASD)	dB ratio of DamageB Vs. DamageA
1050	1.54E+00	2.47E-03	7.35E-03				
1158	1.63E+00	2.41E-03	7.47E-03	8.10E+15	6.16E+15	6.30E+15	-0.36
1276	1.73E+00	2.36E-03	7.62E-03	1.18E+16	8.88E+15	1.02E+16	-0.37
1407	1.84E+00	2.31E-03	7.81E-03	1.68E+16	1.38E+16	1.29E+16	-0.17
1551	1.96E+00	2.25E-03	8.04E-03	2.46E+16	1.70E+16	1.28E+16	-0.76
1710	1.78E+00	2.20E-03	6.01E-03	1.70E+16	7.00E+15	6.60E+15	-1.16
1886	1.43E+00	2.15E-03	3.52E-03	7.54E+15	2.67E+15	2.78E+15	-1.35
2079	1.04E+00	2.10E-03	1.70E-03	2.70E+15	1.68E+15	1.74E+15	-0.62
2292	1.00E+00	2.05E-03	1.42E-03	2.89E+15	1.95E+15	1.94E+15	-0.62
2527	1.03E+00	2.01E-03	1.38E-03	3.54E+15	3.45E+15	3.45E+15	-0.18
2786	1.37E+00	1.96E-03	2.18E-03	9.53E+15	1.01E+16	1.08E+16	0.07
3072	1.73E+00	1.92E-03	3.18E-03	2.61E+16	2.23E+16	2.22E+16	-0.20
3386	1.97E+00	1.87E-03	3.72E-03	5.11E+16	3.57E+16	3.51E+16	-0.47

Table 6 Accurate Value of R_f for Various Ratio of rms Values

Ratio	0	0.2	0.4	0.6	1	1.4	1.8	2.2	2.8	3.4	4	5
$R_{f-b} = 6.67$	1	1.03	1.07	1.11	1.18	1.24	1.28	1.31	1.35	1.37	1.39	1.41
$R_{f-b} = 4.0$	1	1.01	1.03	1.05	1.09	1.12	1.14	1.15	1.17	1.18	1.19	1.20

Results shown in Table 7 (last column) demonstrate that the band-specific R_f significantly reduces the differences to a maximum of 0.27dB, and significantly lower in most cases. Note that the variation in Damage-A in each of Table 5, 6 and 7 are only due to random process error (generation of acceleration vs. time from ASD) and subsequent processing. Convergence of Damage in 5 had shown how the simulation can fall-short when the duration is less than 100sec (for 1kHz mid-frequency band) because the underlying statistical basis for conversion is only valid when there is sufficient length

Table 7 Results of Damage Emulation using Appropriate R_f for Each Octave Band

Mid-band Frequency (Hz)	Sine-tone Peak (g)	Original ASD (g^2/Hz)	Sine-tone Equivalent ASD (g^2/Hz) $b = 6.67$	Damage-A (sinetone + Original ASD)	Damage-B (Equivalent ASD + ASD)	Damage-C (Equivalent ASD - Original ASD)	dB ratio of DamageB Vs. DamageA
1050	1.54E+00	2.47E-03	8.40E-03				
1158	1.63E+00	2.41E-03	8.49E-03	8.72E+15	8.89E+15	8.56E+15	0.03
1276	1.73E+00	2.36E-03	8.62E-03	1.24E+16	1.30E+16	1.25E+16	0.06
1407	1.84E+00	2.31E-03	8.79E-03	1.66E+16	2.04E+16	1.83E+16	0.27
1551	1.96E+00	2.25E-03	9.03E-03	2.53E+16	2.98E+16	2.62E+16	0.20
1710	1.78E+00	2.20E-03	6.94E-03	1.75E+16	1.82E+16	1.82E+16	0.05
1886	1.43E+00	2.15E-03	4.34E-03	7.52E+15	8.52E+15	7.92E+15	0.16
2079	1.04E+00	2.10E-03	2.31E-03	2.85E+15	3.05E+15	3.02E+15	0.09
2292	1.00E+00	2.05E-03	2.05E-03	2.89E+15	3.02E+15	3.08E+15	0.01
2527	1.03E+00	2.01E-03	1.92E-03	3.82E+15	4.12E+15	4.12E+15	0.07
2786	1.37E+00	1.96E-03	2.84E-03	9.77E+15	1.12E+16	1.26E+16	0.16
3072	1.73E+00	1.92E-03	3.92E-03	2.62E+16	2.79E+16	3.00E+16	0.08
3386	1.97E+00	1.87E-03	4.47E-03	5.08E+16	5.31E+16	5.59E+16	0.06

of data. These two statistical issues (generation of random signals from an ASD and sufficient data to fit the Rayleigh distribution of peaks) are therefore responsible for any remaining error $<0.3\text{dB}$.

Simulation #7 Full-Simulation Concurrent Over Entire Frequency Band (1-4kHz)

Figure 7 shows the full-up simulation process and associated steps where all the frequency bands are occurring concurrently. Time-domain inputs are generated for each input set (original ASD, sine-tone at each frequency, equivalent ASDs). The sine-tone varies in frequency and amplitude for each dwell duration (based on the octave bands) and have to be added to the random signal at each point in time. The signals are convolved with the transient impulse response for the modes of interest (or sum of impulse functions for various modes) to obtain corresponding time-domain responses. The resulting response signal is processed using MATLABTM 'rainflow' to count the number of fatigue cycles. The number and amplitude of fatigue cycles are accumulated into a scalar *damage* parameter using Miner's rule and the fatigue exponent of $b = 6.67$ as discussed earlier.

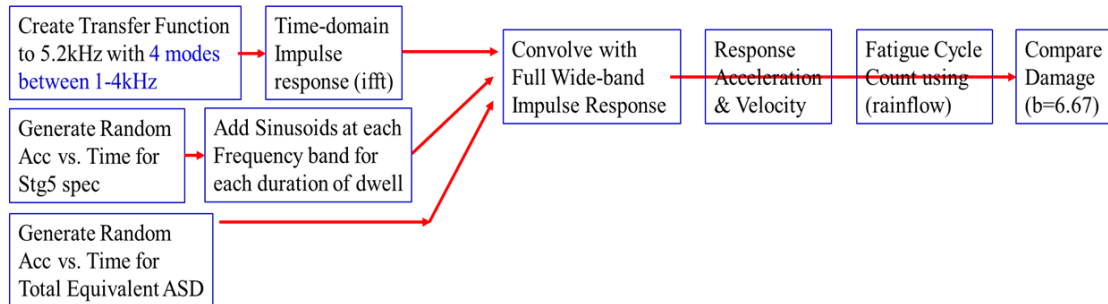


Fig. 7 Sequence of steps in the full concurrent (for all bands) simulation

Equivalent vbe specs (includes both random-vibe and sine-sweep per [Eq (1)] were first calculated using different values of R_f (1.0 for conservative and 1.17 for closer approximation). The simulation evaluates the damage from the different inputs using the sequence of steps shown in Figure 7. In order to simulate the response of the component to inputs, transfer function for an arbitrary system was assumed. The transfer function used the same damping constant ($\xi = 0.05$) as the equivalencing calculations for consistency. The transfer function shown in Figure 8a shows four resonance peaks between 1 and 4kHz. The associated time-domain Impulse response is shown in Figure 8b.

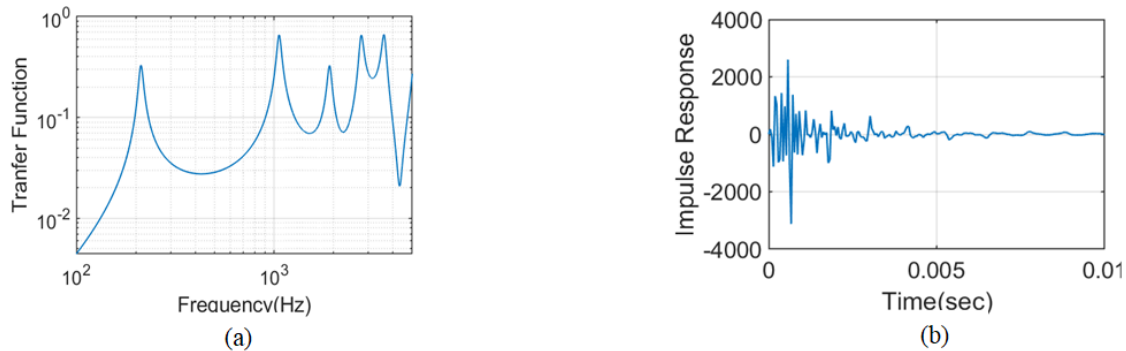


Fig. 8 Transfer function (a) and associated impulse function (b)

The three time-domain inputs that were convolved with the impulse function (Figure 8b) were the following:

- Original random-vibe signal added to the sine-sweep signal.
- Time-domain signal corresponding to the Safe Equivalent (very conservative).
- Time-domain signal corresponding to the Closer Equivalent (less conservative) ASD.

Each of the three signals were convolved with the impulse function (Figure 8b) to obtain the response. The response was then subject to a 'rainflow' cycle counting followed by determination of total damage using the fatigue coefficient ($b = 6.67$). Results are shown in Table 8 (columns 2 and 3). The damage from the equivalent ASDs is conservative by approximately 3dB due to the use of conservative values of $R_f = 1.0$ and $R_f = 1.17$.

The response and consequent damage were then calculated using a damping factor $\xi = 0.01$ (instead of 0.05) and the results are shown in the last two columns in Table 8. Lower damping increases the response and hence the consequent damage. More importantly, the effect of each response frequency is more contained near that frequency due to the lower damping. The reduced impact of one band over adjacent bands significantly decreases the variation in damage. The overly conservative damage values from the equivalent ASDs can also be attributed to the low sampling rate ($\approx 20000\text{Hz}$) relative to the highest frequency of the transfer function in Figure 8a.

Table 8 Comparison of Damage from the Three Simulations, Variation of Damping

Type of Input	Damage ($\xi = 0.05$)	dB variation	Damage ($\xi = 0.01$)	dB variation
Sinetone+Stg5 ASD	8.11E+36	0	1.84E+38	0
Equivalent Random ($R_f = 1.17$)	5.73E+37	2.5	3.42E+38	0.8
Equivalent Random ($R_f = 1.0$)	1.32E+38	3.6	8.97E+38	2

Conclusion

Converting sine-sweep test specifications to equivalent random vibration ASDs and vice-versa are necessary for test execution as well as for evaluating consequences of various test in terms of imparted damage. Detailed process for converting a concurrent sine-sweep and random vibration into an equivalent random vibration test was outlined in a previous IMAC paper [2]. This paper presented the associated theoretical approximations and provided uncertainty quantifications associated with each step in the process. A sequence of numerical simulations with progressively greater complexity were undertaken to capture the gradual reduction of uncertainty based on greater understanding and quantification of the contributing factors.

References

1. Fackler, Warren C., 1972, "Equivalence Techniques for Vibration Testing", SVM-9, *Shock and Vibration Monograph Series* (DTIC, 1987).
2. Maji A., "Equivalent of Sine-Sweep and Random Vibration Specification with Considerations of Nonlinear Statistics", *Proceedings of IMAC-XLI, Austin, TX, Feb 2023, V7, Ch 5, #14228*.

