



Chapter 11

Model Initialization in Real Time Hybrid Testing for Experimental Detection of Isolated Branches

A. Mario Puhwein and Markus J. Hochrainer

Abstract Due to complex interactions nonlinear structures exhibit a variety of difficult and unexpected phenomena. Isolated branches are of technical relevance and they typically occur at moderate to high vibration amplitudes. In a numerical model, their systematic detection assumes steady state conditions, and is e.g. based on the systematic detection of bifurcation points until the isolated branch appears. However, since a nonlinear system cannot be investigated experimentally at instable operating points, this approach is not feasible in experimental testing. In this work, a physical Duffing type absorber is constructed and coupled to a simulated Duffing type host structure, in a real-time hybrid testing configuration. The experimental study is limited to steady state conditions, however, satisfactory results are obtained in this configuration. Only small and slow excitation adjustments are possible to keep the coupled system in steady state. Therefore, it takes a long time to guide the system to an isolated branch along a predefined trajectory. In this work, an alternative to this cumbersome process is proposed. If the desired vibration amplitudes of the coupled systems are known, the physical absorber is driven to the desired configuration, before it is coupled to the virtual model. Similarly, the simulated structure can be brought directly into the corresponding state by prescribing correct initial conditions, before permanently linking the models. The developed switchover technique allows to explore isolated branches experimentally without time-consuming path following strategies. It is demonstrated that RTHT experiments can be used to investigate stable parts of isolated branches, which can be reached fast and reliably.

Keywords Real-time hybrid testing · Nonlinear oscillation · Isolated branches · Dynamic vibration absorber

Introduction

Nonlinear mechanical systems exhibit a wide variety of effects which are often analyzed in numerical studies because they are difficult to reproduce in real physical experiments. These include bifurcations, multiple solutions at different energy levels, hysteresis, quasi-stationary oscillation ranges, sub- and superharmonic as well as chaotic solutions. Due to material or structural optimizations many technical systems have non-linear behavior, and if they possess low damping, non-linear resonant vibrations can occur. This results in high vibration amplitudes, which are desirable in the case of energy harvesting, but should be avoided in the majority of applications e.g. by vibration damping measures like the traditional vibration absorber. Since linear absorbers are inefficient in non-linear systems, the dynamic absorbers must also behave non-linear [1].

Since the construction and investigation of non-linear systems with predefined properties is complex, expensive and sometimes dangerous, experimental studies use either model tests or significantly simpler alternatives such as real-time hybrid testing (RTHT). The latter allows accurate testing of physical components, and has become popular in research and industry due to advances in simulation techniques and the availability of real-time hardware. The basic idea is to construct only a part of the overall system (physical model), and implement the remaining parts in a computer simulation (virtual model). Coupling the models at carefully selected interfaces is crucial, because all kinematic and dynamic boundary conditions must be fulfilled to reproduce the correct system dynamics. For mechanical applications the interface often requires a

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“transfer system”, which transmits the simulated forces or movements to the physical model. It typically uses actuators with robust and high-precision control, which also limit the possible frequency and amplitude range of the RTHT experiment. The transfer system and its control directly influence the quality and even the stability of the entire setup, see. e.g. [2–5]. Since dynamic properties are often unknown, different control schemes have been proposed, e.g. phase shift [6],[7] and inverse compensation [8],[9], adaptive control with actuator identification [10]–[13], or AI based approaches [14]. Another way to reduce the adverse effects of the transfer system is the restriction to repeatable processes. To reduce inaccuracies due to coupling errors, the entire experiment can be repeated while adapting the transfer system control until the required accuracy is achieved, see e.g. [15]. If the experiment is focused on quasi-stationary periodic processes, iterative learning control (ILC) or repetitive control (RC) [16]–[19] can be applied successfully. ILC used in this project works very accurately and robustly, does not require prior knowledge of the dynamics of physical model and transfer system, dead times are perfectly compensated and it has proven to operate reliably for quasi-stationary processes.

In RTHT the focus is generally on the physical model, but the simulation model plays an equally important role as it must represent all relevant components that are not tested experimentally. In recent years the development of virtual models is well supported by high end simulation tool which have developed significantly. Today several reliable tool chains are commercially available that automatically convert the simulation model into the required representation for the real-time application.

To study nonlinear dynamic absorption, the physical oscillator used in this work is of Duffing type, see Fig 1a). The nonlinear restoring forces result from the interaction of a strong base magnet with a small magnet attached to the absorber mass, see Fig 1b). For the host structure, it is assumed that it can also be represented as a Duffing type SDOF oscillator, see again Fig 1a). According to the RTHT methodology, host structure and absorber are separated, the absorber is tested physically, whereas the host structure (virtual model, simulated model) is simulated in real time.

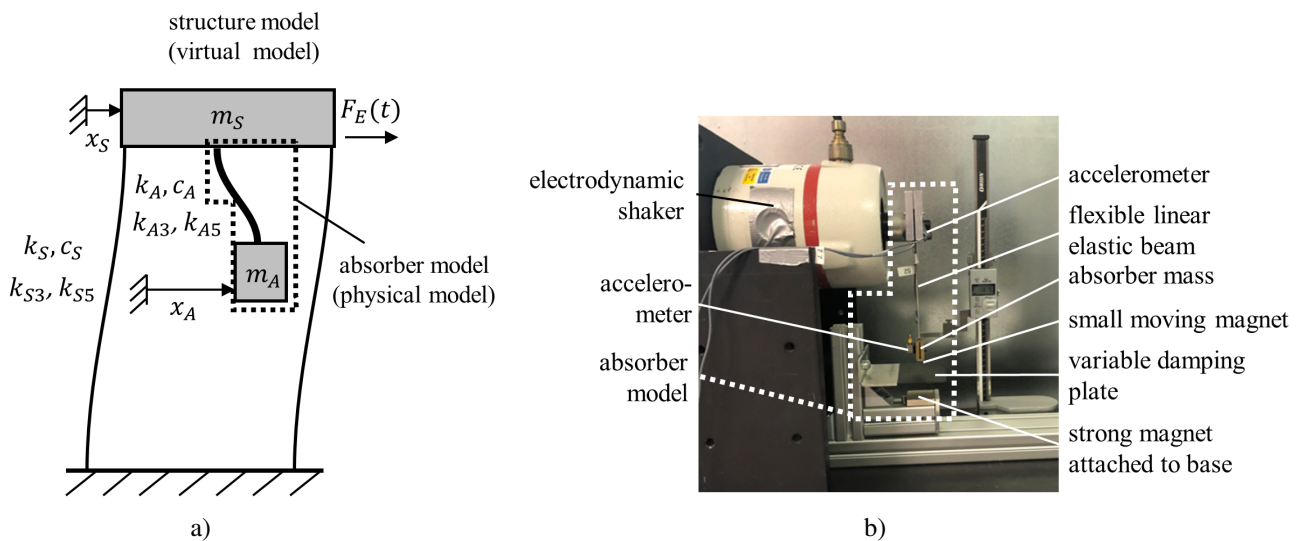


Fig. 1 a) Model of the coupled system b) laboratory setup of test bench

The simulated host structure can be regarded as “digital twin” of the real structure which can be configured easily in a wide parameter range. Any dynamics or nonlinearity can be implemented, as long as the motion can be calculated in real time. Therefore, the selection of nonlinear host structure parameters which reflect a proper physical system is crucial. For the dynamic absorber, in the contrary, it is rather difficult to develop a setup with adjustable dynamic parameters. For oscillators of Duffing type this can be achieved either by geometric or material nonlinearities or the application of magnetic interaction. The latter approach is quite convenient, because permanent magnets are commercially available in any size and shape. It has been demonstrated in [20], that a duffing type absorber can be obtained by adding repelling or attracting magnets to the mass of a linear oscillator. Since magnetic interaction forces depend on the magnetic field strength, it is convenient to attach a small light magnet onto the moving mass while providing a strong magnetic field by a much larger permanent magnet which is rigidly attached to the base. Furthermore, almost linear damping can be added to the oscillator by inserting an aluminum plate close to the moving magnet. It creates eddy current damping effects which are proportional to the relative velocity of the magnet with respect to the damping plate. By changing the distance between moving magnet and plate the almost linear viscous damping can be adjusted.

As already discussed, the coupling of virtual and physical model is a challenge in RTHT. In the given configuration a transfer system is required to transform the motion of the simulated model into a real displacement of the physical absorber's base. To keep the coupled structure in dynamic equilibrium, the interaction forces must be determined and applied to the virtual model as additional forcing. If both effects are incorporated correctly, the RTHT setup behaves exactly like the corresponding physical model. Considering the setup in some more detail two cascaded feedback loops become apparent. One is formed by the feedback control necessary for the transfer system. It assures that the shaker exciting the dynamic absorber exactly behaves like the simulated structure. This inner control loop must be robust, fast and with negligible phase lag, because otherwise any additional delay might lead to instabilities for lightly damped resonant mechanical systems. The second, much slower control loop results from the RTHT force feedback. It contains the host structure reference acceleration \ddot{x}_{Sref} , the electrodynamic shaker excitation \ddot{x}_S , and the interaction feedback force F_A which is calculated from the measured absorber acceleration, see Fig 2. Because a properly configured RTHT setup is able to reproduce any stable dynamics of the corresponding physical system, it can, e.g. be used to follow stable paths to investigate isolated branches if the periodically excited system response remains stable and quasi stationary. Therefore, it can take quite a time to experimentally drive the coupled system to a desired configuration. Isolated branches, e.g. can only be reached via rather complicated paths on the frequency-excitation response surface. However, if a desired steady state response is known from simulations, it can also be reached directly in a RTHT experiment by first treating the physical and virtual model as separated systems which are independently guided to the predefined steady state. Only if both systems are in equilibrium, they are coupled, thereby closing the RTHT loop. Using this methodology, the physical absorber is treated as SDOF system, and stable points on the nonlinear frequency response curve can be reached by varying either excitation force or frequency. For the simulated model, any desired model state can directly be specified at the instant of coupling, by prescribing system states as initial conditions.

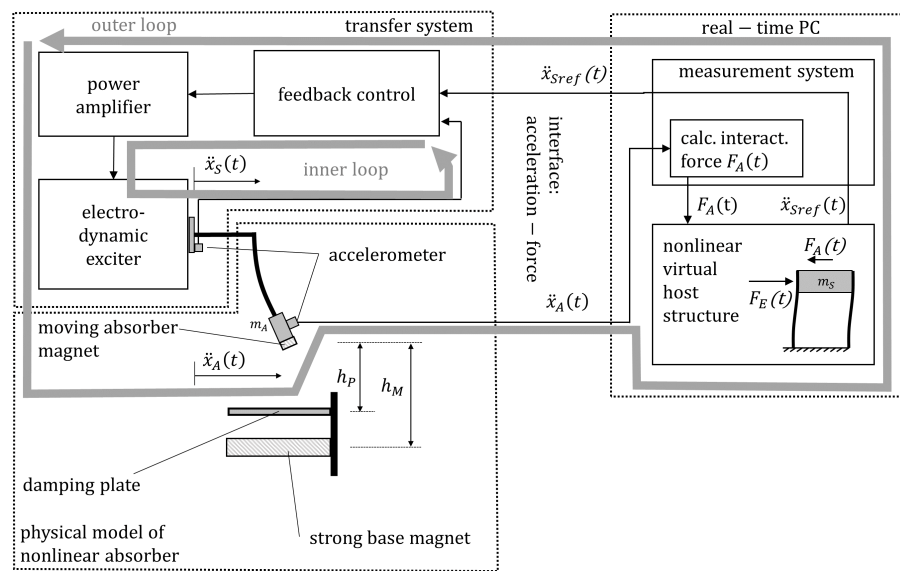


Fig. 2 Schematics of the RTHT setup with feedback loops and relevant interface quantities.

Nonlinear systems in e.g. resonant regimes can be very sensitive to parameter variations. To obtain a desired vibration absorption behavior it is essential that numerical and physical models are known and described accurately, and that the absorber parameters are selected carefully. Thus, several steps are suggested to obtain a reliable RTHT configuration, see Fig 3. Starting point is the definition of the host structure properties like mass, damping and linear as well as nonlinear stiffnesses. Based on these assumptions, the absorber can be designed according to published design rules, see e.g. [1]. After its construction the absorber is tested experimentally to identify its actual parameters. Then, the RTHT experiment is performed to measure the coupled system response. If it deviates from the desired equal peak behavior, the physical absorber is modified and tuned until the coupled response behaves as desired. Then, isolated branches and a possible stable path to them can be established from numerical simulations. If the Isolas identified turn out to be outside the physical limits of the test bench, the absorber model must be redesigned e.g. for a different mass ratio or according to the expected vibration amplitudes. In case of a redesign, however, the entire process must be repeated again. Only if the numerical simulations provide a possible stable path to Isolas, the RTHT experiment can be carried out successfully. Even if the proposed procedure

appears rather complicated, it has proven effective in laboratory operation because the nonlinear absorption investigated has a high parameter sensitivity. Studies in the laboratory confirm, that the iterations indicated in Fig 3 typically must be carried several times before the performance of the test-setup turns out to be optimal.

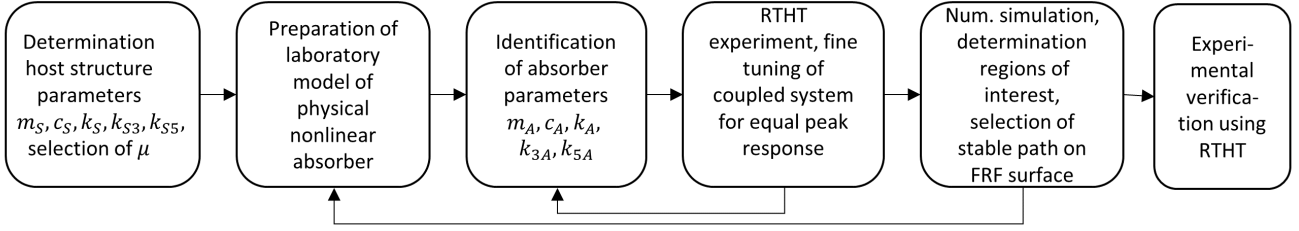


Fig. 3 Flowchart for successful implementation of RTHT; modified from [22].

Analytical Model and Coupled Simulation

When analyzing the motion of complex structures, it is common to decompose the response of complex structures into modal components, thereby identifying the dominant degrees of freedom in a frequency range of interest. In many situations it is even possible to approximate the system dynamics using only a single degree of freedom. If this simplification is acceptable for host structure and absorber, their coupled dynamics can be described by Duffing type oscillators with cubic and quintic stiffness terms [21]

$$\begin{aligned}
 m_S \ddot{x}_S + c_S \dot{x}_S + k_S x_S + k_{S3} x_S^3 + k_{S5} x_S^5 - c_A (\dot{x}_A - \dot{x}_S) - k_A (x_A - x_S) - k_{A3} (x_A - x_S)^3 - k_{A5} (x_A - x_S)^5 \\
 = F_E \cos(\omega t) \\
 m_A \ddot{x}_A + c_A (\dot{x}_A - \dot{x}_S) + k_A (x_A - x_S) + k_{A3} (x_A - x_S)^3 + k_{A5} (x_A - x_S)^5 = 0
 \end{aligned} \tag{1}$$

$m_S, c_S, k_S, k_{S3}, k_{S5}, F_E$ describe the mass, damping coefficient, linear, cubic and quintic stiffness as well as the excitation force of the host structure and $m_A, c_A, k_A, k_{A3}, k_{A5}$ denote the mass, damping coefficient, linear, cubic and quintic stiffness of the absorber. Introducing the the equivalent static deformation $x_{S,stat} = F_E/k_S$, structure and absorber displacements can be expressed dimensionless as $q_S = x_S/x_{S,stat}$ and $q_A = (x_S - x_A)/x_{S,stat}$, the external forcing amplitude is commonly described by the equivalent acceleration amplitude $a_F = F_E/m_S$. The natural frequencies $\omega_{nS} = \sqrt{k_S/m_S}$, $\omega_{nA} = \sqrt{k_A/m_A}$, and the damping ratios $\zeta_S = c_S/(2m_S\omega_{nS})$, $\zeta_A = c_A/(2m_A\omega_{nA})$ of host structure and absorber can be used to describe the underlying linear two degree of freedom dynamics. The normalized excitation frequency is given by $\gamma = \omega/\omega_{nS}$ and the mass ratio is denoted $\mu = m_A/m_S$. Although the equations of motion, eq. (1), can be solved numerically by time integration, analytical solutions can be given by the harmonic balance method (HBM) for steady state conditions. Comparisons with exact numerical results have revealed, that in the current work single term approximations for both, structure and absorber, are sufficiently accurate. The HBM approach yields a set of nonlinear coupled equations, which must be solved numerically to obtain the desired response amplitudes.

In [21] a systematic method is described to discover and identify Isolas by evaluating a turning point condition to generate and follow the turning point curve (TP-curve), see Fig 4. Numerically, it is also possible to determine the closed curve of an isolated branch using a path following algorithm for constant excitation force (FRF-Isola). However, these results cannot be reproduced experimentally because unstable branches of the coupled system cannot be followed. Consequently, it is also not possible to follow the turning point curve experimentally. An Isola can only be reached by keeping the system states on the stable regions of the generally complex shaped response surfaces. This often requires a simultaneous adjustment of amplitude a_F and frequency γ of the structural excitation. Typically, a desired isolated state can be reached experimentally by starting from an initial stable configuration with a higher excitation force, followed by force reductions and frequency corrections until the desired state is reached. In the experiment, however, the actual time to drive the system to an isolated state can take up to several minutes, because the dynamic system must remain in quasi periodic conditions during the entire process. This time can be reduced significantly by the proposed variable system configuration.

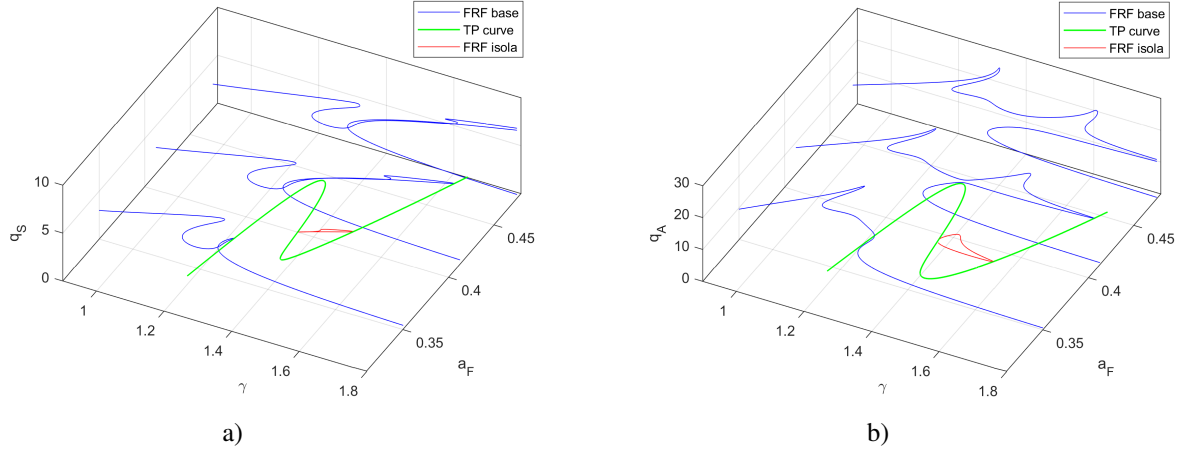


Fig. 4 Isolated branches (red) detected by numerical computations of the turning point curve (TP-curve) a) host structure b) dynamic absorber [22].

Variable System Configuration

The path following just described must be strictly followed in a traditional laboratory experiment of the coupled 2DOF nonlinear structure. It is not possible to leave or circumvent the stable path to get to an isolated branch. However, in the RTHT setup only the absorber is physically tested, the host structure is simulated numerically and consequently its initial conditions can be prescribed. If it is possible to guide the isolated physical absorber to a desired vibration state while completely neglecting the virtual model, an accurate RTHT experiment can still be carried out if the corresponding initial conditions are prescribed to the virtual model before coupling both systems. The only prerequisite for a valid experiment is knowledge of the true states of the virtual model. They must be either determined beforehand or calculated while setting up the vibration of the physical absorber. When implementing this concept, it becomes clear that the moment of coupling the systems is critical, since a very smooth transition is essential. This requires both subsystems to be in almost perfect equilibrium before the switchover, otherwise transient beating phenomena will be induced. Once the physical model has reached the desired vibration, the effective virtual host structure forcing $F_A + F_E$ can be derived from eq. (1)

$$F_A + F_E = m_S \ddot{x}_S + c_S \dot{x}_S + k_S x_S + k_{S3} x_S^3 + k_{S5} x_S^5, \quad (2)$$

and the structure-absorber interaction force F_A can be defined by

$$F_A = m_A \ddot{x}_A = c_A (\dot{x}_A - \dot{x}_S) + k_A (x_A - x_S) + k_{A3} (x_A - x_S)^3 + k_{A5} (x_A - x_S)^5. \quad (3)$$

Since only the accelerations \ddot{x}_S and \ddot{x}_A are recorded, the velocities and displacements must be determined from analytical integration. Since steady state conditions are considered, the measured signal is least squares approximated by a Fourier series up to the fifth harmonics. This is a smart way to filter periodic signals, see [23], [24], with the advantage of eliminating noise and drift while maintaining excellent accuracy due to the exact integration. Thus the r.h.s. of eq. (2) is known and since F_A can be determined using eq. (3) the unknown structural force excitation F_E can be estimated

$$F_E = m_S \ddot{x}_S + c_S \dot{x}_S + k_S x_S + k_{S3} x_S^3 + k_{S5} x_S^5 - m_A \ddot{x}_A. \quad (4)$$

This signal is again least squares approximated

$$F_E = A_E \cos \omega t + B_E \sin \omega t, \quad (5)$$

to minimize measurement noise and errors as well as inaccuracies caused by transient processes. With the variable system configuration the absorber can be directly forced into its final condition. During this process, eqs. (2)–(5) are permanently updated and the switchover to the coupled RTHT configuration can take place once the absorber has reached the intended steady state operating point. The simulation of the virtual model only begins just after the switchover at time t_s , with the initial conditions $x_S(t_s)$ and $\dot{x}_S(t_s)$ derived from the filtered shaker acceleration \ddot{x}_S . If all tasks are carried out correctly, the switching is smooth, and the system states do not change significantly. However, if the coupled system is not in a stable equilibrium, it will converge to a different state. All major steps of the developed switching process are illustrated in Fig 5.

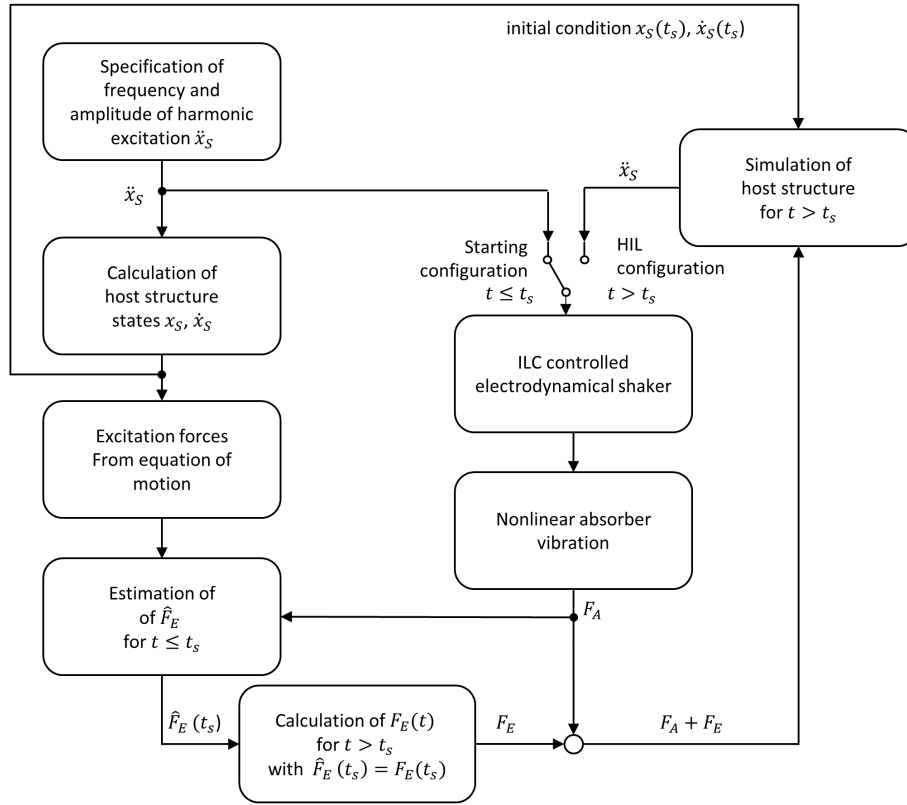


Fig. 5 Flowchart of variable system configuration. For $t \leq t_s$ the absorber is driven to the predefined steady state, before switching to the RTHS configuration for $t > t_s$.

Experimental Results

The setup and identification of the RTHT experiment is described in [22]. For the frequency range of interest $6\pi \leq \omega \leq 10\pi \text{ rad/s}$, the essential parameters can be summarized by $m_S = 3.78 \text{ kg}$, $c_S = 4.68 \text{ Ns/m}$, $k_S = 1.68 \cdot 10^3 \text{ N/m}$, $k_{S3} = 8.20 \cdot 10^7 \text{ N/m}^3$, $k_{S5} = 0 \text{ N/m}^5$, $\mu = 1/45$, $m_A = 0.084 \text{ kg}$, $k_A = 32.4313 \text{ N/m}$, $c_A = 0.256 \text{ Ns/m}$, $k_{A3} = 1.54 \cdot 10^5 \text{ N/m}^3$ and $k_{A5} = -1.20 \cdot 10^8 \text{ N/m}^5$. With this configuration, the nonlinear dynamic system has isolated branches within the amplitude range $0.15 < a_F < 0.215 \text{ m/s}^2$ and the frequency range $4.7 < f < 6.6 \text{ Hz}$ which is appropriate for the RTHT experiment. For this setup, Fig 6 illustrates the experimental determination of the Isola surface for both, structure and absorber by combining the results of 18 path following experiments.

Starting from the stable point A the Isolas can be reached by reducing the excitation force amplitude to move to point B. Then, the excitation frequency is decreased (experiment 1) or increased (experiment 2) until the stable surface is left. To obtain the remaining response curves 3 to 18 the experiment is repeated with the separation point B shifted slightly for each run. The isolated surface is clearly indicated by the amplitude jumps which occur at the border of the surface. Once the Isola has been left by jumping to a lower amplitude stable solution, it cannot be reached again by varying the excitation frequency only. For the experiments shown in Fig 6 a single run typically takes about 15min because the system has to be driven through the quasi stable region quite carefully. The approach proposed in this work is much faster, because the Isola can be reached directly. In Fig 7 the developed technique appears as a direct shortcut from an initial condition I the desired point i on the isolated surface. Subsequently, it is possible to navigate on the Isola surface by varying the excitation amplitude and frequency. To demonstrate the flexibility and robustness of the method, the experiment is repeated for slightly different initial conditions, see point II in Fig 7. Again the same Isola is reached at a different point ii and in this experiment a different path is prescribed and followed.

The “shortcut” paths directly connecting point I and i as well as II and ii , demonstrate, that the proposed switchover works as intended. However, to assess its quality, the time response of the accelerations should be considered. In Fig.8 two different switching processes are illustrated for the same operating point. Up to the switching time $t_s = 19 \text{ s}$ the absorber is excited with a predefined acceleration \ddot{x}_S .

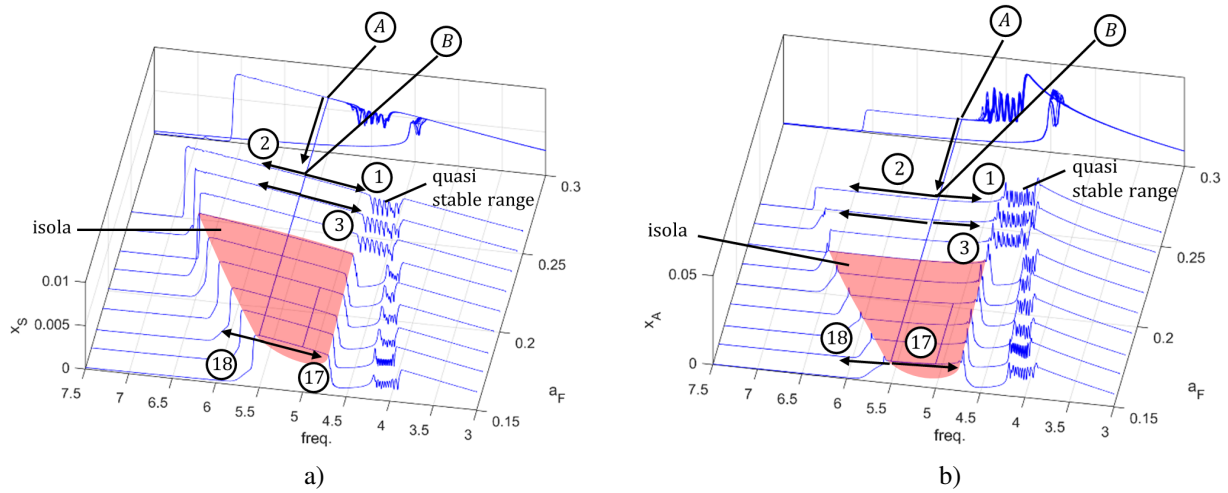


Fig. 6 RTHT experiment to confirm isolated branches experimentally a) structural response, b) dynamic absorber response; modified from [22].

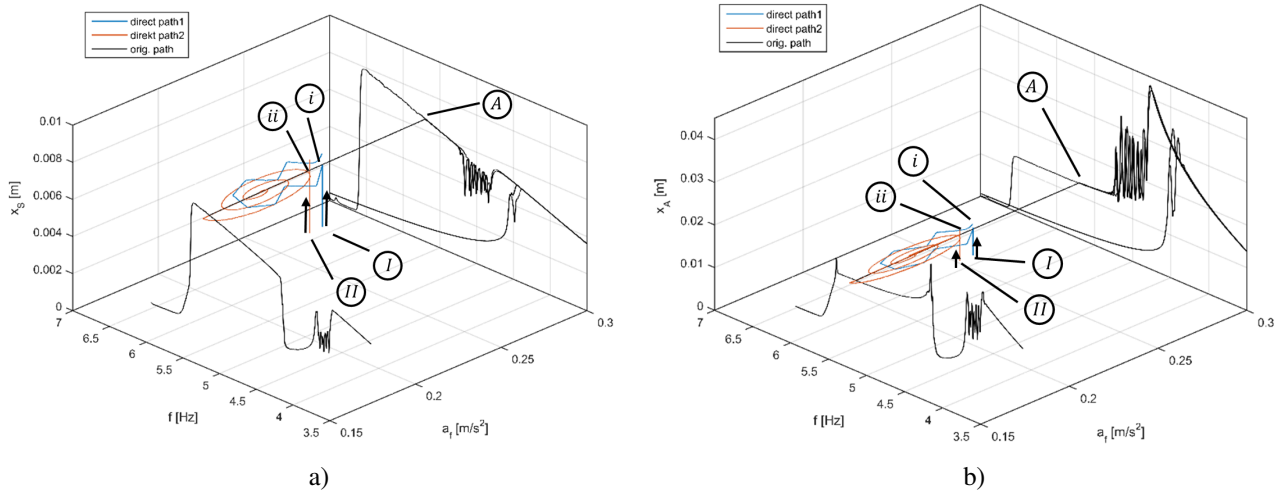


Fig. 7 Comparison of traditional and proposed paths to reach the isolated branch. Displacements of a) the simulated host structure b) the physical absorber.

After the dynamic absorber has reached steady state conditions, the switchover can take place. In Fig 8 a)-b) a suboptimal vibration pattern results from an inaccurate estimation of the external structural excitation F_E using eq. (4). Apparently it takes about 10s until steady state conditions are established again. Obviously, rather large fluctuations occur in the measured accelerations, which are tolerable because the desired state on the Isola can still be maintained. In general, however, such errors result in a jump of the system to a stable state with much lower vibration energy and amplitudes. In Fig 8 c)-d) the same switching situation is illustrated for favorable conditions. Again, a transient beating process is recognizable after switching, but with significantly reduced settling times and amplitude variations. It takes about 5s to reach stable steady state conditions on the isolated branch. Although improvements are still possible in the experimental results presented, they are already satisfactory, because the estimation of the excitation force from eq. (4) is difficult in the case of resonant vibrations. It turns out that transient effects from the switchover are depending on the chosen system state, and they reduce for smaller vibration amplitudes and energy levels. The experiments have revealed, that the structural acceleration \ddot{x}_S of the host structure must be known with the best possible accuracy to guide the coupled system to a desired state. \ddot{x}_S can be either determined from the numerical model or from measurements, if a condition of interest has already been reached experimentally before.

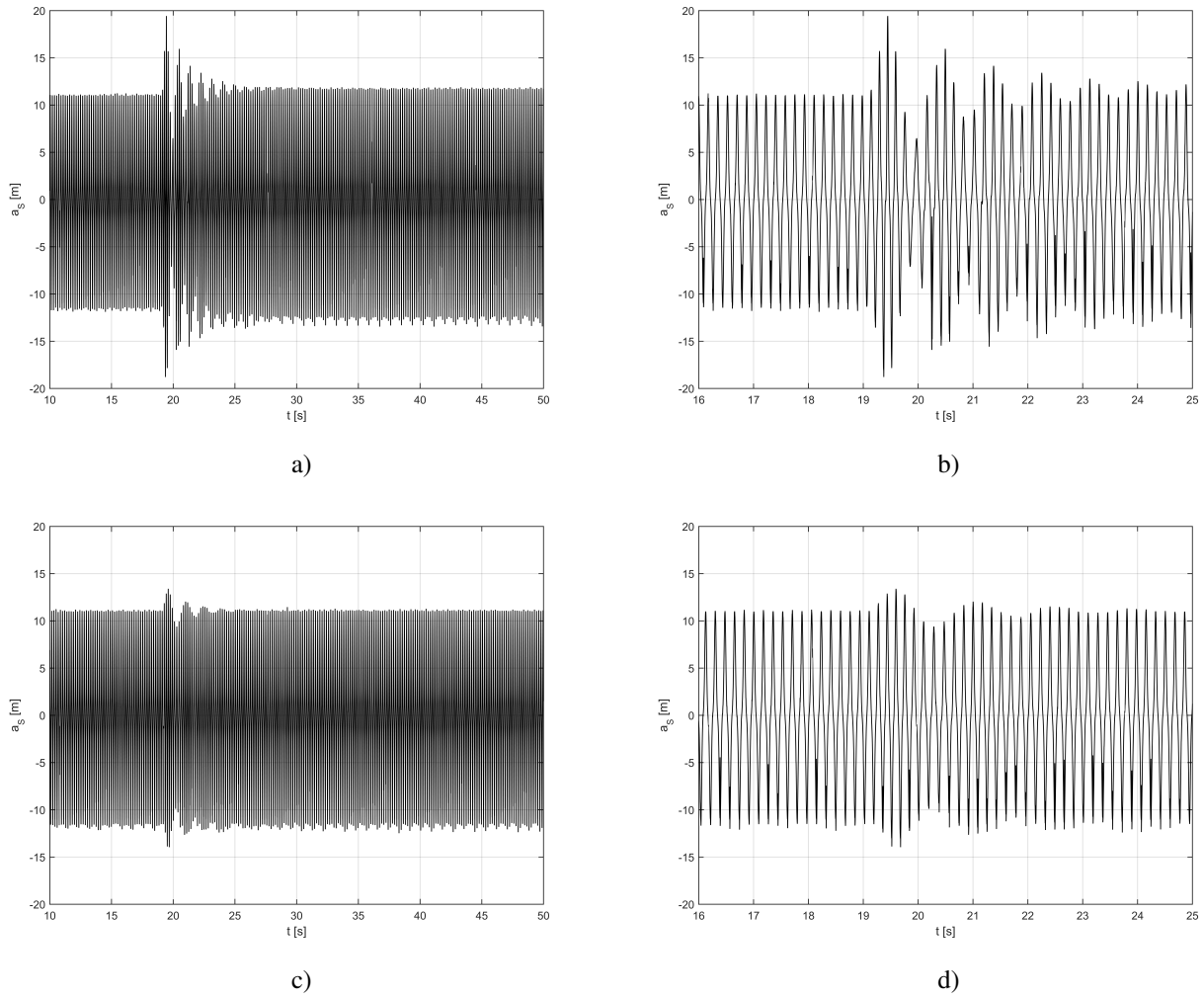


Fig. 8 Transient effect (RTHT experiment) of the switchover at time $t_s = 19$ s on the acceleration $a_S = \ddot{x}_S$ a) inaccurate determination of the external excitation for $10\text{ s} < t < 50\text{ s}$ and b) for $16\text{ s} < t < 25\text{ s}$ c) favorable switching conditions with minor transient effects for $10\text{ s} < t < 50\text{ s}$ and d) for $16\text{ s} < t < 25\text{ s}$.

Conclusion

Isolated branches of nonlinear systems can be of interest for several studies. Generally, they are investigated numerically using path following strategies which slowly guide the dynamic system in to a desired condition, often following turning point curves or other sections of unstable paths. However, this approach is not possible in an experimental setup, because close to boundaries of stability, any physical system immediately jumps to a stable low-energy configuration. Once an isolated branch has been detected, it is always possible to reproduce steady state vibrations in a numerical simulation using proper initial conditions. A similar approach, which avoids path following methods by directly guiding the dynamic system into its final state, is proposed for RTHT. In the hybrid experiment a physical nonlinear dynamic absorber is attached to a virtual nonlinear host structure to mitigate its vibration. Such a coupled system creates fairly complex response characteristics including bifurcations, internal resonances and isolated branches. The latter can be investigated experimentally using a two-step procedure if the desired steady state conditions are known with sufficient accuracy. First, the absorber is driven to its desired point of operation, before it is coupled to the virtual system in a matching vibration state. If the configurations of the individual systems at the switching moment are close to a stable branch, the experiment will converge to the desired state within several oscillations. When subsequently guiding the oscillators along a predefined trajectory on the isolated branch, the system must always remain in steady state, and consequently only small and slow adjustments in the excitation are permissible. In this work it is undoubtedly demonstrated that RTHT experiments can be used to investigate stable parts of isolated branches, which can be reached fast and reliably.

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