Chapter 14

On the Use of Frequency-Dependent Modal Basis for Interface Modeling in Frequency-Based Substructuring

Domen Ocepek, Miha Pogačar, Jure Korbar, and Gregor Čepon

Abstract In experimental dynamic substructuring, coupling of substructures sharing a line- or surface-like interface proves to be a challenge due to the difficulties in interface modeling. Modeling a high number of degrees of freedom at the common interface can be too stringent when imposing compatibility and equilibrium conditions, thereby causing redundancy and ill-conditioning. To mitigate the effects of over-determination and experimental errors several techniques have been developed, proposing different reduction spaces to weaken the interface conditions. In the context of component-mode synthesis (also known as modal substructuring), modal constraints for fixture and subsystem is a commonly used approach the use of which was also suggested for frequency-based substructuring. A flexible fixture is introduced to the coupling workflow and the reduction basis is defined as a truncated set of physical mode shapes of the fixture. Both the selection and the number of modes considered in the reduction step can significantly affect the quality of the substructuring prediction. This work investigates the benefits of applying frequency-dependent reduction spaces to weaken the interface conditions in the context of frequency-based substructuring. The approach is investigated using an experimental case study on the Dynamic Substructuring Round Robin Benchmark.

Keywords Dynamic Substructuring · Frequency-based Substructuring · Transmission Simulator · Modal Constrain for Fixture and Subsystems · Round Robin Benchmark

Introduction

In the context of experimental frequency-based substructuring (FBS) [1], the main challenge for a successful substructure-coupling implementation remains the modeling of the common interface between the substructures. Coupling of the substructures requires compatibility of the displacements along the interface between two substructures and the equilibrium of interface forces to be satisfied. Practical difficulties relate to the measurement of an appropriate number of degrees of freedom (DoFs) at which interface conditions are imposed. Coupling too many DoFs can results in redundancy and consequently bad conditioning of the interface coupling equations. Ill-conditioning combined with inevitably present measurement errors (random or systematic in nature) can lead to a high error amplification. Limiting the number of interface measurements and imposing weak compatibility avoids those problems, but can significantly deteriorate the accuracy of the coupled dynamics.

A solution can be applied by projecting the measured dynamics into a representative subspace [2]. This mitigates the effects of measurement errors since the interface conditions are now imposed in the reduced space and are thus related to as weak. Redundant and insignificant dynamic information that is not included in the reduced subspaces is left uncoupled. This is beneficial since this part is commonly strongly affected by measurement errors, is badly controlled or badly observed.

In the context of component-mode synthesis (CMS), modal constraints for fixture and subsystem (MCFS) [3] is a commonly used approach [4] the use of which was also suggested for FBS in [5]. A flexible fixture is introduced to the coupling workflow and the reduction basis is defined as a truncated set of physical mode shapes of the fixture. A method developed with the aim to handle flexible interface behavior in the frequency domain is the singular vector transformation (SVT) [6], where force and displacement reduction spaces are extracted from the measured response models by the means of singular value decomposition.

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This work can be seen as the extension of [7], where authors investigated MCFS and SVT to couple experimental response models of the substructures sharing a continuous interface. In [7], authors suggested the benefits of applying frequency-dependent reduction spaces to weaken the interface conditions in the context of frequency-based substructuring and particular MCFS, which is elaborated within this contribution.

Theoretical Background and Notation

Consider two substructures A and B assembled at the interface DoFs $(\star)_2^{\mathrm{A}}$ and $(\star)_2^{\mathrm{B}}$, as depicted in Fig. 1.

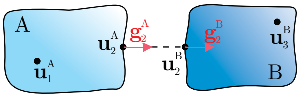

Fig. 1 Substructures A and B to be coupled at the common interface.

With admittances of the individual subsystems known ($\mathbf{Y}^{\mathrm{A}}$ and $\mathbf{Y}^{\mathrm{B}}$) and partitioned in internal ($(\star)_1^{\mathrm{A}}$ and $(\star)_3^{\mathrm{B}}$) and interface DoFs, the governing equation of motion for the uncoupled system can be written as:

$$\mathbf{u} = \mathbf{Y}^{\mathrm{A|B}}\left(\mathbf{f} + \mathbf{g}\right). \tag{1}$$

The vector $\mathbf{u}$ represents the displacements to the external force vector $\mathbf{f}$, and $\mathbf{g}$ is the vector of interface forces between the substructures that exist only at the interface DoFs, keeping the substructures together. $\mathbf{Y}^{\mathrm{A|B}}$ is a block-diagonal matrix of subsystem's admittances. All subsystems considered are assembled through the proper application of interface conditions[1]. The compatibility of the displacements at the common boundary is recast in the general formulation:

$$\mathbf{B}\,\mathbf{T}_{\mathrm{u}}\,\mathbf{u} = \mathbf{0} \quad \text{where} \quad \mathbf{B} = \begin{bmatrix}\mathbf{0} & -\mathbf{I} & \mathbf{I} & \mathbf{0}\end{bmatrix}. \tag{2}$$

The equilibrium condition is imposed by replacing the interface forces using a set of Lagrange multiplier vectors $\boldsymbol{\lambda}$:

$$\mathbf{g} = -\mathbf{T}_{\mathrm{f}}^{\mathrm{H}}\,\mathbf{B}^{\mathrm{T}}\boldsymbol{\lambda}. \tag{3}$$

By eliminating the Lagrange multiplier vector from the set of Eqs. (1 - 3) we obtain:

$$\mathbf{u} = \underbrace{\left[\mathbf{I} - \mathbf{Y}^{\mathrm{A|B}}\,\mathbf{T}_{\mathrm{f}}^{\mathrm{H}}\,\mathbf{B}^{\mathrm{T}}\left(\mathbf{B}\,\mathbf{T}_{\mathrm{u}}\,\mathbf{Y}^{\mathrm{A|B}}\,\mathbf{T}_{\mathrm{f}}^{\mathrm{H}}\,\mathbf{B}^{\mathrm{T}}\right)^{-1}\mathbf{B}\,\mathbf{T}_{\mathrm{u}}\right]\mathbf{Y}^{\mathrm{A|B}}}_{\mathbf{Y}^{\mathrm{AB}}}\,\mathbf{f}. \tag{4}$$

where $\mathbf{Y}^{\mathrm{AB}}$ is the admittance of the assembled system, $\mathbf{T}_{\mathrm{u}}$ is the transformation matrix to project interface displacements to the reduced domain, and $\mathbf{T}_{\mathrm{f}}$ is the transformation matrix for the interface forces which fulfill a weak compatibility in a space of lower dimension. Note that different Boolean matrices can be used to apply the compatibility $\mathbf{B}_{\mathrm{u}}$ and equilibrium $\mathbf{B}_{\mathrm{f}}$ conditions.

A flexible fixture (named transmission simulator or TS) can be introduced to the coupling workflow and attached to substructure B (Fig. 2). The combined (sub)structure is denoted as BTS in the following which is coupled to the substructure A from which the TS structure is then decoupled to obtain the response model for AB. The procedure is schematically depicted in Fig. 2.

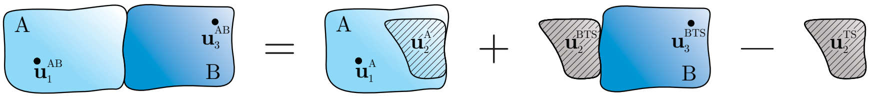

Fig. 2 Coupling application with a TS substructure. Common interface for all substructures is presented as hatched.

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[1] Weak compatibility is considered here. For more information, an interested reader is refereed to [7]

Case Study

The coupling approach with the MCFS and SVT method adopted the workflow presented in Fig. 2. The coupling prediction in [7] where the number of mode shapes and singular modes retained in the reduction step varied from 8 to 10 are presented in Fig. 3. A reasonably higher number of DoFs retained in the reduction improves prediction in a higher frequency range and vice versa.

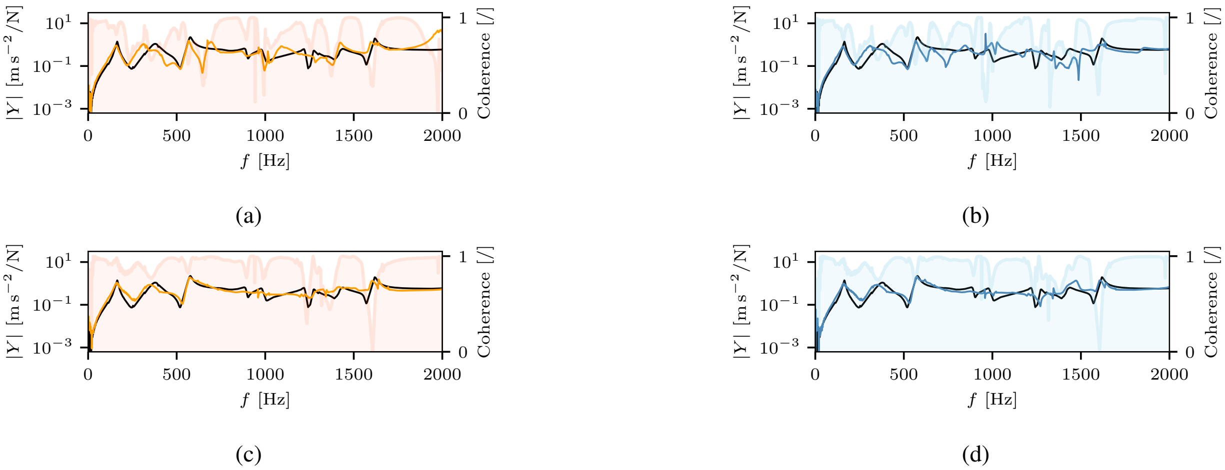

Fig. 3 Amplitude and coherence of a FRF of $\mathbf{Y}^{\mathrm{AB}}$ using MCFS and SVT approach with various number of mode shapes/singular modes retained in the reduction bases; a) first 8 mode shapes, b) first 8 singular modes, c) first 10 mode shapes, d) first 10 singular modes. (— reference, — TS approach, — SVT approach, — coherence on TS approach, — coherence on SVT approach)

The authors argue that the use of frequency-dependent reduction bases for MCFS and SVT, involving a varying number of mode shapes and singular modes respectively, would likely result in more consistent response prediction. The aim of this approach is to define reduction bases that adjust for the dominant/representative physical modes at each frequency line, hence provide a representative interface description yet avoid redundancy in terms of the number of interface DoFs.

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